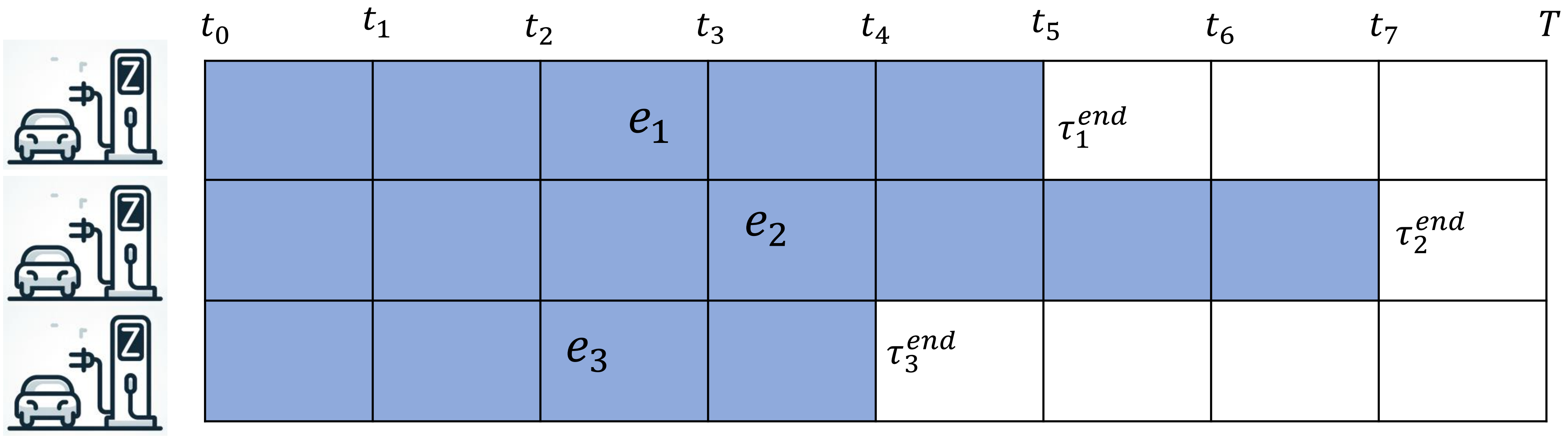
Problem Statement

- We have a set of **Electric Vehicles (EVs)** that are allocated to **Electric Vehicle Supply Equipment (EVSE)**, commonly known as **charging ports**.
- Each EV *i* has a specific **energy requirement ($\boldsymbol{e_i}$)** and a required **charging duration ($\boldsymbol{\tau_i^{end}}$)**

<u>Objective:</u>

Over a defined time horizon ***T*** with discrete time steps, establish a charging schedule that:

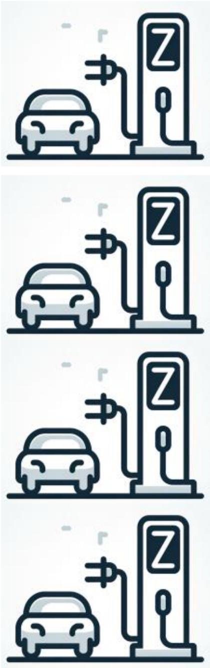
- Ensures each EV meets its energy requirement within the specified deadline.
- Minimizes the energy consumption at each time step.



Toy Example

- Let's assume we are optimizing for a time **horizon *T = 4 hours***, with a timestep ***ΔT = 1 hour***, a constant ***voltage V= 240V***, and a **set of allowable charging rate currents *ρ={8,16,32,48,64} A***
- Consider that we have **4 EVs** where EV 1, 2, and 3 each have an energy requirement $\boldsymbol{e_1} = \boldsymbol{e_2} = \boldsymbol{e_3} = \mathbf{26880}\ \boldsymbol{kWh}$ and a required charging duration of **3 hours**. EV 4 has an energy requirement $\boldsymbol{e_4} = \mathbf{7680}\boldsymbol{kWh}$ and a required charging duration of **1 hour**.

t_0 t_1 t_2 t_3 T



16 A	**48 A**	**48 A**	**0**	$26880/26880\,kWh$
32A	**48A**	**32A**	**0**	$26880/26880\,kWh$
32A	**32A**	**48A**	**0**	$26880/26880\,kWh$
32A	**0**	**0**	**0**	$7680/7680\,kWh$

