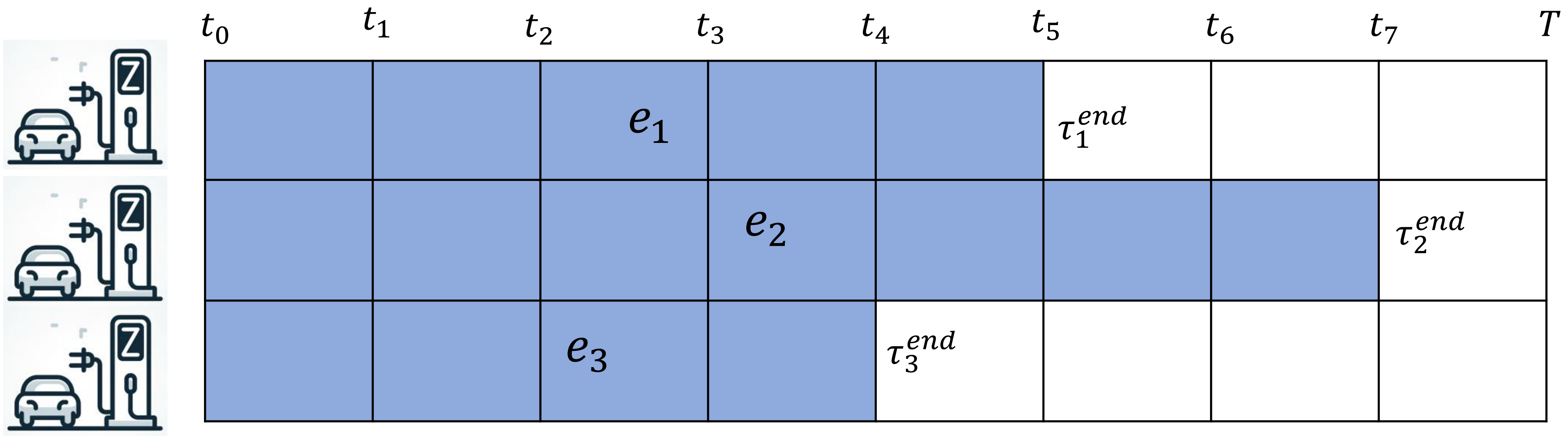
Problem Statement

- We have a set of **Electric Vehicles (EVs)** that are allocated to **Electric Vehicle Supply Equipment (EVSE)**, commonly known as **charging ports**.
- Each EV *i* has a specific **energy requirement ($\boldsymbol{e_i}$)** and a required **charging duration ($\boldsymbol{\tau_i^{end}}$)**

<u>**Objective:**</u>

Over a defined time horizon ***T*** with discrete time steps, establish a charging schedule that:

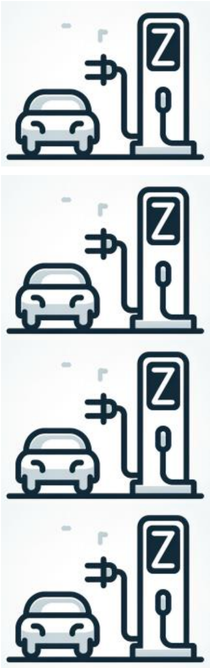
- Ensures each EV meets its energy requirement within the specified deadline.
- Minimizes the energy consumption at each time step.



Toy Example

- Let's assume we are optimizing for a time **horizon *T = 4 hours***, with a timestep ***ΔT = 1 hour***, a constant ***voltage V= 240V***, and a **set of allowable charging rate currents *ρ={8,16,32,48,64} A***
- Consider that we have **4 EVs** where EV 1, 2, and 3 each have an energy requirement $\boldsymbol{e_1 = e_2 = e_3 = 26880\ kWh}$ and a required charging duration of **3 hours**. EV 4 has an energy requirement $\boldsymbol{e_4 = 7680kWh}$ and a required charging duration of **1 hour**.

