Mathematics | Grade 3

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 3rd grade.

Operations and Algebraic Thinking
Students build on their understanding of addition and subtraction to develop an understanding of the meanings of multiplication and division of whole numbers. Students use increasingly sophisticated strategies based on properties of operations to fluently solve multiplication and division problems within 100 (See Table 3 - Properties of Operations). Students interpret multiplication as finding an unknown product in situations involving equal-sized groups, arrays, area and measurement models, and division as finding an unknown factor in situations involving the unknown number of groups or the unknown group size. Students use these interpretations to represent and solve contextual problems with unknowns in all positions. By the end of 3rd grade, students should know all products of two one-digit numbers and related division facts.

Students use all four operations to solve two-step word problems and use place value, mental computation, and estimation strategies to assess the reasonableness of solutions. They build number sense by investigating numerical representations, such as addition or multiplication tables for the purpose of identifying arithmetic patterns. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations).

Number and Operations in Base Ten
Students generalize place value understanding to read and write numbers to 100,000, using standard form, word form, and expanded form. Students begin to develop an understanding of rounding whole numbers to the nearest ten or hundred. Students fluently add and subtract within 1000 using strategies and algorithms. Students multiply one-digit whole numbers by multiples of 10.

Number and Operations in Fractions
This domain builds on the previous skill of partitioning shapes in geometry. This is the first time students are introduced to unit fractions. Students understand that fractions are composed of unit fractions and they use visual fraction models to represent parts of a whole. Students build on their understanding of number lines to represent fractions as locations and lengths on a number line. Students use fractions to represent numbers equal to, less than, and greater than 1 and are able to generate simple equivalent fractions by using drawings and/or reasoning about fractions. Students understand that the size of a fractional part is relative to the size of the whole.

Measurement and Data
In 2nd grade, students tell time in five minute increments, measure lengths, and create bar graphs, pictographs, and line plots with whole number units. In 3rd grade, students tell and write time to the nearest minute and solve contextual problems involving addition and subtraction. They use appropriate tools to measure and estimate liquid volume and mass. Students draw pictographs and bar graphs and answer two-step questions about these graphs. Students generate measurement data and represent the data on line plots marked with whole number, half, or quarter units. Students recognize area as an attribute of two-dimensional shapes and measure the area of a shape using the standard unit (a square) by finding the total number of same-sized units required to cover the shape without gaps or overlaps. Students connect area to multiplication and use multiplication to justify the area of a rectangle by decomposing rectangles into rectangular arrays of squares.

Geometry
Students understand that shapes in given categories have shared attributes and they identify polygons. Students continue their understanding of shapes and fractions by partitioning shapes into parts with equal areas and identify the parts with unit fractions.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Represent and solve problems involving multiplication and division.	**3.OA.A.1** Interpret the factors and products in whole number multiplication equations (*e.g., 4 x 7 is 4 groups of 7 objects with a total of 28 objects or 4 strings measuring 7 inches each with a total length of 28 inches*). **3.OA.A.2** Interpret the dividend, divisor, and quotient in whole number division equations (*e.g., 28 ÷ 7 can be interpreted as 28 objects divided into 7 equal groups with 4 objects in each group or 28 objects divided so there are 7 objects in each of the 4 equal groups*). **3.OA.A.3** Multiply and divide within 100 to solve contextual problems, with the unknown in any positions, in situations involving equal groups, arrays/area, and measurement quantities using strategies based on place value, the properties of operations, and the relationship between multiplication and division (*e.g., contexts including computations such as 3 x ? = 24, 6 x 16 = ?, ? ÷ 8 = 3, or 96 ÷ 6 = ?*). (See Table 2 - Multiplication and Division Situations). **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers within 100. *For example, determine the unknown number that makes the equation true in each of the equations: 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 =?.*
B. Understand properties of multiplication and the relationship between multiplication and division. (See Table 3 - Properties of Operations)	**3.OA.B.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (commutative property of multiplication). 3 x 5 x 2 can be solved by (3 x 5) x 2 or 3 x (5 x 2) (associative property of multiplication). One way to find 8 x 7 is by using 8 x (5 + 2) = (8 x 5) + (8 x 2). By knowing that 8 x 5 = 40 and 8 x 2 = 16, then 8 x 7 = 40 + 16 = 56 (distributive property of multiplication over addition).* **3.OA.B.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
C. Multiply and divide within 100.	**3.OA.C.7** Fluently multiply and divide within 100, using strategies such as the properties of operations or the relationship between multiplication and division (*e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8*). By the end of 3rd grade, know all products of two one-digit numbers and related division facts.

