Intro to Artificial Neural Networks

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talks

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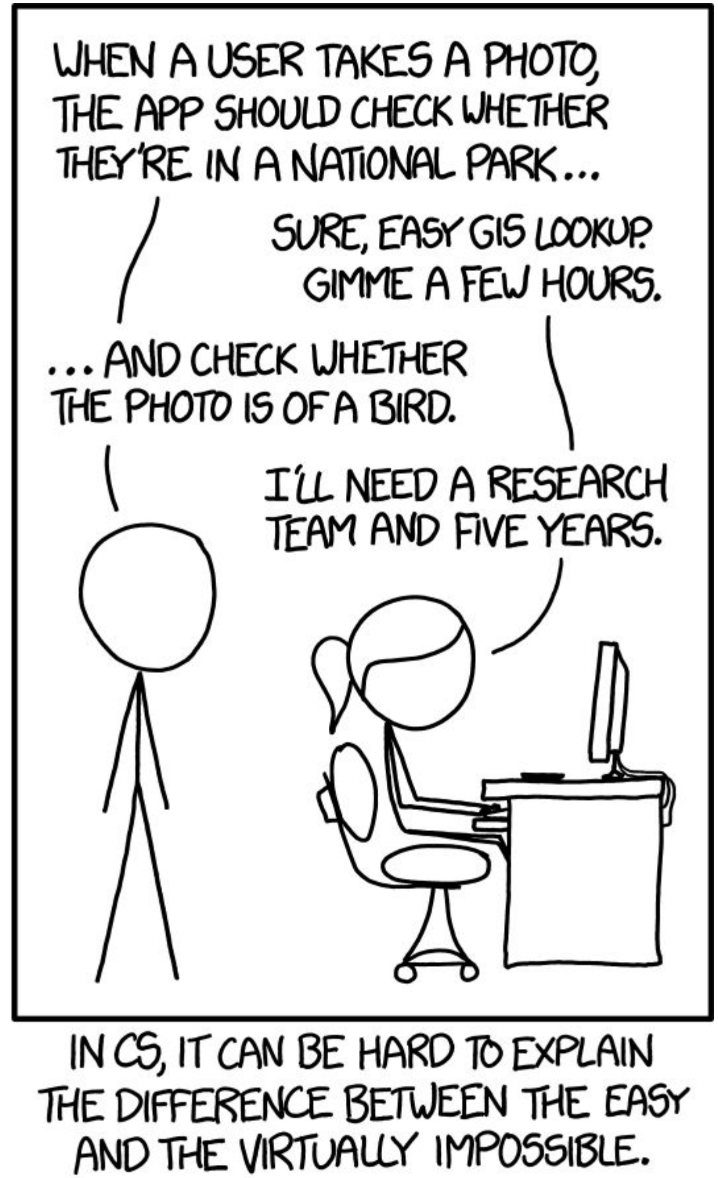
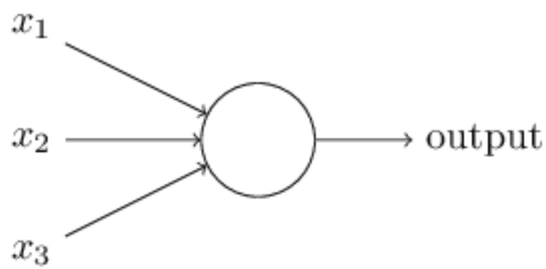


https://xkcd.com/1425/

Perceptrons

- A perceptron is a function that takes several binary inputs, *x1,x2,…*, and produces a single binary output:



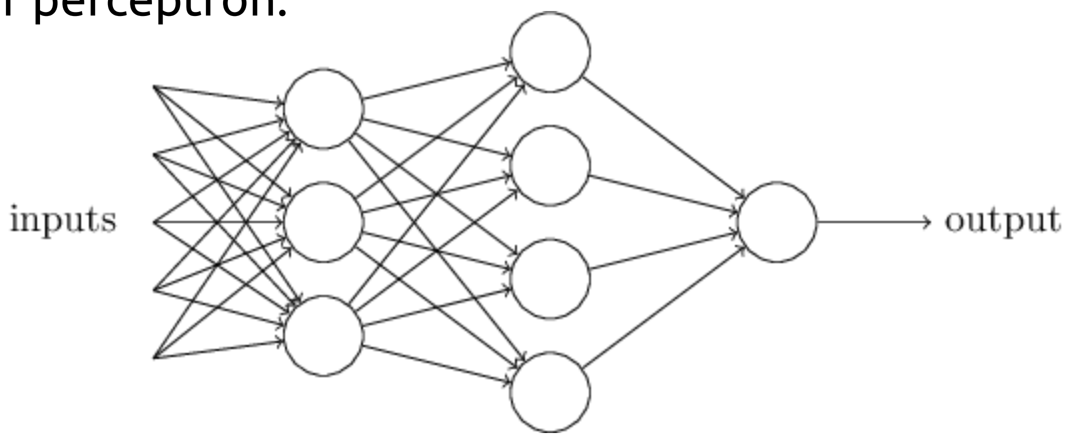
- Weights, *w1,w2,…*, are real numbers expressing the importance of the respective inputs to the output.

- The neuron's output, 0 or 1, is determined by whether the weighted sum $\sum_j w_j x_j$ is less than or greater than some *threshold* value:

$$\begin{cases} 0 & \text{if } \sum_j w_j x_j \leq \text{ threshold} \\ 1 & \text{if } \sum_j w_j x_j > \text{ threshold} \end{cases}$$

Perceptrons

- Multi-layer perceptron:



- We can write $\sum_j w_j x_j$ as a dot product, $w \cdot x \equiv \sum_j w_j x_j$, and replace the threshold by a *bias*, $b \equiv -\text{threshold}$. The perceptron rule can be rewritten as:

$$\begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

Sigmoid Neurons

- The sigmoid neuron has inputs, *x1,x2,…*. But instead of being just 0 or 1, these inputs can also take on any values between 0 and 1.
- Just like a perceptron, the sigmoid neuron has weights for each input, *w1,w2,…*, and an overall bias, *b*.
- But the output is not 0 or 1. Instead, it's $\sigma(w \cdot x + b)$, where σ is called the *sigmoid function*, defined by:

$$\sigma(z) \equiv \frac{1}{1+e^{-z}}$$ (nonlinearity)

- Putting it all together, the output of a sigmoid neuron with inputs *x1,x2,…*, weights *w1,w2,…*, and bias *b* is:

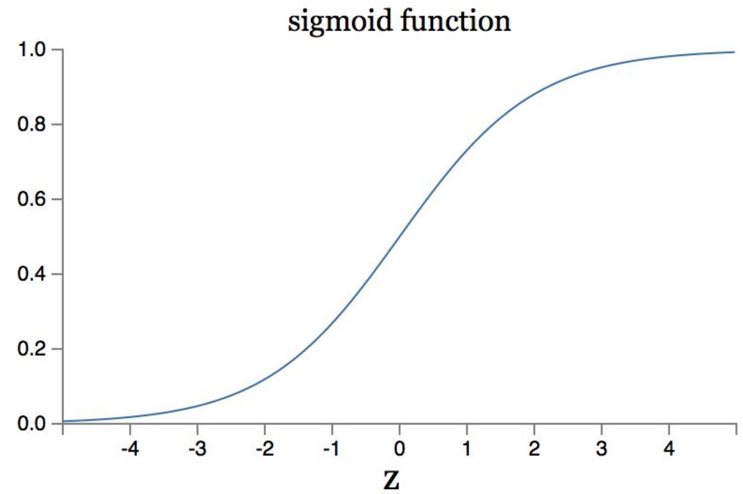
$$\frac{1}{1+\exp(-\sum_j w_j x_j - b)}$$

