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The generalized nested logit model

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Abstract

The generalized nested logit (GNL) model is a new member of the generalized extreme value family of models. The GNL provides a higher degree of flexibility in the estimation of substitution or cross-elasticity between pairs of alternatives than previously developed generalized extreme value (GEV) models. The GNL model includes the paired combinatorial logit (PCL) and cross-nested logit (CNL) models as special cases. It also includes the product differentiation (PD) model, which represents the elasticity structure associated with multi-dimensional choices, and the ordered generalized extreme value model, which represents the elasticity structure associated with ordered alternatives, as special cases. The GNL model includes the two-level nested logit (NL) model as a special case and can approximate closely multi-level nested logit models. It accommodates differential cross-elasticity among pairs of alternatives through the fractional allocation of each alternative to a set of nests, each of which has a distinct logsum or dissimilarity parameter. An empirical example of intercity mode choice confirms the statistical superiority of the GNL model to the paired combinatorial logit, cross-nested logit and nested logit models and indicates important differences in cross-elasticity relationships across pairs of alternatives.

Keywords: Discrete choice; Random utility models; Travel demand; Logit; Intercity travel

1. Introduction

Choice models are used in transportation and other fields to represent the selection of one among a set of mutually exclusive alternatives. The multinomial logit (MNL) model (McFadden, 1973) is the most widely used choice model due to its simple mathematical structure and ease of estimation. However, the MNL imposes the restriction that the distribution of the random error terms is independent and identical over alternatives. This restriction leads to the independence of

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irrelevant alternatives property which causes the cross-elasticities between all pairs of alternatives to be identical. This representation of choice behavior produces biased estimates and incorrect predictions in cases that violate these strict conditions.

The most widely known relaxation of the MNL model is the nested logit (NL) model (Williams, 1977), which can be derived from McFadden's (1978) generalized extreme value (GEV) model. The NL model allows the error terms of pairs or groups of alternatives to be correlated. However, the remaining restrictions on the equality of cross-elasticities between pairs of alternatives in or not in common nests may be unrealistic in important cases.

Other relaxations of the MNL model, which allow different cross-elasticity between pairs of alternatives, have been derived from McFadden's GEV model. These include

- the paired combinatorial logit (PCL) model (Chu, 1989;Koppelman and Wen, 2000), which allocates each alternative in equal proportions to a nest with each other alternative and estimates a logsum (dissimilarity parameter) for each nest;
- the cross-nested logit (CNL) model (Vovsha, 1997), which allocates a fraction of each alternative to a set of nests with equal logsum parameters across nests;
- the ordered generalized extreme value (OGEV) model (Small, 1987), which allocates alternatives to nests based on their proximity in an ordered set; and
- the product differentiation (PD) model (Bresnahan et al., 1997), which allocates each alternative to one nest along each of a set of pre-selected dimensions with allocation parameters associated with each dimension and logsum parameters constrained to be equal for each nest along each choice dimension.

This paper introduces the generalized nested logit (GNL) model, which includes these models and the MNL model as special cases and closely approximates the NL model. The GNL accommodates differential cross-elasticity of pairs of alternatives through the fractional allocation of each alternative to a set of nests, each of which has a distinct logsum or dissimilarity parameter.

The remainder of this paper is organized as follows. Section 2 presents the formulation, description and estimation approach for the GNL model and shows that the NL, PCL, CNL, OGEV and PD models are special cases. Section 3 describes the data for four intercity travel modes in the Toronto–Montreal corridor (KPMG Peat Marwick and Koppelman, 1990) and estimation results for the MNL, NL, PCL, CNL and GNL models. [2] Section 4 suggests further developments in the search for a preferred structural form and directions for additional model flexibility. Section 5 provides a summary and conclusions.

2. The generalized nested logit model

2.1. Model formulation

The GNL model is a GEV model (McFadden, 1978) derived from the function

$$G(Y_1, Y_2, \ldots, Y_n) = \sum_m \left(\sum_{n' \in N_m} (\alpha_{n'm} Y_{n'})^{1/\mu_m} \right)^{\mu_m}, \tag{1}$$

[2] The OGEV and PD models are not included in this comparison since the alternatives neither are ordered nor fall into categorical groupings along dimensions.

where N_m is the set of all alternatives included in nest m, α_{nm} the allocation parameter which characterizes the portion of alternative n assigned to nest m (α_{nm} must satisfy the condition $\alpha_{nm} \geqslant 0$, the additional condition $\sum_m \alpha_{nm} = 1,\ \forall n$ provides a useful interpretation with respect to allocation of each alternative to each nest), μ_m is the logsum or dissimilarity parameter for nest m ($0 < \mu_m \leqslant 1$) and Y_n characterizes the value for each alternative.

