ECE 433 Industrial Electronics and Control Systems

Lab #3 Modeling and Simulation of the SRV02 Motor

Objective:

1. To derive dynamic equations (mathematical model) and transfer function for the SRV02 motor using the first-principles.
2. To implement the model of the SRV02 motor in simulation and analyze the performance of the system.

Introduction:

The SRV02 motor armature circuit schematic and gear train is illustrated in Fig. 1. The model of the SRV02 is consisted of two parts: electrical equations and mechanical equations.

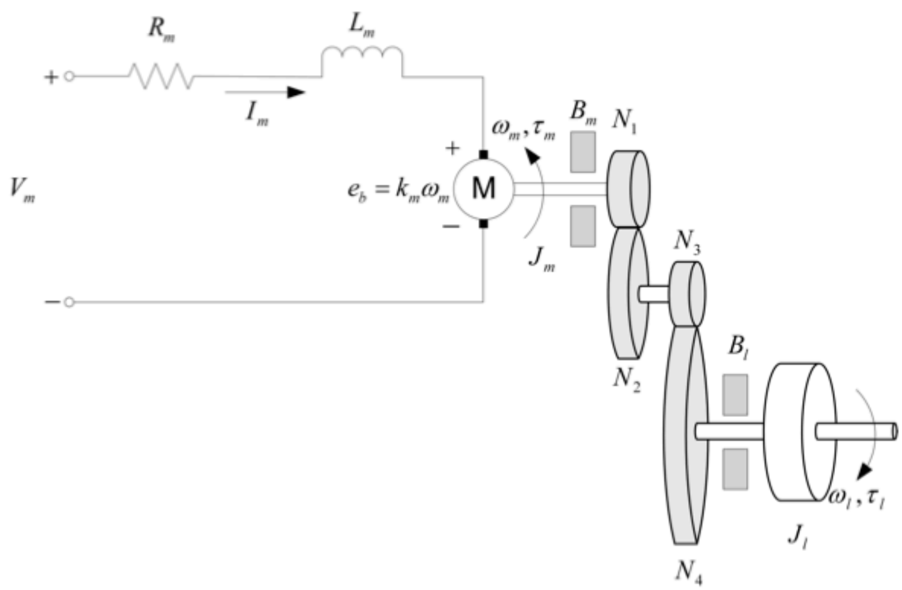

Figure 1: SRV02 DC motor armature circuit and gear train

1. Electrical Equations

   The back-emf (electromotive) voltage $e_b(t)$ depends on the rotational speed of the motor shaft $\omega_m(t)$ and the back-emf constant of the motor k_m. It opposes the current flow. The back-emf voltage is given by:

$$e_b(t) = k_m \cdot \omega_m(t) \tag{1}$$

   Using KVL, one can write the following equation:

$$V_m(t) = R_m \cdot \mathrm{I_m}(t) + L_m \cdot \frac{d\mathrm{I_m}}{dt} + k_m \cdot \omega_m(t), \tag{2}$$

   where R_m and L_m are the resistance and inductance of the armature circuit of the motor, respectively. Rearranging (2) results in

$$\frac{d\,\mathrm{I_m}}{dt} = -\frac{R_m}{L_m}\cdot \mathrm{I_m}(t) - \frac{k_m}{L_m}\cdot \omega(t) + \frac{1}{L_m}V_m(t)\,. \tag{3}$$

2. Mechanical Equations
   This section demonstrates the derivation of the motion equation, represented by the speed of the load shaft $\omega_l(t)$, with respect to the applied motor torque $\tau_m(t)$. According to the Newton's Second Law of Motion, the load equation of motion on the SRV02 can be described as

$$J_l \frac{d\omega_l(t)}{dt} + B_l \cdot \omega_l(t) = \tau_l(t)\,, \tag{4}$$

where J_l and B_l are the moment of inertia of the load and the viscous friction acting on the load, respectively. The variable $\tau_l(t)$ is the total torque applied on the load. The load inertia includes the inertia from the gear train and from any external loads attached, such as disc and bar. Similarly, the motion equation of the motor shaft can be written as

$$J_m \frac{d\omega_m(t)}{dt} + B_m \cdot \omega_m(t) + \tau_{ml}(t) = \tau_m(t)\,, \tag{5}$$

where J_m and B_m are the moment of inertia of the motor shaft and the viscous friction on the motor shaft, respectively. The variable $\tau_{ml}(t)$ is the resulting torque acting on the motor shaft from the load torque. The torque at the load shaft from an applied motor torque can be written as

$$\tau_l(t) = \eta_g K_g \tau_{ml}(t)\,, \tag{6}$$

where η_g is the gear box efficiency and K_g is the gear ratio. The gearbox that is directly mounted on the SRV02 motor is represented by the N_1 and N_2 gears in Fig. 1 and has a gear ratio of

