Rollout-Based Charging Strategy for Electric Trucks with Hours-of-Service Regulations

Ting Bai, Yuchao Li, Karl H. Johansson, *Fellow, IEEE*, and Jonas Mårtensson

***Abstract*— Freight drivers of electric trucks need to design charging strategies for where and how long to recharge the truck in order to complete delivery missions on time. Moreover, the charging strategies should be aligned with drivers' driving and rest time regulations, known as hours-of-service (HoS) regulations. This letter studies the optimal charging problems of electric trucks with delivery deadlines under HoS constraints. We assume that a collection of charging and rest stations is given along a pre-planned route with known detours and that the problem data are deterministic. The goal is to minimize the total cost associated with the charging and rest decisions during the entire trip. This problem is formulated as a mixed integer program with bilinear constraints, resulting in a high computational load when applying exact solution approaches. To obtain real-time solutions, we develop a rollout-based approximate scheme, which scales linearly with the number of stations while offering solid performance guarantees. We perform simulation studies over the Swedish road network based on realistic truck data. The results show that our rollout-based approach provides near-optimal solutions to the problem in various conditions while cutting the computational time drastically.**

I. Introduction

Vehicle electrification is becoming mainstream globally to reduce carbon emissions and achieve sustainable transportation [1]. In particular, road freight electrification is crucial for reducing greenhouse gas emissions caused by diesel-powered trucks in freight operations, which are responsible for around 25% of vehicle-related carbon emissions in Europe [2]. However, the process of freight truck electrification today is lagging far behind that of electric passenger vehicles [3]. A major concern with electrifying trucks, among others, is their limited driving ranges, known as *range anxiety*. Currently, the average travel range of a commercial electric truck on a full battery varies between 200 and 600 kilometers, depending on diverse truckloads and battery capacities [4]. This is typically insufficient to sustain trucks to complete their delivery missions without stopping and refilling batteries, especially for long-haul journeys. To diminish range anxiety, increase electric truck adoption, and accelerate road freight electrification, reliable and efficient charging strategies are needed. In addition to charging batteries, truck drivers also need to stop and take rests during trips to avoid driving fatigue. The so-called hours-of-service (HoS) regulations [5] address exactly this issue and put restrictions on how long one can drive consecutively without rest, as well as during one day. As a result, charging strategies for electric trucks should be designed not only for mission completion but also to align with the HoS regulations.

To date, there have been extensive works developing viable charging strategies. A majority of these approaches integrate charging stops into conventional routing problems and minimize the travel time or energy required on the route, as in [6], [7], [8]. The authors in [9] propose an optimal driving and charging strategy for electric vehicles. It includes the driving speed as an additional control variable when minimizing the total travel time. However, none of these works incorporates the HoS regulations in optimal charging problems. To the best of our knowledge, [10] is the first to incorporate today's HoS regulations in their charging strategy, which is obtained via a genetic algorithm. Our work differs from the approach in [10] in two major ways. Firstly, we model the route and optimal charging problem in a more general framework, allowing for multiple rests within the maximum daily driving time before delivery deadlines. Secondly, we develop an online solution scheme that allows real-time optimization to deal with travel time uncertainties or model mismatches, as opposed to the genetic algorithm, which is offline and time-consuming.

To enable a real-time solution, we rely on the idea of *rollout*, which refers to the process of simulating a known solution. Proposed first in [11] for addressing backgammon, rollout has since been extended for combinatorial optimization [12] and trajectory-constrained problems [13], to name a few. Our scheme is modified from the methods introduced in [13] and [14, Section 3.4], where an improved solution is computed based on a known solution. As shown in [14], the rollout scheme can be viewed as one step of Newton's method applied to solve the optimization problem with the initial guess supplied by the known solution. In view of the fast convergence rate of Newton's method, the solution provided by rollout has substantial improvement from the known solution, which is consistent with many empirical studies [14, p. 136].

In this letter, we study the optimal charging strategy for electric trucks with realistic HoS regulations. In particular, we consider an electrified transportation system where every electric truck has a pre-planned route for a delivery task. With the knowledge of a collection of charging and rest stations available along the route, the truck driver could design an optimal charging strategy to determine where and how long to recharge the truck and take rests so that the

This work was supported in part by the Swedish Research Council Distinguished Professor under Grant 2017-01078); in part by the Knut and Alice Wallenberg Foundation; and in part by the Swedish Strategic Research Foundation CLAS under Grant RIT17-0046).

