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ACCOMMODATING VARIATIONS IN RESPONSIVENESS TO LEVEL-OF-SERVICE MEASURES IN TRAVEL MODE CHOICE MODELING

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Abstract—An individual's responsiveness to level-of-service variables affects her or his travel mode choice for a trip. This responsiveness will, in general, vary across individuals based on observed (to an analyst) and unobserved (to an analyst) individual characteristics. The current paper formulates a multinomial-logit based model of travel mode choice that accommodates variations in responsiveness to level-of-service measures due to both observed and unobserved individual characteristics in a comprehensive manner. The choice probabilities in the resulting model are evaluated using Monte Carlo simulation techniques and the model parameters are estimated using a maximum simulated likelihood approach. The model is applied to examine the impact of improved rail service on weekday, business travel in the Toronto—Montreal corridor. The empirical results show that not accounting adequately for variations in responsiveness across individuals leads to a statistically inferior data fit and also to inappropriate evaluations of policy actions aimed at improving inter-city transportation services. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: intercity travel, maximum simulated likelihood function, multinomial logit model, taste heterogeneity, random-coefficients.

1. INTRODUCTION

Travel mode choice modeling is an important component of urban and intercity travel demand analysis. Mode choice models provide the tool to undertake an *a priori* evaluation of the effectiveness of alternative traffic congestion alleviation strategies by estimating the shifts in mode shares in response to such strategies. Accurate estimations of the displacements of shares among modes requires the development of models which include policy-sensitive variables and which capture differences in intrinsic preferences and differences in sensitivity to level-of-service changes across individuals in the population (Hensher, 1981).

Travel mode choice models are, in general, based on the utility maximization hypothesis which assumes that an individual's mode choice is a reflection of underlying preferences for each of the available alternatives and that the individual selects the mode with the highest preference or utility. The utility that an individual associates with a mode may be viewed as comprising two components from the perspective of a travel demand analyst. The first component is the intrinsic bias of the individual toward the mode due to observed and unobserved (to an analyst) individual factors (such as sex, lifestyle and culture) and the individual's evaluation of unobserved (to an analyst) characteristics of the mode (such as comfort and privacy). The second component is the utility that the individual derives from observable (to an analyst) level-of-service characteristics offered by the mode for the individual's trip. Ideally, we should obtain individual-specific parameters for the intrinsic mode biases (or preferences) and for the subjective evaluations of modal level-of-service attributes. However, the data used for mode choice estimation is usually cross-sectional. This precludes estimation at the individual level and constrains the modeler to pool the data across individuals. In such pooled estimations, the analyst should accommodate differences in intrinsic mode preferences (preference heterogeneity) and differences in responsiveness to level-of-service attributes (response heterogeneity) across individuals. In particular, if the assumption of homo-

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geneity is imposed when there is heterogeneity, the result is biased and inconsistent parameter and choice probability estimates (see Chamberlain, 1980).

Preference heterogeneity and response heterogeneity may each be decomposed into two components. The first component is associated with heterogeneity due to observed (to an analyst) individual factors and the second component corresponds to heterogeneity due to unobserved (to the analyst) individual factors. We will label the former as systematic heterogeneity and the latter as random heterogeneity.

In choice modeling, it is important to capture both systematic and random components of preference and response heterogeneity. Systematic heterogeneity allows differential sensitivity (to policy actions) of different groups in the population. As a first step in choice modeling, the analyst must always strive to attribute as much preference and response heterogeneity to systematic variations as possible (Bhat, 1996). However, it is very unlikely that we will be aware of, or will have information on, all individual factors affecting intrinsic biases and responsiveness to level-of-service variables. In particular, there may be appreciable variation in tastes even after developing the best possible systematic specification for preference and response heterogeneity (Fischer and Nagin, 1981). Consequently, we must consider superimposing random heterogeneity over the best systematic specification.

Systematic preference heterogeneity is generally incorporated in travel mode choice models by introducing observed individual (socio-economic and trip) characteristics as alternative-specific variables. Random preference heterogeneity is accommodated by including an additive stochastic error term in the utility function of each alternative.

Systematic response heterogeneity is commonly included in mode choice models by interacting level-of-service variables with individual characteristics. These interactions are usually limited to one or two specifications such as 'travel cost over income' or 'out-of-vehicle time over distance'. Interactions of the cost and out-of-vehicle time variables with other individual characteristics, and interactions of other level-of-service variables (such as in-vehicle time and frequency of service) with relevant individual attributes, are not explored extensively. Alternatively, some studies have attempted to capture systematic response heterogeneity by using a market segmentation scheme. To keep the estimation manageable and also from parameter stability considerations, typically only one or two individual-related variables are considered for segmentation. This again implies limited accommodation of systematic response heterogeneity. Mode choice studies which accommodate systematic response heterogeneity in a limited fashion using one of the two approaches mentioned above also do not account for random response heterogeneity. On the other hand, some studies in the travel mode choice and econometric literature have adopted a (pure) random coefficients specification for the level-of-service (or equivalent) variables without accommodating systematic response heterogeneity (see Ben-Akiva *et al.*, 1993; Revelt and Train, 1997; Train, 1997).*

In this paper, we formulate a multinomial-logit based model of travel mode choice that accommodates systematic response heterogeneity in an efficient and comprehensive manner. In addition, the model allows random response heterogeneity and ensures the correct sign on the level-of-service measures for all individuals. The model (which we label the random-coefficients logit or RCL model) is applied to an empirical analysis of inter-city travel mode choice in the Toronto–Montreal corridor.