The function, Eq. (1) which is non-negative, homogeneous of degree one, approaches infinity with any Y_i and has kth cross-partial derivatives which are non-negative for odd k and non-positive for even k. The resultant GEV probability function, after substituting $e^{V_{n'}}$ to ensure positive $Y_{n'}$, is

$$P_n = \frac{\sum_m \left[(\alpha_{nm} e^{V_n})^{1/\mu_m} \left(\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m} \right)^{\mu_m - 1} \right]}{\sum_m \left(\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m} \right)^{\mu_m}}$$
$$= \sum_m \left(\frac{(\alpha_{nm} e^{V_n})^{1/\mu_m}}{\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m}} \frac{\left(\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m} \right)^{\mu_m}}{\sum_m \left(\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m} \right)^{\mu_m}} \right). \tag{2}$$

This equation can be decomposed into components and rewritten as

$$P_n = \sum_m P_{n/m} P_m, \tag{3}$$

where P_m, the probability of nest m, is

$$P_m = \frac{\left(\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m} \right)^{\mu_m}}{\sum_m \left(\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m} \right)^{\mu_m}} \tag{4}$$

and $P_{n/m}$, the probability of alternative n if nest m is selected, is

$$P_{n/m} = \frac{(\alpha_{nm} e^{V_n})^{1/\mu_m}}{\sum_{n' \in N_m} (\alpha_{n'm} e^{V_{n'}})^{1/\mu_m}}. \tag{5}$$

The GNL model is consistent with random utility maximization if the conditions, $0 < \mu_m \leqslant 1$, are satisfied. The direct elasticity of an alternative, n, which appears in one or more nests with logsum, μ_m, less than one, is

$$\frac{\sum_m P_m P_{n/m} \left[(1 - P_n) + \left(\frac{1}{\mu_m} - 1 \right) (1 - P_{n/m}) \right]}{P_n} \beta X_n. \tag{6}$$

The terms in the summation evaluate to zero for any nest which does not include alternative n. The elasticity reduces to the MNL elasticity, $(1 - P_n)\beta X_n$, if the alternative does not share a nest with any other alternative or is assigned only to nests for which the logsum value equals one.

The corresponding cross-elasticity of a pair of alternatives, n and n', which appear in one or more common nests, is

$$-\left[P_n + \frac{\sum_m \left(\frac{1}{\mu_m} - 1\right) P_m P_{n/m} P_{n'/m}}{P_{n'}}\right] \beta X_n. \tag{7}$$

In this case, the terms in the summation evaluate to zero for any nest which does not include both alternatives, n and n', and reduces to the MNL cross-elasticity, $-P_n \beta X_n$, if the alternatives do not share any common nest. These elasticities are independent of the elasticities for any other alternative or pair of alternatives.

Swait (2000) recently proposed the general logit (GenL) model, in which nest represents a possible choice set so that the marginal probability represents the selection or availability of the choice set and the conditional probability represents the choice of an alternative given that choice set. The GenL model is similar to the GNL except that the allocation parameters are associated with individuals rather than alternatives. Vovsha (1999) reports development and application of the fuzzy nested logit model, which is identical to the GNL, except that it allows multiple levels of nesting. While the additional levels of nesting appear to increase the flexibility of the model, they raise complex problems of identification since the GNL can represent the same differential sensitivities within its two level nesting structure.

2.2. Structural relationships between the GNL and other GEV models

The PCL, CNL, OGEV and PD models are restricted versions of the GNL model. The NL model is not a restricted case of the GNL model, but it can be approximated closely by a suitably specified GNL model.