The authors are with the Integrated Transport Research Lab, the Division of Decision and Control Systems, and the Digital Futures, KTH Royal Institute of Technology, 100 44 Stockholm, Sweden. E-mails: {tingbai, yuchao, kallej, jonas1}@kth.se

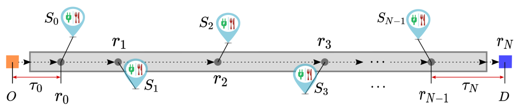

Fig. 1. A simplified route model of electric trucks, where each charging and rest station, denoted by S_k, provides both charging and rest services. Each ramp in the route leading to S_k with the shortest detour is denoted by r_k and shown by a grey node, where $k=0,\ldots,N-1$.

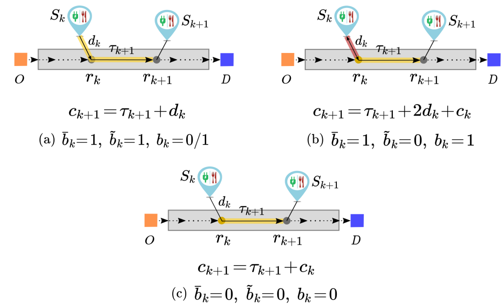

Fig. 2. The consecutive driving time when first arriving at r_{k+1}.

extra operational costs due to the charging and rest decisions are minimized. The main contributions of this letter are: *i)* we model the optimal charging problems of electric trucks as mixed integer programs with *bilinear constraints* that incorporate the delivery deadlines and HoS regulations; *ii)* a rollout-based approximate solution method, as well as its variants, is developed for addressing the problem, through which the computational demands required by exact solution approaches are significantly decreased while offering solid performance guarantees. Simulation studies performed over the Swedish road network using realistic truck data illustrate the effectiveness of the developed method.

II. Problem Formulation

A. Route Model

We consider simplified route models of electric trucks. As illustrated in Fig. 1, given a pre-planned route of a truck between its origin O and destination D, we assume that N *charging and rest stations* are available along the route, denoted as S_k, $k=0,\ldots,N-1$. The ramp along the pre-planned route leading to station S_k with the shortest detour is denoted as r_k. For simplicity, the destination is referred to as ramp r_N. The travel time required for the truck to take a detour between its ramp r_k and the station S_k (one-way) is denoted as d_k. Moreover, in the pre-planned route, the travel time on the route segment connecting r_k and r_{k+1} is denoted by τ_{k+1},k=$0,\ldots,N-2$. Particularly, the truck's travel times from its origin to its first ramp r_0, and from its ramp r_{N-1} to the destination are denoted by τ_0 and τ_N, respectively.

Note that in our route model, each station provides both charging and rest services, allowing drivers to take a rest while the truck is charging. This is aligned with the charging infrastructure building plans nowadays [15] as it saves time in logistics. Nevertheless, our problem formulation and solution approach to be introduced later also apply to other cases where some stations provide only charging or rest service.

B. Battery Energy and Consecutive Driving Time

Given the travel and detour times on route, the initial battery energy, and the delivery deadline, a truck driver could design a charging strategy to complete the delivery mission on time subject to the HoS regulations. The decisions involved in the charging strategy include: (i) whether to charge the truck at S_k, $k=0,\ldots,N-1$; (ii) whether to rest at S_k, $k=0,\ldots,N-1$; and (iii) how long to charge the truck at S_k once decided to charge there. These decisions can be represented by the variables

$$b_k,\tilde{b}_k\in\{0,1\},\ t_k\in\Re_+,\quad k=0,\ldots,N-1, \tag{1}$$

where $b_k=1$ means charging at S_k and 0 otherwise, t_k is the planned charging time at S_k if $b_k=1$, and $\Re_+$ contains nonnegative reals. Similarly, $\tilde{b}_k=1$ means resting at S_k and 0 otherwise. These decisions affect how the battery energy and the consecutive travel time vary when arriving at different ramps, as we introduce next.

To describe the battery dynamics, we denote by $e_k\in\Re_+$ the *remaining energy in the battery* of the truck when it *first* arrives at r_k, $k=0,\ldots,N$. In addition, let $e_{\rm ini}$ be the initial energy in the battery at the origin, and $\bar{P}$ be the battery consumption of the truck on route per travel time unit. The remaining energy e_k can then be characterized as

$$e_{k+1}=e_k+b_k\Delta e_k-\bar{P}\big(2\bar{b}_kd_k+\tau_{k+1}\big), \tag{2a}$$

$$\bar{b}_k=b_k\vee\tilde{b}_k, \tag{2b}$$

for $k=0,\ldots,N-1$, with $e_0=e_{\rm ini}-\bar{P}\tau_0$. In (2b), $\bar{b}_k$ reflects whether the truck visits S_k for charging or rest, where $\vee$ is logical or operator. In (2a), Δe_k denotes the charged energy at S_k. As a linear approximation to the charging process, as adopted in [16], Δe_k is modeled as

$$\Delta e_k=t_k\min\big\{P_k,P_{\max}\big\},\quad k=0,\ldots,N-1, \tag{3}$$

where P_k denotes the charging power provided by S_k, and $P_{\max}$ is the maximum charging power that can be accepted by the battery of the truck.