The rest of this paper is organized as follows. The next section develops the model structure for the RCL model. Section 3 discusses the estimation procedure for the RCL model. Section 4 presents the empirical results obtained from applying the model to intercity mode choice modeling. The final section provides a summary of the research findings.

2. MODEL STRUCTURE

We develop the model structure assuming that all individuals have all alternatives available to them. Extension of the formulation to the case where some individuals have only a subset of alternatives available to them is straightforward.

*Ben-Akiva *et al.* (1993) point out that their model is a random trade-off model and should not be confused with a random-coefficients model. In fact, their model can be considered as a special case of a random-coefficients model where the coefficient on the travel cost variable is specified to be fixed (see section 2).

Let the utility U_{qi} that an individual q associates with an alternative i be the sum of an intrinsic bias term Λ_{qi} and a term Δ_{qi} that represents the individual's evaluation of observable (to an analyst) level-of-service characteristics offered by the alternative. We assume a linear-in-parameters specification for the second term so that it may be written as, $\Delta_{qi} = \eta'_q x_{qi}$, where η_q is a vector representing the responsiveness of individual q to a corresponding vector of level-of-service variables x_{qi}. Thus we can write:

$$U_{qi} = \Lambda_{qi} + \eta'_{qi} x_{qi} \tag{1}$$

To allow systematic and random preference heterogeneity, we can parameterize the intrinsic bias term Λ_{qi} as the sum of (a) an individual-invariant bias constant α_i, (b) a systematic preference heterogeneity term that is a function of a vector of observed individual characteristics z_q (assuming a linear-in-parameters specification for this term, we may write it as $\delta'_i z_q$, where δ_i is a vector of parameters to be estimated), and (c) a random preference heterogeneity term ε_{qi} representing idiosyncracies in preferences. We can then write eqn (1) as:

$$U_{qi} = \alpha_i + \delta'_i z_q + \varepsilon_{qi} + \eta'_q x_{qi} \tag{2}$$

Conditional on η_q, and assuming that the ε_{qi} terms have a type I extreme value distribution and are independent and identically distributed across individuals and alternatives, we get the familiar multinomial logit form for the choice probabilities:

$$P_{qi}|\eta_q = \frac{\exp(\alpha_i + \delta'_i z_q + \eta'_q x_{qi})}{\sum_{j=1}^{I} \exp(\alpha_j + \delta'_j z_q + \eta'_q x_{qj})} \tag{3}$$

The unconditional choice probabilities corresponding to the conditional choice probabilities in eqn (3) depend on the response heterogeneity specification adopted for the η_q vector. Three different specifications are possible for the η_q vector, giving rise to three different models for the unconditional choice probabilities.

The first (and simplest) specification is to maintain the same value of the response vector η_q across all individuals; that is, $\eta_q = \eta$ for all q. This corresponds to response homogeneity and the conditional choice probabilities in eqn (3) also become the unconditional probabilities after replacing η_q with η on the right side (we will refer to this model as the multinomial logit or MNL model).

The second specification allows for systematic response heterogeneity by writing each element η_{qk} of the vector η_q as a function of a vector w_{qk} of relevant observed individual characteristics: $\eta_{qk} = \gamma_k + \beta_k f(w_{qk)}$ (η_{qk} represents the response coefficient of the qth individual to the kth level-of-service variable; k is an index for the level-of-service variables). If we pre-specify a functional form for $f(w_{qk})$, then the unconditional choice probabilities take the multinomial logit form after replacing $\eta'_q x_{qi}$ by $\sum(\gamma_k[x_{qik}] + \beta_k[f(w_{qk})x_{qik}])$ in eqn (3). A problem with the specification of the form $\eta_{qk} = \gamma_k + \beta_k\, {}^k f(w_{qk})$, however, is that it does not guarantee the correct sign of the response coefficient η_{qk} for all individuals*. An alternative method that accommodates systematic response heterogeneity and, at the same time, ensures the appropriate sign on the response coefficients is to specify $\eta_{qk} = \pm\exp(\gamma_k + \beta'_k w_{qk})$. The '+' sign is applied for a non-negative response coefficient and the '$-$' sign is applied for a non-positive response coefficient. The unconditional choice probabilities with this exponential specification are the same as the conditional probabilities in eqn (3) after replacing η_{qk} with $\pm\exp(\gamma_k + \beta'_k w_{qk})$. The resulting model does not have the usual multinomial logit form (we will refer to this model as the fixed-coefficients logit or FCL model to indicate that the response coefficients are deterministic given the observed individual characteristics).

*In the intercity empirical context of this paper, we found that a specification of the form $\eta_{qk} = \gamma_k + \beta'_k w_{qk}$ led to negative response coefficients for the frequency of service variable and positive response coefficients for the time and cost variables for some individuals in the sample.

The third (and most general) specification for the response coefficients superimposes random response heterogeneity over the systematic response heterogeneity of the second specification: $\eta_{qk} = \pm\exp(\gamma_k + \beta'_k w_{qk} + \nu_{qk})$, where ν_{qk} is a term representing random taste variations across individuals with the same observed characteristics w_{qk}. In the rest of this section, we discuss the assumptions made in the current paper regarding the ν_{qk} terms and present the structure for the resulting unconditional choice probabilities (we will label this model as the random-coefficients logit or RCL model).