2.2.1. The PCL model

Comparison between the GNL and PCL models requires adoption of a special case of the GNL model that includes one nest for each pair of alternatives, as in the PCL model. Such a paired GNL (PGNL) model has the form

$$P_{n,\mathrm{PGNL}} = \sum_{n' \neq n} \left[\frac{(\alpha_{n,nn'} \mathrm{e}^{V_n})^{1/\mu_{nn'}}}{(\alpha_{n,nn'} \mathrm{e}^{V_n})^{1/\mu_{nn'}} + (\alpha_{n',nn'} \mathrm{e}^{V_n})^{1/\mu_{nn'}}}\right] \left[\frac{\left((\alpha_{n,nn'} \mathrm{e}^{V_n})^{1/\mu_{nn'}} + (\alpha_{n',nn'} \mathrm{e}^{V_n})^{1/\mu_{nn'}}\right)^{\mu_{nn'}}}{\sum\limits_{\substack{\forall k \\ k \neq k'}} \left((\alpha_{k,kk'} \mathrm{e}^{V_k})^{1/\mu_{kk'}} + (\alpha_{k',kk'} \mathrm{e}^{V_{k'}})^{1/\mu_{kk'}}\right)^{\mu_{kk'}}}\right]. \tag{8}$$

The PGNL model, Eq. (8), restricted so that all allocation parameters, $\alpha_{n,(nn')}$, are equal, is equivalent to the PCL model. [3] The non-equal allocation to nests in the PGNL model allows

[3] The allocation parameters equal the inverse of the number of alternatives minus one so that the sum of allocation parameters equals one. This differs from, but has the same effect as, the original PCL model for which all allocation parameters are equal to one.

greater freedom in the magnitude of cross-elasticity than is allowed by the corresponding PCL model. Further, the PGNL allows an allocation of zero for an alternative to a nest and the elimination of nests for which both alternatives have zero allocation.

2.2.2. The CNL model

The CNL model is a straightforward restriction of the GNL model. That is, the restriction that all logsum parameters, μ_m, are equal in the GNL model, Eq. (2), results in the CNL model.

2.2.3. The OGEV model

The OGEV model allows cross-elasticity between pairs of alternatives in an ordered choice to be related to their proximity in that order. Each alternative is a member of nests with one or more adjacent alternatives. The general OGEV model allows different levels of cross-elasticity by changing the number of adjacent alternatives in each nest (and therefore the number of common nests shared by each pair of alternatives), the allocation weights of each alternative to each nest and the dissimilarity parameters for each nest. The choice probability for alternative m is

$$P_i = \sum_{m=i}^{i+L} P_{i/m} \cdot P_m = \sum_{m=i}^{i+L} \left[\frac{(w_{m-i}\mathrm{e}^{V_i})^{1/\mu_m}}{\sum_{j\in N_m}(w_{m-j}\mathrm{e}^{V_j})^{1/\mu_m}} \frac{\left(\sum_{j\in N_m}(w_{m-j}\mathrm{e}^{V_j})^{1/\mu_m}\right)^{\mu_m}}{\sum_{s=1}^{J+L}\left(\sum_{j\in N_s}(w_{s-j}\mathrm{e}^{V_j})^{1/\mu_s}\right)^{\mu_s}} \right], \tag{9}$$

where L is a positive integer that defines the maximum number of contiguous alternatives in a nest, w the allocation weight of the alternative to the nest and $\mathrm{e}^{V_i/\mu}$ is equal to zero for $i < 1$ and $i > J$.

This is equivalent to the GNL model with the constraint that the weights associated with the assignment of each alternative to a nest are associated with its ordered position in the nest.

2.2.4. The PD model

The PD model is based on the notion that markets for differentiated products (alternatives) exhibit increased cross-elasticity due to clustering (nesting) relative to dimensions, which characterize attributes of the product. Such dimensions could include, in the case of transportation modeling, mode and destination or number of cars, residential location and mode to work. The choice probability equation for a PD model with D dimensions is given by

$$P_i = \sum_{d\in D} \left(\alpha_d \frac{\mathrm{e}^{V_i/\mu_d}}{\sum_{k\in d}\mathrm{e}^{V_k/\mu_d}} \frac{\left(\sum_{k\in d}\mathrm{e}^{V_k/\mu_d}\right)^{\mu_d}}{\sum_{d'\in D}\left(\sum_{k'\in d'}\mathrm{e}^{V_{k'}/\mu_{d'}}\right)^{\mu_{d'}}} \right), \tag{10}$$

where α_d is the portion of each alternative allocated to dimension d and μ_d is the logsum parameter for all groups (nests) along dimension d.