To ensure that the HoS regulations are followed under the designed strategy, we also keep track of consecutive driving time upon arriving at each ramp. For this purpose, let us denote by c_k, k=$0,\ldots,N$, the consecutive driving time when arriving at r_k for the first time. Its dynamics are given by

$$c_{k+1}=\tau_{k+1}+\bar{b}_kd_k+\big(1-\tilde{b}_k\big)\big(c_k+b_kd_k\big), \tag{4}$$

for $k=0,\ldots,N-1$, with $c_0=\tau_0$. Here, $\tau_{k+1}+\bar{b}_kd_k$ is the driving time from S_k to r_{k+1} when $\bar{b}_k=1$. If the driver takes a rest at S_k (i.e., $\tilde{b}_k=1$), S_k becomes the start of a new consecutive driving period. Thus, the previous accumulated driving time c_k will not be accounted for, as illustrated by the yellow lines in Fig. 2(a). Otherwise (i.e., $\bar{b}_k=1$, $\tilde{b}_k=0$), the accumulated travel time upon arriving at r_k will be taken into account when computing the accumulated travel time at r_{k+1}, and a round-way detour between r_k and S_k will be taken, as illustrated by the red and yellow lines in Fig. 2(b). If the driver neither drive to nor rest at S_k (i.e., $\bar{b}_k=0, b_k=0$), the consecutive driving time c_{k+1} is then shown in Fig. 2(c).

C. Constraints on the Problem

In what follows, we introduce the constraints imposed on the charging strategy due to the battery dynamics, HoS regulations, and the delivery deadline.

1) Battery Constraints: Let e_f be the energy of a truck with a full battery. Due to the capacity limitation, the total energy that the truck can be charged at S_k is restricted by

$$0 \leq \Delta e_k \leq e_f - \big(e_k - \bar{P} d_k\big), \quad k=0,\ldots,N-1, \tag{5}$$

where $e_k - \bar{P} d_k$ is the remaining energy in the battery when the truck arrives at S_k.

Furthermore, to ensure that there is sufficient energy for reaching S_k, each e_k with $k=0,\ldots,N$ shall fulfill

$$e_k \geq e_s + \bar{P} d_k,\ k=0,\ldots,N-1, \quad e_N \geq e_s, \tag{6}$$

where e_s denotes a constant safety margin.

2) HoS Regulations Constraints: The HoS regulations involve three quantities, namely, the maximum consecutive driving time, denoted as T_d, the maximum daily driving time, denoted as $\bar{T}_d$, and the minimum mandatory rest time before starting a new consecutive driving period, denoted as T_r.

In line with the HoS regulations, the maximum consecutive driving time shall be bounded by T_d. That is, for k=$0,\ldots,N$, the consecutive driving time c_k is restricted by

$$c_k + d_k \leq T_d,\ k=0,\ldots,N-1, \quad c_N \leq T_d. \tag{7}$$

Moreover, as the driver's daily driving time is no more than $\bar{T}_d$, we have that

$$\sum_{k=0}^{N} \tau_k + \sum_{k=0}^{N-1} 2\bar{b}_k d_k \leq \bar{T}_d, \tag{8}$$

where, as defined above, $\bar{b}_k = b_k \vee \tilde{b}_k$, so that $\bar{b}_k = 1$ if the truck visits S_k and 0 otherwise.

When charging at S_k, there is a preparation time p_k before the battery can get charged. In addition, we consider staying at S_k over T_r as taking a rest. As a result, when $b_k=1$ and $\tilde{b}_k=0$, the sum t_k+p_k should be less than T_r, i.e., $t_k+p_k < T_r$. On the other hand, no such restriction is needed if $\tilde{b}_k=1$. These constraints can be described compactly as

$$b_k\big(t_k + p_k\big) \leq \big(1-\tilde{b}_k\big)\big(T_r - \underline{\delta}\big) + \tilde{b}_k \bar{\delta}, \tag{9}$$

for $k=0,\ldots,N-1$, where $\underline{\delta}$ is some small positive constant so that constraint $t_k+p_k<T_r$ is approximated by $t_k+p_k \leq T_r - \underline{\delta}$. The large constant $\bar{\delta}$ is introduced to approximate unboundedness above.

