The p4ott P4 Formalization

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June 14, 2021

This is a description of the p4ott formalization of P4, which includes a syntax and a strictly small-step style semantics. It is based on the official P4 specification and inspired by Core P4 [1].

p4ott is constructed using the ott tool. ott files can then be exported to LATEX commands (used in this document) as well as to the HOL4, Isabelle/HOL and Coq interactive theorem provers (of which only the first is currently supported).

1 Syntax

1.1 Types

 $\begin{array}{lll} x,\,f,\,msg & \text{string} \\ b & \text{boolean} \\ n_w & \text{integer} \\ i & \text{natural number} \\ m,\,n,\,o & \text{indices} \end{array}$

Figure 1: Variables

The variables shown in Figure 1 are standard designations for variables of P4 base types included in p4ott, plus the numerals i and the indices m, n, o which are not part of the P4 syntax, but used on a meta-level throughout this formalization. Depending on the context, strings are denoted with x (variable name), f (function or field name) or msg (error message). The integer n_w is a 64-bit word.

Types are sometimes explicitly referenced in the syntax, e.g. in declaration statements. The notation for this is shown in Figure 2. Subscript t is used to clarify the notation refers to a type, as opposed to a variable of that type. Instances of composite types also have names.

$$\begin{array}{ccc} t & & ::= & & types \\ & | & bool_t \\ & | & int_t \end{array}$$

Figure 2: Types

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1.2 Expressions

p4ott includes a subset of the full set of P4 expressions found in Section 8 of the P4 specification, shown in Figure 3.

e	::=	expression	
		v	constant value
	j	x	variable name
	İ	$\ominus e$	unary operation
	İ	$e_1 \oplus e_2$	binary operation
	İ	$\operatorname{\mathbf{call}} f(e_1, \ldots, e_n)$	function call
	j	$\mathbf{exec}stmt$	function execution
	Ì	(e)	

Figure 3: P4 Expressions

First, an expression can be a Boolean or an integer (collectively referred to as constant values v), or a string. There exist unary and binary arithmetic operations, where the semantics of the individual operations are defined on some subset of the constants¹. The function call is built from the function name f, and a list of arguments (expressions). In-progress execution of the body of a called function, **exec** stmt, is not a part of the P4 syntax, but is rather an artifact of our small-step semantics.

 $^{^{1}}$ The concrete syntax of unary and binary operations is found in Appendix A

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1.3 Statements

p4ott includes a subset of the full set of P4 statements found in Section 11 of the P4 specification, shown in Figure 4.

stmt	::=		statement
		$\emptyset_{ ext{stmt}}$	empty statement
		x := e	assignment
		if $e \operatorname{\mathbf{then}} stmt_1 \operatorname{\mathbf{else}} stmt_2$	conditional
		$\operatorname{\mathbf{decl}} x t$	declaration
		$\{stmt\}$	block
		[stmt]	block in progress
		${f return}\ e$	return
		$stmt_1; stmt_2$	sequence
		$\mathbf{verify}\;e\;e'$	verify
		${\bf transition}p$	transition

Figure 4: P4 Statements

TODO: Describe statements.

1 SYNTAX 4

1.4 Execution State

The P4 execution state is shown in Figure 5. Note that nothing like this is described in the P4 specification, so it is entirely an artifice of the p4ott implementation. In short, the execution state s is a tuple of the state memory σ and the state status t. The state memory σ consists of a tuple (ε, E) , where ε is a stack of scopes γ which hold the values of variables which are currently visible, and E holds variable mappings which belong to previous caller contexts.

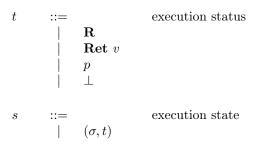


Figure 5: P4 Execution State

More formally, a scope $\gamma: X \hookrightarrow V$ is a partial function from variable names $x \in X$ to constant values $v \in V$. The following operations can be performed on γ :

- $dom(\gamma)$: Gets the domain of γ : obtains the set of variable names $x \in X$ which are mapped to values in γ .
- $(x \mapsto v) \gamma$: Updates a variable mapping in γ : yields the scope γ' , which is just γ where x instead maps to v. By writing $\forall i \leq n$. $(x_i \mapsto v_i) \gamma$ we extend this to lists of mappings from variable names to values.

