① $\displaystyle\int_0^3 \underset{u}{x}\cdot\underset{v'}{(x-3)^5} = \left[\underset{u}{x}\cdot\underset{v}{\tfrac{1}{6}(x-3)^6}\right]_0^3 - \int_0^3 \tfrac{1}{6}(x-3)^6\,dx$

$\displaystyle = 0 - \left[\tfrac{1}{42}(x-3)^7\right]_0^3 = -\frac{3^7}{42} = -\frac{3^6}{14} = -\frac{729}{14}$

② $\displaystyle\int_0^\pi \underset{u}{x^2}\cdot\underset{v'}{\cos x}\,dx = \left[x^2\sin x\right]_0^\pi - \int_0^\pi 2x\cdot\sin x\,dx = 0 - 2\int_0^\pi x\cdot\sin x\,dx$

$\displaystyle\int_0^\pi \underset{u}{x}\cdot\underset{v'}{\sin x}\,dx = \left[-x\cdot\cos x\right]_0^\pi + \int_0^\pi \cos x\,dx = -\pi\cdot(-1) + \left[\sin x\right]_0^\pi = \pi$

$\displaystyle\Rightarrow \int_0^\pi x^2\cdot\cos x\,dx = -2\pi$

③ $\displaystyle\int_0^3 \frac{6}{2x+5}\,dx = \left[\frac{6}{2}\ln|2x+5|\right]_0^3 = 3(\ln 11 - \ln 5)$

④ $\displaystyle\int_0^1 \frac{2x}{1+x^2}\,dx = \left[\ln|1+x^2|\right]_0^1 = \ln 2$

⑤ $$\int_0^4 \frac{4}{1+2\sqrt{x}}\,dx \qquad u = 1+2\sqrt{x} \qquad 2\sqrt{x} = u-1$$

$$\frac{du}{dx} = \frac{1}{\sqrt{x}} \qquad \sqrt{x} = \frac{1}{2}(u-1)$$

$$dx = du \cdot \sqrt{x}$$

$$= \int_1^5 \frac{4\cdot\sqrt{x}}{u}\,du = 2\int_1^5 \frac{u-1}{u}\,du = 2\cdot\int_1^5 1-\frac{1}{u}\,du = 2\cdot\left[u - \ln|u|\right]_1^5$$

$$= 2\cdot\left(5-\ln 5 - 1 + 0\right) = 8 - 2\cdot\ln 5$$

⑥

$$\int_1^2 \frac{2x+14}{x^2-x-6}\,dx = (*) \qquad x^2-x-6=0$$

$$x_{1/2} = \frac{1\pm\sqrt{1+24}}{2} = \frac{1\pm 5}{2}$$

$$x_1 = 3, \quad x_2 = -2$$

$$\frac{2x+14}{x^2-x-6} = \frac{2x+14}{(x-3)(x+2)} = \frac{A}{(x-3)} + \frac{B}{(x+2)}$$

$$A = \frac{20}{5} = 4, \quad B = \frac{10}{-5} = -2$$

$$(*) = \int_1^2 \frac{4}{(x-3)} - \frac{2}{(x+2)}\,dx = \left[4\ln|x-3| - 2\ln|x+2|\right]_1^2$$

$$= 4\ln 1 - 2\ln 4 - (4\ln 2 - 2\ln 3)$$

$$= -2\ln 4 - 4\ln 2 + 2\ln 3$$

$$= -8\ln 2 + 2\ln 3$$