3) Delivery Deadline Constraint: Let the total time allowed to complete the trip be $\Delta T + \sum_{k=0}^{N} \tau_k$, where ΔT provides an upper bound on the extra time spent due to charging and rest. Then the constraint imposed by the deadline is

$$\sum_{k=0}^{N-1} \max\Big\{b_k\big(2d_k + p_k + t_k\big), \tilde{b}_k\big(2d_k + T_r\big)\Big\} \leq \Delta T. \tag{10}$$

III. Exact Solution to the Optimal Charging Problem with HoS Regulations

This section presents the optimization problem for determining the optimal charging strategy while fulfilling the HoS regulations. We start by introducing the optimal charging problem, followed by the exact solution and the computational complexity analysis of the problem.

A. Optimal Charging Problem

1) Cost Function: Our goal is to complete the delivery mission on time under the HoS regulations while saving operational costs. This includes the cost of charging and economic loss due to extra labor costs. Specifically, the expenses from charging at selected stations along the route are defined as

$$F_1\big(b_0, t_0, \ldots, b_{N-1}, t_{N-1}\big) = \sum_{k=0}^{N-1} \xi_k b_k t_k,$$

where ξ_k represents the electricity price per charging time unit in accordance with the charging power at S_k, and t_k is the charging time at S_k.

In addition, the cost due to the extra travel time during the entire trip is represented as

$$\begin{aligned} &F_2\big(b_0, \tilde{b}_0, t_0, \ldots, b_{N-1}, \tilde{b}_{N-1}, t_{N-1}\big) \\ &= \sum_{k=0}^{N-1} \max\Big\{b_k\big(2d_k + p_k + t_k\big), \tilde{b}_k\big(2d_k + T_r\big)\Big\}\varepsilon, \end{aligned} \tag{11}$$

where, as previously defined, T_r represents the minimum mandatory rest time specified by the HoS regulations. The monetary loss per extra travel time unit is denoted by ε.

The cost function of the optimal charging problem is then of the following form

$$\begin{aligned} &F\big(b_0, \tilde{b}_0, t_0, \ldots, b_{N-1}, \tilde{b}_{N-1}, t_{N-1}\big) \\ =&F_1\big(b_0, t_0, \ldots, b_{N-1}, t_{N-1}\big) + \\ &F_2\big(b_0, \tilde{b}_0, t_0, \ldots, b_{N-1}, \tilde{b}_{N-1}, t_{N-1}\big), \end{aligned} \tag{12}$$

which includes the cost of charging and the cost of extra travel time for completing the delivery mission.

2) Optimization Problem: Based on the battery dynamics, consecutive driving times, HoS regulations, and delivery deadline constraints formulated in Section II, as well as the cost function given above, the optimal charging strategy can be obtained by solving the following optimization problem

$$\begin{aligned} \min_{\{(b_k, \tilde{b}_k, t_k)\}_{k=0}^{N-1}} \quad & F\big(b_0, \tilde{b}_0, t_0, \ldots, b_{N-1}, \tilde{b}_{N-1}, t_{N-1}\big) \\ \text{s.t.} \quad & (1)-(10), \end{aligned}$$

where (1) defines the domains of the decision variables b_k, $\tilde{b}_k$, and t_k, (2) and (3) characterize the battery dynamics during driving and charging, and (4) describes the consecutive driving times upon arriving at each ramp. The constraints imposed by the battery capacity and its safety margin are (5) and (6). The HoS regulations are characterized by (7)-(9). The constraint related to the delivery deadline is (10). The sufficient conditions under which the problem is feasible are given in Appendix E of the extended version [17].

Note that the proposed formulation is flexible to incorporate various modifications, such as taking the sum of $\bar{b}_k$ as the cost function (12) for a sparse selection of the stations, or replacing the linear approximation of battery dynamics (3) to nonlinear ones. For simplicity, we focus on the present setting.

B. Exact Solution

The optimal charging problem formulated above is a mixed integer program with bilinear constraints. Thus, it cannot be directly addressed by many standard solvers. To obtain the exact solution to the problem, one could iterate over all possible combinations of integer variables. Since the integer variables b_k and $\tilde{b}_k$ admit 4 combinations at each station, i.e., $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, there are in total 4^N charging and rest choices, where N is the number of stations. Therefore, the exact solution requires solving 4^N linear programs, which leads to high computational demands and is not practical.

Note that the bilinear constraints can be transformed into linear ones so that the problem becomes a standard mixed integer linear program. We demonstrate this transformation in Appendix F of [17]. However, the exact solution to the transformed problem may still require an exponential number of iterations; see [18, p. 480]. Moreover, if the linear approximation of charging in (3) is replaced by nonlinear functions of t_k, such transformations would become obsolete.

To obtain tractable charging strategies, especially for long-haul trips with many candidate charging and rest stations, a rollout-based approximate solution to the optimal charging problem is proposed in the following section.