In the RCL model, we assume that the ν_{qk} terms $(k = 1, 2, ..., K)$ are independent across response coefficients and normally distributed with mean zero and variance σ_k^2 (the ν_{qk} terms are also assumed to be independently and identically distributed across individuals). The independence assumption for the ν_{qk} terms across response coefficients may not be very restrictive since we are allowing systematic response heterogeneity. For example, high income may lead to low cost sensitivity and high time sensitivity. If we do not allow the response coefficients to be a function of individual characteristics (as in a pure random coefficients approach), then there would be a negative correlation between the cost and time response coefficients. In our framework, we can include income in the w_{qk} vector for the cost and time coefficients. After controlling for the effect of income, there may be little correlation remaining between the cost and time coefficients. The normal distribution assumption for the ν_{qk} terms implies a log-normal distribution (across individuals with the same observed vector w_{qk}) for the response coefficients with the following properties (see Johnson and Kotz, 1970, p. 115): (a) Median$=\exp(\omega_{qk})$; (b) mode$=\exp(\omega_{qk})/\mu_k$; (c) mean$=\exp(\omega_{qk})\mu_k^{1/2}$ and (d) variance$=\exp(2\omega_{qk})\mu_k(\mu_k - 1)$, where $\omega_{qk} = \gamma_k + \beta'_k w_{qk}$ and $\mu_k = \exp(\sigma_k^2)$. A useful property of the log-normal distribution is that the ratio of two independent log-normally distributed variables is also log-normally distributed. Therefore, a log-normal distribution assumption for the response coefficients implies a log-normal distribution for the money value of time (which is obtained as the ratio of the travel time and travel cost coefficients). If we obtain the parameters ω_{qc} and ω_{qt} (with corresponding variance parameters σ_c^2 and σ_t^2) for the cost and time coefficients, respectively, for individuals with observed vector w_{qk}, then the distribution of the money value of time across these individuals has a log-normal distribution with (a) Median$=\exp(\omega_{qtc})$; (b) mode$=\exp(\omega_{qtc})/\mu_{tc}$; (c) mean$=\exp(\omega_{qtc})\mu_{tc}^{1/2}$ and (d) variance$=\exp(2\omega_{qtc})\mu_{tc}(\mu_{tc} - 1)$, where $\omega_{qtc} = \omega_{qt} - \omega_{qc}$ and $\mu_{tc} = \exp(\sigma_t^2 + \sigma_c^2)$*.

The utility function of eqn (2) can be written in the RCL model as:

$$U_{qi} = \alpha_i + \delta'_i z_q + \varepsilon_{qi} + \sum_k [\exp(\gamma_k + \beta'_k w_{qk} + \nu_{qk})] x_{qik} \tag{4}$$

The ν_{qk} terms in the above equation represent the random tastes of person q and are common to the utility of all alternatives i. Therefore, variance in the ν_{qk} terms across individuals induces a correlation among the utility of different alternatives (see Revelt and Train, 1997). As a result, the RCL model does not exhibit the restrictive independence from irrelevant alternatives (IIA) property of the multinomial logit model. This can also be noticed by writing the expressions for the RCL choice probabilities. The choice probabilities conditional on the random variables $\nu_{qk}, k = 1, 2, \ldots K$, are given by:

$$P_{qi}|(\nu_{q1}, \ldots, \nu_{qk}) = \frac{\exp\left(\alpha_i + \delta'_i z_q + \sum_{k=1}^{K} \left[\exp(\gamma_k + \beta'_k w_{qk} + \nu_{qk})\right] x_{qik}\right)}{\sum_{j=1}^{I} \exp\left(\alpha_j + \delta'_j z_q + \sum_{k=1}^{K} \left[\exp(\gamma_k + \beta'_k w_{qk} + \nu_{qk})\right] x_{qjk}\right)} \tag{5}$$

To obtain the unconditional probability of choosing alternative i, we integrate eqn (5) with respect to the K independent random variables $\nu_{q1}, \nu_{q2}, \ldots, \nu_{qK}$. The resulting expression has the following form:

*The study by Ben-Akiva *et al.* (1993) imposes the *a priori* assumption that the cost coefficient is fixed; that is $\sigma_c^2 = 0$. They allow random variation only in one level-of-service parameter (i.e. the total travel time parameter). Ben-Akiva *et al.* also do not accommodate systematic response heterogeneity in their model.

$$P_{qi} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\left\{\frac{\exp\left(\alpha_i + \delta_i' z_q + \sum_{k=1}^{K}\left[\exp\left(\gamma_k + \beta_k' w_{qk} + \nu_{qk}\right)\right]x_{qik}\right)}{\sum_{j=1}^{I}\exp\left(\alpha_j + \delta_j' z_q + \sum_{k=1}^{K}\left[\exp\left(\gamma_k + \beta_k' w_{qk} + \nu_{qk}\right)\right]x_{qjk}\right)}\right\} \cdot f(\nu_{q1})f(\nu_{q2})\ldots f(\nu_{qK})d\nu_{q1}d\nu_{q2}\ldots d\nu_{qK} \quad (6)$$

From eqn (6), we note that the ratio of P_{qi} to P_{ql} for $l \neq i$ depends on all I alternatives in the choice set; that is the RCL model is not saddled with the IIA property.

