

Distributed control in water distribution network



Water distribution network

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Model Predictive Controller

Separable Controller

Method of Multipliers

Simulation of syste

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Conclusion

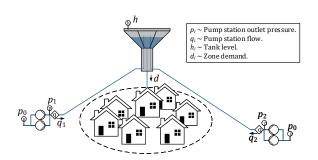


Figure: Layout of a water distribution network with an elevated reservoirs supplied from two pumping stations. The measured variables are the level of water in the elevated reservoir h, the outlet pressures and flows of the pumping stations p_i and q_i , respectively, and the water consumption in the zone, also denoted the demand d.



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$$\min_{u_1, \dots, u_N} \sum_{i=1}^{N} \sum_{k=0}^{M-1} \left(c(t_k) E_i(u_i(t_k), \bar{p}_i(t_k)) + K_i u_i(t_k) \right) \\
+ \kappa ||V(t_0) - V(t_M)||^2, \tag{1}$$

s.t
$$\sum u_i(t_k) \leq \overline{U}_i$$
 (2)

$$0 \le u_i(t_k) \le \overline{u}_i \tag{3}$$

$$V(t_{k+1}) = V(t_k) + \sum_{i=1}^{N} u_i(t_k) - g(t_k), V(t_0) = f(h(t_0))$$
(4)

 $V \le V(t_k) \le \overline{V} \tag{5}$

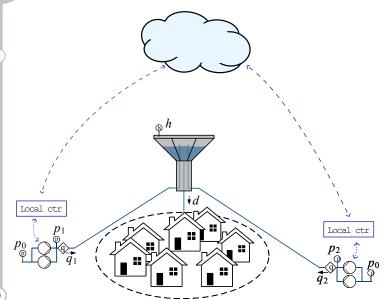
 Global problem that require coordination between the pumps.



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Define

$$\mathbf{u}_i = [u_i(t_0), \dots, u_i(t_M)]^\top, \tag{6}$$

$$\boldsymbol{U} = [\boldsymbol{u}_1, \dots, \boldsymbol{u}_N] \tag{7}$$

$$\mathbf{V} = [V(t_0), \dots, V(t_M)]^{\top}$$
 (8)

$$\boldsymbol{g} = [g(t_0), \dots, g(t_M)]^{\top}$$
 (9)

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix}$$
 (10)

(**A** is the lower uni-triangular matrix with all ones below the diagonal.)



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Matrix formulation

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$$\min_{\boldsymbol{U}} \sum_{i=1}^{N} y_i(\boldsymbol{U}[:,i]) + \kappa \|\mathbb{1}_{M}^{\top} (\boldsymbol{U}\mathbb{1}_{N} - \boldsymbol{g})\|^2$$
 (11)

$$s.t.$$
 (12)

$$\underline{\boldsymbol{V}} \leq \mathbb{1}_{M} f(h(t_0)) + \boldsymbol{A}(\boldsymbol{U} \mathbb{1}_{N} - \boldsymbol{g}) \leq \overline{\boldsymbol{V}}$$
 (13)

$$\mathbb{1}_{M}^{\top}U[:,i] \leq \overline{U}_{i} \tag{14}$$

$$0 \le U[k,i] \le \overline{u}_i \quad \forall k \tag{15}$$

(16)

- Still not separable due to **U**.



Separable formulation of Controller

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$$\min_{\widehat{\boldsymbol{U}}_i} \sum_{i=1}^N y_i(\widehat{\boldsymbol{U}}_i[:,i]) + \kappa \|\mathbb{1}_M^\top (\widehat{\boldsymbol{U}}_i \mathbb{1}_N - \boldsymbol{g})\|^2$$

s.t.

$$egin{aligned} & \underline{V} \leq \mathbb{1}_M f(h(t_0)) + m{A}(\widehat{m{U}}_i \mathbb{1}_N - m{g}) \leq \overline{V} \ & \mathbb{1}_M^{ op} \widehat{m{U}}[:,i] \leq \overline{U}_i \ & 0 \leq \widehat{m{U}}[k,i] \leq \overline{u}_i \quad orall k \ & \widehat{m{U}}_i = m{U} \quad orall i, \end{aligned}
ight.$$
 Local



Method of Multipliers

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1. Minimization with respect to local variables.

$$\widehat{\boldsymbol{U}}_{i}^{j+1} = \underset{\widehat{\boldsymbol{U}}_{i}}{\operatorname{argmin}} \sum_{i}^{N} y_{i}(\widehat{\boldsymbol{U}}_{i}[:,i]) + \kappa \|\mathbb{1}_{M}^{\top}(\widehat{\boldsymbol{U}}_{i}\mathbb{1}_{N} - \boldsymbol{g})\|^{2} + \lambda_{i}^{\top}(\widehat{\boldsymbol{U}}_{i} - \boldsymbol{U}^{j}) + \frac{\rho}{2}(\|\widehat{\boldsymbol{U}}_{i} - \boldsymbol{U}^{j}\|_{2}^{2})$$
s.t. local constraints.

2. Minimization with respect to consensus variable.

$$\boldsymbol{U}^{j+1} = \frac{1}{N} \sum_{i=1}^{N} \left(\widehat{\boldsymbol{U}}_{i}^{j+1} + \frac{1}{\rho} \lambda_{i}^{j} \right)$$

3. Update Lagrange multipliers

$$\boldsymbol{\lambda}_{i}^{j+1} = \boldsymbol{\lambda}^{j} + \rho(\widehat{\boldsymbol{U}}_{i}^{j+1} - \boldsymbol{U}^{j+1})$$



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Comparison with global solution

On average 175 iterations of M of M.

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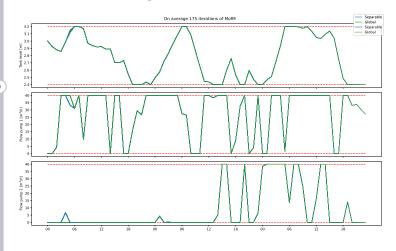
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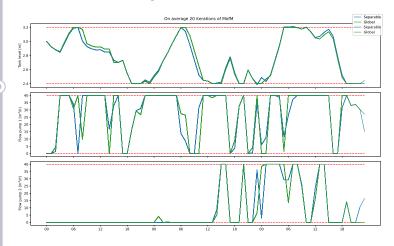
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On average 20 iterations of M of M.





Comparison with global solution

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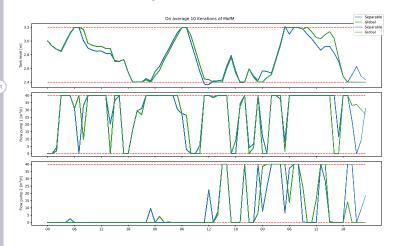
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On average 10 iterations of M of M.



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- Still some work to be done on implementing the encrypted part.
- ► Figuring out the trade-off between accuracy of the solution (iterations of MofM) and the time to compute the solution.
 - ► How many iterations are enough? How to best quantify when to stop iterations?
- ► Implementation in lab :-)



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