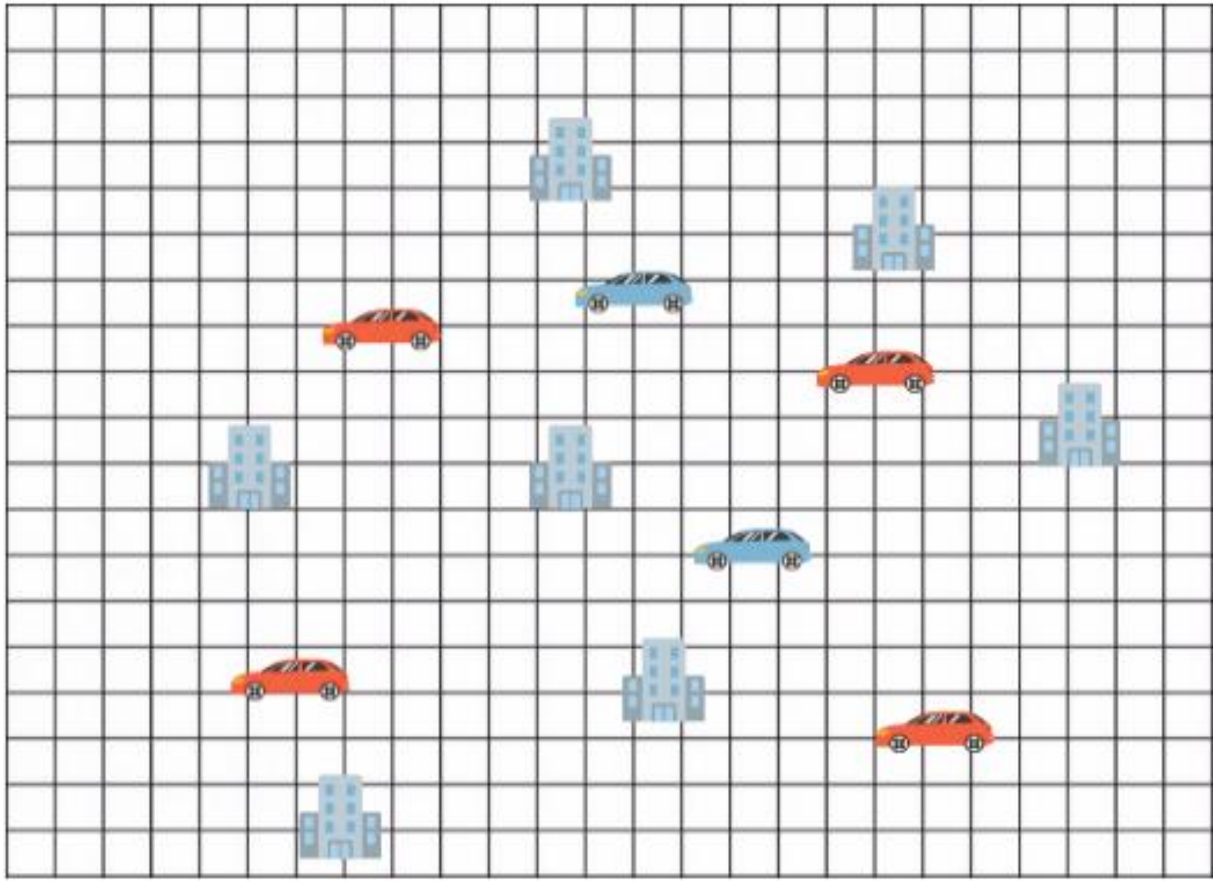
t_0 t_1 t_2 t_3 T



16 A	**48 A**	**48 A**	**0**	$26880/26880\,kWh$
32A	**48A**	**32A**	**0**	$26880/26880\,kWh$
32A	**32A**	**48A**	**0**	$26880/26880\,kWh$
32A	**0**	**0**	**0**	$7680/7680\,kWh$

EV charging station placement problem

Aman Chandra, Jitesh Lalwani, Babita Jajodia: *"Towards an Optimal Hybrid Algorithm for EV Charging Stations Placement using Quantum Annealing and Genetic Algorithms"*



- **Buildings**: Point of Interest (POI)
- Blue Cars: Existing charging stations
- Red Cars: New charging stations

EV charging station placement problem

Aman Chandra, Jitesh Lalwani, Babita Jajodia: *"Towards an Optimal Hybrid Algorithm for EV Charging Stations Placement using Quantum Annealing and Genetic Algorithms"*

Objective:

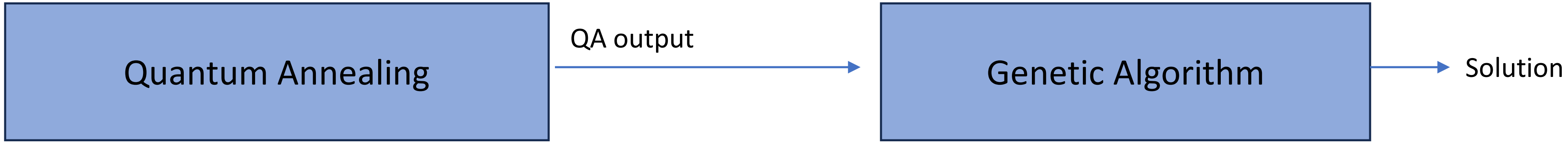
- Install the new chargers at the **minimum possible distance from the point of interest**.
- Ensure that the chargers are placed at **the maximum possible distance from the existing chargers** and at the **maximum distance from each other**.

$$H_1 = +\sum_{i=1}^{N} x_i d_i^p$$

$$H_2 = -\sum_{i=1}^{N} x_i d_i^c$$

$$H_3 = -\sum_{i=1}^{N} x_i d_i^l$$

Solved: Quantum Annealing combined with Genetic Algorithm



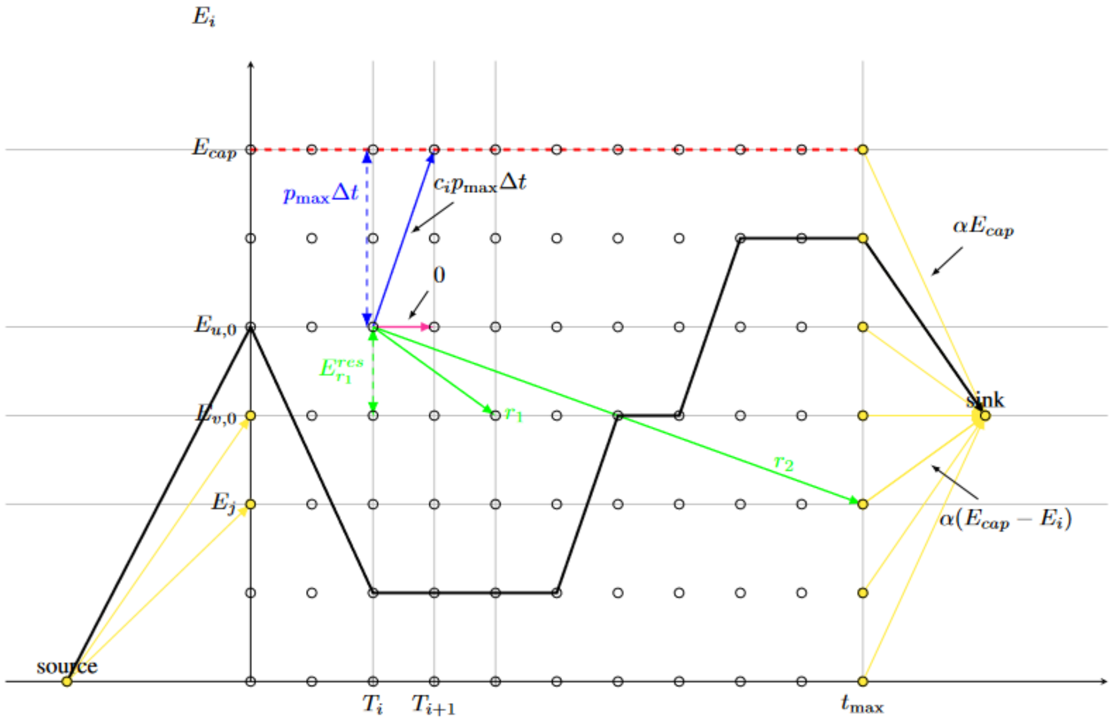
Electric Mobility Problem

Margarita Veshchezerova, Mikhail Somov, David Bertsche, Steffen Limmer, Sebastian Schmitt, Michael Perelshtein, Ayush Joshi Tripathi: *"A Hybrid Quantum-Classical Approach to the Electric Mobility Problem"*

Problem Statement

Suppose we have rental EVs. We use each EV to satisfy reservations.