IV. Approximate Solution to the Optimal Charging Problem via Rollout

In this section, we introduce the proposed rollout scheme for the optimization problem formulated in Section III. We first describe a basic form of the method within the context of a general mixed integer program, which is modified from the methods introduced in [13] and [14, Section 3.4]. It is followed by a variant of the scheme. Then we demonstrate how the basic form, as well as its variant, can be applied to obtain an approximate solution to the optimal charging problem. Further analysis and variants of our scheme are provided in [17].

A. Rollout for Mixed Integer Program

Let us consider the following mixed integer program:

$$\min_{(u,v)} \; G(u,v) \quad \mathrm{s.t.}\; (u,v) \in \overline{C} \tag{13}$$

where $u = (u_0,\dots,u_{N-1})$ is composed of discrete elements, with each element u_k belonging to a finite discrete set U_k, i.e., $u_k \in U_k$, $k=0,\dots,N-1$, and $v \in \Re^m$ wheere $\Re^m$ is the m-dimensional Euclidean space. The function G maps elements in $U \times \Re^m$ to real numbers with $U = U_0 \times \cdots \times U_{N-1}$, and $\overline{C}$ is a nonempty subset of $U \times \Re^m$.

When favorable structures are absent, problem (13) can be difficult to address. A naive approach is to enumerate all possible values of u, and then solve just as many optimization problems that involve only the continuous variable v. However, the number of such problems could increase exponentially as the dimension of u increases. On the contrary, the number of continuous optimization problems involved in our scheme grows only linearly with N, as we will see shortly.

For our proposed scheme to find a feasible solution in theory, we assume that there is a known $\bar{u} = (\bar{u}_0,\dots,\bar{u}_{N-1})$, referred to as the *base solution*, such that $(\bar{u},\bar{v}) \in \overline{C}$ for some $\bar{v} \in \Re^m$. In other words, if we define a set C as

$$C = \left\{ u \in U \mid (u,v) \in \overline{C} \text{ for some } v \in \Re^m \right\}, \tag{14}$$

then our scheme relies on the assumption that some $\bar{u} \in C$ is known. Based on this condition, the proposed method focuses on the discrete variables *one at a time*. In particular, it first computes the $\tilde{u}_0$ via solving

$$\begin{aligned} \tilde{u}_0 \in \arg\min_{u_0 \in U_0} \min_{v \in \Re^m} \; & G(u_0,\bar{u}_1,\dots,\bar{u}_{N-1},v) \\ \mathrm{s.t.}\; & (u_0,\bar{u}_1,\dots,\bar{u}_{N-1},v) \in \overline{C}. \end{aligned} \tag{15}$$

Having computed $\tilde{u}_0$, it proceeds by solving

$$\begin{aligned} \tilde{u}_1 \in \arg\min_{u_1 \in U_1} \min_{v \in \Re^m} \; & G(\tilde{u}_0,u_1,\bar{u}_2,\dots,\bar{u}_{N-1},v) \\ \mathrm{s.t.}\; & (\tilde{u}_0,u_1,\bar{u}_2,\dots,\bar{u}_{N-1},v) \in \overline{C}. \end{aligned} \tag{16}$$

At last, it solves

$$\begin{aligned} \tilde{u}_{N-1} \in \arg\min_{u_{N-1} \in U_{N-1}} \min_{v \in \Re^m} \; & G(\tilde{u}_0,\dots,\tilde{u}_{N-2},u_{N-1},v) \\ \mathrm{s.t.}\; & (\tilde{u}_0,\dots,\tilde{u}_{N-2},u_{N-1},v) \in \overline{C}. \end{aligned} \tag{17}$$

Denoting as $\tilde{u}$ the solution $(\tilde{u}_0,\dots,\tilde{u}_{N-1})$ computed above, referred to as the *rollout solution*, the approximate solution obtained via our scheme is $(\tilde{u},\tilde{v})$ where

$$\tilde{v} \in \arg\min_{(\tilde{u},v) \in \overline{C}} G(\tilde{u},v). \tag{18}$$

We have the following result for the proposed scheme.

Proposition 4.1: Let $\bar{u} \in C$ and consider $(\tilde{u},\tilde{v})$ obtained via (15)-(18). We have that $(\tilde{u},\tilde{v}) \in \overline{C}$ and

$$G(\tilde{u},\tilde{v}) \leq \min_{(\bar{u},v) \in \overline{C}} G(\bar{u},v). \tag{19}$$

Proof: See Appendix B of [17]. ■

Remark 4.1: Denote as n the maximum number of elements contained in U_k. The naive scheme involves solving as many as n^N continuous optimization problems, while our rollout scheme requires solving at most nN such problems. Supposing that polynomial-time algorithms are used for continuous problems, our scheme can be executed in polynomial time.

Remark 4.2: The rollout scheme (15)-(17) can be carried out even starting from $\bar{u} \notin C$. In this case, the resulting rollout solution $\tilde{u}$ may still be feasible.