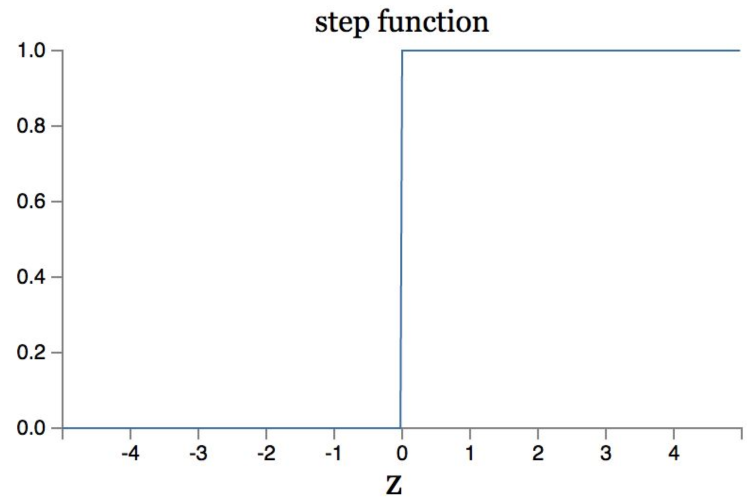
Sigmoid Neurons

- How does σ look?
- This shape is a smoothed out version of a *step function*.
- In fact, If σ had been a step function, then the **sigmoid neuron** would be a **perceptron**!