QUBO Formulation of the Problem

Let $\boldsymbol{r_i}(\boldsymbol{t_k})$ be the charging current rate for EV ***i*** at time $\boldsymbol{t_k}$, ***N*** the number of currently plugged EVs and $\boldsymbol{t_n}$ the last time step of the Horizon

Optimization Variables

$$\boldsymbol{r} = \begin{bmatrix} r_1(t_1) & \cdots & r_1(t_n) \\ \vdots & \ddots & \vdots \\ r_n(t_1) & \cdots & r_n(t_n) \end{bmatrix}$$

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Cost Function

Non – Completion Penalty

$$Q^{NC}(\boldsymbol{r}) = \sum_{i=1}^{N} \left(\sum_{t_k=t_1}^{\tau_i^{end}} (V \cdot r_i(t_k) \cdot \Delta T) - e_i \right)^2$$

Load Variation

$$Q^{LV}(\boldsymbol{r}) = \sum_{k=0}^{T} \left(\sum_{i=1}^{N} r_i(t_k) \right)^2$$

QUBO Formulation of the Problem

$$minimize\ C(\boldsymbol{r}) := w^{LV} \sum_{t_k=t_1}^{T} \left(\sum_{i=1}^{N} r_i(t_k) \right)^2 + w^{NC} \sum_{i=0}^{N} \left(\sum_{t_k=t_1}^{\tau_i^{end}} (V \cdot r_i(t_k) \cdot \Delta T) - e_i \right)^2$$

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$Subject\ to:\ r_i(t_k) \in \{0, 8, 16, 32, 48, 64\}$

$$\sum_{i=1}^{N} V \cdot r_i(t_k) \leq C \quad \forall t_k$$

Where C *is the Network's power capacity*

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We have to find a binary encoding for the set {0, 8, 16, 32, 48, 68}

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Simplest one I could think of $\longrightarrow$ $$r_i(t_k) = x_{i,k,2} \cdot 16 \cdot \sum_{q=0}^{1} 2^q x_{i,k,q} + (1 - x_{i,k,2}) \cdot 8 \cdot x_{i,k,1}$$

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Increases the order of the cost function

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Instead, we simplify the set of allowable currents to : {0, 16, 32, 48 }