The objective is : to satisfy all reservations while spending the least amount of energy possible.



$$\min \sum_{p \in \mathcal{P}} c_p \lambda_p + c^{uncov} \sum_{r \in R} E^{res}_r y_r$$

$$\sum_{\substack{p \in \mathcal{P}: \\ r \in p}} \lambda_p + y_r = 1, \qquad \forall r \in R$$

$$\sum_{\substack{p \in \mathcal{P}: \\ v \in p}} \lambda_p = 1, \qquad \forall v \in V$$

$$\lambda_p \in \{0, 1\}, \qquad \forall p \in \mathcal{P}$$

Electric Mobility Problem

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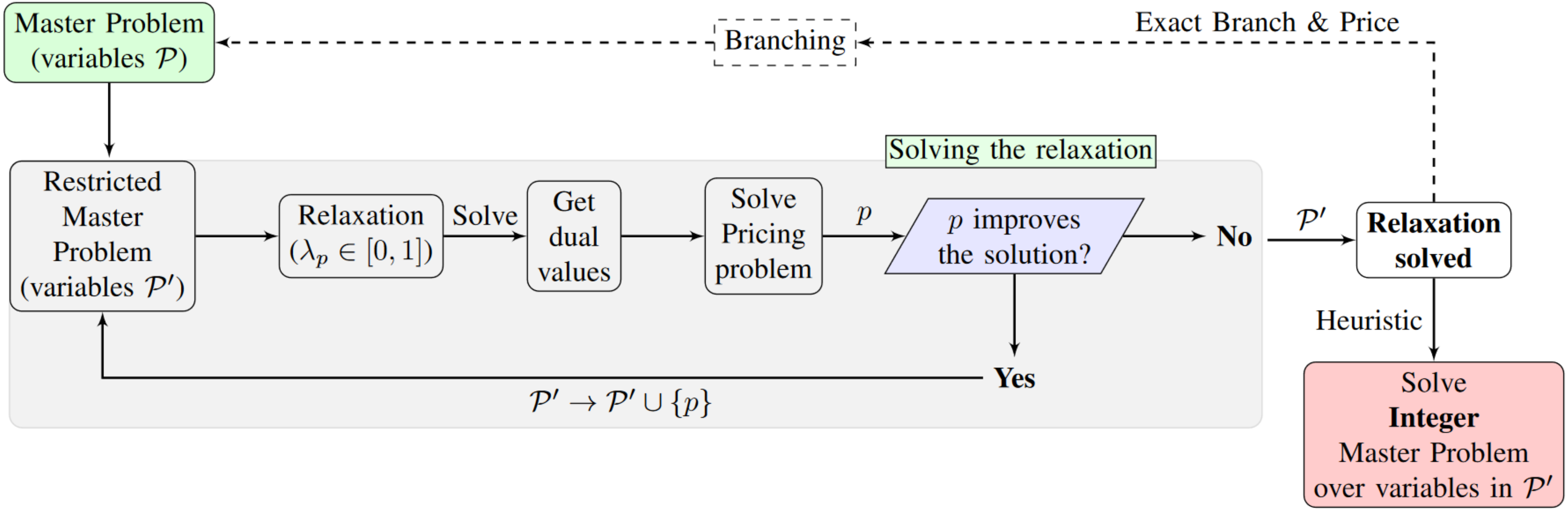


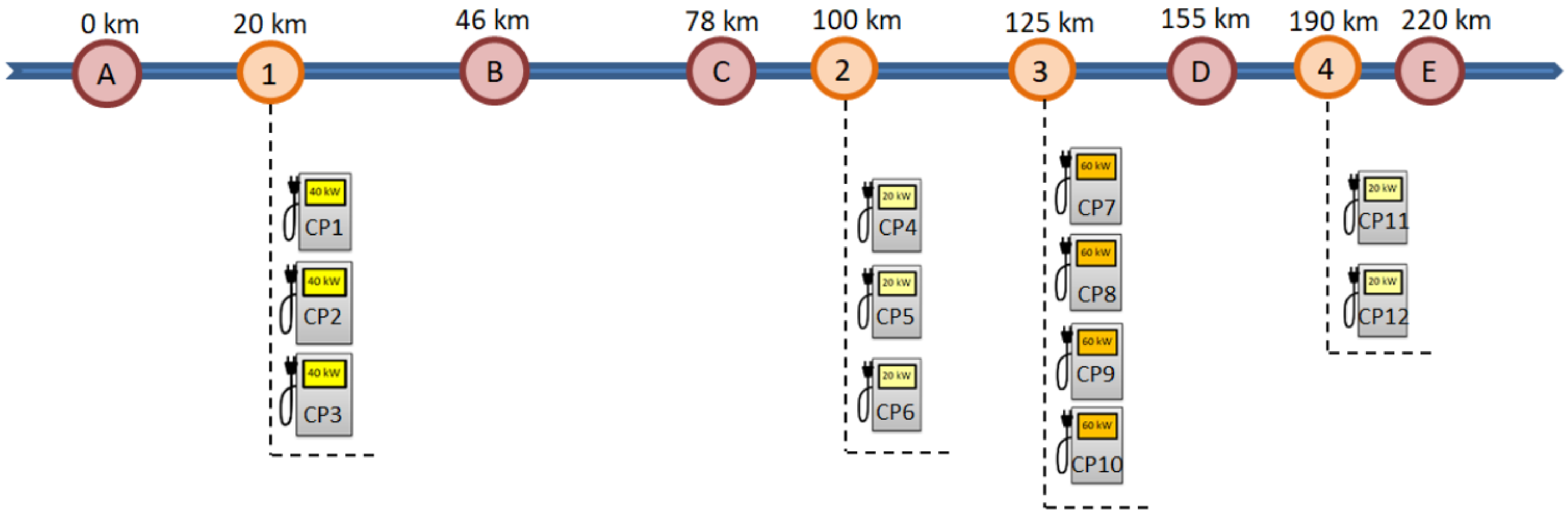
Charging Electric Cars on a Motorway

Różycki, R.; Józefowska, J.; Kurowski, K.; Lemański, T.; Pecyna, T.; Subocz, M.; Waligóra, G. A Quantum Approach to the Problem of Charging Electric Cars on a Motorway. Energies 2023, 16, 442. https://doi.org/10.3390/en16010442