This model restricts the GNL so that all alternatives have the same allocation to each dimension and the nests along each dimension have the same logsum parameters.

2.2.5. The NL model

As stated earlier, the two-level NL model is a special case of the GNL; that is, a GNL with each alternative allocated to a single nest. More importantly, the GNL model can approximate any multi-level nested logit model by including a nest, which corresponds to each node in the nested logit. This can be seen in Fig. 1, which shows, in part (a), a three-level nested logit structure with four nodes. Part (b) shows the corresponding GNL approximation in which the alternatives grouped under each node in the nested logit structure are assigned to a common nest. Alternatives, which are nested at multiple levels, are assigned to all nests represented by nodes between the alternative and the root of the NL tree. The self- and cross-elasticities, and substitution patterns, in the GNL model are based on the logsum parameters associated with

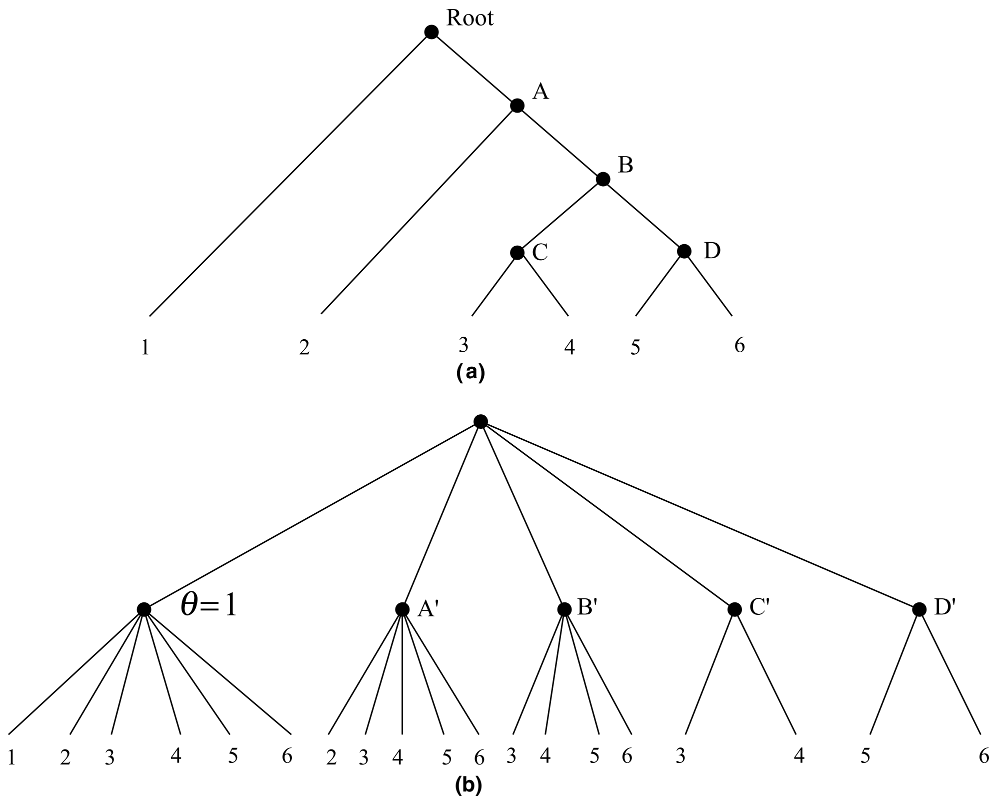

Fig. 1. NL approximation within GNL model structure: (a) three-level nested logit model structure; (b) GNL approximation of three-level nested logit structure.

