(A1)

a) $(5x^3 + 21x^2 - 56x - 12) : (x+6) = 5x^2 - 9x - 2$

$$\underline{5x^3 + 30x^2}$$
$$ -9x^2 - 56x - 12$$
$$ \underline{-9x^2 - 54x}$$
$$ -2x - 12$$

b) $(2x^3 + 2x^2 - 21x + 12) : (x+4) = 2x^2 - 6x + 3$

$$\underline{2x^3 + 8x^2}$$
$$ -6x^2 - 21x + 12$$
$$ \underline{-6x^2 - 24x}$$
$$ 3x + 12$$

c) $(2x^3 - 7x^2 - x + 2) : (2x - 1) = x^2 - 3x - 2$

$$\underline{2x^3 - x^2}$$
$$ -6x^2 - x + 2$$
$$ \underline{-6x^2 + 3x}$$
$$ -4x + 2$$

(A2)

a) $3x^3 - 15x^2 - 36x + 108 \quad | : 3$

$$x^3 - 5x^2 - 12x + 36$$

$x = 2 \quad 8 - 20 - 24 + 36 = 0$

$(x^3 - 5x^2 - 12x + 36) : (x-2) = x^2 - 3x - 18$

$$\underline{x^3 - 2x^2}$$
$$ -3x^2 - 12x$$
$$ \underline{-3x^2 + 6x}$$
$$ -18x + 36$$

(Vieta)

$\overset{\text{II}}{(x+3)(x-6)}$

$(3x^3 - 15x^2 - 36x + 108) = 3 \cdot (x-2)(x+3)(x-6)$

b) $(2x^4 - 9x^3 - 10x^2 + 27x - 10) : (x-1) = 2x^3 - 7x^2 - 17x + 10 \qquad \frac{d}{a} = 5$

$$5(50 - 35 - 17) + 10 = 0$$

$$\underline{2x^4 - 2x^3}$$
$$ -7x^3 - 10x^2$$
$$ \underline{-7x^3 + 7x^2}$$
$$ -17x^2 + 27x$$
$$ \underline{-17x^2 + 17x}$$
$$ 10x - 10$$

$(2x^3 - 7x^2 - 17x + 10) : (x-5) = 2x^2 + 3x - 2$

$$\underline{2x^3 - 10x^2}$$
$$ 3x^2 - 17x$$
$$ \underline{3x^2 - 15x}$$
$$ -2x + 10$$

$x_{1/2} = \dfrac{-3 \pm \sqrt{9 + 16}}{4} = \dfrac{-3 \pm 5}{4}$

$x_1 = \frac{1}{2}$, $x_2 = -2$

$(2x^4 - 9x^3 - 10x^2 + 27x - 10) = 2(x-1)(x-5)(x - \frac{1}{2})(x+2)$

(A2)

c) $x^3 + 19x^2 + 55x - 363$ $363 = 3 \cdot 11 \cdot 11$

$-11 \cdot (121 - 19 \cdot 11 + 55) - 363 = 0$
  $\underbrace{176 - 209}_{} = -33$

$(x^3 + 19x^2 + 55x - 363) : (x+11) = x^2 + 8x - 33$
$\underline{x^3 + 11x^2}$ $\|$
     $8x^2 + 55x$ Viela: $(x-3)(x+11)$
     $\underline{8x^2 + 88x}$
          $-33x - 363$

$(x^3 + 19x^2 + 55x - 363) = (x+11)^2 \cdot (x-3)$

(A3) $f(x) = 5x^6 - 42x^5 + \dfrac{165}{2}x^4 + 70x^3 - 180x^2$

$f'(x) = 30x^5 - 210x^4 + 330x^3 + 210x^2 - 360x$

$= 30x(x^4 - 7x^3 + 11x^2 + 7x - 12)$ $x_1 = 0.$ $x_2 = 1$ (erraten)