Problem Statement

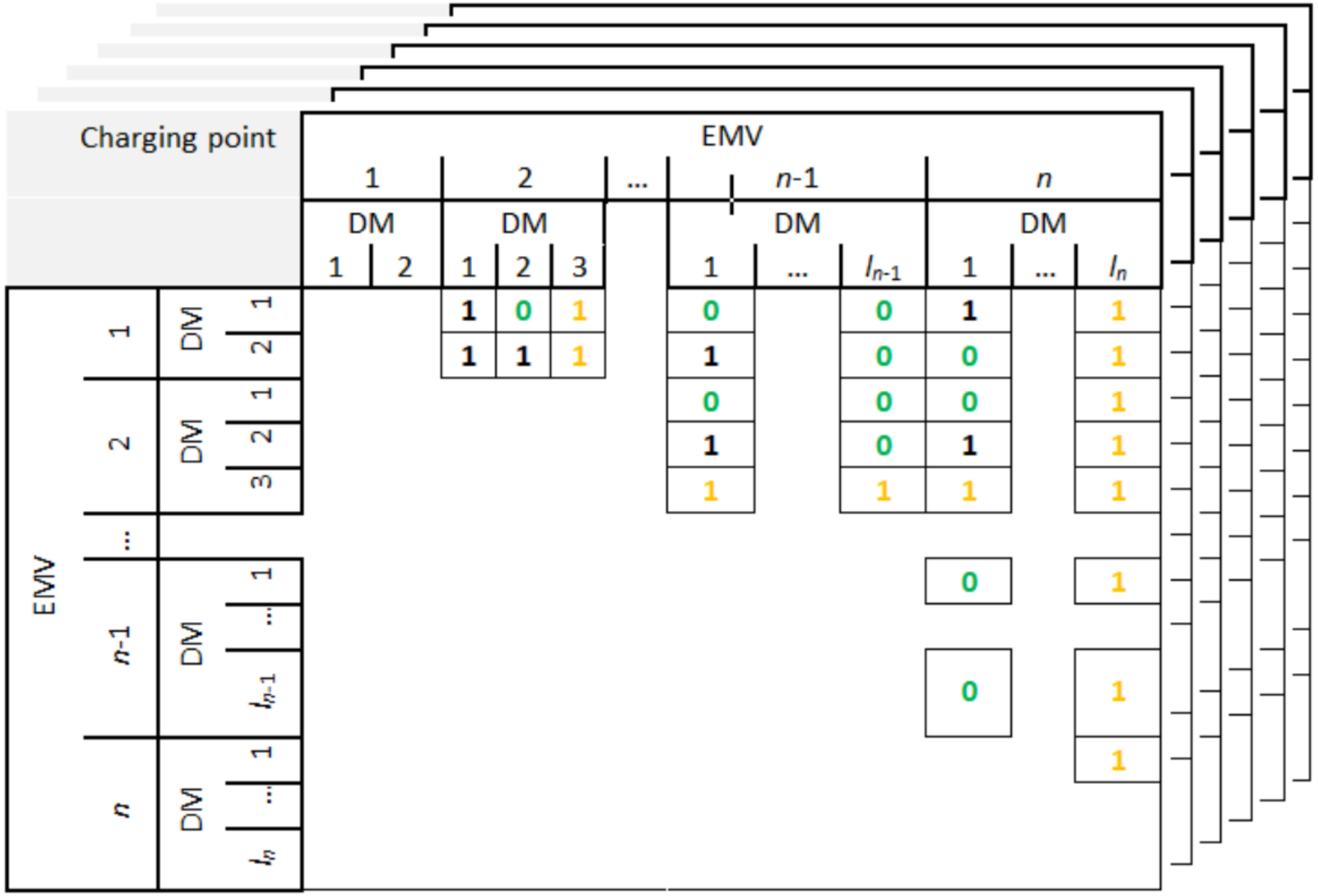
We have a number of EVs and a number of charging stations along a motorway. Each EV operates in a specific driving mode.

The objective is: Find all the cases (driving mode and charging station to be reached by each EV) so that there will be no conflicts at the station.



Charging Electric Cars on a Motorway

Różycki, R.; Józefowska, J.; Kurowski, K.; Lemański, T.; Pecyna, T.; Subocz, M.; Waligóra, G. A Quantum Approach to the Problem of Charging Electric Cars on a Motorway. Energies 2023, 16, 442. https://doi.org/10.3390/en16010442



QUBO Formulation of the initial Problem

Let $\boldsymbol{r_i(t_k)}$ be the charging current rate for EV $\boldsymbol{i}$ at time $\boldsymbol{t_k}$ and $\boldsymbol{N}$ the number of currently plugged EVs

Cost Function

$$C(r) = \sum_{k=0}^{T} \left(\sum_{i=0}^{N} r_i(t_k) \right)^2 + \rho \sum_{i=0}^{N} \left(\sum_{t_k=0}^{\tau_i^{end}} (V \cdot r_i(t_k) \cdot \Delta T) - e_i \right)^2$$

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Let $\delta_{ik} = 1$ if $t_k \leq \tau_i^{end}$
Else $\delta_{ik} = 0$

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Else $\delta_{ik} = 0$

$$\Downarrow$$

$$C(r) = \sum_{k=0}^{T}\left(\sum_{i=0}^{N} r_i(t_k)\right)^2 + \rho \sum_{i=0}^{N}\left(\sum_{t_k=0}^{T} (V \cdot \delta_{ik} \cdot r_i(t_k) \cdot \Delta T) - e_i\right)^2$$

QUBO Formulation of the initial Problem

$$\Rightarrow \quad C(r) = \sum_{i=0}^{N} \sum_{k=0}^{T} \left(r_i(t_k)\right)^2 + 2 \sum_{i,j=0}^{N} \sum_{k}^{T} r_i(t_k) r_j(t_k) + \rho \sum_{i=0}^{N} \sum_{k=0}^{T} (V\delta_{ik}\, r_i(t_k)\Delta\mathrm{T})^2 +$$

$$+\rho \sum_{i=0}^{N} 2 \sum_{k,l=0}^{T} V^2 \Delta\mathrm{T}^2 \delta_{ik} r_i(t_k) \delta_{il} r_i(t_l) + \rho \sum_{i=0}^{\mathrm{N}} 2e_i \sum_{k=0}^{T} V\Delta\mathrm{T}\delta_{ik} r_i(t_k)$$

QUBO Formulation of the initial Problem

$$\Rightarrow \quad C(r) = \sum_{i=0}^{N}\sum_{k=0}^{T}\left(r_i(t_k)\right)^2 + 2\sum_{i,j=0}^{N}\sum_{k}^{T} r_i(t_k)r_j(t_k) + \rho\sum_{i=0}^{N}\sum_{k=0}^{T}(V\delta_{ik}\, r_i(t_k)\Delta\mathrm{T})^2 +$$

$$+\rho\sum_{i=0}^{N}2\sum_{k,l=0}^{T}V^2\Delta\mathrm{T}^2\delta_{ik}r_i(t_k)\delta_{il}r_i(t_l) + \rho\sum_{i=0}^{\mathrm{N}}2e_i\sum_{k=0}^{T}V\Delta\mathrm{T}\delta_{ik}r_i(t_k)$$

<u>Convert to binary.</u>

Simplify set of allowable current to {0,16,32,48} $\longrightarrow$ $r_i(t_k) = 16\sum_{q=0}^{1}2^q x_{ikq}$