A frame ε is a stack of scopes where the global scope γ_G is located at the bottom; that is, in location $\varepsilon[0]$. The current scope - that which was most recently entered by execution - is stored on the top of ε (note that this indexing is the reverse of what you would expect from a list). Whenever a new block (delineated by $\{\}$) is entered, a new fresh scope γ_\emptyset is pushed onto the frame ε . The following operations can be performed on a frame ε :

- $\gamma :: \varepsilon$: Pushes a scope γ on top of ε .
- $(i \mapsto \gamma) \varepsilon$: Updates the scope located at index i of ε by setting it to γ .

The call stack E is a stack of frames used whenever a function call occurs. When a function call is executed, the frame ε (minus the global scope γ_G) of the caller will be pushed onto E. When the callee function finishes execution and returns, ε will be popped from E and pushed onto a frame containing only γ_G . Note that this means that the same γ_G is kept throughout function calls, and updates to it are passed along accordingly. The following operations can be performed on E:

• $\varepsilon :: E$: Pushes a frame ε onto the call stack E.

The status **R** represents that the program is executing under regular circumstances. **Ret** v is used when the **return** statement returns a constant v at the end of a function call. The status p signifies transition to a new parser state inside the parser - a named state in the case of **Trans** x, or

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a final state (p_{fin}) in the case of **Accept** or **Reject**. \perp represents a crash or undefined behaviour, for example caused by some badly-typed part of the program.

In addition to the above, there's also a function map F mapping function names to tuples of their bodies and argument names, a parser map P mapping parser state names to their bodies. These are assumed to be static, and are therefore not part of the execution state.

2 Semantics

2.1 Expressions

$$[e](\sigma)\leadsto[e'](\sigma') \qquad \text{expression semantics}$$

$$(stmt,x_1,\ldots,x_n)=F(f) \\ \gamma'=\forall i\leq n.\ (x_i\mapsto v_i)\ \gamma_\emptyset \\ \gamma=\varepsilon[0] \\ \varepsilon'=\gamma'::[\gamma] \\ E'=\operatorname{tl}(\varepsilon)::E \\ \hline {[\operatorname{call}\ } f(v_1,\ldots,v_n)]((\varepsilon,E),\mathbf{R})\leadsto[\operatorname{exec}\ stmt]((\varepsilon',E'),\mathbf{R}) } \quad \text{E_FUNC_CALL_NEWFRAME}$$

$$\frac{[stmt](\sigma,\mathbf{R})\to[stmt'](\sigma',\mathbf{R})}{[\operatorname{exec}\ stmt](\sigma,\mathbf{R})\leadsto[\operatorname{exec}\ stmt'](\sigma',\mathbf{R})} \quad \text{E_FUNC_EXEC}$$

$$\overline{[\operatorname{exec}\ \emptyset_{\operatorname{stmt}}](\sigma,\operatorname{Ret}\ v)\leadsto[v](\sigma',\mathbf{R})} \quad \text{E_FUNC_RET_E}$$

$$i=\max\{j.\ x\in\operatorname{dom}(\varepsilon[j])\} \\ \gamma=\varepsilon[i] \\ v=\gamma(x) \\ \overline{[x]((\varepsilon,E),\mathbf{R})\leadsto[v]((\varepsilon,E),\mathbf{R})} \quad \text{E_LOOKUP}$$

Figure 6: P4 Expression Evaluation Semantics

The semantics of expressions is shown in Figure 6^{23} .

The E_FUNC_CALL_NEWFRAME rule is used when all of the function arguments have been reduced to constants. The constants are assigned to their respective argument names in a fresh scope, after which this scope is put on top of the global scope γ_G in order to form the new current frame ε' . The old current frame ε (minus γ_G) is then saved on top of the call stack E to be used later when returning from the function call, and the function call statement is reduced to **exec** stmt - in-progress execution of the function body stmt (obtained from the function map F, which holds mappings between function names f and tuples of function bodies and lists of their argument names). Note that this rule also covers the case of a function call with no arguments.