$(x^4 - 7x^3 + 11x^2 + 7x - 12) : (x-1) = x^3 - 6x^2 + 5x + 12$
$\underline{x^4 - x^3}$ $x_3 = -1$ (erraten)
    $-6x^3 + 11x^2$
    $\underline{-6x^3 + 6x^2}$
         $5x^2 + 7x$
         $\underline{5x^2 - 5x}$
              $12x - 12$
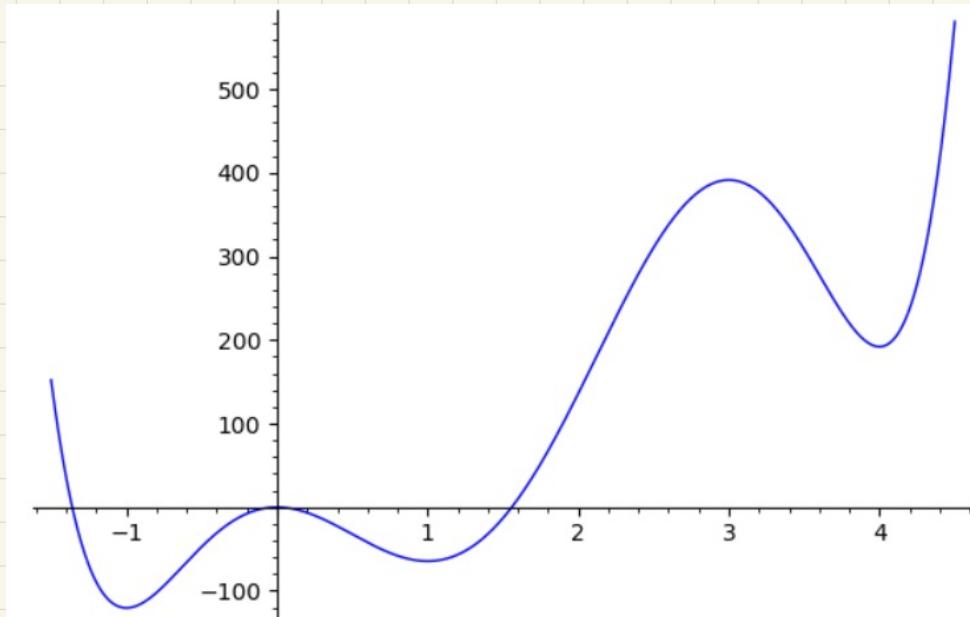
$(x^3 - 6x^2 + 5x + 12) : (x+1) = x^2 - 7x + 12$
$\underline{x^3 + x^2}$ $\|$
    $-7x^2 + 5x$ (Viela) $(x-3)(x-4)$
    $\underline{-7x^2 - 7x}$
         $12x + 12$

Die Extremstellen sind bei $x_1 = 0, \; x_2 = 1, \; x_3 = -1, \; x_4 = 3, \; x_5 = 4$

(A4) Beispiel:

$$(x^3 - 2x^2 + x - 2) : (x^2 + 1) = x - 2$$

$$\underline{x^3 \qquad + x}$$

$$-2x^2 - 2$$

a) $(x^5 + x^4 - x^3 + 2x + 1) : (x^2 + 2x + 1) = x^3 - x^2 + 1$

$$\underline{x^5 + 2x^4 + x^3}$$

$$- x^4 - 2x^3 + 2x + 1$$

$$\underline{- x^4 - 2x^3 - x^2}$$
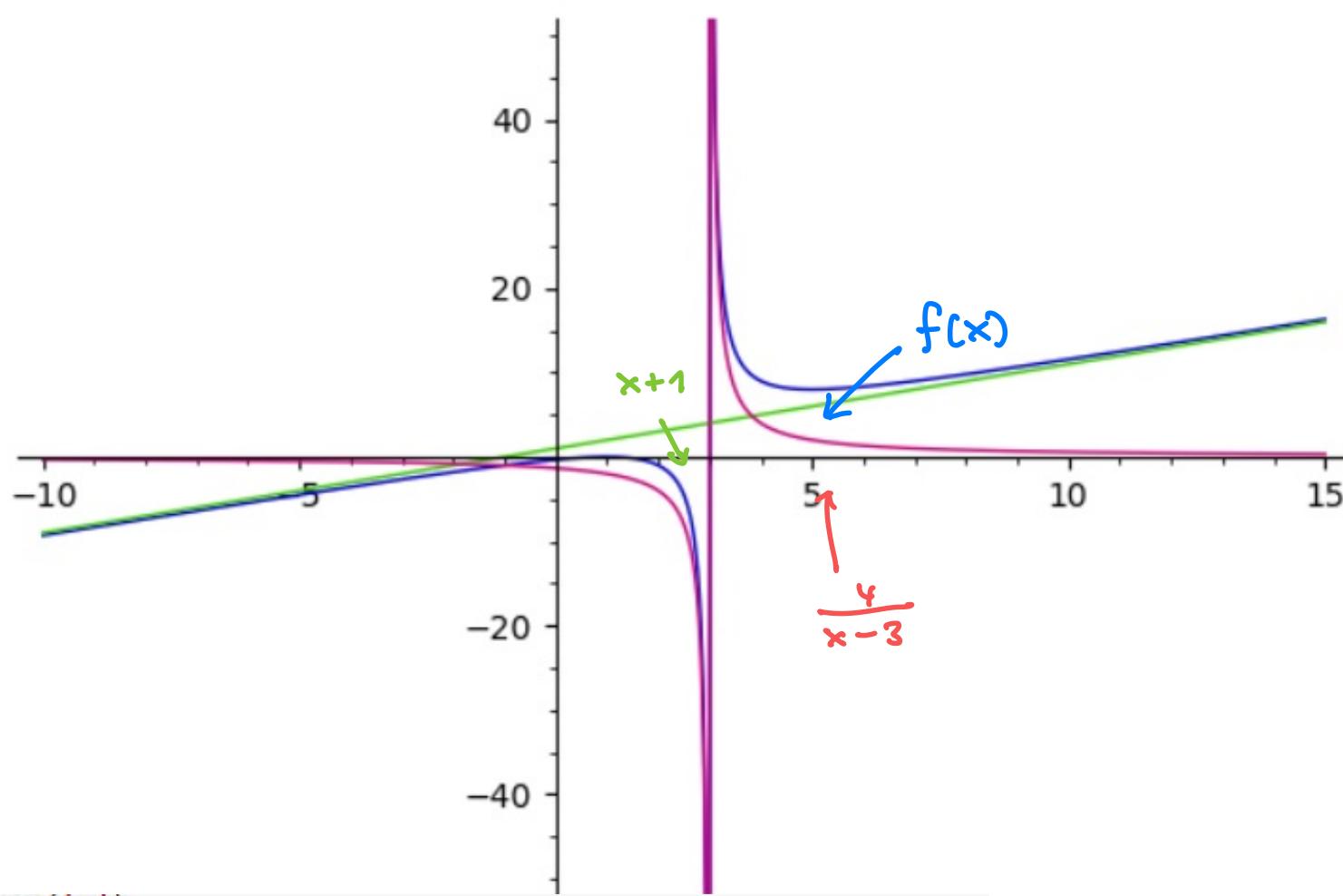
$$x^2 + 2x + 1$$

b) $(2x^5 + 3x^4 + 8x^3 + 2x^2 + 3x + 8) : (x^3 + 1) = 2x^2 + 3x + 8$

$$\underline{2x^5 + 2x^2}$$

$$3x^4 + 8x^3 + 3x + 8$$

$$\underline{3x^4 + 3x}$$

$$8x^3 + 8$$

(A5) Beispiel:

$$(2x^4 + 4x^3 + 8x) : (x^3 + 1) = 2x + 4 + \frac{6x - 4}{(x^3 + 1)}$$

$$\underline{2x^4 + 2x}$$

$$4x^4 + 6x$$

$$\underline{4x^3 + 4}$$

$$6x - 4$$

a) $(x^2 - 2x + 1) : (x - 3) = x + 1 + \frac{4}{(x - 3)}$

$$\underline{x^2 - 3x}$$

$$x + 1$$

$$\underline{x - 3}$$

$$4$$


```
var('x')
f(x) = (x^2-2*x+1)/(x-3)
plot([f(x),(x+1),(4/(x-3))],(x,-10,15),ymin=-50,ymax=50)
```

(A5)

b) $(x^2 + x - 6) : (x+2) = x - 1 - \frac{4}{(x+2)}$

$\underline{x^2 + 2x}$

$\quad - x - 6$

$\quad \underline{- x - 2}$

$\qquad - 4$

```
var('x')
f(x) = (x^2+x-6)/(x+2)
plot([f(x),(x-1),(-4/(x+2))],(x,-6,2),ymin=-50,ymax=50)
```