Distributed control in water distribution network

Water distribution network

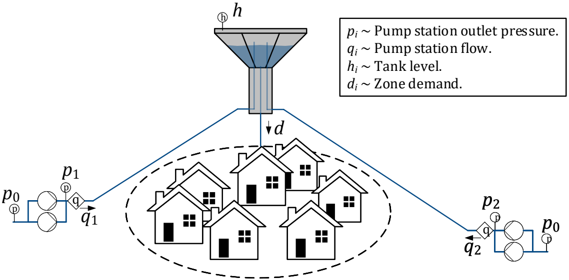


Figure: Layout of a water distribution network with an elevated reservoirs supplied from two pumping stations. The measured variables are the level of water in the elevated reservoir h, the outlet pressures and flows of the pumping stations p_i and q_i, respectively, and the water consumption in the zone, also denoted the demand d.

Model Predictive Control of the pumps

$$\min_{u_1,\cdots,u_N} \sum_{i=1}^{N} \sum_{k=0}^{M-1} \left(c(t_k)E_i(u_i(t_k),\bar{p}_i(t_k)) + K_i u_i(t_k)\right) + \kappa||V(t_0) - V(t_M)||^2, \tag{1}$$

$$\text{s.t} \quad \sum_{k=0}^{M-1} u_i(t_k) \leq \overline{U}_i \tag{2}$$

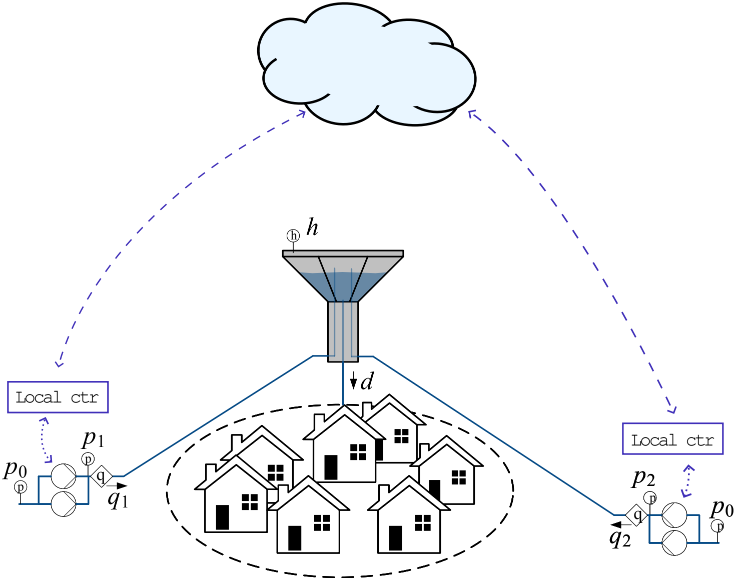
$$0 \leq u_i(t_k) \leq \overline{u}_i \tag{3}$$

$$V(t_{k+1}) = V(t_k) + \sum_{i=1}^{N} u_i(t_k) - g(t_k), V(t_0) = f(h(t_0)) \tag{4}$$

$$\underline{V} \leq V(t_k) \leq \overline{V} \tag{5}$$

- *Global* problem that require coordination between the pumps.

Architecture



Define

$$\boldsymbol{u}_i = [u_i(t_0), \ldots, u_i(t_M)]^\top, \tag{6}$$

$$\boldsymbol{U} = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_N] \tag{7}$$

$$\boldsymbol{V} = [V(t_0), \ldots, V(t_M)]^\top \tag{8}$$

$$\boldsymbol{g} = [g(t_0), \ldots, g(t_M)]^\top \tag{9}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 1 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \tag{10}$$

($\boldsymbol{A}$ is the lower uni-triangular matrix with all ones below the diagonal.)

$$\min_{\boldsymbol{U}} \sum_{i=1}^{N} y_i(U[:,i]) + \kappa \|\mathbb{1}_M^\top(\boldsymbol{U}\mathbb{1}_N - \boldsymbol{g})\|^2 \tag{11}$$

$$s.t. \tag{12}$$

$$\underline{\boldsymbol{V}} \leq \mathbb{1}_M f(h(t_0)) + \boldsymbol{A}(\boldsymbol{U}\mathbb{1}_N - \boldsymbol{g}) \leq \overline{\boldsymbol{V}} \tag{13}$$

$$\mathbb{1}_M^\top U[:,i] \leq \overline{U}_i \tag{14}$$

$$0 \leq U[k,i] \leq \overline{u}_i \quad \forall k \tag{15}$$

$$\tag{16}$$

- Still not separable due to $\boldsymbol{U}$.

Separable formulation of Controller

$$\min_{\widehat{\boldsymbol{U}}_i} \sum_{i=1}^{N} y_i(\widehat{U}_i[:, i]) + \kappa \| \mathbb{1}_M^\top (\widehat{\boldsymbol{U}}_i \mathbb{1}_N - \boldsymbol{g}) \|^2$$

s.t.

$$\left.\begin{aligned} \underline{V} \leq \mathbb{1}_M f(h(t_0)) + \boldsymbol{A}(\widehat{\boldsymbol{U}}_i \mathbb{1}_N - \boldsymbol{g}) \leq \overline{V} \\ \mathbb{1}_M^\top \widehat{\boldsymbol{U}}[:, i] \leq \overline{U}_i \\ 0 \leq \widehat{\boldsymbol{U}}[k, i] \leq \overline{u}_i \quad \forall k \end{aligned}\right\} \text{Local}$$

$$\left.\widehat{\boldsymbol{U}}_i = \boldsymbol{U} \quad \forall i, \right\} \text{Consensus}$$

Method of Multipliers

1. Minimization with respect to local variables.

$$\widehat{\boldsymbol{U}}_i^{j+1} = \underset{\widehat{\boldsymbol{U}}_i}{\operatorname{argmin}} \sum_{i}^{N} y_i(\widehat{U}_i[:, i]) + \kappa \| \mathbb{1}_M^\top (\widehat{\boldsymbol{U}}_i \mathbb{1}_N - \boldsymbol{g}) \|^2 + \boldsymbol{\lambda}_i^\top (\widehat{\boldsymbol{U}}_i - \boldsymbol{U}^j) + \frac{\rho}{2} (\| \widehat{\boldsymbol{U}}_i - \boldsymbol{U}^j \|_2^2)$$

s.t. local constraints.