²The semantics for reducing concrete arithmetic operations is standard and covers everything found in Appendix A ³Rules for reducing expressions in all contexts can be found in Appendix B

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The E_FUNC_EXEC rule reduces the function body of in-progess execution with one statement reduction.

The E_FUNC_RET_E rule reduces finished (empty) in-progress execution with status **Return** e to e, provided e is a constant. This also changes the status to **Running**.

In the E_LOOKUP rule, the first antecedent states that $i = \max\{j. x \in \text{dom}(\varepsilon[j])\}$, which ensures that the variable name x is evaluated in the uppermost (i.e. most recently entered) scope of ε where it can be found. This agrees with the description in Sections 6.8 and 10.2 of the P4 specification. The value of this variable is then resolved, and checked to be a constant.

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2.2 Statement Execution

The semantics of the statements is shown in Figure 7.

The STMT_RET_E rule implements one reduction to the expression at a time, to simplify the expression until it reduces to a constant. Once the **return** statement appears to return a constant the rule STMT_RET_CONST contains the antecedents that are required for such operation. The global scope γ_G is always stored at the bottom (index zero) in the stack of scopes, i.e. $\varepsilon[0]$. It is fetched and concatenated with most recent caller that was stored on top of the call stack E. Thus, this concatenation will generate a ε that has the same shape to the one before the function being called (of course before reaching the **return** statement some global variables could have been changed during function evaluation, that is the reason why we say that the shape is the same but not the variable mappings in the global scope γ_G). The status will be changed to **Return** v where it will be handled later in an other rule.

The STMT_ASS_E rule implements one reduction to the expression at a time, to simplify the expression until it reduces to a constant. The STMT_ASS_CONST rule handles assignment statement. In general, the variables that the program can assign values to it should be in the global scope γ_G or the current scope of the frame, thus we need to look up into the current frame ε , but never into E. So the antecedent $i = max\{j.\ x \in \text{dom}(\varepsilon[j])$ fetches the proper index that locates the variable location, it should be the one in the uppermost part of the the current frame ε (i.e. oldest entry). In the last antecedent, $(x \longrightarrow v)scope$ updates the mapping of the variable name x to the new value (constant v) in the proper frame location, that indeed lies in the current frame. One step reduction in this rule results an updated current frame, and an empty statement to execute.

The STMT SEQ1 and STMT SEQ2 rules are trivial to understand, they are pretty standard.

The STMT_SEQ3 handles the **return** statement when it does not occur at the end of the the function code. The next statement to reduce will be empty and the status will be changed to **Return** that will be handled later.

The STMT COND1, STMT COND2 and STMT COND3 rules are pretty trivial to understand.

The STMT_DECL rule is a transition reduction for the declaration statements. Whenever a variable is declared, it will updated in the most recent scope and reduce the transition to an empty statement. The most recent/newest scope is fetched simply by checking the length of the current frame ε .

The $\{\}$ brackets indicates a block, while the $[\]$ brackets indicates a block in progess of being executed. Whenever entering a block, in rule STMT_BLOCK_ENTER, a new empty scope γ_{\emptyset} is pushed onto the current frame ε , and then the $\{\}$ brackets are switched to the in-progress ones $[\]$ to signify that the block is currently being executed.

STMT BLOCK EXEC rule shows a small step reduction for the statements inside a block.

STMT_BLOCK_EXIT rule handles the case where the end of a block is reached, i.e. whenever a block contains only an empty statement. It pops the scope corresponding to the block (the most

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recent one) from the frame ε .

2.3 Parser

The parser is a part of the P4 language which is generally used to parse packets from bit-string representations to structures of parsed headers, described in Section 13 of the P4 specification. It can be thought of as describing a state machine with three unique states: a *start* state, an **Accept** state and a **Reject** *msg* state. A parser state *p* (including *start*, but not the abstract final states of **Accept** and **Reject** *msg*) consists of a list of statements to be executed, with a transition statement at the end which decides the parser state to jump to next.

The parser semantics is shown in Figure 8. Note the separate judgment form for the final step of the parser, which goes to a state with status **Accept** or **Reject** (representing the abstract accepting and rejecting states).