The disaggregate-level self- and cross- marginal effects take the following form in the RCL model:

$$\begin{aligned}\frac{\partial P_{qi}}{\partial x_{qik}} &= \int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\left\{P_{qi}|\left(\nu_{q1},\ldots,\nu_{qK}\right)\cdot\left[1 - P_{qi}|\left(\nu_{q1},\ldots,\nu_{qK}\right)\right]\cdot\exp\left(\gamma_k + \beta_k' w_{qk} + \nu_{qk}\right)\right\} \\ &\qquad f(\nu_{q1}),\ldots,f(\nu_{qK})d\nu_{q1},\ldots,d\nu_{qK} \\ \frac{\partial P_{qi}}{\partial x_{qlk}} &= -\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{-\infty}\left\{P_{qi}|\left(\nu_{q1},\ldots,\nu_{qK}\right)\cdot P_{ql}|\left(\nu_{q1},\ldots,\nu_{qK}\right)\cdot\exp\left(\gamma_k + \beta_k' w_{qk} + \nu_{qk}\right)\right\} \\ &\qquad f(\nu_{q1}),\ldots,f(\nu_{qK})d\nu_{q1},\ldots,d\nu_{qK}\end{aligned} \quad (7)$$

The disaggregate-level self- and cross-elasticities can be obtained from the marginal effects. The aggregate-level elasticities may be computed from the individual-level elasticities in the usual way (see Ben-Akiva and Lerman, 1985, p. 113).

3. MODEL ESTIMATION

The parameters to be estimated in the random-coefficients logit model include the vector $\zeta_i = (\alpha_i, \delta_i')'$ for each i (this vector is normalized to zero for one of the alternatives due to identification considerations) and the vector $\xi_k = (\gamma_K, \beta_K', \sigma_K')$ for each level of service variable k. Define $u_{qk}(q = 1,\ldots,Q, k = 1,\ldots,K)$ as a standard-normal variate so that $u_{qk} = \nu_{qk}/\sigma_k$. Then, the log-likelihood function for a given value of the parameter vector $\theta = (\zeta_1', \zeta_2', \ldots, \zeta_I', \xi_1', \xi_2', \ldots, \xi_K')'$ takes the form:

$$\begin{aligned}\mathscr{L}(\theta) &= \sum_{q=1}^{Q}\sum_{i=1}^{I} y_{qi}\log P_{qi}(\theta) \\ &= \sum_{q=1}^{Q}\sum_{i=1}^{I} y_{qi}\log\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\left\{\frac{\exp\left(\alpha_i + \delta_i' z_q + \sum_{k=1}^{K}\left[\exp\left(\gamma_k + \beta_k' w_{qk} + \sigma_k u_{qk}\right)\right]x_{qik}\right)}{\sum_{j=1}^{I}\exp\left(\alpha_j + \delta_j' z_q + \sum_{k=1}^{K}\left[\exp\left(\gamma_k + \beta_k' w_{qk} + \sigma_k u_{qk}\right)\right]x_{qik}\right)}\right\}\right. \\ &\qquad \left.\cdot\phi(u_{ql})\phi(u_{q2})\ldots\phi(u_{qK})du_{q1}du_{q2}\ldots du_{qK}\right],\end{aligned} \quad (8)$$

where $\phi(.)$ represents the standard normal density function and

$$y_{qi} = \begin{cases} 1 \text{ if the } q\text{th individual chooses alternative } i \\ 0 \text{ otherwise,}\end{cases} \qquad (q = 1, 2, \ldots Q, i = 1, 2, \ldots, I) \quad (9)$$

The log-likelihood function in eqn (8) involves the evaluation of a K-dimensional integral (K is the number of level-of-service variables). The K-dimensional integral cannot be evaluated analytically since it does not have a closed-form solution. When $K = 1$ (as in Ben-Akiva *et al.*, 1993) or $K = 2$

(as in Train *et al*., 1987), the integration may be performed using general purpose numerical methods such as quadrature. However, when K is greater than 2, quadrature techniques cannot compute the integrals with sufficient precision and speed for estimation via maximum likelihood (see Revelt and Train, 1997 and Hajivassiliou and Ruud, 1994).*

We apply Monte Carlo simulation techniques to approximate the choice probabilities in the log-likelihood function of eqn (8) and maximize the resulting simulated log-likelihood function. The use of simulation techniques to evaluate multi-dimensional integrals has received substantial attention in recent years and these methods have been used in empirical applications in the economics field (see, for example, Börsch-Supan *et al*., 1992; Train, 1997; and Corres *et al*., 1993). However, transportation researchers have yet to take advantage of simulation methods to specify and estimate more realistic models of travel-demand choices that accommodate random response heterogeneity.

Our implementation of simulation methods to estimate the RCL model of mode choice takes the same form as the procedure adopted by Revelt and Train (1997). The simulation technique approximates the choice probabilities by computing the integrand in eqn (8) at randomly chosen values for each u_{qk}. Since the u_{qk} terms are independent across individuals and level-of-service variables, and are distributed standard normal, we generate a matrix u of standard normal random numbers with Q^*K elements (one element for each u_{qk}) and compute the corresponding choice probabilities for a given value of the parameter vector θ. We then repeat this process R times for the given value of the parameter vector θ. Let $\tilde{P}^r_{qi}(\theta)$ be the realization of the choice probability in the rth draw ($r = 1, 2, \ldots, R$). The choice probabilities are then approximated by averaging over the $\tilde{P}^r_{qi}(\theta)$ values:

$$\tilde{P}_{qi}(\theta) = \frac{1}{R}\sum_{r=1}^{R} \tilde{P}^r_{qi}(\theta), \tag{10}$$

where $\tilde{P}_{qi}(\theta)$is the simulated choice probability of the qth individual choosing alternative i given the parameter vector θ. $\tilde{P}_{qi}(\theta)$is an unbiased estimator of the actual probability $P_{qi}(\theta)$. It's variance decreases as R increases. It also has the appealing properties of being smooth (i.e. twice differentiable) and being strictly positive for any realization of the finite R draws. The former property is important since it implies that conventional gradient-based optimization methods can be used in the maximization of the simulated log-likelihood function. The latter property ensures that the simulated log-likelihood function (which involves the logarithm of the choice probabilities) is always defined.