$$r_i(t_k) = 16 \cdot \sum_{q=0}^{1} 2^q x_{i,k,q}$$

Incorporating Inequality Constraints

Convert Inequality Constraint to Equality Constraint

$$\sum_{i=1}^{N} V \cdot r_i(t_k) \leq C \quad \forall t_k$$

$$\Downarrow$$

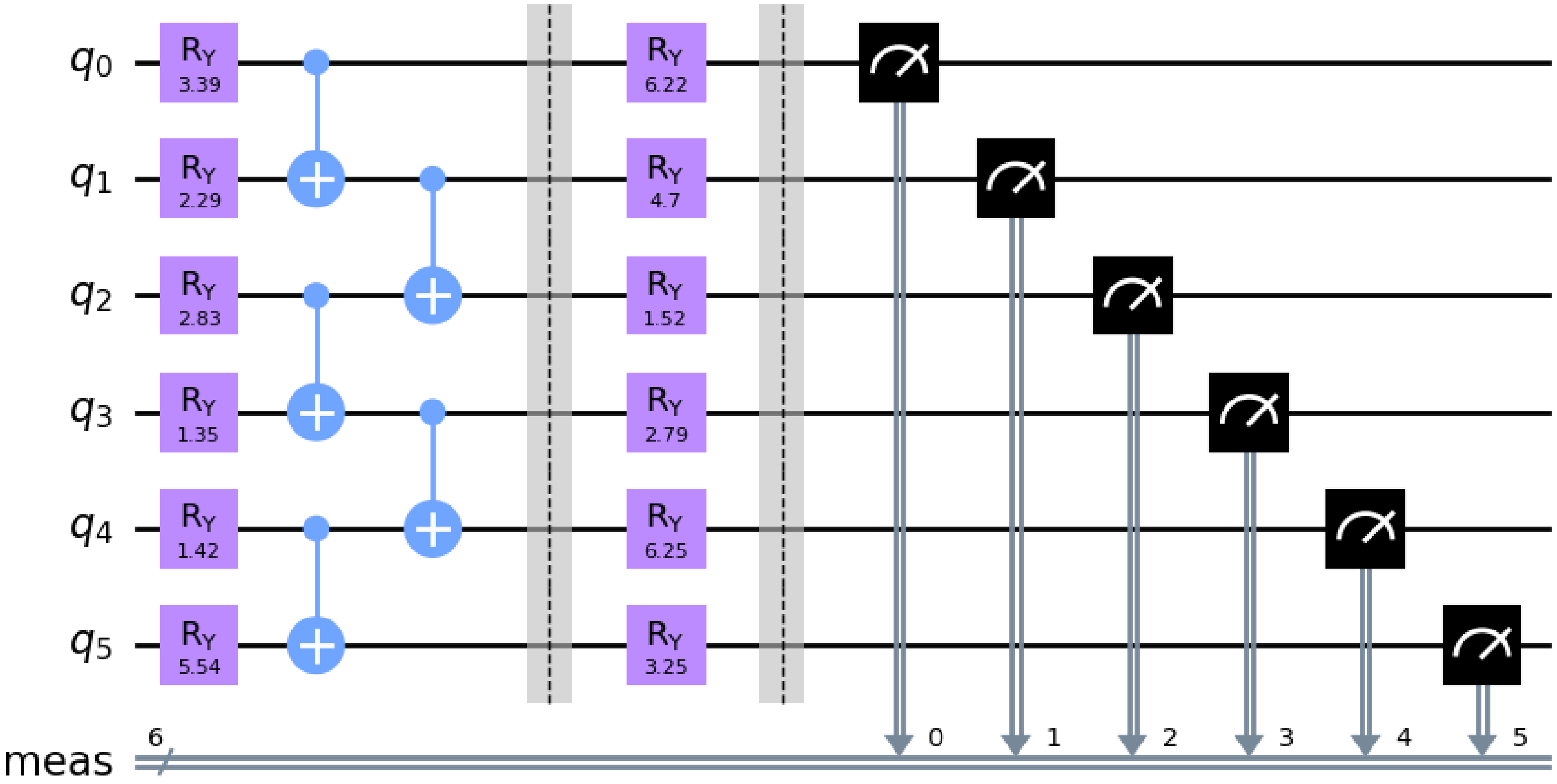
$$\sum_{i=1}^{N} V \cdot r_i(t_k) + s_k = C \quad \forall t_k$$

"**Slack Variables**" $\boldsymbol{s_k}$ are positive integers where $\boldsymbol{s_k \leq C}$

To encode one of them we need $\lceil \mathbf{log_2} \boldsymbol{C} \rceil$ binary variables

For a $\boldsymbol{n}$ timestep Horizon we need additional $\mathbf{n} \lceil \mathbf{log_2} \boldsymbol{C} \rceil$ binary variables