B. Variant of the Rollout Scheme

The proposed scheme admits a few variants. Here we discuss one that is particularly relevant to our application. Additional variants are given in Appendices C and D of [17].

Suppose that ℓ different base solutions $\bar{u}^1,\dots,\bar{u}^\ell \in U$ are known. We can obtain their respective rollout solutions $\tilde{u}^1,\dots,\tilde{u}^\ell$ as well as the corresponding minimizing $\tilde{v}^1,\dots,\tilde{v}^\ell$. We then select $(\tilde{u}^{i^*},\tilde{v}^{i^*})$ where $i^* \in \arg\min_i \{G(\tilde{u}^i,\tilde{v}^i)\}_{i=1}^{\ell}$. Clearly, we have the following performance bound

$$G(\tilde{u}^{i^*},\tilde{v}^{i^*}) \leq \min_{i \in \{1,\dots,\ell\}} \min_{(\bar{u}^i,v) \in \overline{C}} G(\bar{u}^i,v). \tag{20}$$

However, this is at the expense of the increased computational demands, which are ℓ-fold of that of the original scheme.

C. Rollout-Based Charging Strategy

In what follows, we show that the charging problem formulated in Section III belongs to the class of generic problem (13). As a result, the rollout scheme and its variant developed thus far can be applied to provide charging strategies.

To this end, let us define as u_k the pair $(b_k, \tilde{b}_k)$, $k = 0, \ldots, N-1$, and $u = (u_0, \ldots, u_{N-1})$. Accordingly, $U_k = \{(0,0), (1,0), (0,1), (1,1)\}$. We lump all the continuous variables involved in the charging problem as v, namely,

$$v = (t_0, e_0, \Delta e_0, c_0, \ldots, t_{N-1}, e_{N-1}, \Delta e_{N-1}, c_{N-1}, e_N, c_N).$$

Then the function F defined in (12) can be written as a function of (u, v), which we denote as G. Moreover, let $\overline{C}$ denote the set of (u, v) that fulfills the conditions (1)-(10). Via the change of variables introduced here, the charging problem can be seen as an instance of the generic problem (13).

To obtain charging strategies via the rollout scheme, we use two different base solutions. The first solution $\bar{u}^1$ is referred to as the *greedy solution*. Intuitively, the greedy solution sets $\bar{u}_k^1 = (1,1)$ if the battery energy e_{k+1} upon arriving at r_{k+1} does not fulfill constraint (6) without charging at S_k. Moreover, once $\bar{u}_k^1 = (1,1)$, the battery is fully charged at S_k. Another base solution $\bar{u}^2$, referred to as the *relaxed solution*, is obtained via solving a relaxation of the original problem, where the binary constraints are replaced by closed intervals $[0,1]$. If the optimal value for binary variables is nonzero, the relaxed solution sets respective binary variables to 1. Apart from a base solution, the relaxation of the original problem also provides a lower bound of the optimal cost of the original problem. Together with the upper bounds (19) and (20), we obtain a certificate for the optimality gap of the rollout scheme.

Note that either one of the two base solutions may not be feasible as they involve approximations of the original problem. Due to the presence of the HoS constraint (8) and the delivery deadline (10), computing a feasible base solution can be as hard as solving the original problem. On the other hand, owing to reasons discussed in Remark 4.2, both $\bar{u}^1$ and $\bar{u}^2$ are used in our simulation studies, and together they suffice for the practical needs.

V. Simulation Studies

A. Setup

1) Transport Route: We consider the Swedish road network with 105 real road terminals, where the coordinates of the road terminals are obtained from the SAMGODS model [19]. The delivery missions for trucks are generated by randomly selecting the origin and destination pairs (i.e., OD pairs) from the set of road terminals. Since today only very few charging stations for electric trucks are in operation, some other real road terminals obtained from the SAMGODS model, except for those considered as origins and destinations, are used as potential charging and rest stations in the simulation, as shown by the grey nodes in Fig. 3. For each OD pair, the shortest path between the OD is pre-planned via *OpenStreetMap*[20], and the charging

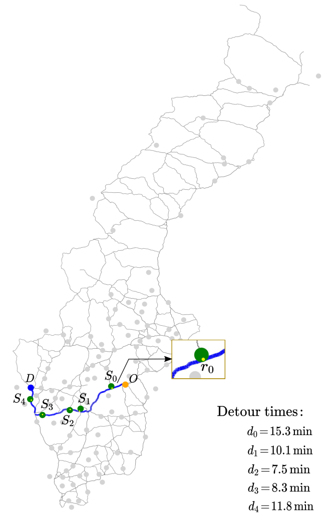

Fig. 3. The transport route model of one truck, where 5 charging and rest stations are available along the route, as shown by the green nodes. Ramps leading to the shortest detours are shown by the yellow nodes.

and rest stations along the route are identified with a given search range. Accordingly, the travel times $\{\tau_k\}$ and the detour times $\{d_k\}$ are accessible from *OpenStreetMap*. The transport route model of one truck is shown in Fig. 3.

