THE SPLIT CONTROL METHOD FOR MIMO SERVO DESIGN

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ABSTRACT

A new servo design methodology for MIMO linear systems subject to unknown disturbances with output feedback which we the call 'Split Control Method' is presented. In this method the control action is decomposed into two parts. One part is designed using Variable Structure Systems techniques and produces a rough motion around the desired state space trajectory. The other part is designed using the Model Based Compensator - Loop Transfer Recovery method, and acts as a fine tuner that brings the system output to the desired trajectory. Both designs are independent and stability of each design guarantees at least nominal stability of the combined system. Finally a switching disturbance is introduced to compensate for unknown external disturbances. For a nth order plant the resulting compensator is of order 2n. A 4th order, 2 input - 2 output example is shown. This servo design technique is restricted to minimum phase invertible plants and its extention to nonlinear systems is very promising.

1. Introduction

MIMO servo design has been the object of extensive research in the last few years. Two central techniques, sliding mode control (SMC) and model based compensator-loop transfer recovery (MBC-LTR), stand out for their adaptability to the servo problem, and for the performance they provide.

Each one has its own intrinsic limitations and drawbacks which restrict their use. SMC provides good tracking of arbitrary reference inputs to linear and nonlinear plants in the presence of structured modelling errors and external disturbances, the main constraint for its use is the requirement of full state knowledge. MBC-LTR gives good tracking of simple reference inputs to linear plants with unstructured modelling errors in the presence of external disturbances with limited frequency band, using output feedback. Tracking of more complex reference inputs can be achieved with MBC-LTR controllers that use the linear quadratic servo structure which includes a feedforward term. However, this feedforward term has to be calculated offline with previous knowledge of the command references and extensive memory is required to store it.

The purpose of this paper is to introduce a new servo design methodology, the "split control method", which incorporates SMC and MBC-LTR design techniques with the objective of combining some of their useful features to solve a different class of control problems. Specifically the split control method solves the problem in which a linear plant with known parameters is required to track an unspecified complex reference command using output feedback and in the presence of unknown external disturbances.

Consider the linear time invariant plant

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t) \tag{1.1a}$$

$$y(t) = Cx(t) \tag{1.1b}$$

where $x(t)$ is the n-dimensional state vector, $u(t)$ and $y(t)$ are the m-dimensional control input and output vectors respectively, and $d(t)$ is a n-dimensional vector which represents external disturbances. The goal is to design u such that y follows as close as possible the command reference signal $r(t)$. The tracking error $e(t)$ is defined as:

$$e = r - y \tag{1.2}$$

We start with a brief description of SMC, [1]-[7], and MBC-LTR, [11]-[14].

1.1 Sliding Mode Control: The purpose of SMC is to bring the system (1.1) from any initial state to the $(n-m)$-dimensional space defined by [7]:

$$s(x,t) = s = P(d/dt)e = P(d/dt)r - Nx = 0 \tag{1.3a}$$

$$d/dt\,[P(d/dt)] \triangleq Q(d/dt) \tag{1.3b}$$

where the operator $Q(d/dt)$ is such that acting on y produces a signal that directly contains u. If (1.1) is invertable and d satisfies the matching condition [7]:

$$d(t) = B\delta(t) \tag{1.4}$$

for some $\delta(t)$ with known bound in any norm, there exists a u that brings x to (1.3). In SMC d can also be a function of x that represents model errors and nonlinearities. If a positive semidefinite Lyapunov like scalar function is defined as:

$$V(s) = \frac{1}{2}s^T \Xi s\,, \qquad \Xi > 0 \tag{1.5}$$

a sufficient condition to bring s to zero is:

$$\dot{V}(s) = s^T \Xi \dot{s} < 0\,, \qquad \forall t \tag{1.6}$$

which, since $NB = I$, can be achieved if

$$u = -NAx + R_K + (\eta_R \|R_u\|_{\max} + \eta_\delta \|\delta\|_{\max})\Xi^{-1}1_s \tag{1.7}$$

R_K and R_u are the known and unknown parts of $Q(d/dt)r$ respectively, η_R, $\eta_\delta \geq 1$ are "safety factors" and

$$1_s \triangleq s/\|s\| \tag{1.8}$$

The system dynamics during sliding in (1.3) can be determined by the equivalent control method [3], and is given by

$$\dot{x} = (I - BN)Ax + BQ(d/dt)r \tag{1.9}$$

If (1.1) has ℓ infinite zeros the dynamics of (1.9) is governed by m poles at zero that correspond to the sliding condition $s = 0$, $n - \ell$ poles at the zeros of (1.1) and $\ell - m$ poles which are the roots of the $\det[P(\lambda)] = 0$ and are freely assignable [7]. Stability robustness in the presence of unstructured modelling errors is not well addressed by SMC, and its performance in the presence of sensor noise is not well defined.