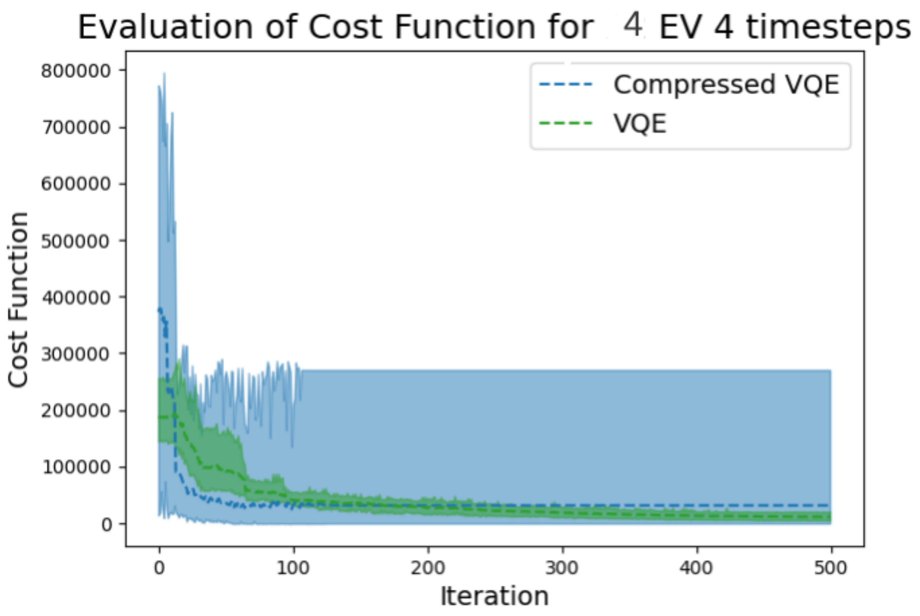
Circuit: HEA with 2 layers

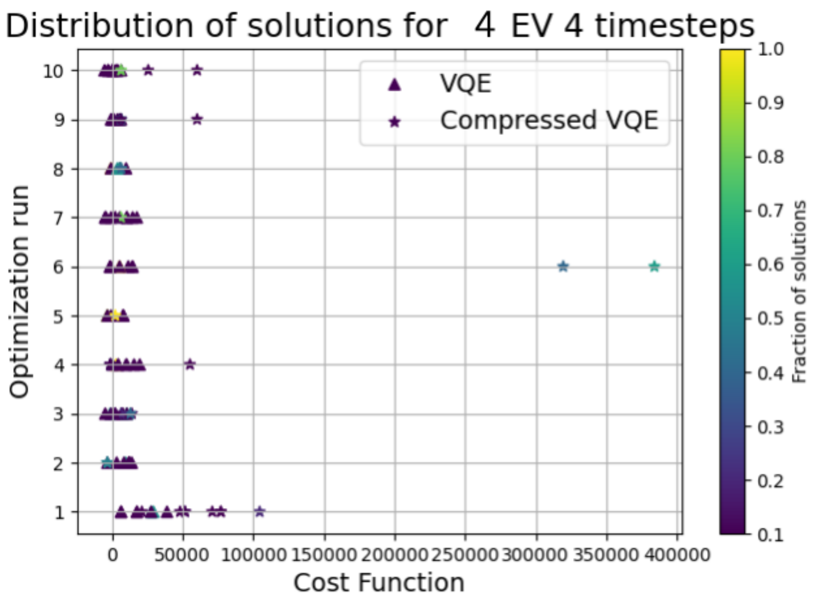


Experimental Procedure: Toy Example

For the toy example with ***T=4 hours*** , ***Evs = 4*** and **2 layer** HEA

Algorithm	#Classical Variables	#Qubits	#Shots	#parameters θ	#Ry gates	#CNOT gates	#gates	#iterations
VQE	16	32	5000	64	64	31	95	400+
Compressed VQE	16	6	10000	12	12	5	17	130





Toy Example

CompressedVQE

$$\begin{bmatrix} 32 & 32 & 48 & 0 \\ 32 & 48 & 32 & 0 \\ 32 & 48 & 32 & 0 \\ 32 & 0 & 0 & 0 \end{bmatrix}$$