QUBO Formulation of the initial Problem

$$C(r) = \sum_{i=0}^{N}\sum_{k=0}^{T}\left(r_i(t_k)\right)^2 + 2\sum_{i,j=0}^{N}\sum_{k}^{T} r_i(t_k)r_j(t_k) + \rho\sum_{i=0}^{N}\sum_{k=0}^{T}(V\delta_{ik}\, r_i(t_k)\Delta\mathrm{T})^2 + $$
$$+\rho\sum_{i=0}^{N} 2\sum_{k,l=0}^{T} V^2\Delta\mathrm{T}^2\delta_{ik}r_i(t_k)\delta_{il}r_i(t_l) + \rho\sum_{i=0}^{\mathrm{N}} 2e_i\sum_{k=0}^{T} V\Delta\mathrm{T}\delta_{ik}r_i(t_k)$$

<u>Convert to binary.</u>

Simplify set of allowable current to {0,16,32,48} $\longrightarrow$ $r_i(t_k) = 16\sum_{q=0}^{1} 2^q x_{ikq}$

$$C(x) = \sum_{i=0}^{N}\sum_{k=0}^{T}\left(16\sum_{q=0}^{1} 2^q x_{ikq}\right)^2 + 2\sum_{i,j=0}^{N}\sum_{k}^{T} 256\sum_{q=0}^{1} 2^q x_{ikq}\sum_{q=0}^{1} 2^q x_{jkq} + \rho\sum_{i=0}^{N}\sum_{k=0}^{T}\left(V\delta_{ik}16\sum_{q=0}^{1} 2^q x_{ikq}\,\Delta\mathrm{T}\right)^2 +$$
$$+\rho\sum_{i=0}^{N} 2\sum_{k,l=0}^{T} 256V^2\Delta\mathrm{T}^2\delta_{ik}\sum_{q=0}^{1} 2^q x_{ikq}\,\delta_{il}\sum_{q=0}^{1} 2^q x_{ilq} + \rho\sum_{i=0}^{\mathrm{N}} 2e_i\sum_{k=0}^{T} V\Delta\mathrm{T}\delta_{ik}16\sum_{q=0}^{1} 2^q x_{ikq}$$

ClusterVQE

Split vector $\boldsymbol{x}$ to *n* clusters of equal length $\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}$

$$\text{Then } x^T Q x = [x_1^T\ x_2^T\ \ \dots x_n^T] \begin{bmatrix} Q_{11} & Q_{12} & \dots & Q_{1n} \\ 0 & Q_{22} & \dots & Q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Q_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \rightarrow$$

ClusterVQE

Split vector $\boldsymbol{x}$ to *n* clusters of equal length $\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}$

Then $x^T Q x = [x_1^T \; x_2^T \; \dots x_n^T] \begin{bmatrix} Q_{11} & Q_{12} & \dots & Q_{1n} \\ 0 & Q_{22} & \dots & Q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Q_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \rightarrow$

$$x^T Q x = x_1^T Q_{11} x_1 + x_2^T Q_{22} x_2 + \cdots + x_n^T Q_{nn} x_n + x_1^T Q_{12} x_2 + \cdots + x_1^T Q_{1n} x_n + \cdots + x_2^T Q_{2n} x_n + \cdots$$

ClusterVQE

Split vector $\boldsymbol{x}$ to n clusters of equal length $\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}$

$$\text{Then } x^T Q x = [x_1^T \; x_2^T \; \dots x_n^T] \begin{bmatrix} Q_{11} & Q_{12} & \dots & Q_{1n} \\ 0 & Q_{22} & \dots & Q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Q_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \rightarrow$$

$$x^T Q x = x_1^T Q_{11} x_1 + x_2^T Q_{22} x_2 + \cdots + x_n^T Q_{nn} x_n + x_1^T Q_{12} x_2 + \cdots + x_1^T Q_{1n} x_n + \cdots + x_2^T Q_{2n} x_n + \cdots$$

$$\Downarrow$$

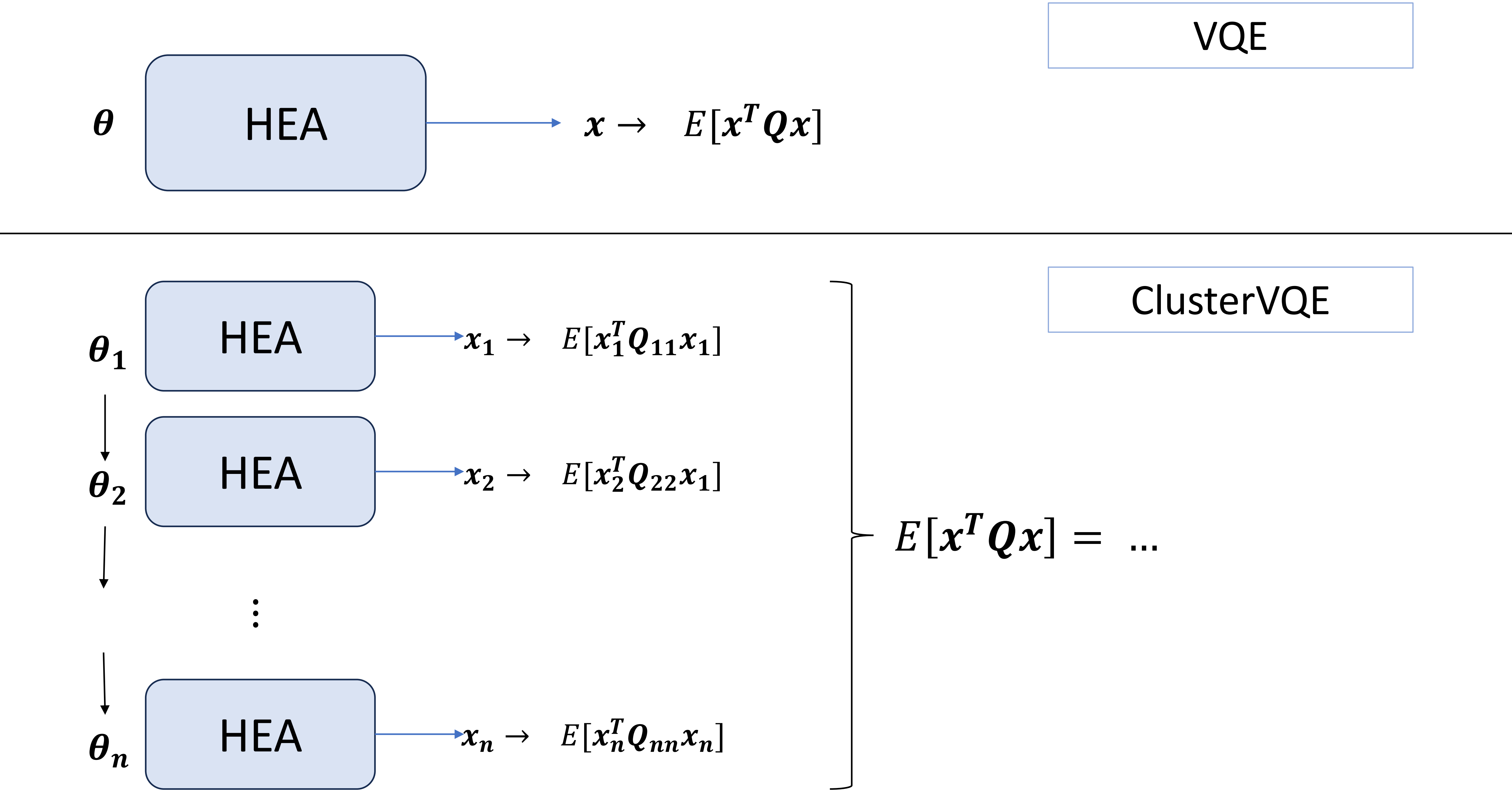
$$E[x^T Q x] = E[x_1^T Q_{11} x_1] + E[x_2^T Q_{22} x_2^T] + \cdots + E[x_n^T Q_{nn} x_n] + E[\, x_1^T Q_{12} x_2] + \cdots + E[x_1^T Q_{1n} x_n] + \cdots + E[x_2^T Q_{2n} x_n] + \cdots$$