The PARS_STMT rule performs a single small-step reduction of the current statement (the body of the current parser state), while the PARS_STATE rule governs transition to the next parser state: if the current statement stmt is reduced to stmt' with the status being **Trans** x, the next statement is the body of the parser state with name x, obtained from the map P from parser state names to parser bodies.

The PARS_T_FIN rule says that when reduction using the statement semantics of the current statement results in a status with a final parser state p_{fin} , this is also set as the status in the parser semantics. The PARS_T_EMPTY rule covers the special case when the statement semantics runs out of statements in a parser state, in which case the status is set to **Reject** ParserStateEnd.

2.4 Control Blocks

The control block is a part of the P4 language which is generally used to decide which actions to take (typically forwarding) based on the metadata (headers) which was extracted by the parser, as described in Section 12 of the P4 specification. The two main components of a control block is the match-action table and the actions themselves. Note that part of the functionality is separated into the control plane, which is interfaced with here using the $\operatorname{ctrl}(table_name, v, m_kind)$ function that takes a table name, constant value and matching kind and obtains an action name f and a list of function arguments $v_1, ..., v_n$. Actions can be thought of roughly as functions with no return values. The action can be called implicitly from the match-action process (i.e. in the table application), or explicitly from another action or a control block, as described in Section 13.1.1 of the P4 specification.

The APPLY_TABLE_E rule performs small-step evaluation of the header expression used for the matching.

The APPLY_TABLE_V looks up the table name in the table name map, then uses the result together with the header to be looked up to obtain an action (together with action arguments) from the control plane.

References

[1] Ryan Doenges et al. "Petr4: formal foundations for p4 data planes". In: *Proceedings of the ACM on Programming Languages* 5.POPL (2021), pp. 1–32.

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$$|stmt|s \rightarrow |stmt'|s' |$$
 statement semantics
$$\gamma = \varepsilon[0]$$

$$\varepsilon' : E' = E$$

$$\varepsilon'' = (e') + +([\gamma])$$
 [return $v]((\varepsilon, E), \mathbb{R}) \rightarrow [0]_{\text{stmt}}[(\varepsilon'', E'), \text{Ret } v)$ STMT_RET
$$i = \max\{j. \ x \in \text{dom}(\varepsilon[j])\}$$

$$\gamma = \varepsilon[i]$$

$$\varepsilon' = (i \mapsto (x \mapsto v)\gamma)\varepsilon$$
 [stmt_1[(\sigma, \text{R}) \rightarrow |\text{STMT}_ASS_V | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{stmt_1}[(\sigma, \text{R}) \rightarrow |\text{STMT}_seq1 | \text{STMT}_seq1 | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{stmt_1}[\sigma', \text{R}) \text{[stmt_1](\sigma', \text{R})} \text{STMT}_seq2 | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{stmt_1}[\sigma', \text{R}) \rightarrow |\text{STMT}_seq2 | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{STMT}_seq2} | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{STMT}_seq2} | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{STMT}_seq2} | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{STMT}_seq3} | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{STMT}_seq3} | \text{STMT}_cond3 | \text{[stmt_1](\sigma, \text{R}) \rightarrow |\text{Stmt_1}[\sigma', \text{R}) \rightarrow |\text{STMT}_seq3} | \text{STMT}_cond3 | \text{if true then } \text{stmt_1}_1 \text{else } \text{stmt_2}[\sigma', \text{R}) \rightarrow |\text{stmt_1}[\sigma', \text{R}) \rightarrow |\text{STMT}_seq3} | \text{STMT}_cond3 | \text{if true then } \text{stmt_1}_1 \text{else } \text{stmt_2}[\sigma', \text{R}) \rightarrow |\text{stmt_1}[\sigma', \text{R}) \rightarrow |\text{STMT}_sep3 | \text{STMT}_cond3 | \text{if true } \text{else} \text{stmt_1}[\sigma', \text{R}) \rightarrow |\text{stmt_1}[\sigma', \text{R}) \rightarrow |\text{STMT}_sep4 | \text{STMT}_cond3 | \text{if true } \text{else} \text{stmt_1}[\sigma', \text{R}) \rightarrow |\text{stmt_1}[\sigma', \text{R}) \rightarrow |\text{STMT}_sep4 | \text{Exec} \\ \end{e's et al.} \text{else} \\ \text{else} \\ \end{e's et al.} \text{else} \\ \text{else} \\ \end{e's et al.} \text{else} \\ \text{else} \\ \end{e's et al.} \text{else} \\ \text{else} \\ \end{e's et al.} \text{e