Table 1
Direct and cross-elasticities of the MNL, GNL, CNL and PCL models

Model	Direct elasticity	Cross-elasticity
MNL	$(1-P_n)\beta X_n$	$-P_n\beta X_n$
GNL	n assigned to a single nest with no other alternatives $(1-P_n)\beta X_n$	n and n' not in any common nest $-P_n\beta X_n$
	n in one or more nests $\dfrac{\sum_m P_m P_{n/m}\left[(1-P_n)+\left(\frac{1}{\mu_m}-1\right)(1-P_{n/m})\right]}{P_n}\beta X_n$	n and n' in one or more common nests $-\left[P_n+\dfrac{\sum_m\left(\frac{1}{\mu_m}-1\right)P_m P_{n/m}P_{n'/m}}{P_{n'}}\right]\beta X_n$
CNL	n assigned to a single nest with no other alternatives $(1-P_n)\beta X_n$	n and n' not in any common nest $-P_n\beta X_n$
	n in one or more nests $\dfrac{\sum_m P_m P_{n/m}\left[(1-P_n)+\left(\frac{1}{\mu}-1\right)(1-P_{n/m})\right]}{P_n}\beta X_n$	n and n' in one or more common nests $-\left[P_n+\dfrac{\sum_m\left(\frac{1}{\mu}-1\right)P_m P_{n/m}P_{n'/m}}{P_{n'}}\right]\beta X_n$
PCL	$\dfrac{\sum_{n'\neq n} P_{nn'}P_{n/nn}\left[(1-P_n)+\left(\frac{1}{\mu_{nn'}}-1\right)(1-P_{n/nn'})\right]}{P_n}\beta X_n$	$-\left[P_n+\dfrac{\left(\frac{1}{\mu_{nn'}}-1\right)P_{nn'}P_{n/nn'}P_{n'/nn'}}{P_{n'}}\right]\beta X_n$

each nest in which an alternative or pair of alternatives is (are) included. Thus, for example, the cross-elasticity between alternatives 3 and 4 will be greater than between 3 and 5 or 6 and these are greater than the cross-elasticity between 3 and 2. The estimation is somewhat more complex since the GNL requires estimation of allocation parameters in addition to the four logsum parameters.

2.3. Direct and cross-elasticities

The differences between the GNL model and the MNL, PCL, CNL, OGEV and PD models can be examined further by comparison of direct and cross-elasticities of probabilities with respect to changes in attributes of any alternative (Table 1).

The direct-elasticity formula for the MNL model is identical for all alternatives depending only on the probability of the alternative. [4] The direct-elasticity formulae for the other models are greater than for the MNL model for alternatives in a common nest with logsum less than one and the same as the MNL model for other alternatives. [5] However, the similarity among the GNL, CNL and PCL elasticities is somewhat misleading as they do not explicitly show the effect of the allocation parameters which are embedded in the probabilities as shown in Eqs. (4) and (5) for the GNL model.

The cross-elasticity formulae of the MNL model depend exclusively on the probability of the changed mode, which gives the commonly observed equal proportional effect of the addition, deletion or change of any alternative on all other alternatives. The cross-elasticity for pairs of alternatives in the other models are greater in magnitude than for the MNL model if the pair is in a common nest with logsum less than one and equal to the MNL model otherwise. The elasticity increases in magnitude as μ_m decreases from one, with the magnitude of the impact related to the probability of the nest and the conditional probabilities of the alternatives in the nest. As with the direct elasticities, the similarity among the GNL, CNL and PCL elasticities is somewhat misleading as they do not explicitly show the effect of the allocation parameters which are embedded in the probabilities.

An alternative perspective on the relationships among pairs of alternatives is the implied correlation between the error terms for pairs of alternatives. Table 2 reports the correlations for different combinations of allocation and logsum parameters in the CNL and GNL models. The important point of this table is that the correlations can achieve very high values if such values are supported by the observed behavior. However, the correlations of the CNL model are not as flexible as this table suggests since the logsum parameters in the CNL are limited by the requirement that all logsum parameters be equal.

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[4] All the elasticities include the variable of change and the utility function parameter associated with that variable.

[5] Empirical experience indicates that utility function parameters are smaller in magnitude for these models than for the MNL model so that the direct elasticities decrease for alternatives not in any nest with logsum less than one but increase for all other alternatives. Similarly, the cross-elasticities decrease for alternatives not in a common nest and increase for alternatives in one or more common nests with logsum less than one.