Since s is defined as a differential operator acting on the error, in some cases first or higher order derivatives of the reference signals will be required, if they are not available, they can be generated by filtering. The major constraint of SMC in servo design is the requirement of full state feedback. Some research has been done on SMC servos with output feedback [8]–[10]. In [8] and [9] it is shown that for the regulator problem, if an asymptotic observer exists, sliding is still achieved. However, an observer might not be a good solution to the precise tracking problem with output feedback, since the correction term in the observer structure in general does not satisfy a matching condition of the type (1.4) the effect of sensor noise, disturbances and unknown initial conditions appears not only as an estimation error but also as a disturbance that inhibits sliding of the observer state vector $z(t)$ on $s(z,t)$.

1.2 MBC-LTR: A model based compensator is a dynamic compensator that has as input the plant output error and as output the control signal u, which structure resembles that of the plant model:

$$\begin{aligned} \dot{w} &= Aw + Bu + H(y - r - Cw) \\ &= (A - BG - HC)w - He && (1.10a) \\ u &= -Gw && (1.10b) \end{aligned}$$

Under stabilizability and detectability assumptions, stability of the closed-loop system is reduced to the problem of finding G and H such that

$$Re[\lambda_i(A - BG)] < 0$$

$$Re[\lambda_i(A - HC)] < 0$$

In the MBC-LTR methodology the first step is to design a target loop of the form

$$\begin{aligned} \dot{x} &= (A - BG)x + Br && (1.11a) \\ u &= -Gx && (1.11b) \end{aligned}$$

or

$$\begin{aligned} \dot{x} &= (A - HC)x + Hr && (1.12a) \\ y &= Cx && (1.12b) \end{aligned}$$

that meets the design specifications, given in the frequency domain. Different approaches can be taken to generate the target loop, but usually optimal regulator techniques (Linear Quadratic regulator) for (1.11) and optimal filtering theory (Kalman filter) for (1.12) are used, because of their built-in performance guarantees. Once a target loop has been chosen, if (1.1) is minimum phase, the LTR method makes the frequency response of the closed loop system approach that of the target loop in the frequency range of interest.

There are two LTR methods, LTR at the plant input, when the target loop is given by (1.11) and LTR at the plant output, when the target loop is given by (1.12). In both cases the problem reduces to the solution of an algebraic Riccati equation that gives H (LTR at plant input) or G (LTR at plant output).

The performance of MBC-LTR designs depend primarily on the choice of the target loop. If the purpose is to design a regulator, (1.11) should be used, with modelling errors and disturbances reflected at the plant input. For the servo problem, (1.12) is the best choice and modelling errors and disturbances must be reflected at the plant output.

In general, by augmenting the plant and the MBC with integrators, good command following can be achieved for simple reference inputs such as steps or very low frequency sinusoids. Good tracking of more complex reference signals require augmenting the plant and the MBC with more complex SISO controllers, increasing the design bandwidth or what is more preferable introducing feedforward control action.

2. Split Control Method

Consider the problem of finding a control input $u(t)$ for the linear plant (1.1) such that $y(t)$ tracks the reference signal $r(t)$ with very small error $e(t)$. Assume that all conditions required to bring (1.1) to the sliding surface (1.3) are satisfied, except for the fact that only y is available for measurement. A first approach could be to use a fast enough asymptotic observer of the form

$$\dot{z} = Az + Bu + H(y - Cz)$$

However, since the correction term $H(y - Cz)$ acts as a disturbance to the observer dynamics and in general H is not in the range space of B, sliding of $z(t)$ on $s(z,t) = 0$ is not attainable. Therefore even when $s(x,t)$ approaches $s(z,t)$, because $s(z,t) \neq 0$, good tracking of r will in general not be achieved.