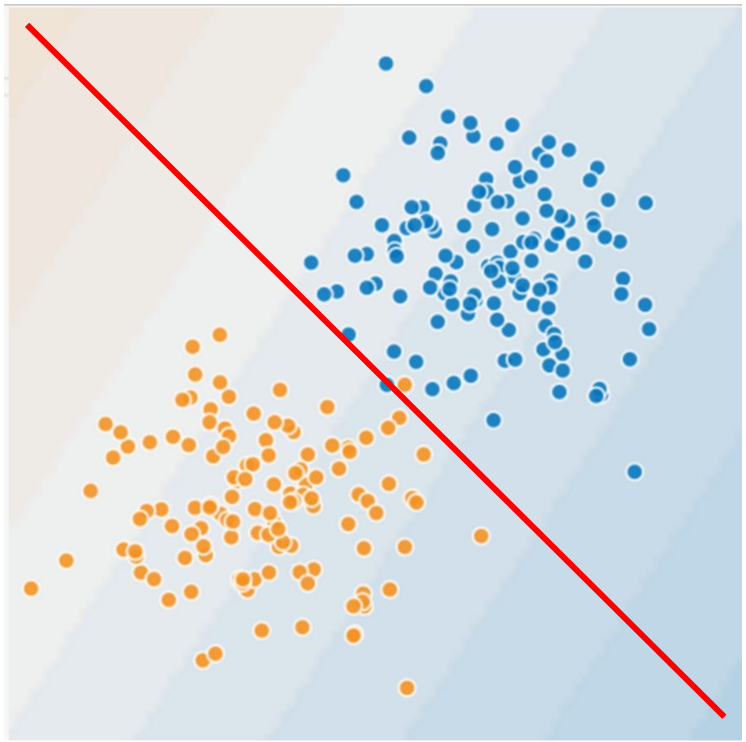
$$output = \sigma(w \cdot x + b)$$

$$\sigma(z) \equiv \frac{1}{1 + e^{-z}}$$

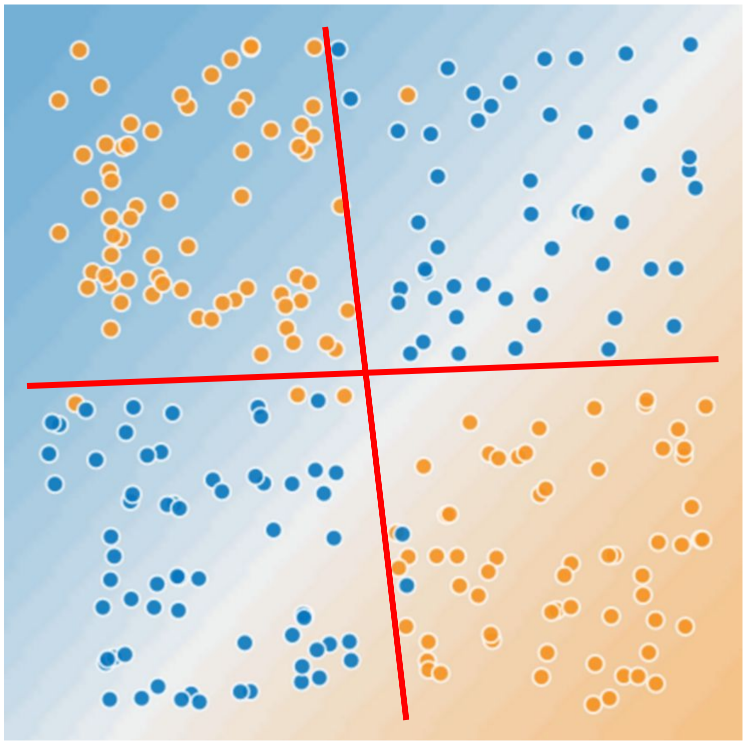




Linear vs Nonlinear Classification



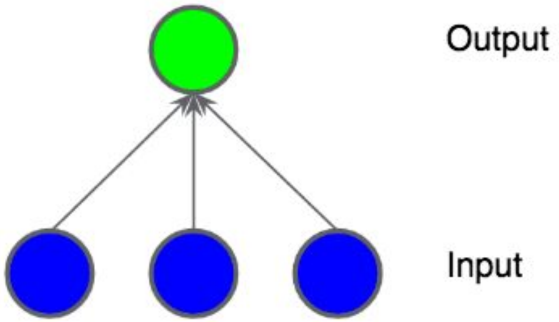
Linear classification problem



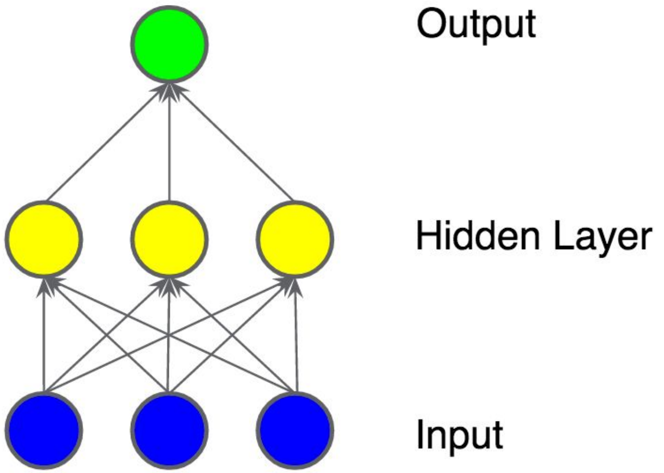
Nonlinear classification problem

Neural Networks Anatomy

- We can represent a linear model as a graph.