Table 2
Correlation between pairs of alternatives in a nest implied by the CNL and GNL models as a function of the logsum and allocation parameters

Allocation parameter (α_n)	Logsum parameter	Allocation parameter ($\alpha_{n'}$)		
		0.1	0.5	1.0
0.1	0.1	0.09	0.17	0.21
	0.3	0.08	0.16	0.20
	0.5	0.07	0.14	0.17
	0.7	0.04	0.10	0.12
	0.9	0.02	0.04	0.05
	1.0	0.00	0.00	0.00
0.5	0.1		0.45	0.64
	0.3		0.42	0.59
	0.5		0.35	0.50
	0.7		0.24	0.34
	0.9		0.09	0.13
	1.0		0.00	0.00
1.0	0.1			0.99
	0.3			0.91
	0.5			0.75
	0.7			0.51
	0.9			0.19
	1.0			0.00

2.4. Estimation

The GNL model requires joint estimation of the utility, logsum and allocation parameters. This paper employs constrained maximum likelihood (Aptech Systems, 1995) to estimate the three sets of parameters, simultaneously, taking account of the restrictions that the logsum and allocation parameters are bounded by zero and one and that the allocation parameters for each alternative sum to one. The number of logsum parameters that can be identified is one less than the number of pairs of alternatives. This limitation and the general flexibility of the model structure require the analyst to make judgements about the clustering of alternatives into nests. This is similar to the problem of selecting one among a large set of alternative nesting structures when estimating a nested logit model or imposing restrictions on the covariance matrix in the MNP model. The GNL model requires similar judgements to be made.

Analyst judgement can be implemented in a variety of ways. First, the analyst can limit the nesting options a priori, based on judgement about the likely elasticity or substitution relationships among pairs or groups of alternatives. Second, structural relationships can be imposed on the cross-elasticities among pairs of alternatives to reduce the number of independent allocation and/or logsum parameters. For example, logsum parameters can be constrained to be equal for groups along choice dimensions and allocations to each dimension can be constrained to be equal as in the PD model (Bresnahan et al., 1997). Third, the analyst can search over all or most of the possible structures. Additional options include using various constrained versions of the GNL model such as the PCL, CNL or PGNL to obtain preliminary estimates of the relative magnitude of elasticity/substitution relationships among pairs of alternatives. Fourth, the Hessian of the log-likelihood function for the GNL model is not negative semi-definite over its whole range. It may be required to repeat optimization with different starting points to locate the global optimum.

3. Empirical analysis

The data used in this study were assembled by VIA Rail in 1989 (KPMG Peat Marwick and Koppelman, 1990) to estimate the demand for high-speed rail in the Toronto–Montreal corridor and to support future decisions on rail service improvements in the corridor. The data includes 4324 individuals, whose choice set includes two or more of four intercity travel modes (air, train, bus and car) in the corridor. The fractions of the sample which had each alternative available are train (4299, 99.4%), air (3626, 83.9%), bus (3271, 75.6%) and car (4324, 100%) and the distribution of choices is train (623, 14.41%), air (1472, 34.04%), bus (16, 0.37%) [6] and car (2213, 51.18%). This dataset has been used for a variety of model formulation and estimation studies including Forinash and Koppelman (1993), Koppelman and Wen (2000, 1999, 1998), Bhat (1995, 1997a,b) and others.

The utility function specification includes mode-specific constants, frequency, travel cost, and in- and out-of-vehicle travel times. [7] The estimation results for the MNL, two NL models (Koppelman and Wen, 1998) and the PCL model (Koppelman and Wen, 2000) are reported in Table 3. The NL models have almost identical goodness of fit, neither is able to reject the other, but they both reject the MNL model and lead to very different behavioral interpretations and different forecasts of the effect of changes in the alternatives. The train–car nested model represents a higher level of competitiveness between train and car than between other modes and the air–car nested model represents a higher level of competitiveness between air and car than between other modes. The PCL model, which allows increased competitiveness for both the train–car and air–car pairs, rejects the MNL model and both NL models at high levels of significance as shown in the table.

Estimation results for the CNL and GNL models are reported in Table 4. Exploratory estimation, limited to a maximum of two alternatives per nest, is used to select among different nesting structures. The resultant nests, for both the CNL and GNL models (columns 1 and 2), are bus alone, train alone, car alone, train–car and air–car. CNL Model 1 obtains a significant (with respect to one) logsum parameter that applies to both the train–car and air–car nests; the logsum parameters for single alternative nests (train, car and bus) are set to one. This model rejects the MNL, both NL and the PCL models at very high levels of significance, in excess of 0.001, using the nested hypothesis test for the MNL model and the non-nested hypothesis test for the NL and PCL models (Horowitz, 1983). GNL Model 1 obtains logsum parameters (0.05 for train–car and 0.32 for air–car) that are significantly different from one and from each other; as with the CNL

[6] The small number of cases for which bus is chosen limits the estimability of allocation and logsum parameters associated with the bus alternative.