The simulated log-likelihood function is constructed as:

$$S\mathscr{L}(\theta) = \sum_{q=1}^{Q}\sum_{i=1}^{I} y_{qi}\log\left[\tilde{P}_{qi}(\theta)\right] \tag{11}$$

The parameter vector θ is estimated as the vector value that maximizes the above simulated function. Under rather weak regularity conditions, the maximum simulated likelihood (MSL) estimator is consistent, asymptotically efficient, and asymptotically normal (see Hajivassiliou and Ruud, 1994 and Lee, 1992). However, the MSL estimator will generally be a biased simulation of the maximum likelihood (ML) estimator because of the logarithmic transformation of the choice probabilities in the log-likelihood function (the logarithm of the simulated probability will not be an unbiased estimate of the logarithm of the true probability even though the simulated probability is an unbiased estimate of the true probability). The bias of the SML estimator decreases with the variance of the probability simulator; that is, it decreases as the number of repetitions increase. Brownstone and Train (1996) have shown the bias to be rather negligible with 250 repetitions. In the current paper, we use 1000 repetitions for accurate simulations of the choice probabilities and to reduce simulation variance of the MSL estimator.

All estimations were carried out using the GAUSS programming language. Gradients of the simulated log-likelihood function with respect to the parameters were coded.

*In the empirical analysis of the current paper, we use four level-of-service variables and so $K = 4$.

4. EMPIRICAL ANALYSIS OF INTERCITY MODE CHOICE

4.1. Background

The cost–benefit evaluation of alternative proposals to improve intercity transportation services is an important input to making informed infrastructure investment decisions in inter-city corridors. Among other things, such an evaluation entails the estimation of reliable inter-city mode choice models to estimate ridership share on the proposed new (or improved) inter-city service and to identify the modes from which existing inter-city travelers will be diverted to the new (or upgraded) service.

Previous inter-city mode choice models have either not accommodated systematic differences in response sensitivity to level-of-service variables or have accommodated such differences in a very restrictive manner by selective interaction of individual characteristics with level-of-service measures (as discussed in section 1). Further, to the author's knowledge, no previous inter-city mode choice study has accommodated both systematic and random variation in responsiveness to level-of-service measures.

In the empirical analysis of this paper, we apply the multinomial logit, the fixed-coefficients logit, and the random coefficients logit models to examine the travel mode choice behavior of weekday business travelers in the Toronto–Montreal corridor of Canada. The data used in the analysis were assembled by VIA Rail (the Canadian national rail carrier) in 1989 to develop travel demand models to forecast future inter-city travel in the Toronto–Montreal corridor and to estimate shifts in mode split in response to a variety of potential rail service improvements. The data includes socio-demographic and general trip-making characteristics of the traveler, and detailed information on the current trip (purpose, party size, origin and destination cities, etc.). The travel mode alternatives are air, train, and car. Level of service data were generated for each available mode and for each trip based on the origin/destination information of the trip (for detailed information on the procedure to compute level-of-service measures, the reader is referred to KPMG Peat Marwick and Koppelman, 1990). The sample used comprises 2000 weekday-business travelers whose aggregate mode choice shares reflect the market mode shares in the inter-city corridor.

The level-of-service variables used to model choice of mode include frequency of service, total cost, in-vehicle time and out-of-vehicle time. Table 1 presents the market shares of each mode and the descriptive statistics for the level-of-service measures in the sample.

Four variables associated with individual socio-demographics and trip characteristics were considered for accommodating systematic preference heterogeneity and systematic response heterogeneity. The variables are: income, sex (female or male), travel group size (traveling alone or traveling in a group), and a large city indicator which identified whether a trip originated and/or terminated in a large city. The mean values for these exogenous variables are: large city indicator (0.70), income (Canadian $54,800), Female dummy variable (0.18), and dummy variable for traveling alone (0.71).

4.2. Empirical results

The final model specification results for the multinomial logit (MNL) model, the fixed-coefficients logit (FCL) model, and the random-coefficients logit (RCL) model are shown in Table 2.* We maintain the car mode as the base in accommodating differences in intrinsic mode preferences across individuals.

The multinomial logit (MNL) model shows that higher income favors air travel relative to the car and train modes. Individuals who travel alone prefer common carrier modes more so than individuals traveling in groups (who prefer the car mode). Women are more likely to use common carrier modes (especially the train mode) compared to men (see KPMG Peat Marwick *et al.*, 1993 for a similar result). There is a preference for the common carrier modes (air and train) over the

*As indicated earlier, direct interaction of the level-of-service variables with the socio-demographic/trip variables to accommodate systematic heterogeneity within the framework of the MNL structure led to inappropriate values on the cost and time variables (for example, the coefficient on the cost variable became positive for high income travlers in the sample). Further, the direct-interaction specification did not fare as well as the FCL model in terms of log-likelihood value at convergence. Other specifications such as the cost over income specification instead of the simple cost specification in the MNL model were also examined, but provided worse results than the MNL specification used here.