$$\Downarrow$$

$$\boxed{E[x^T Q x] = \sum_{i=1}^{n} E[x_i^T Q_{ii} x_i] + \sum_{i=1}^{n} \sum_{j \neq i}^{n} E[x_i^T Q_{ij} x_j]}$$

VQE

$\boldsymbol{\theta}$ HEA $\rightarrow$ $\boldsymbol{x} \rightarrow \quad E[\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x}]$



VQE
$\boldsymbol{\theta}$
HEA
$\boldsymbol{x} \rightarrow \quad E[\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x}]$
ClusterVQE
$\boldsymbol{\theta}_1$
HEA
$\boldsymbol{x}_1 \rightarrow \quad E[\boldsymbol{x}_1^T \boldsymbol{Q}_{11} \boldsymbol{x}_1]$
$\boldsymbol{\theta}_2$
HEA
$\boldsymbol{x}_2 \rightarrow \quad E[\boldsymbol{x}_2^T \boldsymbol{Q}_{22} \boldsymbol{x}_1]$
$\vdots$
$\boldsymbol{\theta}_n$
HEA
$\boldsymbol{x}_n \rightarrow \quad E[\boldsymbol{x}_n^T \boldsymbol{Q}_{nn} \boldsymbol{x}_n]$
$E[\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x}] = \ \ldots$

Apply ClusterVQE to EV charging problem

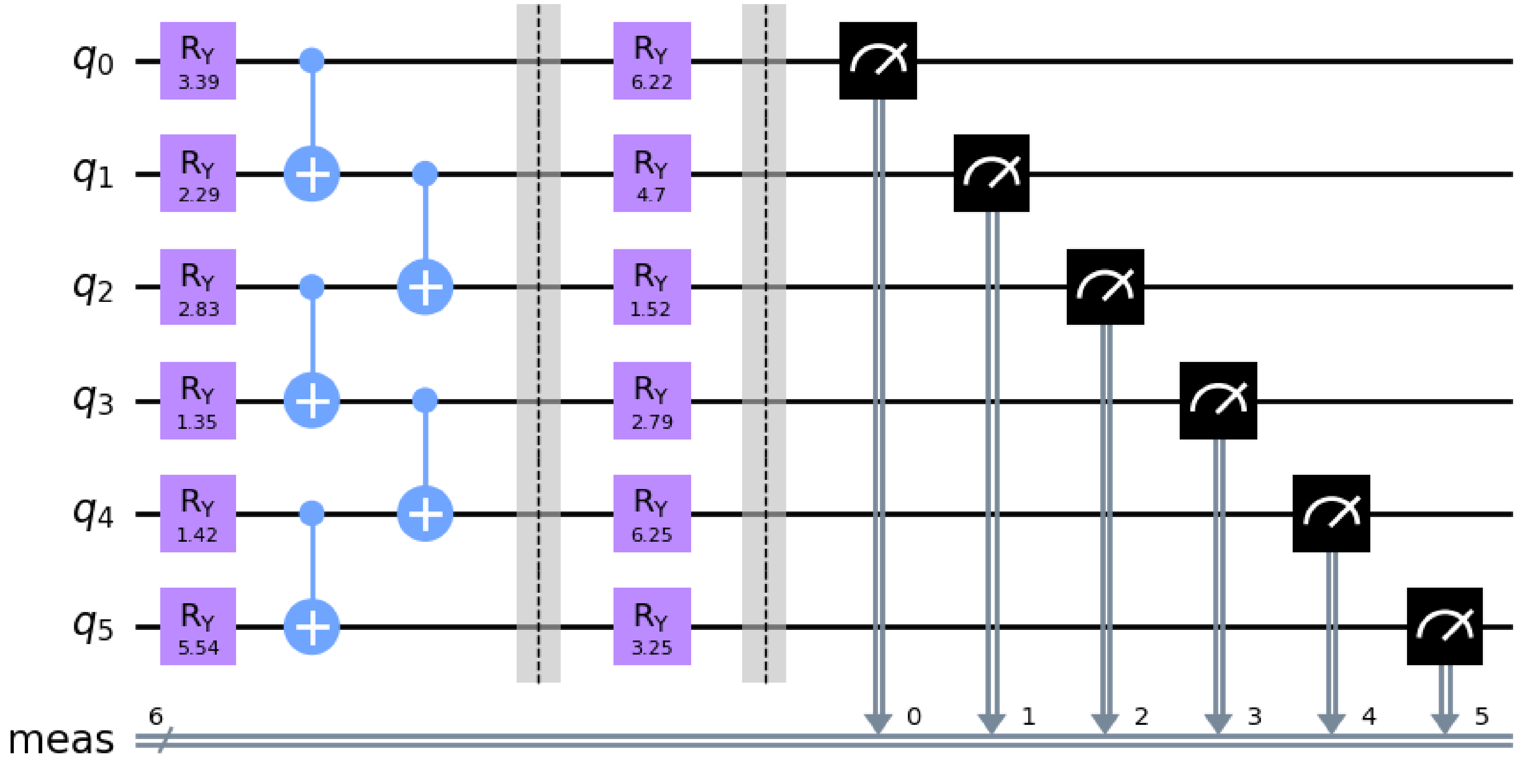
$$\text{Num of EVs} = N \left\{ \overbrace{\begin{bmatrix} r_1(t_1) & r_1(t_2) & r_1(t_3) & r_1(t_4) \\ r_2(t_1) & r_2(t_2) & r_2(t_3) & r_2(t_4) \\ r_3(t_1) & r_3(t_2) & r_3(t_3) & r_3(t_4) \\ r_4(t_1) & r_4(t_2) & r_4(t_3) & r_4(t_4) \end{bmatrix}}^{Horizon} \right. \begin{array}{c} \\ r_2 \\ \\ \\ \end{array}$$