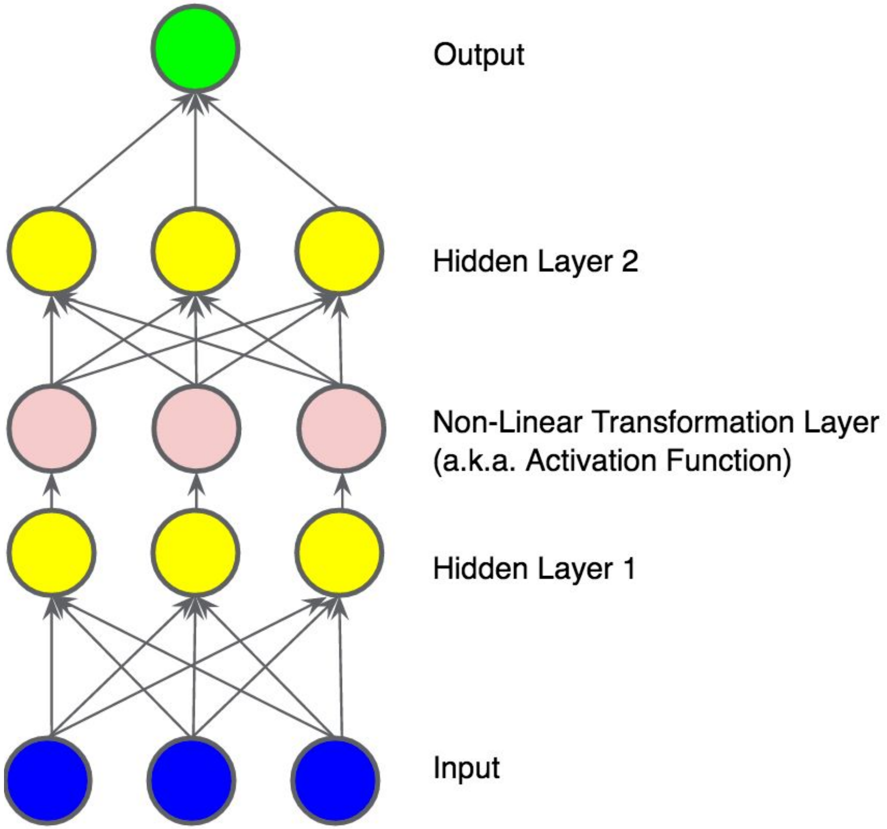


- We can add a "hidden layer" of intermediary values. This is still a linear model.



Neural Networks Anatomy

- To model a nonlinear problem, we can directly introduce a nonlinearity.
- We can pipe each hidden layer node through a nonlinear function.
- The *sigmoid function* is a common activation function.
- Stacking nonlinearities on nonlinearities lets us model very complicated relationships between the inputs and the predicted outputs.



Neural Networks Anatomy

- Summary: a Neural Network is a classifier model consisting of:
    - A set of nodes, analogous to neurons, organized in layers.
    - A set of weights representing the connections between each neural network layer and the layer beneath it.
    - A set of biases, one for each node.
    - An activation function that transforms the output of each node in a layer. Different layers may have different activation functions

Training and Loss

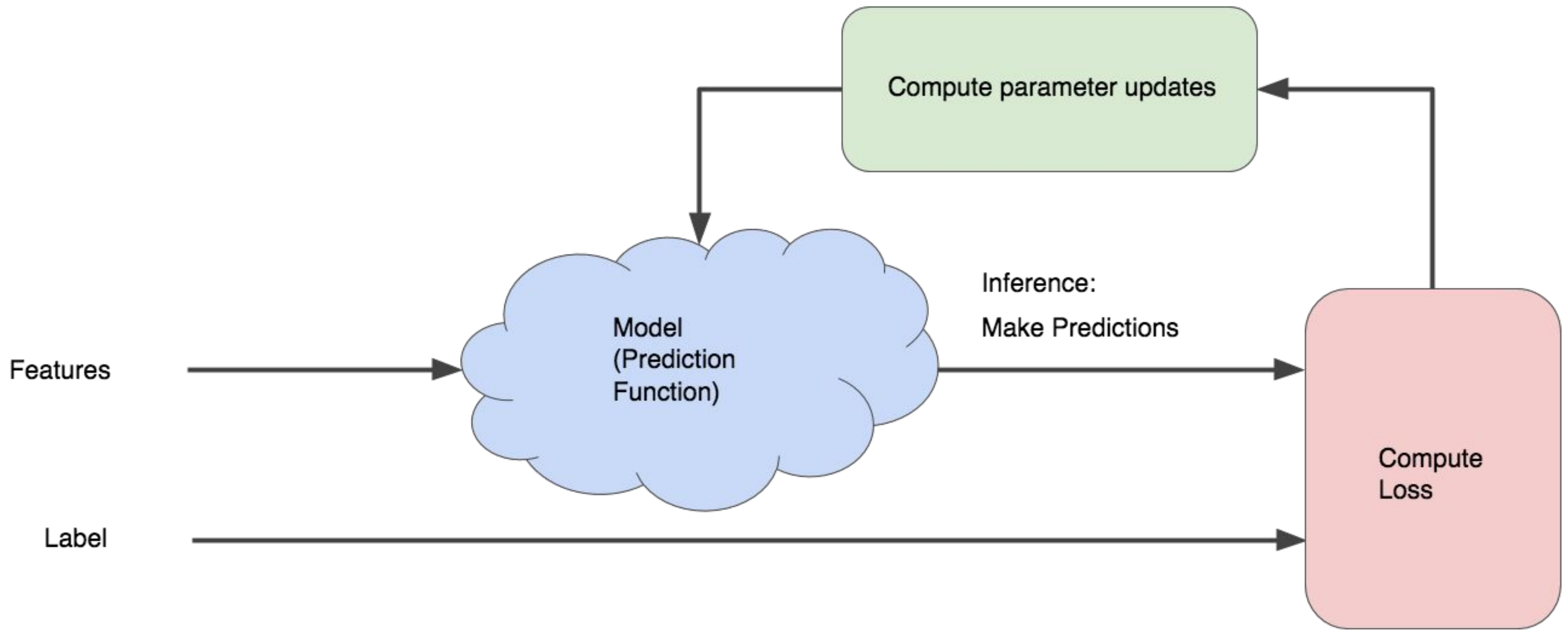
- **Training** a model simply means learning (determining) good values for all the weights and the bias from labeled examples.
- In **supervised learning**, a machine learning algorithm builds a model by examining many examples and attempting to find a model that minimizes loss.
- **Loss** is the penalty for a bad prediction. It is a number indicating how bad the model's prediction was on a single example (0 for a perfect prediction, greater than 0 otherwise).