To overcome this difficulty we use an open loop observer, which will be regarded as the plant model, and whose structure is:

$$\begin{aligned} \dot{z} &= Az + Bu_0 && (2.1a) \\ y_z &= Cz && (2.1b) \end{aligned}$$

The control input to the plant model u_0 is not the same as the control input to the plant, but is related to it by

$$u = u_0 + u^* \qquad (2.2)$$

The separation of the control action u into two parts u_0 and u^* is the origin to the name split control method. The idea behind this method is to use u_0 as a feedforward control action and u^* as a regulating control action that makes x asymptotically converge to z or to remain sufficiently close in the presence of the disturbance $d(t)$.

Since $r(t)$ is not previously known and some of its derivatives can not be either measured or estimated by proper filtering u_0 is designed as a sliding mode controller for the plant model. All conditions, including full state knowledge, are given to bring (2.1) from any initial state to the sliding surface.

$$\begin{aligned} e_z &= r - y_z && (2.3a) \\ s_z &= s(z,t) = P(d/dt)e_z = P(d/dt)r - Nz = 0 && (2.3b) \end{aligned}$$

if u_0 is given by

$$\begin{aligned} u_0 &= -NAz + R_K + \eta_R \|R_u\|_{max} \Xi^{-1} 1_{s_z} && (2.4a) \\ 1_{s_z} &= s_z / \|s_z\| && (2.4b) \end{aligned}$$

To the plant (1.1) u_0 is an open loop control and by itself does not guarantee any performance. Even if a correction term $BH(y - y_s)$ is included in (2.1), in general it would not be enough to make x converge to z sufficiently fast. Dynamic compensation through u^* is therefore required. Let the estimation error ϵ be defined as

$$\epsilon = x - z \tag{2.5}$$

its dynamics is described by

$$\dot{\epsilon} = A\epsilon + Bu^* + d \tag{2.6a}$$
$$y_\epsilon = C\epsilon \tag{2.6b}$$

Note that (2.6) is independent of u_0. Since u_0 is designed to make y_s to track r, a sufficient condition for y to do the same is that y_ϵ goes to zero or remains very small when $d \neq 0$. The servo design problem has been reduced to a regulator design problem with output feedback.

There are several methods that can be used in designing u^*, including the most recently developed H_∞ methodology [16]. However, given that the ultimate goal is to extend the split control method to servo design for nonlinear plants, the MBC-LTR which already has been extended to nonlinear plants [15], is chosen.

The structure of the model based compensator for (2.6) is:

$$\dot{w} = (A - BG - HC)w + Hy_\epsilon \tag{2.7a}$$
$$u^* = -Gw \tag{2.7b}$$

The most popular version of the MBC-LTR methodology is LQG-LTR, [13]-[14], where the target loop is an optimal regulator like (1.11) or an optimal filter like (1.12). Either one can be used in designing (2.7), but the fact that the disturbances have been reflected at the plant input, while the goal is to regulate the output y_ϵ makes it impossible to use some of the loop shaping techniques of the LQG-LTR method. The main requirement of the estimation error dynamics (2.6) is that it must be faster than the sliding surface dynamics which is given by the roots of $\det[P(\lambda)] = 0$.

Even though as stated above either type of LQG-LTR can be used to design u^*, more insight in the design process is obtained using LQG-LTR at the estimation error input.

For recovery at the input the target loop has the form

$$\dot{\epsilon} = (A - BG)\epsilon + d \tag{2.8a}$$
$$y_\epsilon = C\epsilon \tag{2.8b}$$

Let $-\alpha$ be the real part of the fastest root of $\det[P(\lambda)] = 0$, the target loop will be faster than the sliding surface dynamics if G is calculated from [11]:

$$\begin{aligned} 0 = &-K(A + \alpha I) - (A + \alpha I)^T K \\ &- M^T M + \frac{1}{\rho} K B B^T K \end{aligned} \tag{2.9a}$$

$$G = \frac{1}{\rho} B^T K \tag{2.9b}$$

The design parameters are M and ρ. M is used to shape the frequency response of (2.8) and ρ to adjust its bandwidth. Equation (2.9) minimizes the performance index

$$\int_0^\infty e^{2\alpha t}(\epsilon^T M^T M \epsilon + \rho {u^*}^T u^*)\, dt \, ,$$

in particular if $M = C$, (2.9b) can be interpreted as minimizing the estimation error output, and if $M = N$ it can be interpreted as minimizing the difference between $s(x,t)$ and $s(z,t)$. When N or any other left inverse of B is chosen for M the singular values of the transfer function from the disturbance d to $N\epsilon = s(x,t) - s(z,t)$ coincide at high frequencies.