$$K_{gi} = \frac{N_2}{N_1}\,. \tag{7}$$

This is the <u>internal</u> gearbox ratio. The motor gear N_3 and the load gear N_4 are directly meshed together and are visible from the outside. These gears comprise the <u>external</u> gear box which has an associated gear ratio of

$$K_{ge} = \frac{N_4}{N_3}\,. \tag{8}$$

The gear ratio of the SRV02 gear train is then given by

$$K_g = K_{ge} K_{gi} = \frac{N_2 N_4}{N_1 N_3}. \tag{9}$$

From (6) and (9), the torque seen at the motor shaft through the gears can be expressed as

$$\tau_{ml}(t) = \frac{\tau_l(t)}{\eta_g K_g}. \tag{10}$$

Intuitively, the motor shaft must rotate K_g times for the output shaft to rotate one revolution, i.e.

$$\theta_m(t) = K_g \cdot \theta_l(t). \tag{11}$$

And therefore,

$$\omega_m(t) = K_g \cdot \omega_l(t). \tag{12}$$

Now substituting (4), (10), and (12) into (5), one has

$$J_m K_g \frac{d\omega_l(t)}{dt} + B_m K_g \cdot \omega_l(t) + \frac{1}{\eta_g K_g}\left[J_l \frac{d\omega_l(t)}{dt} + B_l \cdot \omega_l(t) \right] = \tau_m(t), \tag{13}$$

and thus,

$$\left[\eta_g K_g^2 J_m + J_l \right] \frac{d\omega_l(t)}{dt} + \left[\eta_g K_g^2 B_m + B_l \right] \cdot \omega_l(t) = \eta_g K_g \cdot \tau_m(t). \tag{14}$$

Equation(14) can be written as

$$J_{eq} \frac{d\omega_l(t)}{dt} + B_{eq} \cdot \omega_l(t) = \eta_g K_g \cdot \tau_m(t). \tag{15}$$

where $J_{eq} = \eta_g K_g^2 J_m + J_l$ and $B_{eq} = \eta_g K_g^2 B_m + B_l$.

3. Combining the Electrical and Mechanical Equations
The motor torque in (15) is proportional to the voltage applied and is described as

$$\tau_m(t) = \eta_m k_t \cdot \mathrm{I_m}(t), \tag{16}$$

where k_t is the current-torque constant (N·m/A), η_m is the motor efficiency, and $\mathrm{I}_m(t)$ is the armature current. As the result, the dynamics of the SRV02 can be described by the following system of differential equations:

$$\begin{aligned}
&\frac{d}{dt}\theta_l(t) = \omega_l(t) \\
&\frac{d}{dt}\omega_l(t) = K_A \cdot \mathrm{I}_\mathrm{m}(t) - \left[\frac{B_{eq}}{J_{eq}}\right] \cdot \omega_l(t) \\
&\frac{d}{dt}\mathrm{I}_\mathrm{m}(t) = \frac{1}{L_m}\left[V_m(t) - R_m \cdot \mathrm{I}_\mathrm{m}(t) - K_B \cdot \omega_l(t)\right]
\end{aligned}, \tag{17}$$

where $K_A = \dfrac{\eta_g K_g \eta_m k_t}{J_{eq}}$ and $K_B = k_m K_g$.

4. Model Parameters:
To implement the dynamic equations in (17) in computer simulation, one must obtain the model parameters associated with the equations. The model parameters of SRV02 available from the manufacturer are listed in Table 1 and 2.

For SRV02 set up as high-gear configuration, $K_g = 70$ and $B_{eq} = 0.015$ N·m/(rad/s). The moment of inertia about the motor shaft $J_m = J_{m,rotor} + J_{tach} = 4.0606 \times 10^{-7}$ Kg·m^2. The load attached to the motor shaft includes a 24-tooth gear, two 72-tooth gears, and a single 120-tooth gear along with any other external load that is attached to the load shaft. Thus, for the gear moment of inertia J_g and the external load moment of inertia $J_{l,ext}$, the load inertia is

$$J_l = J_g + J_{l,ext}. \tag{18}$$

For a disk with its mass m and radius r, the moment of inertia about the disk is

$$J = \frac{1}{2} m \cdot r^2. \tag{19}$$

By applying the gear head specifications from Table 2 into (19),

$$J_g = \frac{1}{2} m_{24} \cdot r_{24}^2 + 2\left[\frac{1}{2} m_{72} \cdot r_{72}^2\right] + \frac{1}{2} m_{120} \cdot r_{120}^2 = 5.28 \times 10^{-5} \text{ Kg·m}^2. \tag{20}$$

For a disk load, the moment of inertia of the external load is $J_{l,ext} = \frac{1}{2} m_d \cdot r_d^2 = 5\times10^{-5}$ Kg·m². Therefore, $J_l = J_g + J_{l,ext} = 1.025\times10^{-4}$ Kg·m². As the result, $J_{eq} = \eta_g K_g^2 J_m + J_l = 0.00213$ Kg·m².

