p atomic proposition
l proof entry label
n index variable (subscript)

claim	::=			claim
	|	$judgment\ \rho$		
judgment	::=			judgment
	|	$\overline{\phi} \vdash \phi$		
ϕ	::=			proposition
	|	p		atomic
	|	$\neg\phi$		negation
	|	$\phi \land \phi'$		conjunction
	|	$\phi \lor \phi'$		disjunction
	|	$\phi \to \phi'$		implication
	|	$\bot$		contradiction
	|	(ϕ)	S	
$\overline{\phi}$	::=			list of propositions
	|	$\phi_1, \ .. \ , \phi_n$	M	
ρ	::=			proof
	|	$entry_1\ ..\ entry_n$		
	|	$()$	M	
	|	$entry\ \rho$	M	
entry	::=			proof entry
	|	*derivation*		line
	|	$[\rho]$		box
	|	invalid		
	|	last (ρ)	M	last entry in proof
derivation	::=			derivation in proof
	|	$l\ \phi\ \ reason$		
reason	::=			
	|	assumption		
	|	*justification*		
justification	::=			derivation justification
	|	premise		premise
	|	LEM		law of excluded middle
	|	copy l		copying
	|	$\land i\ l, l'$		conjunction introduction
	|	$\land e_1\ l$		conjunction elimination
	|	$\land e_2\ l$		conjunction elimination
	|	$\lor i_1\ l$		disjunction introduction
	|	$\lor i_2\ l$		disjunction introduction
	|	$\to e\ l, l'$		implication elimination
	|	$\neg e\ l, l'$		negation elimination

	|	$\bot e\, l$	contradiction elimination
	|	$\neg\neg i\, l$	double negation introduction
	|	$\neg\neg e\, l$	double negation elimination
	|	$\mathrm{MT}\, l, l'$	modus tollens
	|	$\rightarrow i\, l - l'$	implication introduction
	|	$\neg i\, l - l'$	negation introduction
	|	$\vee e\, l_1, l_2 - l_3, l_4 - l_5$	disjunction elimination
	|	$\mathrm{PBC}\, l - l'$	proof by contraduction
terminals	::=		
	|	premise	
	|	LEM	
	|	copy	
	|	$\wedge i$	
	|	$\wedge e_1$	
	|	$\wedge e_2$	
	|	$\vee i_1$	
	|	$\vee i_2$	
	|	$\rightarrow e$	
	|	$\neg e$	
	|	$\bot e$	
	|	$\neg\neg i$	
	|	$\neg\neg e$	
	|	MT	
	|	$\rightarrow i$	
	|	$\neg i$	
	|	$\vee e$	
	|	PBC	
	|	assumption	
	|	$\vdash$	
	|	$\neg$	
	|	$\wedge$	
	|	$\vee$	
	|	$\rightarrow$	
	|	$\bot$	
	|	$\in$	
	|	$\mapsto$	
	|	last	
	|	invalid	
dyadicprop	::=		
	|	ϕ	M
	|	(ϕ, ϕ')	M
Γ	::=		admissible context
	|	$[]$	M
	|	$\Gamma[l \mapsto \phi]$	M

	$\mid$	$\Gamma[(l, l') \mapsto (\phi, \phi')]$	M	
formula	::=			
	$\mid$	*judgement*		judgement
	$\mid$	$\phi \in \overline{\phi}$	M	proposition in set of premises
	$\mid$	$\Gamma(l) = \phi$	M	map label to prop
	$\mid$	$\Gamma(l, l') = (\phi, \phi')$	M	map tuple of labels to tuple of props
	$\mid$	$entry = entry'$	M	entry equality
validity	::=			
	$\mid$	*claim*		valid claim
	$\mid$	$\Gamma, \overline{\phi} \vdash \rho$		valid proof
	$\mid$	$\Gamma, \overline{\phi} \vdash derivation$		valid derivation
judgement	::=			
	$\mid$	*validity*		
user_syntax	::=			
	$\mid$	p		
	$\mid$	l		
	$\mid$	n		
	$\mid$	*claim*		
	$\mid$	*judgment*		
	$\mid$	ϕ		
	$\mid$	$\overline{\phi}$		
	$\mid$	ρ		
	$\mid$	*entry*		
	$\mid$	*derivation*		
	$\mid$	*reason*		
	$\mid$	*justification*		
	$\mid$	*terminals*		
	$\mid$	*dyadicprop*		
	$\mid$	Γ		
	$\mid$	*formula*		

$\boxed{claim}$ valid claim

$$\frac{\begin{array}{l}\mathrm{last}\,(\rho) = l\ \phi\ \ justification\\ [], \overline{\phi} \vdash \rho\end{array}}{\overline{\phi} \vdash \phi\, \rho}\ \text{VC\_CLAIM}$$