[7] Tests of alternative model structures with different utility function specifications, including income and city pair indicator variables, did not substantially affect the comparison among model structures.

Table 3
Estimation results for the MNL, NL and PCL models[a]

Variables	Estimated parameters (standard errors)			
	MNL model	NL with train–car nested	NL with air–car nested	PCL model
Mode constants				
Air	8.238 (0.429)	7.812 (0.450)	7.533 (0.511)	7.157 (0.430)
Train	5.412 (0.267)	5.513 (0.270)	5.061 (0.300)	5.129 (0.267)
Car	4.421 (0.301)	4.446 (0.300)	4.372 (0.303)	4.262 (0.294)
Bus (base)				
Frequency	0.0850 (0.004)	0.0845 (0.004)	0.0722 (0.006)	0.0689 (0.003)
Travel cost	−0.0508 (0.003)	−0.0464 (0.003)	−0.0420 (0.004)	−0.0379 (0.003)
In-vehicle time	−0.0088 (0.001)	−0.0084 (0.001)	−0.0080 (0.001)	−0.0076 (0.001)
Out-of-vehicle time	−0.0354 (0.002)	−0.0339 (0.002)	−0.0310 (0.002)	−0.0305 (0.002)
Logsum parameters				
Train–car		0.8302 (0.059)		0.5200 (0.109)
Air–car			0.8233 (0.063)	0.1922 (0.076)
Log-likelihood at convergence	−2784.6	−2781.2	−2780.9	−2769.1
Likelihood ratio index				
vs. zero	0.4896	0.4903	0.4903	0.4925
vs. market share	0.3205	0.3213	0.3214	0.3243
Value of time (per hour)				
In-vehicle time	C$ 10	C$ 12	C$ 12	C$ 12
Out-of-vehicle time	C$ 42	C$ 48	C$ 48	C$ 48
Significance test rejecting MNL model (χ^2, DF, Sig.)	–	6.8, 1, 0.001	7.4, 1, 0.001	31.0, 2, 0.001

[a] *Note*: The PCL model rejects both NL models at the 0.001 level using the non-nested hypothesis test.

model, the logsum parameters for single alternative nests are set to one. The GNL model rejects the CNL model as well as the MNL, NL and PCL models, at the 0.001 level. Additional CNL and GNL models with an additional air–train–car nest (columns 3 and 4) statistically reject the corresponding models without any three alternative nests. The inclusion of train and car in the train–car and train–air–car nests in the GNL model results in colinearity among the logsum and allocation pa#132;rameters. Nonetheless, this model strongly rejects all the previously estimated models. This problem is avoided in the CNL model due to the equality constraint across the logsum parameters. Nevertheless, the final GNL model appears to be superior to all models previously estimated. Based on limited exploration, these results hold across a variety of utility function parameters.

There are significant differences among the different structural models. These differences are likely to produce important differences in mode forecasts under alternative scenarios for future transportation services, possibly resulting in different investment decisions. The attribute parameters in the utility function decrease in magnitude with increasing complexity in model