Table 1: Main SRV02 Specifications

Symbol	Description	Matlab Variable	Value	Variation
V_{nom}	Motor nominal input voltage		6.0 V	
R_m	Motor armature resistance	Rm	2.6 Ω	± 12%
L_m	Motor armature inductance	Lm	0.18 mH	
k_t	Motor current-torque constant	kt	7.68×10^{-3} N m/A	± 12%
k_m	Motor back-emf constant	km	7.68×10^{-3} V/(rad/s)	± 12%
K_g	High-gear total gear ratio	Kg	70	
	Low-gear total gear ratio	Kg	14	
η_m	Motor efficiency	eta_m	0.69	± 5%
η_g	Geabox efficiency	eta_g	0.90	± 10%
$J_{m,rotor}$	Rotor moment of inertia	Jm_rotor	3.90×10^{-7} kg · m²	± 10%
J_{tach}	Tachometer moment of inertia	Jtach	7.06×10^{-8} kg · m²	± 10%
J_{eq}	High-gear equivalent moment of inerta without external load	Jeq	9.76×10^{-5} kg · m²	
	Low-gear equivalent moment of inerta without external load	Jeq	2.08×10^{-5} N · m / (rad/s)	
B_{eq}	High-gear Equivalent viscous damping coefficient	Beq	0.015 N · m / (rad/s)	
	Low-Gear Equivalent viscous damping coefficient	Beq	1.50×10^{-4} kg · m²	
m_b	Mass of bar load	m_b	0.038 kg	
L_b	Length of bar load	L_b	0.1525 m	
m_d	Mass of disc load	m_d	0.04 kg	
r_d	Radius of disc load	r_d	0.05 m	
m_{max}	Maximum load mass		5 kg	
f_{max}	Maximum input voltage frequency		50 Hz	
I_{max}	Maximum input current		1 A	
ω_{max}	Maximum motor speed		628.3 rad/s	

Table 2: SRV02 Gear Head Specifications

Symbol	Description	Matlab Variable	Value
K_{gi}	Internal gearbox ratio	Kgi	14
$K_{ge,low}$	Internal gearbox ratio (low-gear)	Kge	1
$K_{ge,high}$	Internal gearbox ratio (high-gear)	Kge	5
m_{24}	Mass of 24-tooth gear	m24	0.005 kg
m_{72}	Mass of 72-tooth gear	m72	0.030 kg
m_{120}	Mass of 120-tooth gear	m120	0.083 kg
r_{24}	Radius of 24-tooth gear	r24	6.35×10^{-3} m
r_{72}	Radius of 72-tooth gear	r72	0.019 m
r_{120}	Radius of 120-tooth gear	r120	0.032 m

Pre-Lab Exercise: Provide clear derivation to find answers for the following questions.

1. Use the differential equations in (17) to find the transfer functions $G_1(s) = \dfrac{\omega_l(s)}{V_m(s)}$ and $G_2(s) = \dfrac{\mathrm{I_m}(s)}{V_m(s)}$.

2. Apply the final value theorem,
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s),$$
to determine the angular velocity speed of the motor shaft ω_m and the motor current $\mathrm{I_m}$ under steady-state by using the transfer function $G_1(s)$ and $G_2(s)$ with the input voltage V_m as a unit-step function.

Lab Assignments:

1. Prepare a Matlab m-script to assign model parameters in (17). The values of these parameters are available either from Table 1 and 2 (ex. R_m, L_m, K_g, etc.) or from algebraic calculation (ex. K_A, K_B, J_{eq}, etc.).

2. Implement the motor equation in (17) in Simulink to simulate the motor dynamics. Apply a step input $V_m(t)$ volt to the motor and plot $\theta_l(t)$, $\omega_l(t)$, and $\mathrm{I_m}(t)$ obtained from the simulation.

3. Set $V_m(t)$ from 1 to 10 volts with an increment of 1 volt. At each voltage setting, record the steady state values of $\omega_l(t)$ and $\mathrm{I_m}(t)$. Do these correspond to the steady state values predicted by the final value theorem? Plot the steady state $\omega_l(t)$ and $\mathrm{I_m}(t)$ versus the input voltage $V_m(t)$ and find the trend-line for each plot. Can you predict the slopes of the trend-lines from the transfer functions derived previously? What other sanity checks can you perform to verify that the simulation matches the transfer functions?