Table 1. Mode market shares and descriptive statistics of modal level-of-service variables

Mode	Share	Frequency (departures/day)	Total cost (in Canadian $)	In-vehicle time (min)	Out-of-vehicle time (min)
Train	0.10	4.13 (2.2)	56.32 (17.2)	230.08 (109.1)	86.78 (22.4)
Air	0.38	24.52 (13.2)	154.89 (20.6)	55.76 (18.3)	107.79 (26.0)
Car	0.52	Not applicable	66.55 (31.7)	237.42 (104.9)	0.00 (0.0)

The numbers in the table for the level-of-service measures denote one-way mean (no parenthesis) and standard deviation (parenthesis) values. Air and train frequency refer to total departures by all carriers.

Table 2. Intercity mode choice estimation results

Parameter affecting	Parameter on	MNL		FCL		RCL	
		Parm.	*t*-stat.	Parm.	*t*-stat.	Parm.	*t*-stat.
Intrinsic mode preferences	Mode constants						
	Train	−1.172	−2.57	−0.996	−2.77	−0.341	−0.75
	Air	−0.887	−1.30	1.256	1.98	1.466	1.88
	Income ($\times$x10^4)						
	Train	0.011	0.22	–	–	–	–
	Air	0.390	8.21	–	–	–	–
	Traveling alone						
	Train	0.366	1.97	–	–	–	–
	Air	0.436	2.72	–	–	–	–
	Female						
	Train	1.281	6.78	2.313	3.68	2.489	3.26
	Air	0.919	4.70	1.444	1.83	1.417	1.39
	Large city indicator						
	Train	1.105	5.11	1.058	4.90	0.994	4.41
	Air	0.672	3.40	0.699	3.50	0.857	2.88
Response to level-of-service variables	Freq. of service						
	Constant*	−2.435	–	−2.459	–	−2.163	–
	Traveling alone	–	–	−0.154	−1.42	−0.035	−0.34
	Female	–	–	0.395	2.89	0.395	2.47
	Std deviation	–	–	–	–	0.260	4.14
	Travel cost						
	Constant*	−3.165	–	−2.909	–	−2.727	–
	Income ($\times 10^4$)	–	–	−0.077	−3.22	−0.059	−2.45
	Std deviation	–	–	–	–	0.028	0.22
	In-vehicle time						
	Constant*	−4.610	–	−5.174	–	−5.089	–
	Income ($\times 10^4$)	–	–	0.064	1.90	0.115	2.89
	Traveling alone	–	–	0.359	2.63	0.228	1.72
	Std. deviation	–	–	–	–	0.546	6.51
	Out-of-vehicle time						
	Constant*	−3.453	–	−3.543	–	−3.201	–
	Female	–	–	0.428	2.16	0.373	2.02
	Std deviation	–	–	–	–	0.005	0.07

*We do not report any *t*-statistics for the constants because the only reasonable test of the constant parameters would be against a value of negative infinity. The standard errors of the constant parameters are very small ranging from 0.10 to 0.30.

car mode, and for the train mode over the air mode, for trips which originate and/or end in a large city. The entries for the level-of-service (LOS) constants for the MNL model in Table 2 reflect parameter estimates whose positive exponent for the frequency entry and negative exponent for the cost and time entries correspond to the actual estimated LOS parameter values. We adopt this presentation structure to be consistent with the entries for the constants in the FCL and RCL models. The actual level-of-service parameter estimates and associated *t*-statistics (with respect to the value of zero) for the MNL model are as follows: (a) frequency parameter $= \exp(-2.435) = 0.088$ (*t*-statistic of 13.78), (b) Cost parameter $= -\exp(-3.165) = -0.042$ (*t*-statistic of −8.18), (c) In-vehicle time parameter $= -\exp(-4.61) = -0.010$ (*t*-statistic of −9.99), and out-of-vehicle parameter $= -\exp(-3.453) = -0.032$ (*t*-statistic of −9.64). All the level-of-service parameters have the expected signs and are highly significant. The larger negative coefficient on the out-of-vehicle travel time variable compared to the in-vehicle travel time variable reveals that out-of-vehicle time is more onerous than in-vehicle time.

The fixed-coefficients logit (FCL) model shows that income and traveling alone affect the responsiveness to level-of-service variables, but do not affect the intrinsic mode preferences. Thus, the apparent effect of these variables on preference heterogeneity in the MNL is actually an inappropriate manifestation of the effect of the variables on response sensitivity. The results corresponding to the systematic heterogeneity specification for responsiveness to the frequency of service variable indicate that individuals traveling together and women are more sensitive to frequency of service than individuals traveling alone and men, respectively. Individuals traveling together may have different schedule conveniences and women may have tighter schedule constraints because of their role in the household, which may explain the higher premium placed on frequency of service by these segments of travelers. The negative sign on the cost and time variables is ensured by specifying the response parameters on these variables as $\eta_{qk} = -\exp(\gamma_k + \beta_k' w_{qk})$, where w_{qk} is a relevant vector of individual socio-demographic/trip attributes. Thus, a negative sign on the coefficient of a socio-demographic/trip variable corresponding to the time and cost response parameters implies a lower response sensitivity and a positive effect indicates a higher response sensitivity. As one would expect, high income travelers are less sensitive to cost than low income travelers. Also, individuals with high incomes and individuals traveling alone are more sensitive to in-vehicle travel time than low income and group travelers, respectively. However, there are no income-based or travel group-based differences in sensitivity to out-of-vehicle travel time. Finally, women are more sensitive to out-of-vehicle travel time than men.