For the frequency response of the combined system (2.6) and (2.7) to approach that of (2.8) H is calculated from:

$$0 = A\Sigma + \Sigma A^T + BB^T + \frac{1}{\mu}\Sigma C^T C \Sigma \tag{2.10a}$$

$$H = \frac{1}{\mu}\Sigma C^T \tag{2.10b}$$
$$\mu \to 0$$

The resulting compensator obtained with the split control method is defined by (2.1), (2.2), (2.4), (2.7) and is of order $2n$, n compensator states correspond to the plant model and n states correspond to the model based compensator.

3. Closed Loop Dynamics and Performance

The closed loop dynamics during sliding of (2.1) on (2.3) is determined by the equivalent control method. The equivalent control is found by setting $\dot{s}_z = 0$ and solving for $u_0 = u_{0_{eq}}$.

$$u_{0_{eq}} = Q(d/dt)r - NAz \tag{3.1}$$

Substituting $u_{0_{eq}}$ for u_0 in (2.1) and (2.2) and defining

$$\varsigma = \epsilon - w \tag{3.2}$$

the closed loop average dynamics can be written as

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\epsilon} \\ \dot{\varsigma} \end{bmatrix} &= \begin{bmatrix} (I - BN)A & B(NA - G) & BG \\ 0 & A - BG & BG \\ 0 & 0 & A - HC \end{bmatrix} \begin{bmatrix} x \\ \epsilon \\ \varsigma \end{bmatrix} \\ &+ \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} Q(d/dt)r + \begin{bmatrix} B \\ B \\ B \end{bmatrix} \delta \end{aligned} \tag{3.3a}$$

$$y = [C \quad 0 \quad 0][x \quad \epsilon \quad \varsigma]^T \tag{3.3b}$$

The poles of the closed loop system (3.3) are the eigenvalues of the matrices $(I - BN)A$, $(A - BG)$ and $(A - HC)$. Among the $3n$ closed loop poles $2n + \ell - m$ are freely assignable, m are at zero and $n - \ell$ are at the plant zeros [7]. Since (1.1) is minimum phase, nominal stability of the closed loop system is assured by proper choice of N, G and H.

The performance of the closed-loop system is measured by the difference between the plant response and that of the plant model, either in terms of their outputs or in terms of their sliding surfaces. The disturbance rejection properties of (3.3) are given by the singular values of the matrix transfer function from δ to the estimation error output y_ϵ, which we call sensitivity $S(\lambda)$. Assuming that recovery is achieved in the frequency band in which δ has most of its energy, $S(\lambda)$ is given by:

$$S(\lambda) = G_p(\lambda)[I + G_t(\lambda)]^{-1} \tag{3.4a}$$
$$G_p(\lambda) = C(\lambda I - A)^{-1}B \tag{3.4b}$$
$$G_t(\lambda) = G(\lambda I - A)^{-1}B \tag{3.4c}$$

when LTR is done at the estimation error input.

$S(\lambda)$ is the product of the plant transfer function $G_p(\lambda)$ and the target loop sensitivity $[I + G_t(\lambda)]^{-1}$. The goal is to make the maximum singular value of $S(j\omega)$ as small as possible when $G_p(j\omega)$ is big. Therefore the disturbance rejection

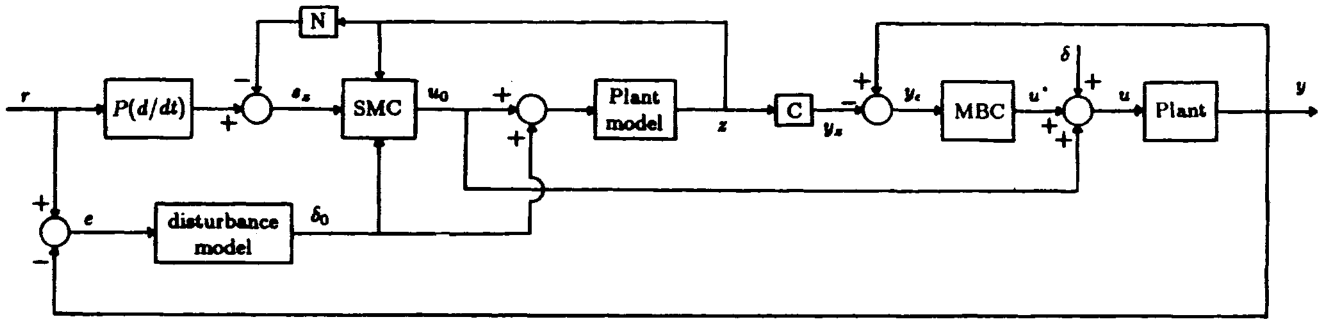

Figure 1 - Split control servo.