Figure 7: P4 Statement Execution Semantics

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$$\begin{array}{c} [stmt]s \longrightarrow [stmt']s' \\ \hline \\ & \underbrace{[stmt](\sigma,\mathbf{R}) \rightarrow [stmt'](\sigma',\mathbf{R})}_{[stmt](\sigma,\mathbf{R}) \longrightarrow [stmt'](\sigma',\mathbf{R})} \quad \text{PARS_STMT} \\ \hline \\ & \underbrace{[stmt](\sigma,\mathbf{R}) \rightarrow [stmt'](\sigma',\mathbf{Trans}\,x)}_{stmt'' = P(x)} \\ \hline \\ & \underbrace{[stmt](\sigma,\mathbf{R}) \rightarrow [stmt'](\sigma',\mathbf{R})}_{[stmt](\sigma,\mathbf{R}) \longrightarrow [stmt''](\sigma',\mathbf{R})} \quad \text{PARS_STATE} \\ \hline \\ & \underbrace{[stmt]s \longrightarrow s'} \quad \text{parser semantics, final step} \\ \hline \\ & \underbrace{[stmt](\sigma,\mathbf{R}) \rightarrow [stmt'](\sigma,p_{\text{fin}})}_{[stmt](\sigma,\mathbf{R}) \longrightarrow (\sigma,p_{\text{fin}})} \quad \text{PARS_T_FIN} \\ \hline \\ & \underbrace{[stmt](\sigma,\mathbf{R}) \rightarrow [\emptyset_{\text{stmt}}](\sigma',\mathbf{R})}_{stmt} \\ \hline \\ & \underbrace{[stmt](\sigma,\mathbf{R}) \rightarrow [\emptyset_{\text{stmt}}](\sigma',\mathbf{R})}_{pars} \quad \text{PARS_T_EMPTY} \\ \hline \\ & \underbrace{[stmt](\sigma,\mathbf{R}) \rightarrow [\emptyset_{\text{stmt}}](\sigma',\mathbf{R})}_{pars} \quad \text{PARS_T_EMPTY} \\ \hline \\ \end{array}$$

Figure 8: Parser Execution Semantics

Figure 9: Match-Action Semantics

A Concrete Syntax of Operations

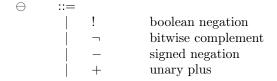


Figure 10: P4 Unary Operations

The unary expressions included are shown in Figure 10. These include all of the unary operations in P4. Boolean negation is only defined on Booleans, the other operations have their standard meanings (note that unary plus is a no-op).

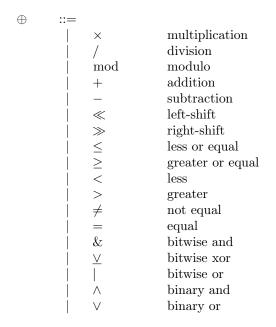


Figure 11: P4 Binary Operations

The binary expressions included are shown in Figure 10. These include all of the binary operations in P4.

B Semantics of Expression Reduction

This appendix describes semantics for reducing expressions in certain contexts. The expression semantics are shown in Figure 12.

The E_FUNC_CALL_ARGS rule reduces the leftmost function argument which has yet to be reduced to a constant with one expression evaluation step. The first two antecedents divide the list of arguments into two sub-lists, where the prefix must contain all constants. The head of the suffix is then reduced with one step, after which the corresponding index in the original list of arguments is update with the resulting expression.

8.1 of the P4 specification states that expressions are evaluated left-to-right. Accordingly, the rules for binary operations - E_BINOP1 and E_BINOP2 - are split up so that reduction of the second operand requires that the first operand has been completely reduced to a constant. This is trivial for unary operations (E_UNOP).

Figure 12: P4 Expression Evaluation Semantics