The parameters in the RCL model are, in general, similar to that of the FCL model. However, there are some differences. The intrinsic mode bias constants are relatively insignificant in the RCL model compared to the other two models, suggesting that the 'significant' mode bias constants in the MNL and FCL models are actually a reflection of random response heterogeneity to the level-of-service variables. The 'traveling alone' variable which has a marginally significant impact on frequency sensitivity in the FCL model fades into insignificance in the RCL model. The effect of income on travel cost sensitivity is less pronounced in the RCL model, while the effect of income on in-vehicle travel time is more pronounced. The estimated standard deviations of the frequency and in-vehicle travel time coefficients are highly significant in the RCL model, indicating that there is random variation in tastes with respect to frequency of service and in-vehicle time even after accounting for systematic sensitivity differences. However, the results show that, in the current empirical context, the cost and out-of-vehicle time sensitivities do not vary significantly across individuals after controlling for sensitivity differences due to observed individual/trip characteristics.

To obtain an intuitive characterization of the responsiveness to level-of-service variables among the different models, we compute the average of the response coefficients and implied money values of travel time across individuals in the sample for the FCL model. For the RCL model, we compute three summary measures to characterize the log-normal distribution of the response coefficients and implied money values of time. Specifically, we compute the mode (point at which the density function peaks), median (50th percentile value), and mean of the log-normally distributed response coefficients and money values of time conditional on the observed characteristics of each individual in the sample and then average these values across individuals. The results are shown in Table 3.

The RCL model shows variation in the average values of the mode, median, and mean for the frequency and in-vehicle time response coefficients, but not for the cost and out-of-vehicle time coefficients. This is a reflection of our earlier finding that there is significant random variation in sensitivity to the frequency and in-vehicle time variables, but not to the cost and out-of-vehicle time variables. The higher value for the mean of the response coefficients relative to the median and mode (and for the median relative to the mode) in the RCL model is a result of the left skew and long right tail of the log-normal distribution.

A comparison of the level-of-service parameters across the three specifications reveals that the RCL model estimates higher magnitudes for the parameters relative to the MNL and FCL models (this is an expected result since the variance before scaling is larger in the MNL and FCL models compared to the RCL model; see Revelt and Train, 1997 for a similar result).

The (average) implied values of in-vehicle and out-of-vehicle travel times from the FCL model are larger than those from the MNL model (see bottom of Table 3). In the RCL model, the mode, median, and mean money values of in-vehicle time show considerable variation. This indicates

Table 3. Comparison of response coefficients and implied money values of time

Level-of-service/money value of time	Average response coefficient values and average implied values of time across individuals				
	MNL	FCL	RCL		
			Mode	Median	Mean
Level-of-service variable					
Frequency of service (dep./day)	0.0876	0.0835	0.1138	0.1218	0.1260
Travel cost (Canadian $)	−0.0422	−0.0399	−0.0525	−0.0526	−0.0526
In-vehicle time (min)	−0.0100	−0.0105	−0.0105	−0.0140	−0.0162
Out-of-vehicle time (min)	−0.0316	−0.0317	−0.0440	−0.0440	−0.0440
Money value of time					
In-vehicle time ($ h)	14.15	16.34	12.14	16.38	19.03
Out-of-vehicle time ($ h)	44.96	48.29	50.62	50.66	50.68

significant random variation in the money value of in-vehicle time for individuals with the same observed characteristics. The (average) median value of the money value of in-vehicle time in the RCL model is about the same as the (average) value of time from the FCL model. Thus, among individuals with the same observed characteristics, the RCL model shows that about 50% of individuals have a value of in-vehicle time greater than the fixed value suggested by the FCL model. Because of the long right-hand tail of the log-normal distribution, the (average) mean value of in-vehicle time from the RCL model is higher than the (average) value of in-vehicle time from the FCL model. The mode, median, and mean money values of out-vehicle time show almost no variation in the RCL model because of the little random variation in the out-of-vehicle and cost coefficients. The values are, however, higher than the value suggested by the FCL model.

The differences in empirical results among the MNL, FCL and RCL models suggest the need to apply formal statistical tests to determine the structure that is most consistent with the data. Table 4 provides the log-likelihood value at convergence and the adjusted likelihood ratio index ($\bar{\rho}^2$) for the different models.*

The FCL model and the MNL model may be statistically compared using the non-nested likelihood ratio test (see Ben-Akiva and Lerman, 1985, p. 172). The result of such a test indicates that the upper bound of the probability that the estimated difference in adjusted likelihood ratio index values between the two models could have occurred by chance is $9 \times 10{-}5$. This result provides strong evidence of the improved fit of the FCL model over the MNL model. The data fit of the FCL and RCL models may be statistically compared using a nested likelihood ratio test. Such a test leads to the clear rejection of the hypothesis of homogeneity in response sensitivity to each (and all) level-of-service measures (the likelihood ratio test value is 30.3 which is larger than the chi-squared statistic with four degrees of freedom at any reasonable level of significance)†. The empirical results, taken together, indicate that both the MNL and FCL models are mis-specified.

4.3. Policy implications for predicting rail mode shares

The objective of the original study for which the data were collected in the Toronto–Montreal corridor was to examine the effect of alternative improvements in rail level of service characteristics (see KPMG Peat Marwick and Koppelman, 1990). Consequently, we focus on an examination of the aggregate-level self and cross-elasticities of the effect of changes in rail level-of-service (see section 2). The aggregate elasticities provide the proportional change in the expected market shares of each mode in response to a uniform percentage change in rail level-of-service measures across all individuals.

*The adjusted likelihood ratio index is defined as follows:

$$\bar{\rho}^2 = 1 - \frac{L(M) - K}{L(C)}$$

where $L(M)$ is the model log-likelihood value, $L(C)$ is the log-likelihood value with only the intrinsic mode bias constants, and K is the number of parameters (besides the mode bias constants) in the model.