<u>Advantages of ClusterVQE</u>

- Reduce the number of qubits by a factor of N
- Increase the number of measurements by a factor of N

*Can also be used with qubit compression

Circuit: HEA with 2 layers

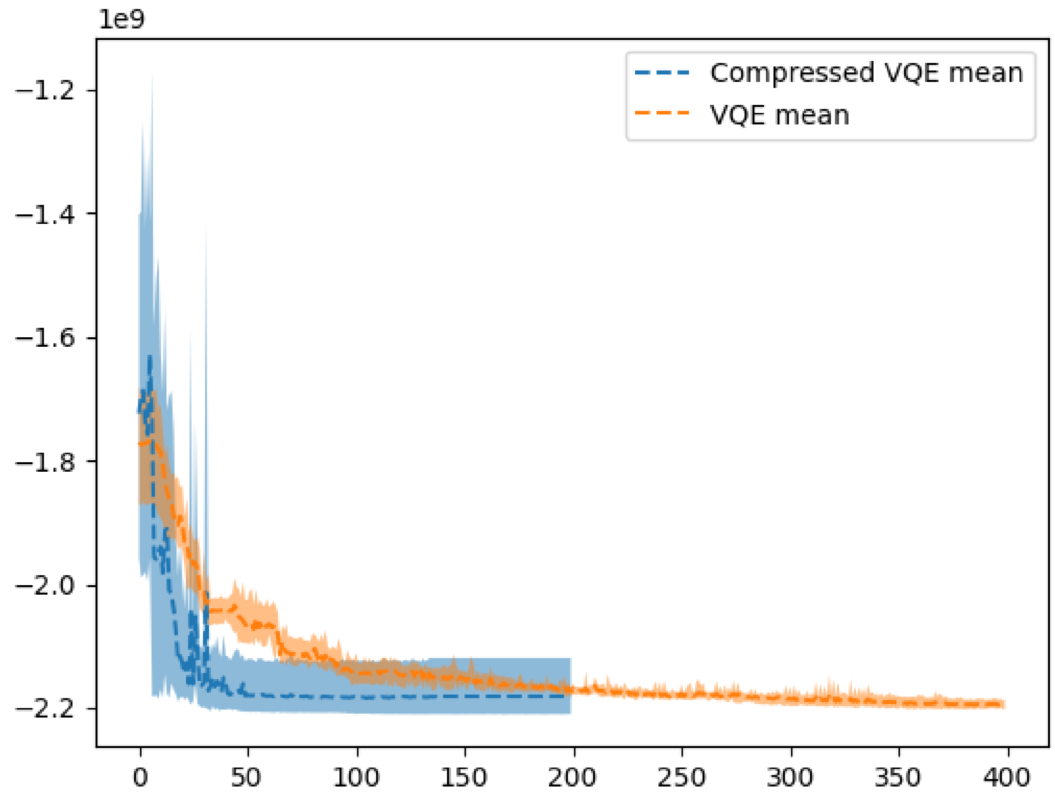


Comparison

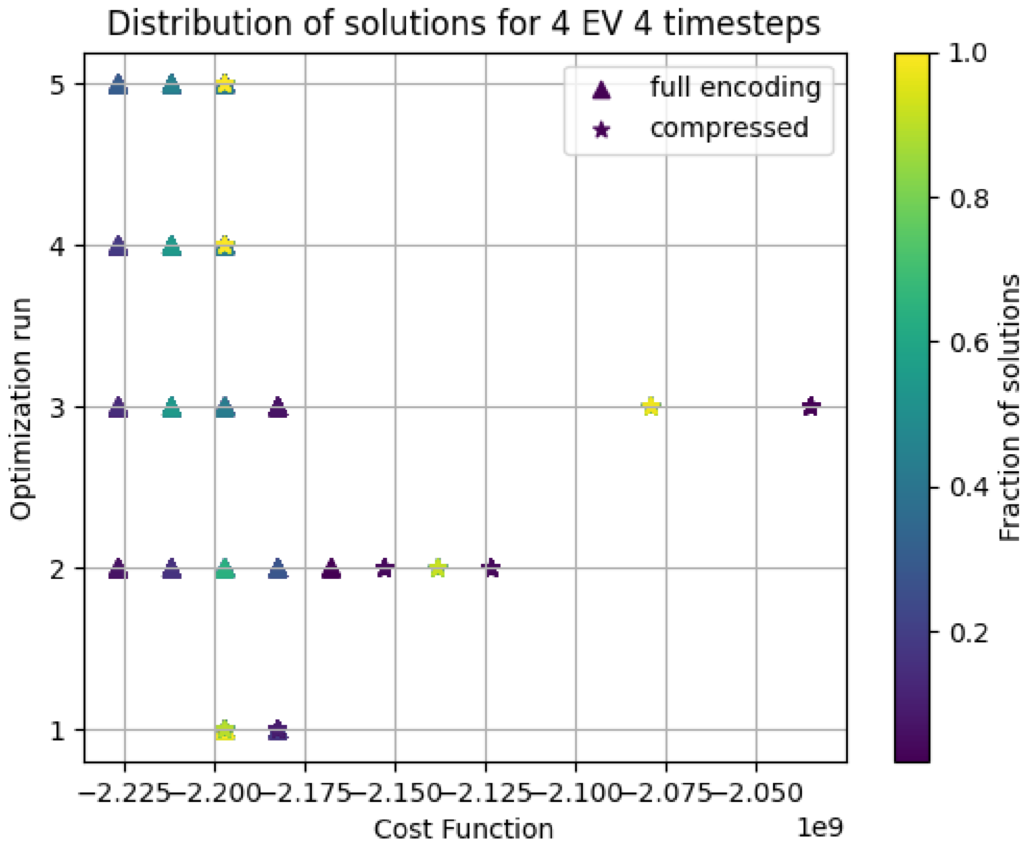
For an example with ***T=4 hours*** , ***Evs = 4*** and **2 layer** HEA