2) Parameter Settings: The latest published data for electric trucks manufactured by Scania [21] is used in setting the parameters. We consider electric trucks at a load capacity of 40 tonnes with a usable battery of 468 kWh, with up to 350 kilometers driving range. The usable battery energy is $e_f - e_s$, which can vary from 0 to 468 kWh. The charging power P_k and $P_{\max}$ are considered as 300 and 375 kW, respectively. We assume trucks drive at a constant speed of 82 km/h, resulting in approximately 1.83 kWh/min of battery consumption. In addition, p_k is 6 min, ξ_k is 0.36€/kWh, and ϵ is set as 0.4€/min, based on truck drivers' salaries per hour in Sweden in 2023. The EU's HoS regulations are applied.

B. Solution Evaluation

To evaluate the rollout-based charging strategy, we conduct simulation studies for trucks in 6 scenarios where N is varied from 5 to 10, and in each scenario, the proportion of the initial battery is changed from 20% to 100%, incremented with 5%. The optimal solution is computed by enumerating all the possible combinations of the binary variables, and the rollout solution is obtained by taking $\bar{u}^1$ and $\bar{u}^2$ as the base solutions. Both solutions use *Gurobi* as the linear program solver. For brevity, we refer to the optimal and rollout-based solutions as OS and RS, respectively, and refer to the lower and upper bounds of the optimal cost of the rollout solution as LB and UB. In Fig. 4, we provide the costs of the OSs and RSs for the selected scenarios $N = 6, 7$, as well as the LBs and UBs obtained using our base solutions. The results show that RSs are near-optimal to the OSs, and could be achieved based on different base solutions. Moreover, the greedy and relaxed base solutions provide good upper bounds for the rollout solution. Further details of parameters, additional simulation results, sensitivity analysis, and the link

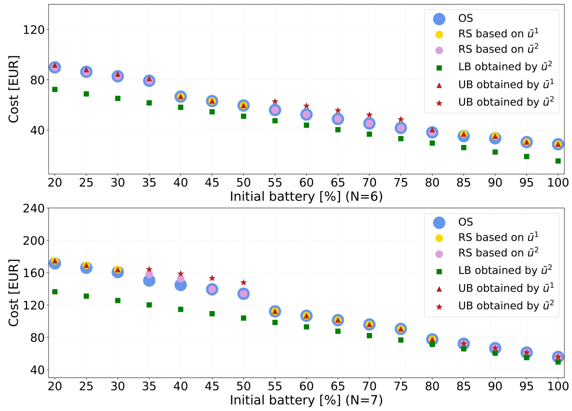

Fig. 4. The costs of the optimal and rollout-based solutions.

TABLE I

COMPARISON BETWEEN THE OS AND RS

N	5	6	7	8	9	10
AOG-RS [%]	0	0.55	0.72	0.49	0.03	0.42
AOG-UB [%]	1.04	5.46	2.23	0.49	4.28	1.72
ACT of RS [s]	0.34	0.42	0.57	0.65	0.84	1.43
ACT of OS [min]	0.32	1.34	5.45	24.02	98.50	413.68

to codes are given in [17].

The optimality gap between RSs and OSs, UBs and OSs, and the computational efficiency of the RS and OS methods are shown in Table I. For each N with a given initial battery, the optimality gap between the RS and OS is computed by $100\times(F(\text{RS})-F(\text{OS}))/F(\text{OS})$, where F is the cost function defined by (12). Similarly, the optimality gap between the UB and OS is computed by $100\times(\text{UB}-F(\text{OS}))/F(\text{OS})$. We show in Table I the average optimality gap (AOG) of 17 situations for each N and the average computational times (ACT) to obtain the RS and OS. It can be seen that the computational demands for obtaining OSs increase exponentially with the increase in N. By employing the proposed RS scheme, the computational time decreases significantly, taking less than 2 seconds, while having an average optimality gap within 1%, which illustrates the desirable properties of our method.

VI. CONCLUSION

This letter investigated the optimal charging strategy for electric trucks, which allows freight drivers to determine where and how long to recharge trucks to complete the delivery task before deadlines while respecting the HoS regulations. We assumed that every truck has a pre-planned route with a given collection of charging and rest stations. The optimal charging problem of each truck was modeled as a mixed integer program integrated with bilinear constraints, which is computationally intractable to be solved exactly. As an approximate scheme, a rollout-based charging strategy was proposed, which provides near-optimal solutions to the problem with solid performance guarantees while reducing the computational load drastically. Compared to the existing literature, our modeling method allows for handling the HoS regulations subject to delivery deadlines. Moreover, the rollout-based solution of high efficiency is promising to be applied in real-time strategy planning to cope with travel time uncertainties. Future work could be developing optimal charging strategies for electric trucks with limited charging resources at stations.