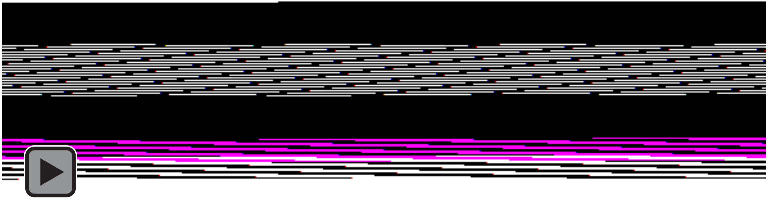
2. Minimization with respect to consensus variable.

$$\boldsymbol{U}^{j+1} = \frac{1}{N} \sum_{i=1}^{N} \left(\widehat{\boldsymbol{U}}_i^{j+1} + \frac{1}{\rho} \lambda_i^j \right)$$

3. Update Lagrange multipliers

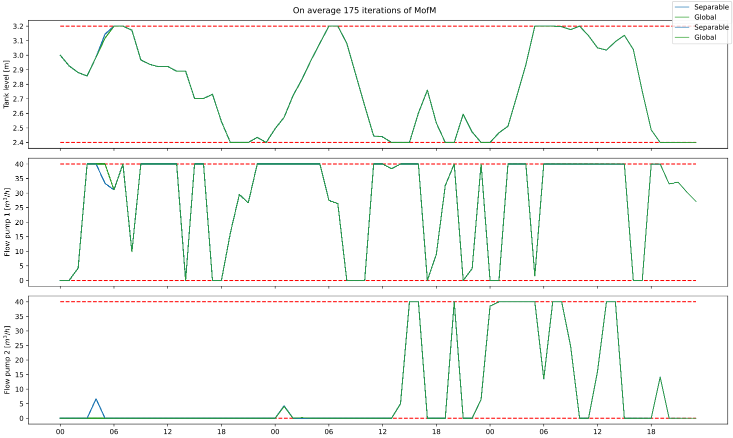
$$\lambda_i^{j+1} = \lambda^j + \rho(\widehat{\boldsymbol{U}}_i^{j+1} - \boldsymbol{U}^{j+1})$$

Simulation



Comparison with global solution

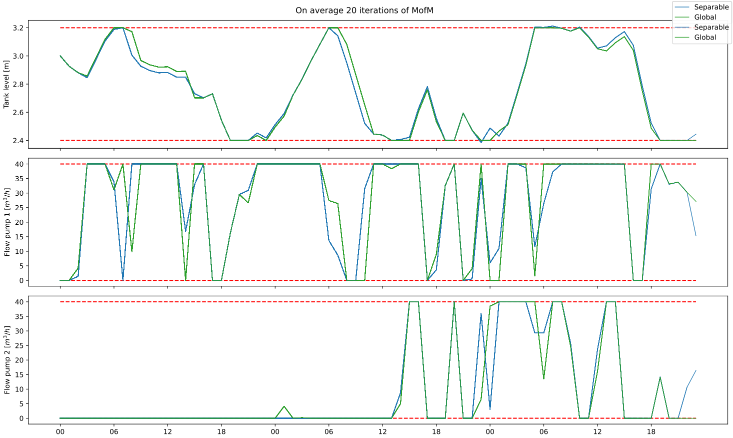
On average 175 iterations of M of M.



(Animation)

Comparison with global solution

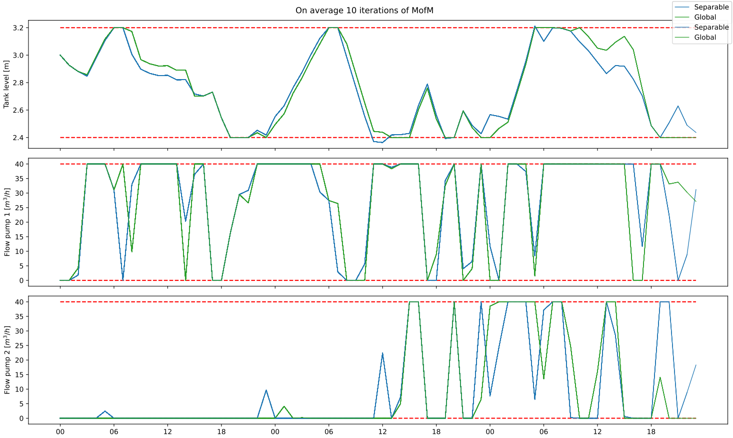
On average 20 iterations of M of M.



(Animation)

Comparison with global solution

On average 10 iterations of M of M.



(Animation)

Conclusion

- Still some work to be done on implementing the encrypted part.
- Figuring out the trade-off between accuracy of the solution (iterations of MofM) and the time to compute the solution.
  - How many iterations are enough? How to best quantify when to stop iterations?
- Implementation in lab :-)

Architecture