Table 4
Estimation results for the CNL and GNL models

Variable	Estimated parameters (standard errors)			
	Without train–car–air nest		With train–car–air nest	
	CNL Model 1	GNL Model 1	CNL Model 2	GNL Model 2
Mode constants				
Air	5.746 (0.429)	5.344 (0.367)	5.476 (0.343)	6.264 (0.321)
Train	4.618 (0.286)	4.460 (0.281)	5.083 (0.256)	4.981 (0.285)
Car	4.455 (0.275)	4.300 (0.267)	4.901 (0.284)	5.133 (0.253)
Bus (base)				
Frequency	0.0460 (0.006)	0.0421 (0.005)	0.0206 (0.009)	0.0288 (0.002)
Travel cost (C$)	−0.0209 (0.004)	−0.0172 (0.003)	−0.0096 (0.004)	−0.0173 (0.002)
In-vehicle time (min)	−0.0059 (0.001)	−0.0060 (0.001)	−0.0023 (0.001)	−0.0031 (0.0002)
Out-of-vehicle time (min)	−0.0201 (0.002)	−0.0198 (0.002)	−0.0088 (0.004)	−0.0110 (0.001)
Logsum parameter				
Train–car	0.3141 (0.041)	0.0463 (0.019)	0.1008 (0.035)	0.0146 (0.002)
Air–car	0.3141 (0.041)	0.3159 (0.042)	0.1008 (0.035)	0.2819 (0.032)
Train–car–air			0.1008 (0.035)	0.01 (–)
Allocation parameter				
Train–car nest				
Train	0.7032 (0.074)	0.4904 (0.046)	0.1547 (0.045)	0.2717 (0.033)
Car	0.2611 (0.047)	0.1896 (0.023)	0.1060 (0.026)	0.1057 (0.012)
Air–car nest				
Air	1.0000	1.0000	0.2287 (0.065)	0.6061 (0.040)
Car	0.5163 (0.059)	0.5664 (0.054)	0.1145 (0.031)	0.4179 (0.046)
Train–car–air nest				
Train			0.7409 (0.069)	0.5286 (0.031)
Car			0.6335 (0.059)	0.2741 (0.029)
Air			0.7713 (0.065)	0.3939 (0.041)
Train nest	0.2226 (0.071)	0.5096 (0.046)	0.1044 (0.037)	0.1998 (0.025)
Car nest	0.2968 (0.074)	0.2440 (0.051)	0.1460 (0.047)	0.2024 (0.032)
Bus nest	1.0000	1.0000	1.0000	1.0000
Log-likelihood at convergence	−2746.6	−2736.3	−2723.1	−2711.3
Likelihood ratio index				
vs. zero	0.4966	0.4985	0.5009	0.5031
vs. market share	0.3298	0.3320	0.3355	0.3382
Value of time (per hour)				
In-vehicle time	C$ 17	C$ 21	C$ 14	C$ 11
Out-of-vehicle time	C$ 57	C$ 69	C$ 55	C$ 38
Significance test rejecting CNL models (χ^2, DF, Sig.)	20.6, 1, <0.0001	–	23.6, 2, <0.001	–

structure. This implies that the cross-elasticities between alternatives in a common nest are reduced while those in common nests are increased, as expected. The relative value of these parameters, as represented by the values of time, are reasonably stable over all the models estimated.

4. Estimation and use of complex structural models

The development of multiple forms of GEV models with potentially large numbers of estimable parameters raises important questions of model selection and use in analysis in both transportation and non-transportation contexts. Models with increased flexibility add to the estimation complexity, the importance of analyst judgement, computational demands and the time required searching for and selecting a preferred model structure. This task is interrelated with the task of searching for and selecting a preferred utility function specification. Horowitz (1991) raised the concern that the increased flexibility of error structure specification of the multinomial probit model might lead to a proliferation of random effects parameters and thereby reduce the incentive for modelers to develop enhanced utility function specifications. The same concern can be applied to the search for and selection among alternative GEV models and the structural parameters that define each model type. Therefore, an important issue for additional research is the analysis and understanding of interrelationships between model structure and parameters and utility function specification. The development of useful rules to guide the search among complex alternative structures would provide the option of guiding the analyst and reducing both the search and computational time associated with obtaining a preferred model.

A further issue is the usefulness of developing more complex GEV models when suitably specified multinomial probit and mixed logit models (Brownstone and Train, 1998; McFadden and Train, 1997) can approximate all such models. Our perspective is that there is a place in the set of analytic tools for models with different levels of complexity in structure, estimation, interpretation and application. Advanced research is likely to employ models with high degrees of complexity. Professional practice, however, may be best served by the use of models, the complexity of which is closely matched to the problem at hand; that is, use the minimally complex model to capture and represent the behavior under study. We believe that the development of models of varying degrees of complexity serves this purpose.

5. Conclusions

The GNL model adds useful flexibility to the family of GEV models by providing a more flexible structure for estimating differential cross-elasticities among pairs of alternatives. It also provides a unifying structure for previously reported GEV models, with the exception of the NL model, and provides a framework for understanding the properties of these models. This paper demonstrates that the GNL model can be feasibly estimated and is useful in applied work.

An additional advantage is that the GNL model provides a structural framework for exploring alternative cross-elasticity structures without necessarily estimating a large number of distinct models as required in the estimation of the NL model.

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