problem for the split control servo is equivalent to shaping the target loop sensitivity.

Let ω_p be the bandwidth of $G_p(j\omega)$ and ω_t the bandwidth of $G_t(j\omega)$, in general $\omega_t > \omega_p$. Since $\|G_p(j\omega)\| < 0\,dB$ for $\omega > \omega_p$ while in general $\|[I + G_t(j\omega)]^{-1}\| < 0\,dB$ for $\omega < \omega_t$, $\|S(j\omega)\| < 0$ for all frequencies, if the target loop is properly designed. Therefore the split control servo in general has good disturbance rejection properties. Improvement can be achieved by adding integrators or some other type of filters to the model based compensator. However, this increases the compensator order which is already $2n$.

When all plant states are available, sliding mode control assures perfect disturbance rejection of disturbances with known bounds that satisfy (1.4). In the split control servo this property is lost and disturbance rejection becomes strictly dependent on the MBC-LTR design. To recuperate some of the SMC properties the plant model can be modified to include a linear or nonlinear model of the disturbance:

$$\dot{z} = Az + Bu_0 + d_0 \tag{3.5a}$$

$$d_0 = B\delta_0 \tag{3.5b}$$

This has no effect on the plant model response because d_0 satisfies the matching condition and u_0 becomes:

$$u_0 = -NAz + R_K - \delta_0 + \eta_R\|R_u\|_{max}1_{s_z} \tag{3.6}$$

but the extra term provides additional control action to the plant. How to find δ_0 such that the closed loop system is stable and improved disturbance rejection is obtained perhaps with δ_0 also representing modelling errors and bounded nonlinearities, is an object of current research. Comparing (1.7) and (3.6) suggests a nonlinear disturbance model:

$$\delta_0 = -\eta_\delta\|\delta\|_{max}\Xi^{-1}1_e \tag{3.7a}$$

$$1_e = e/\|e\| \tag{3.7b}$$

This assumes that recovery is still achieved in the presence of the nonlinear term $B\delta_0$, that $NA\epsilon$ is sufficiently small and that $1_e \approx 1_s$. No formal proof of these statements exists, but significant improvement has been found in simulations of several split control servos when δ_0 given by (3.7) is used.

Noisy output measurements can also be handled by the split control servo. Its effect on performance is also dependent on the MBC design.

A block diagram of the resulting closed loop system including the disturbance model is shown in Figure 1.

4. Example

In order to illustrate the split control method let us consider the trajectory control of the low damped two mass-spring system of Figure 2. The objective is to design a servo such that the position of mass 1 and the distance between both masses follow an arbitrary time trajectory, without velocity measurements, and with motion of the left wall as a disturbance. We assume that r and $\dot{r}$ are measurable and that $\ddot{r}$ as well as d have known bounds.

Let $m_1 = m_2 = 1$, $k_1 = k_2 = k_3 = 1$, $b_1 = b_2 = .1$ in the appropriate units, the state space equations are:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -2 & 1 & -.1 & 0\\ 1 & -2 & 0 & -.1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} + [0 \;\; 0 \;\; 0 \;\; d]^T$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ -1 & 1 & 0 & 0\end{bmatrix}[x_1 \;\; x_2 \;\; x_3 \;\; x_4]^T$$

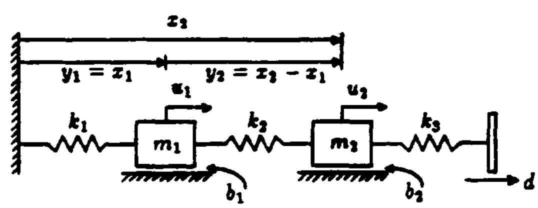

Figure 2 - Two mass-spring system.