26880/26880
26880/26880
26880/26880
7680/7680

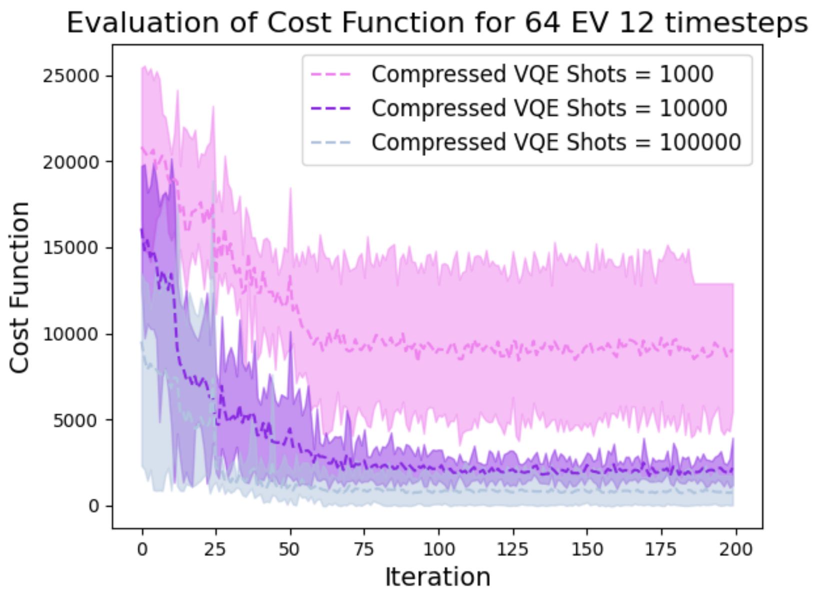
VQE

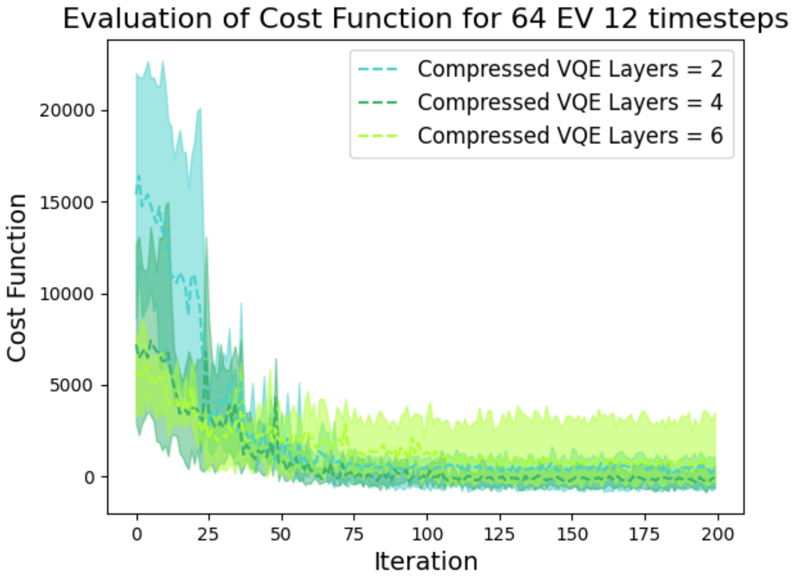
$$\begin{bmatrix} 16 & 48 & 48 & 0 \\ 32 & 32 & 48 & 0 \\ 48 & 32 & 32 & 0 \\ 32 & 0 & 0 & 0 \end{bmatrix}$$

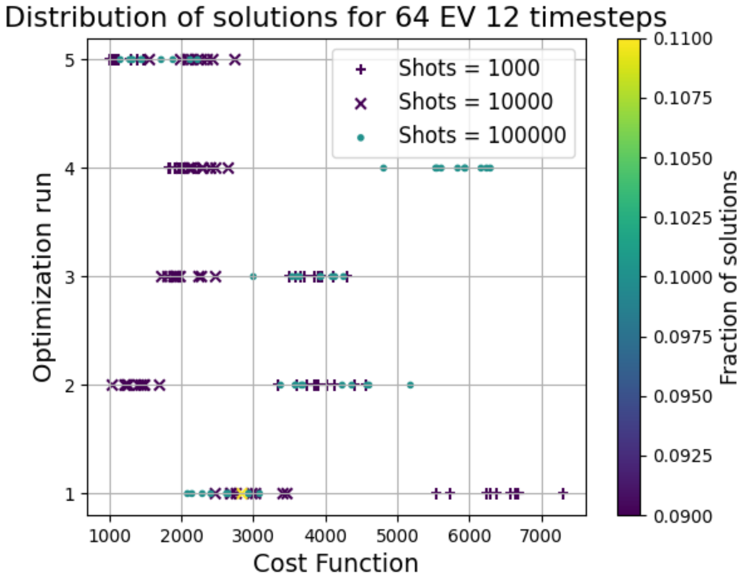
26880/26880
26880/26880
26880/26880
7680/7680