†However, as noted earlier, one cannot reject the hypothesis of response homogeneity to the cost and out-of-vehicle travel time variables in the current empirical context.

Table 4. Measures of data fit

Summary statistic	MNL	FCL	RCL
Log-likelihood value at convergence*	−1104.23	−1096.28	−1081.12
Number of parameters†	12	14	18
Adjusted likelihood ratio index	0.4069	0.4100	0.4159

*The log-likelihood value at zero is −2197.22 and the log-likelihood value with only the intrinsic mode bias constants is −1881.89.
†The number of parameters excludes the intrinsic mode bias constants.

Tables 5–7 show the elasticity effects of incremental improvements in rail level-of-service. The rail self-elasticities in Table 5 indicate that a reduction in out-of-vehicle travel time is the most effective means of increasing rail market share. Between the FCL and MNL models, the MNL model shows a higher cost self-elasticity and a lower out-of-vehicle time self-elasticity. Overall, however, the results from the FCL and MNL models are quite close to each other. The random-coefficients logit (RCL) model suggests larger self-elasticities for all level-of-service variables relative to the MNL and FCL models. The frequency and cost elasticities suggested by the RCL model are about 37 and 16% higher, respectively, than in the other two models. The corresponding figures for the in-vehicle and out-of-vehicle time elasticities are 13 and 26%, respectively. Thus, the MNL and FCL models underestimate the increase in rail market share in response to improvements in rail level-of-service. In summary, using the rail self-elasticities from the MNL and FCL models will make a policy analyst much more conservative than (s)he should be in pursuing rail service improvements.

The car cross-elasticities in Table 6 show that the RCL model suggests a much larger draw of market share from the car mode in response to improvements in rail level-of-service compared to the other two models. The air cross-elasticities in Table 7 do not show much variation among the three models, except for the cost cross-elasticity which is lower in the RCL model. In summary, the cross-elasticities indicate that improvements in rail level-of-service will alleviate auto congestion on highways more so than that estimated by the MNL and FCL models.

Table 5. Aggregate rail self-elasticity matrix in response to improvement in rail service

Rail level of service attribute	Model		
	MNL	FCL	RCL
Increase in frequency	0.329	0.325	0.447
Decrease in cost	1.672	1.607	1.905
Decrease in in-vehicle time	1.532	1.511	1.714
Decrease in out-of-vehicle time	1.956	2.025	2.515

Table 6. Aggregate car cross-elasticity matrix in response to improvement in rail service

Rail level of service attribute	Model		
	MNL	FCL	RCL
Increase in frequency	−0.044	−0.044	−0.065
Decrease in cost	−0.193	−0.192	−0.263
Decrease in in-vehicle time	−0.167	−0.169	−0.220
Decrease in out-of-vehicle time	−0.250	−0.263	−0.365

Table 7. Aggregate air cross-elasticity matrix in response to improvement in rail service

Rail level of service attribute	Model		
	MNL	FCL	RCL
Increase in frequency	−0.028	−0.027	−0.033
Decrease in cost	−0.186	−0.166	−0.161
Decrease in in-vehicle time	−0.184	−0.175	−0.169
Decrease in out-of-vehicle time	−0.184	−0.184	−0.188

5. SUMMARY AND CONCLUSIONS

This paper formulates a multinomial-logit based model of travel mode choice that accommodates systematic variations in responsiveness to level-of-service measures in a simple and comprehensive manner. In addition, the model allows random taste variations and ensures the correct sign on the level-of-service measures for all individuals.

The maximum-likelihood estimation of the random-coefficients mode choice model requires the evaluation of a K-dimensional integral (K being the number of level-of-service variables) for the choice probabilities. The K-dimensional integral cannot be evaluated analytically since it does not have a closed-form solution. In the empirical application of this paper, there are four-level-of-service variables, necessitating the evaluation of a 4-dimensional integral for the choice probabilities. We employ a simulated maximum-likelihood procedure in which the multi-dimensional integral is evaluated using Monte Carlo techniques.

The empirical analysis of the paper applied the MNL without any response heterogeneity (i.e. MNL model), the MNL model with systematic response heterogeneity (i.e the FCL model) and the proposed MNL model with both systematic and random response heterogeneity (i.e RCL model) to the estimation of weekday, business, inter-city travel mode choice in the Toronto–Montreal corridor. The FCL and RCL models showed significant differences in sensitivity to level-of-service variables based on the socio-demographic/trip characteristics of the traveler, thus rejecting the response homogeneity assumption of the MNL model. In addition, the RCL model indicated significant random variation in tastes with respect to frequency of service and in-vehicle time.

In an empirical comparison of data fit, the RCL model rejects the FCL model based on a nested likelihood ratio test. The FCL model, in turn, rejects the MNL model based on a non-nested adjusted likelihood ratio test.

The effects of rail service improvements on mode market shares, expressed in terms of aggregate self- and cross-elasticities of rail service changes, are quite different among the MNL, FCL, and RCL models. The MNL and FCL models underestimate the increase in rail market share in response to improvements in rail level-of-service. These two models also underestimate the decrease in car market share in response to rail level-of-service improvements. These results imply that the simpler MNL and FCL models provide inappropriate evaluations of policy actions aimed at improving inter-city transportation services. In summary, the results underscore the need to accommodate systematic and random variation in responsiveness to level-of-service variables in travel mode choice modeling.

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