Cluster Headings	Content Standards
D. Solve problems involving the four operations and identify and explain patterns in arithmetic.	**3.OA.D.8** Solve two-step contextual problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations). **3.OA.D.9** Identify patterns in a multiplication chart and explain them using properties of operations. *For example, in the multiplication chart, observe that 4 times a number is always even (because 4 x 6 = (2 x 2) x 6 = 2 x (2 x 6), which uses the associative property of multiplication) or, for example, observe that 6 times 7 is one more group of 7 than 5 times 7 (because 6 x 7 = (5 + 1) x 7 = (5 x 7) + (1 x 7), which uses the distributive property of multiplication over addition).* (See Table 3 - Properties of Operations)

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Use place value understanding and properties of operations to perform multi-digit arithmetic.	**3.NBT.A.1** Round whole numbers to the nearest 10 or 100 using understanding of place value and use a number line to explain how the number was rounded. **3.NBT.A.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.NBT.A.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 *(e.g., 9 x 80, 5 x 60)* using strategies based on place value and properties of operations. **3.NBT.A.4** Read and write multi-digit whole numbers (less than or equal to 100,000) using standard form, word form, and expanded form (*e.g., 23,456 can be written as 20,000 + 3,000 + 400 + 50 + 6*).

Number and Operations - Fractions (NF)

Cluster Headings	Content Standards
A. Develop understanding of fractions as numbers.	**3.NF.A.1** Understand a unit fraction, 1/*b*, as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a non-unit fraction, *n/b*, as the quantity formed by *n* parts of size 1/*b*. *For example, 3/4 represents a quantity formed by 3 parts of size 1/4.* **3.NF.A.2** Understand a fraction as a number on the number line. Represent fractions on a number line. **a.** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/*b* and that the endpoint locates the number 1/*b* on the number line. *For example, on a number line from 0 to 1, students can partition it into 4 equal parts and recognize that each part represents a length of 1/4 and the first part has an endpoint at 1/4 on the number line.* **b.** Represent a fraction *n/b* on a number line diagram by marking off *n* lengths 1/*b* from 0. Recognize that the resulting interval has size *n/b* and that its endpoint locates the number *n/b* on the number line. *For example, 5/3 is the distance from 0 when there are 5 iterations of 1/3.* **3.NF.A.3** Explain equivalence of fractions and compare fractions by reasoning about their size. **a.** Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. **b.** Recognize and generate simple equivalent fractions (*e.g., 1/2 = 2/4, 4/6 = 2/3*) and explain why the fractions are equivalent using a visual fraction model. **c.** Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form 3 = 3/1; recognize that 6/1= 6; locate 4/4 and 1 at the same point on a number line diagram*. **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	**3.MD.A.1** Solve contextual problems in time and money. **a.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. **b.** Solve one-step contextual problems involving amounts less than one dollar including quarters, dimes, nickels, and pennies using the ¢ symbol appropriately. Solve contextual problems involving whole number dollar amounts up to $1000 using the $ symbol appropriately. **3.MD.A.2** Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). Estimate the mass of objects and liquid volume using benchmarks. *For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams.*
B. Represent and interpret data.	**3.MD.B.3** Draw a pictograph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in graphs. **3.MD.B.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units: whole numbers, halves, or quarters.

Cluster Headings	Content Standards
C. Geometric measurement: understand and apply concepts of area and relate area to multiplication and to addition.	**3.MD.C.5** Recognize that plane figures have an area and understand concepts of area measurement. **a.** Understand that a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area. **b.** Understand that a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.C.6** Measure areas by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units). **3.MD.C.7** Relate area of rectangles to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. **c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and (*b* + *c*) is the sum of (*a* x *b*) and (*a* x *c*). Use area models to represent the distributive property in mathematical reasoning. *For example, in a rectangle with dimensions 4 by 6, students can decompose the rectangle into 4 x 3 and 4 x 3 to find the total area of 4 x 6.* (See Table 3 - Properties of Operations) **d.** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	**3.MD.D.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exploring rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry (G)

Cluster Headings	Content Standards
A. Reason about shapes and their attributes.	**3.G.A.1** Understand that shapes in different categories may share attributes and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals and recognize examples of quadrilaterals that do not belong to any of these subcategories. **3.G.A.2** Partition shapes into parts with equal areas. Recognize that equal shares of identical wholes need not have the same shape. Express the area of each part as a unit fraction of the whole. **3.G.A.3** Determine if a figure is a polygon.

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1st)**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2nd)**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ **(1st)**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** **(2nd)**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1st)**

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$