Large Scale Example

64 Evs 12 timestep Horizon → 1.536 binary variables , 12 qubits

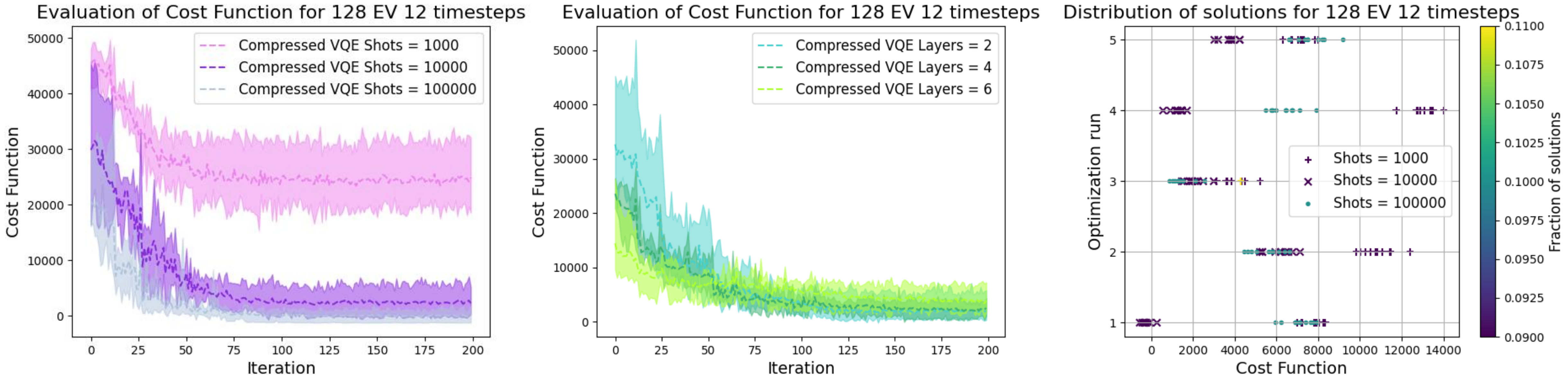






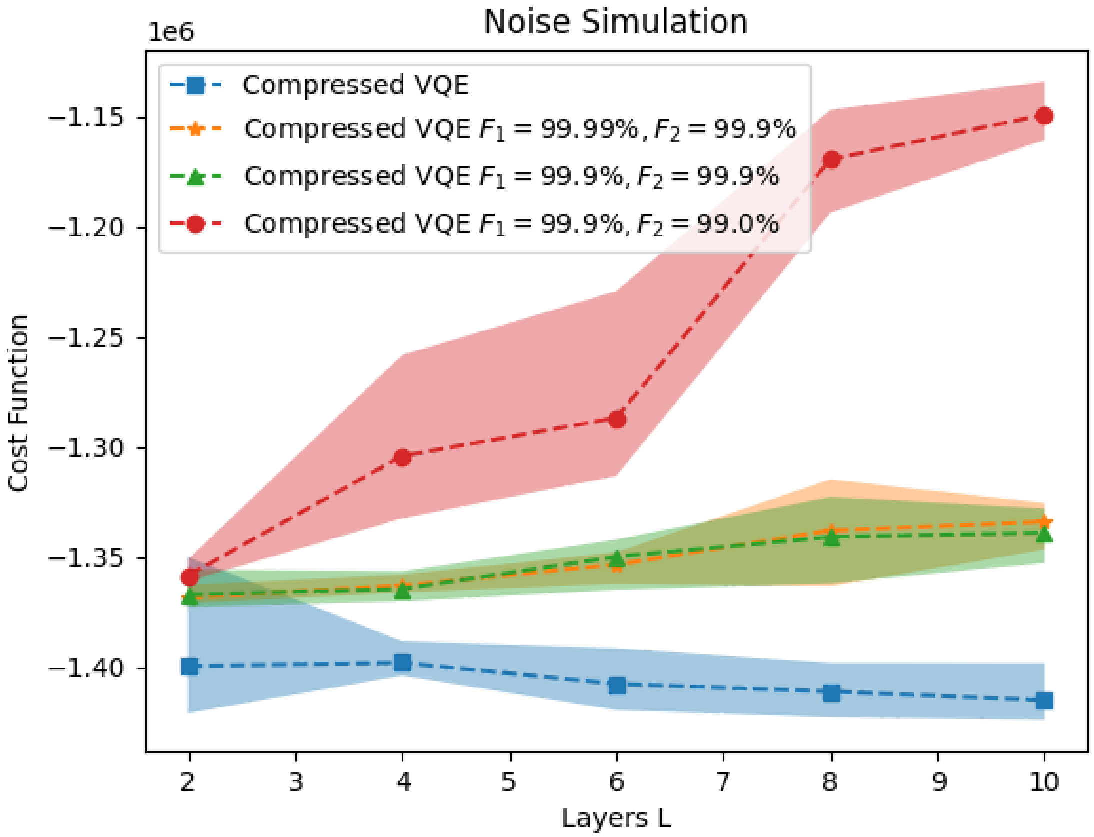
Large Scale Example

128 Evs 12 timestep Horizon → 3.072 binary variables , 13 qubits



Noise Simulation

8 Evs 4 timestep Horizon → 64 binary variables , 7 qubits



Comparison with Classical Algorithms

Simulation of 100 Evs that arrives and depart in a timespan of 24 hours