According to (1.3b) the sliding surface is defined by the differential operator:

$$P(d/dt) = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\frac{d}{dt} + T\begin{bmatrix}-.9975 & .0713\\ -.0713 & -.9975\end{bmatrix}$$

where T is an arbitrary 2×2 matrix whose coefficients are chosen to define the sliding surface dynamics. Choosing T for x_1 and x_2 to be decoupled during sliding with time constants $1/a$ and $1/b$, $P(d/dt)$ becomes:

$$P(d/dt) = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\frac{d}{dt} + \begin{bmatrix}a & 0\\ 0 & b\end{bmatrix} = P_1 d/dt + P_0$$

and N is:

$$N = \begin{bmatrix}a & 0 & 1 & 0\\ -b & b & 0 & 1\end{bmatrix}$$

The plant model control u_0 for $\Xi = I$ is given by:

$$u_0 = \begin{bmatrix} a\dot{r}_1 + 2z_1 - z_2 - (a-.1)z_3 - \delta_{0_1} \\ b\dot{r}_2 - z_1 + 2z_2 + bz_3 - (b-.1)z_4 - \delta_{0_2} \end{bmatrix} \begin{matrix} \\ \end{matrix}$$
$$\qquad \begin{bmatrix} +\eta_R\|P_1 r\|_{\max} 1_{s_{z_1}} \\ +\eta_R\|P_1 r\|_{\max} 1_{s_{z_2}} \end{bmatrix}$$

$$s_z = \begin{bmatrix} \dot{r}_1 + a(r_1 - z_1) - z_3 \\ \dot{r}_1 + \dot{r}_2 + b(r_2 + z_1 - z_2) - z_4 \end{bmatrix}$$

δ_0 as in (3.7)

For this example $a = b = 5$. Solving (2.9) and (2.10) for $M = N$, $\rho = .005$, $\mu = 10^{-9}$ and $\alpha = 5$ gives

$$G = \begin{bmatrix} 182.5 & 11.4 & 29.2 & -.7 \\ -43.8 & 130.7 & -.7 & 26.9 \end{bmatrix}$$

$$H = \begin{bmatrix} 231.4 & 176.7 & 28260.1 & 14123.6 \\ -54.6 & 231.4 & -14135.8 & 28259.1 \end{bmatrix}^T$$

This gives a bandwidth of 20 rad/sec and a maximum sensitivity of -40 dB to the estimation error dynamics.

For comparison a LQG-LTR servo and a sliding mode controller with a Kalman filter were designed. The LQG-LTR servo was designed with integrators in the error signals, with recovery at the plant output and with the same bandwidth and maximum sensitivity of the split control servo. The Kalman filter for the sliding mode controller was also designed for the same bandwidth and maximum sensitivity of the split control servo.

In the split control and sliding mode servos the switching vectors $1_{(\cdot)}$ were substituted by saturation vectors

$$1_{sat(\cdot)} = \begin{cases} (\cdot)/\varphi, & \|(\cdot)\| \le \varphi \\ (\cdot)/\|(\cdot)\|, & \|(\cdot)\| > \varphi \end{cases}$$

with $\varphi = .05$. The simulation results for $r_1 = 1 + \sin(4t - \pi/2)$, $r_2 = -\cos(3t)$ and $d = 2\cos(5t)$ are shown in Figures 3 and 4. The tracking error e_1 is about the same for the split control servo and for the sliding mode servo with some improvement when δ_0 is introduced. The tracking error e_2 for the split control servo is about one third that of the sliding mode servo and further improvement is obtained when δ_0 is used. Both errors are significantly higher for the MBC-LTR servo. Similar results were found for different frequencies of the reference signals and the disturbance.

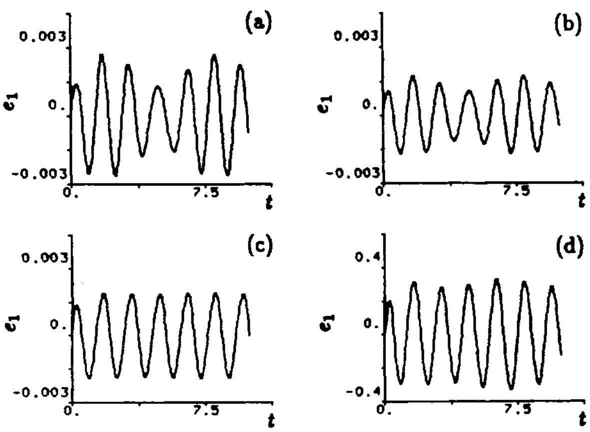

Figure 3 - Tracking error e_1 for:(a) split control servo with $\delta_0 = 0$, (b) split control servo with $\delta_0 = 2\|\delta\|_{\max} 1_e$, (c) sliding mode servo, (d) LQG-LTR.