Algorithm	#Classical Variables	#Qubits	#Shots	#parameters θ	#Ry gates	#CNOT gates	#gates	#iterations
VQE	16	32	5000	64	64	31	95	400+
Compressed VQE	16	6	100000	12	12	5	17	130
ClusterVQE	16	8	40000	64	16	7	23	-
ClusterCompressedVQE	16	4	40000	32	8	3	11	-

Comparison

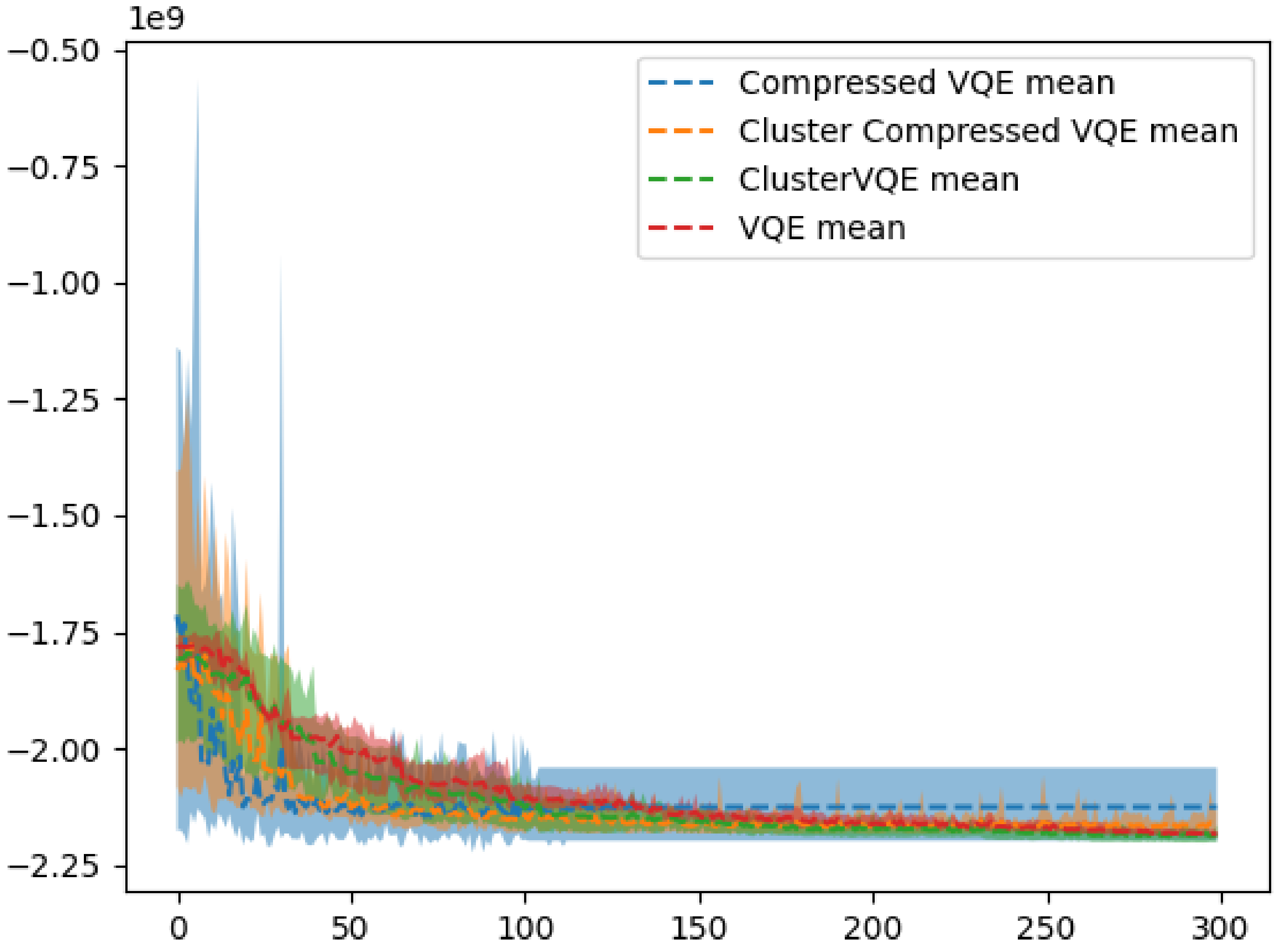


1e9
−1.2
−1.4
−1.6
−1.8
−2.0
−2.2
0
50
100
150
200
250
300
350
400
Compressed VQE mean
VQE mean



Distribution of solutions for 4 EV 4 timesteps
full encoding
compressed
Optimization run
1
2
3
4
5
−2.225
−2.200
−2.175
−2.150
−2.125
−2.100
−2.075
−2.050
Cost Function
1e9
Fraction of solutions
0.2
0.4
0.6
0.8
1.0

Comparison



1e9
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
−2.00
−2.25
0
50
100
150
200
250
300
Compressed VQE mean
Cluster Compressed VQE mean
ClusterVQE mean
VQE mean

Comparison

CompressedVQE

$$\begin{bmatrix} 16 & 48 & 48 & 0 \\ 16 & 48 & 48 & 0 \\ 48 & 16 & 48 & 0 \\ 32 & 0 & 0 & 0 \end{bmatrix}$$

26880/26880
26880/26880
26880/26880
7680/7680

ClusterCompressedVQE

$$\begin{bmatrix} 48 & 48 & 16 & 0 \\ 32 & 32 & 48 & 0 \\ 32 & 48 & 48 & 0 \\ 32 & 0 & 0 & 0 \end{bmatrix}$$

26880/26880
26880/26880
32720/26880
7680/7680

ClusterVQE

$$\begin{bmatrix} 48 & 48 & 16 & 0 \\ 48 & 16 & 48 & 0 \\ 32 & 32 & 48 & 0 \\ 32 & 0 & 0 & 0 \end{bmatrix}$$

26880/26880
26880/26880
26880/26880
7680/7680

VQE

$$\begin{bmatrix} 32 & 48 & 32 & 0 \\ 48 & 48 & 16 & 0 \\ 32 & 32 & 48 & 0 \\ 32 & 0 & 0 & 0 \end{bmatrix}$$

26880/26880
26880/26880
26880/26880
7680/7680

Comparison:64 EVs, 12 time step Horizon