A3L

A3: (Polardarstellung komplexer Zahlen)
Gib die folgenden komplexen Zahlen in Polardarstellung an und berechne jeweils Real und Imaginärteil.
a. $1+i$ b. $8\cos(\frac{\pi}{6}) + 8i\sin(\frac{\pi}{6})$ c. $-\sqrt{3}+3i$
d. $(1+2i)\cdot(3-i)$ e. $i\cdot\overline{3-4i}$ f. $(1+i)^{20}$

Tabelle:

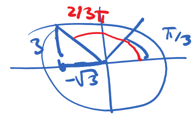
	0	30	45	60	90
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

a. $z = 1 + i \Rightarrow Re(z) = 1, \; Im(z) = 1$

poler: $|z| = \sqrt{2}$ $\quad arg(z) = \frac{\pi}{4} \Rightarrow z = \sqrt{2}\,(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

b. $z = 8\cdot\cos\frac{\pi}{6} + 8i\sin\frac{\pi}{6}$ $\qquad$ polar: $z = 8\cdot(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

$Re(z) = 8\cdot\frac{1}{2}\sqrt{3} = 4\cdot\sqrt{3}$, $\quad Im(z) = \frac{8}{2} = 4$



c. $z = -\sqrt{3} + 3i \Rightarrow Re(z) = -\sqrt{3}, \; Im(z) = 3$

polar: $|z| = \sqrt{3+9} = \sqrt{12} = 2\cdot\sqrt{3}$ $\qquad \cos\alpha = \frac{\sqrt{3}}{2\cdot\sqrt{3}} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

$z = 2\cdot\sqrt{3}\,(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi)$

d. $z = (1+2i)(3-i) = 3-i+6i+2 = 5+5i \Rightarrow Re(z) = Im(z) = 5$

polar: $|z| = \sqrt{50}$, $\; z = \sqrt{50}\cdot(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

e. $z = i\cdot\overline{3-4i} = i(3+4i) = 3i-4 \Rightarrow Re(z) = -4, \; Im(z) = 3$

polar: $|z| = \sqrt{16+9} = 5$ $\qquad \cos\alpha = \frac{4}{5} \Rightarrow \alpha = \cos^{-1}(\frac{4}{5}) \approx 0.6435$

$z = 5\cdot(\cos -\alpha + i\sin -\alpha)$

f. $z = (1+i)^{20} \Rightarrow |z| = (\sqrt{2})^{20} = 2^{10} = 1024$, $\quad \alpha = 20\cdot\frac{\pi}{4} = 5\pi \Rightarrow arg(z) = \pi$

polar: $z = 1024(\cos\pi + i\sin\pi)$, $\; Re(z) = -1024$, $\; Im(z) = 0$