$\boxed{\Gamma, \overline{\phi} \vdash \rho}$ valid proof

$$\frac{}{\Gamma, \overline{\phi} \vdash ()}\ \text{VP\_EMPTY}$$

$$\frac{\begin{array}{l}\Gamma, \overline{\phi} \vdash l\ \phi\ \ justification\\ \Gamma[l \mapsto \phi], \overline{\phi} \vdash \rho\end{array}}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ justification\, \rho}\ \text{VP\_DERIVATION}$$

$$\frac{\begin{array}{l}\text{last}\,(l\ \phi\ \ \text{assumption}\,\rho) = l'\ \phi'\ \ \mathit{reason}\\ \Gamma[l \mapsto \phi], \overline{\phi} \vdash \rho\\ \Gamma[(l, l') \mapsto (\phi, \phi')], \overline{\phi} \vdash \rho'\end{array}}{\Gamma, \overline{\phi} \vdash [l\ \phi\ \ \text{assumption}\,\rho]\,\rho'}\ \text{VP\_BOX}$$

$\boxed{\Gamma, \overline{\phi} \vdash \mathit{derivation}}$ valid derivation

$$\frac{\phi\ \in\ \overline{\phi}}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ \text{premise}}\ \text{VD\_PREMISE}$$

$$\frac{}{\Gamma, \overline{\phi} \vdash l\ \phi \vee \neg\phi\ \ \text{LEM}}\ \text{VD\_LEM}$$

$$\frac{\Gamma(l') = \phi}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ \text{copy}\, l'}\ \text{VD\_COPY}$$

$$\frac{\Gamma(l') = \bot}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ \bot e\, l'}\ \text{VD\_CONTE}$$

$$\frac{\begin{array}{c}\Gamma(l_1) = \phi\\ \Gamma(l_2) = \phi'\end{array}}{\Gamma, \overline{\phi} \vdash l\ \phi \wedge \phi'\ \ \wedge i\, l_1, l_2}\ \text{VD\_ANDI}$$

$$\frac{\Gamma(l') = \phi \wedge \phi'}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ \wedge e_1\, l'}\ \text{VD\_ANDE1}$$

$$\frac{\Gamma(l') = \phi \wedge \phi'}{\Gamma, \overline{\phi} \vdash l\ \phi'\ \ \wedge e_2\, l'}\ \text{VD\_ANDE2}$$

$$\frac{\Gamma(l') = \phi}{\Gamma, \overline{\phi} \vdash l\ \phi \vee \phi'\ \ \vee i_1\, l'}\ \text{VD\_ORI1}$$

$$\frac{\Gamma(l') = \phi'}{\Gamma, \overline{\phi} \vdash l\ \phi \vee \phi'\ \ \vee i_2\, l'}\ \text{VD\_ORI2}$$

$$\frac{\begin{array}{c}\Gamma(l_1) = \phi'\\ \Gamma(l_2) = \phi' \to \phi\end{array}}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ \to e\, l_1, l_2}\ \text{VD\_IMPE}$$

$$\frac{\Gamma(l') = \phi}{\Gamma, \overline{\phi} \vdash l\ \neg\neg\phi\ \ \neg\neg i\, l'}\ \text{VD\_NEGNEGI}$$

$$\frac{\Gamma(l') = \neg\neg\phi}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ \neg\neg e\, l'}\ \text{VD\_NEGNEGE}$$

$$\frac{\begin{array}{c}\Gamma(l_1) = \phi \to \phi'\\ \Gamma(l_2) = \neg\phi'\end{array}}{\Gamma, \overline{\phi} \vdash l\ \neg\phi\ \ \text{MT}\, l_1, l_2}\ \text{VD\_MT}$$

$$\frac{\begin{array}{c}\Gamma(l_1) = \phi\\ \Gamma(l_2) = \neg\phi\end{array}}{\Gamma, \overline{\phi} \vdash l\ \bot\ \ \neg e\, l_1, l_2}\ \text{VD\_NEGE}$$

$$\frac{\Gamma(l_1, l_2) = (\phi, \phi')}{\Gamma, \overline{\phi} \vdash l\ \phi \to \phi'\ \ \to i\, l_1 - l_2}\ \text{VD\_IMPI}$$

$$\frac{\Gamma(l_1, l_2) = (\phi, \bot)}{\Gamma, \overline{\phi} \vdash l\ \neg\phi\ \ \neg i\, l_1 - l_2}\ \text{VD\_NEGI}$$

$$\frac{\begin{array}{c}\Gamma(l_1) = \phi \vee \phi' \\ \Gamma(l_2, l_3) = (\phi, \phi'') \\ \Gamma(l_4, l_5) = (\phi', \phi'')\end{array}}{\Gamma, \overline{\phi} \vdash l\ \phi'' \quad \vee e\, l_1, l_2 - l_3, l_4 - l_5}\ \text{VD\_ORE}$$

$$\frac{\Gamma(l_1, l_2) = (\neg\phi, \bot)}{\Gamma, \overline{\phi} \vdash l\ \phi\ \ \mathrm{PBC}\, l_1 - l_2}\ \text{VD\_PBC}$$