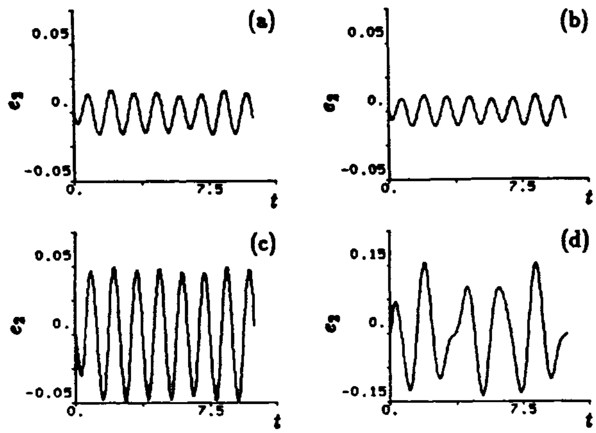

Figure 4 - Tracking error e_2 for servos (a), (b), (c), (d).

5. Conclusions

A hybrid servo design method for MIMO linear time invariant systems with output feedback was introduced. Improvement in the tracking error was achieved w.r.t. the two design methodologies that are involved in the split control servo.

In its structure the split control servo resembles that of a series-parallel adaptive model following controller with signal synthesis [17]. There are however several conceptual differences between both methods. In the split control servo the plant model is used not only as an ideal model which we would like to follow but is also used to generate part of the control action. The MBC combined with the disturbance model δ_0 which are analogous to the adaptation mechanism are required because of lack of full state feedback rather than to compensate for differences between the plant and the model. Finally in the split control servo instead of changing the gain parameters in the plant loop according to the tracking error, this error is used to generate a disturbance to the plant model, therefore modifying the model to best fit the actual plant.

Future research includes synthesis of the plant model disturbance for guaranteed closed loop stability and performance improvement, when it tries to model not only external disturbances but also modelling errors, stability robustness to structured and unstructured modelling errors, and extension of the split control method to at least nonlinear plants linear in the control input.

The main feature of the split control method which makes it attractive for nonlinear plants is the separation of the servo design with output feedback into two, much easier to solve problems, a servo design with full state feedback and a regulator design with output feedback. The split control method also provides an online linearization of the plant w.r.t. an arbitrary trajectory that could greatly simplify the regulator design.

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6. Appendix A

Equation (3.4) is proved using the matrix inversion Lemma (m.i.l.):

$$(F_1 + F_2F_3F_4)^{-1} = F_1^{-1} - F_1^{-1}F_2(F_3^{-1} + F_4F_1^{-1}F_2)F_4F_1^{-1}$$

Taking the Laplace transform of (2.7) we get:

$$u^*(\lambda) = -G[\lambda I - (A - BG - HC)]^{-1}HC\epsilon(\lambda)$$

from loop transfer recovery at the input [12]:

$$G[\lambda I - (A - BG - HC)]^{-1}H \underset{\mu\to 0}{\longrightarrow} G\Phi(\lambda)B[C\Phi(\lambda)B]^{-1}$$

$$\Phi(\lambda) = (\lambda I - A)^{-1}$$

substituting in the Laplace transform of (2.6) gives

$$y_\epsilon(\lambda) = C\left\{\Phi^{-1}(\lambda) + BG\Phi(\lambda)B[C\Phi(\lambda)B]^{-1}C\right\}^{-1}B\delta(\lambda)$$

using the m.i.l. and dropping the λ's:

$$\begin{aligned} y_\epsilon &= C\left\{\Phi - \Phi B[I + (G\Phi B)^{-1}]^{-1}(C\Phi B)^{-1}C\Phi\right\}B\delta \\ &= C\Phi B\left\{I - [I + (G\Phi B)^{-1}]^{-1}\right\}\delta \end{aligned}$$

using the m.i.l. once more:

$$y_\epsilon = C\Phi B[I + G\Phi B]^{-1}\delta$$

which proves (3.4).