REFERENCES

[1] Roland Irle, "Global EV sales for 2022," https://www.ev-volumes.com/, Accessed: 2023.

[2] P. Siskos and Y. Moysoglou, "Assessing the impacts of setting CO2 emission targets on truck manufacturers: A model implementation and application for the EU," *Transportation Research Part A: Policy and Practice*, vol. 125, pp. 123–138, 2019.

[3] "Global EV outlook 2022–Trends in electric heavy-duty vehicles," https://www.iea.org/reports/global-ev-outlook-2022/trends-in-electric-heavy-duty-vehicles#abstract, Accessed: 2023.

[4] N. Wassiliadis, J. Schneider, A. Frank, L. Wildfeuer, X. Lin, A. Jossen, and M. Lienkamp, "Review of fast charging strategies for lithium-ion battery systems and their applicability for battery electric vehicles," *Journal of Energy Storage*, vol. 44, p. 103306, 2021.

[5] M. Poliak, M. Mrnikova, P. Simurkova, P. Medvid, A. Poliakova, and S. Hernandez, "Social law in road transport like tool safety road transport," in *2018 XI International Science-Technical Conference Automotive Safety*. IEEE, 2018, pp. 1–7.

[6] S. Storandt, "Quick and energy-efficient routes: Computing constrained shortest paths for electric vehicles," in *Proceedings of the 5th ACM SIGSPATIAL International Workshop on Computational Transportation Science*, 2012, pp. 20–25.

[7] M. Schneider, A. Stenger, and D. Goeke, "The electric vehicle-routing problem with time windows and recharging stations," *Transportation Science*, vol. 48, no. 4, pp. 500–520, 2014.

[8] G. Huber, K. Bogenberger, and H. Van Lint, "Optimization of charging strategies for battery electric vehicles under uncertainty," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 2, pp. 760–776, 2020.

[9] M. Cussigh and T. Hamacher, "Optimal charging and driving strategies for battery electric vehicles on long distance trips: A dynamic programming approach," in *2019 IEEE Intelligent Vehicles Symposium (IV)*. IEEE, 2019, pp. 2093–2098.

[10] M. Zähringer, S. Wolff, J. Schneider, G. Balke, and M. Lienkamp, "Time vs. capacity–The potential of optimal charging stop strategies for battery electric trucks," *Energies*, vol. 15, no. 19, p. 7137, 2022.

[11] G. Tesauro and G. R. Galperin, "On-line policy improvement using Monte-Carlo search," in *Proceedings of the 9th International Conference on Neural Information Processing Systems*, 1996, pp. 1068–1074.

[12] D. P. Bertsekas, J. N. Tsitsiklis, and C. Wu, "Rollout algorithms for combinatorial optimization," *Journal of Heuristics*, vol. 3, no. 3, pp. 245–262, 1997.

[13] D. P. Bertsekas, "Rollout algorithms for constrained dynamic programming," *LIDS Report*, vol. 2646, 2005.

[14] ——, *Rollout, Policy Iteration, and Distributed Reinforcement Learning*. Athena Scientific Belmont, MA, 2020.

[15] D. Speth, P. Plötz, S. Funke, and E. Vallarella, "Public fast charging infrastructure for battery electric trucks—A model-based network for Germany," *Environmental Research: Infrastructure and Sustainability*, vol. 2, no. 2, p. 025004, 2022.

[16] K. Liu, H. Gao, Z. Liang, M. Zhao, and C. Li, "Optimal charging strategy for large-scale electric buses considering resource constraints," *Transportation Research Part D: Transport and Environment*, vol. 99, p. 103009, 2021.

[17] T. Bai, Y. Li, K. H. Johansson, and J. Mårtensson, "Rollout-based charging strategy for electric trucks with hours-of-service regulations," *arXiv preprint arXiv:2303.08895*, 2023.

[18] D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Optimization*. Athena Scientific Belmont, MA, 1997, vol. 6.

[19] M. Bergquist, V. Bernhardsson, and E. Rosklint, "Representation of the Swedish transport and logistics system in Samgods v. 1.1." 2016.

[20] OpenStreetMap, https://www.openstreetmap.org, Accessed: 2023.

[21] Electric Truck (Scania), https://www.scania.com/group/en/home/products-and-services/trucks/battery-electric-truck.html, Accessed: 2023.