$$Loss \equiv \frac{1}{2n} \sum_x \|y(x) - a(x))\|^2$$

- The goal of training a model is to find a set of weights and biases that have *low* loss, on average, across all examples.

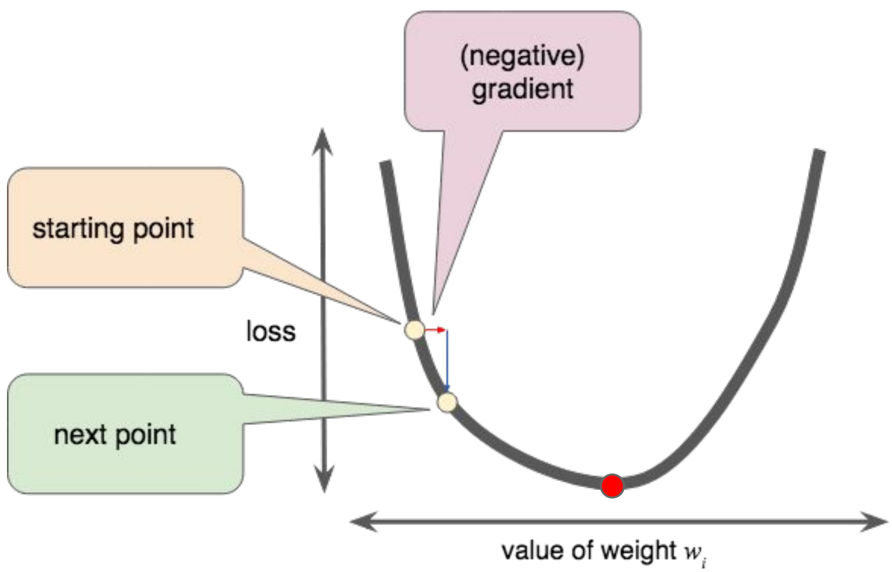
Reducing loss

- Machine learning algorithms use an iterative trial-and-error process to train a model:



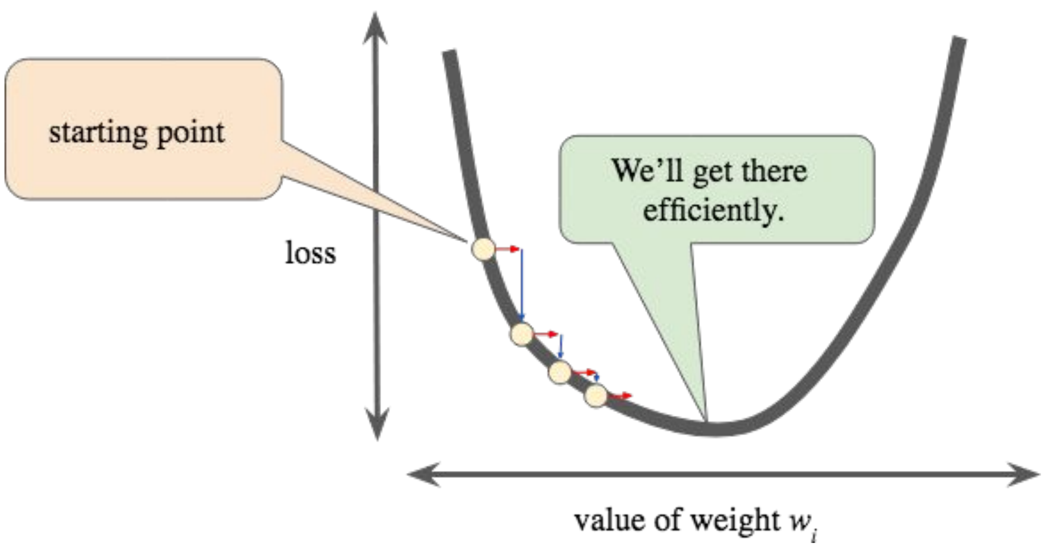
Gradient Descent

- **Gradient descent** is an optimization algorithm used to find a (local) minimum of the loss function, which is parametrized by the weights *w1,w2,…*.
- Algorithm:
  a. Pick a starting point (initialization of weights).
  b. Calculate the gradient (**how?**) of the loss with respect to the weights at the starting point.
  c. Take a step (**how big?**) in the direction of the negative gradient.
  d. Select next point (update weights).
  e. Repeat the process heading towards the minimum of the loss function.



Learning Rate

- Gradient descent algorithms multiply the gradient by a scalar known as the **learning rate** to determine the next point.
- Learning rate too small -> learning will take too long.
- Learning rate too large -> might overshoot the minimum.
- **Hyperparameters** are the knobs that programmers tweak in machine learning algorithms. Most machine learning programmers spend a fair amount of time tuning the learning rate.



Stochastic Gradient Descent

- In gradient descent, a **batch** is the total number of examples you use to calculate the gradient in a single iteration.
- So far, we've assumed that the batch has been the entire data set -> a very large batch may cause even a single iteration to take a very long time to compute.
- By choosing examples at random from our data set, we could estimate (albeit, noisily) a big average from a much smaller one -> in **mini-batch Stochastic Gradient Descent (mini-batch SGD)**, a batch is typically between 10 and 1000 examples, chosen at random.
- **Stochastic Gradient Descent (SGD)** takes this idea to the extreme -> it uses only a single example (a batch size of 1) per iteration. Given enough iterations, SGD works but is very noisy.

Backpropagation

- **Backpropagation** is the algorithm that allows us to compute the gradient of the loss function with respect to the learnable parameters (weights and biases) of the network: $\partial Loss_x / \partial w$, $\partial Loss_x / \partial b$

- Algorithm (high level):
  a. **Input**: set the corresponding activation a^0 for the input layer
  b. **Forward pass**: for each layer l = {1, 2, 3, …, L} compute $z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$
  c. **Output error**: compute the vector $\delta_j^L = \frac{\partial Loss}{\partial a_j^L} \sigma'(z_j^L)$
  d. **Backpropagate the error**: for each layer l = {1, 2, 3, …, L} compute

