

A3

A3 Polynomdivision

Faktorisieren Sie folgende Polynome in Linearfaktoren:

a) $p(x) = x^3 + 3x^2 - 4x - 12, x \in \mathbb{R}$

b) $p(x) = x^3 + x^2 - 2x - 2, x \in \mathbb{R}$

c) $p(x) = x^3 + x^2 - 3x + 1, x \in \mathbb{R}$

a) $x_1 = 2 : 8 + 3 \cdot 4 - 8 - 12 = 0 \checkmark$

$$(x^3 + 3x^2 - 4x - 12) : (x - 2) = x^2 + 5x + 6$$

$$\begin{array}{r} x^3 - 2x^2 \\ \hline 5x^2 - 4x - 12 \\ 5x^2 - 10x \\ \hline 6x - 12 \end{array}$$

$$x^2 + 5x + 6 = 0$$

$$x_{2,3} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm 1}{2}$$

$$x_2 = -3, \quad x_3 = -2$$

$$p(x) = (x - 2)(x + 2)(x + 3)$$

b) $p(x) = x^3 + x^2 - 2x - 2$

$x_1 = -1 : -1 + 1 + 2 - 2 = 0 \checkmark$

$$(x^3 + x^2 - 2x - 2) : (x + 1) = x^2 - 2$$

$$\begin{array}{r} x^3 + x^2 \\ \hline -2x - 2 \end{array}$$

$$p(x) = (x + 1)(x - \sqrt{2})(x + \sqrt{2})$$

c) $p(x) = x^3 + x^2 - 3x + 1$

$x_1 = 1 : 1 + 1 - 3 + 1 = 0 \checkmark$

$$(x^3 + x^2 - 3x + 1) : (x - 1) = x^2 + 2x - 1$$

$$\begin{array}{r} x^3 - x^2 \\ \hline 2x^2 - 3x + 1 \end{array}$$

$$\begin{array}{r}
 \\
 \hline
 2x^2 - 3x + 1 \\
 2x^2 - 2x \\
 \hline
 -x + 1
 \end{array}$$

$$x^2 + 2x - 1 = 0$$

$$x_{2,3} = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

$$x_2 = -1 - \sqrt{2}, \quad x_3 = -1 + \sqrt{2}$$

$$p(x) = (x-1) \cdot (x+1+\sqrt{2}) \cdot (x+1-\sqrt{2})$$