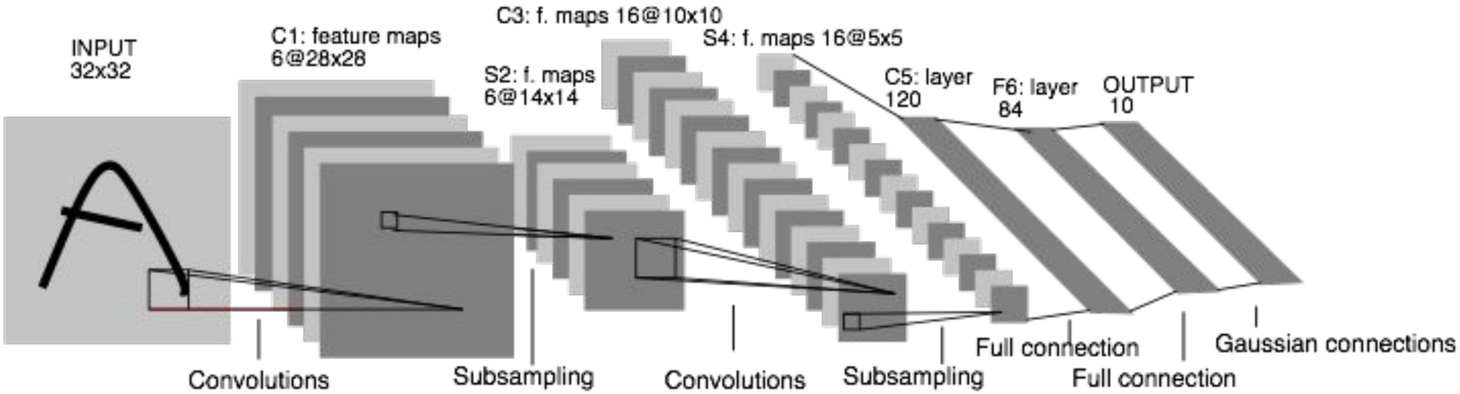
$$\delta_j^l = \sum_k w_{kj}^{l+1} \delta_k^{l+1} \sigma'(z_j^l)$$

  d. **Output**: the gradient of the loss function is given by

$$\frac{\partial Loss}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l \quad \text{and} \quad \frac{\partial Loss}{\partial b_j^l} = \delta_j^l$$

Convolutional Neural Networks

- **Convolutional Neural Networks** are a type of feedforward networks that make use of the *convolution* operation.
- ConvNets make the explicit assumption that the inputs are images.
  - Instead of dense connections use convolutions (with shared weights).
- Convolutional filters are learnt -> no need for feature engineering.

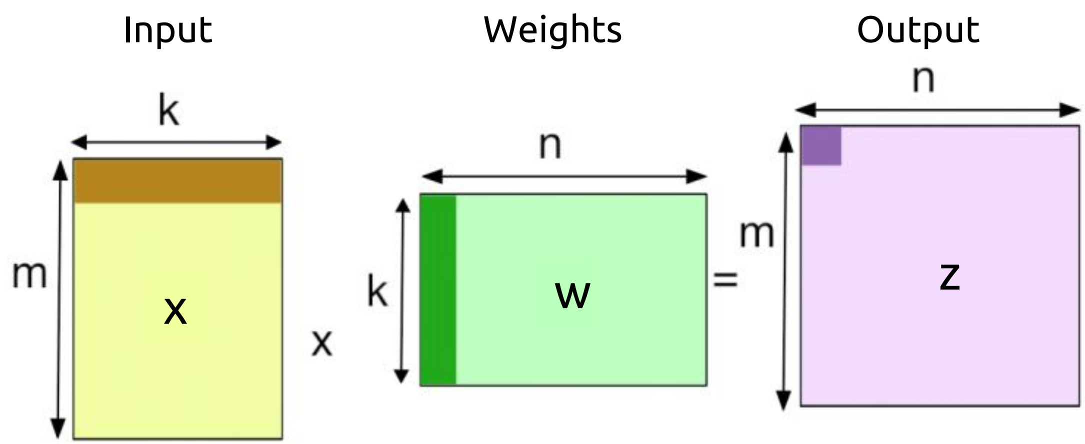


LeNet (1989): a layered model composed of convolution and subsampling operations followed by a classifier for handwritten digits.

Why GEMM is at the heart of Artificial Neural Networks?

- **GEMM** stands for GEneral Matrix to Matrix Multiplication.
- For a fully-connected layer:

$$z = \sum_j w_j x_j$$



- Similar story for convolutional layers (though less obvious).
- GEMM is a highly parallel operation, so it scales naturally to many cores.
    - This kind of workload is well-suited for a **GPU** architecture.

References

- Neural Networks and Deep Learning
- Machine Learning Crash Course
- DIY Deep Learning for Vision: a Hands-On Tutorial with Caffe
- Convolutional Neural Networks (CSE 455)
- Why GEMM is at the heart of deep learning
- Image Classification and Filter Visualization

Thanks!

Questions?

Hands on!

https://goo.gl/Ud2Pws