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Theorem - (Pumping Lemma for regular languages)

Let L be a regular language. Then there is a constant n such that if x is any string in L and $|x| \geq n$, then we may write $x = uvw$ in such a way that

L is infinite language

i) $|v| \geq 1$

ii) $|uv| \leq n$, and

iii) $\forall i \geq 0 \; uv^i w \in L$

Furthermore, n is no greater than the no. of states of the smallest DFA accepting L

$|x| \geq n$

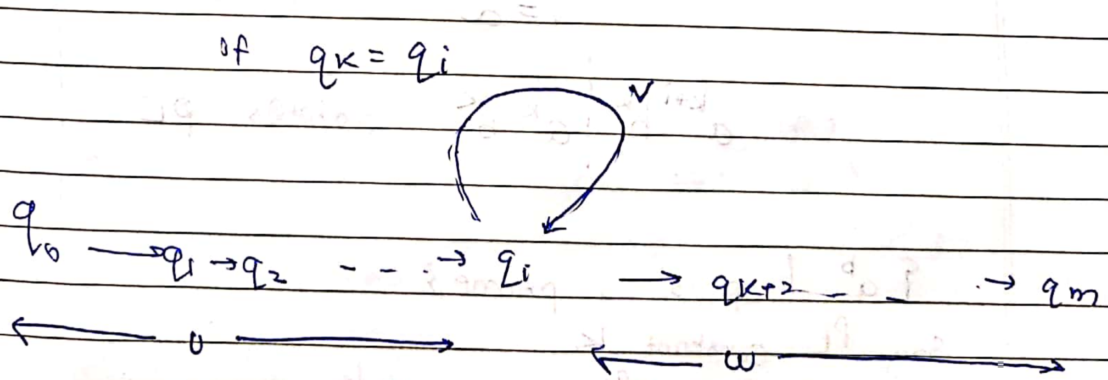
$q_0 \rightarrow q_1 \rightarrow q_2 \;---\; \rightarrow q_i \rightarrow q_{i+1} \rightarrow \;---\; \rightarrow q_k \rightarrow q_{k+1} \;--$

$\rightarrow q_m$

$|x| \geq n$

$n+1$

if $q_k = q_i$



L is regular $\Rightarrow$ L satisfies PL

L doesnot satisfy PL $\Rightarrow$ L is not regular

$\{a^n b^n \mid n \geq 0\}$

Say PL constant is k

$x = a^k b^k \quad v = ab$

$u = a^{k-1}$

$uv = |a^k b| > k$

if $v = a$

$u = a^{k-1}, \quad v = a$ then

$a^{k-1} a^i b^k \Leftarrow$ doesnot accept

$\{ww \mid w \in (a+b)^*\}$

Say PL constant is k

$x = a^k b^k a^k b^k$

$v = a$

$a^{k+i} b^k \mid a^k b^k$ violates PL

if $v = aa$ then couldnot use PL

$\{a^p \mid p \text{ is a prime}\}$

Say PL constant k

$x = a^q \quad q > k \quad q$ is prime

$x = a^k a^{q-k}$

$x = a^m \Rightarrow uvw \quad m$ is prime

$|uvw| = m$

$|v| = i$

$\{ww^R \mid w \in (a+b)^*\}$ R = reverse

$L \subseteq \Sigma^*$

$\bar{L} = \Sigma^* - L$

$\bar{L}$ is regular if L is regular

$L(A) = L$

$A = (Q, \Sigma, \delta, q_0, F)$

$A' = (Q, \Sigma, \delta, q_0, Q-F)$

$x \in L(A) \iff \hat{\delta}(q_0, x) \in F$

$\iff \hat{\delta}(q_0, x) \notin Q-F$

$\iff x \in L(A')$

$L_1 \cap L_2 = \overline{\bar{L_1} \cup \bar{L_2}}$

$\Sigma = \{0, 1\}$

even no of 0s

odd no. of 1s

$A_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ $L_1 = L(A_1)$

$A_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ $L_2 = L(A_2)$

$(L_1 \cap L_2 = \phi)$?

~~A∩A~~ $L(A) = L_1 \cap L_2$

$A = (Q_1 \times Q_2, \Sigma, \delta, (q_1, q_2), F_1 \times F_2)$

$\delta((p,q), a) = (\delta_1(p,a), \delta_2(q,a))$

$(Q_1 \times Q_2) \times \Sigma \rightarrow (Q_1 \times Q_2)$

HW:- $x \in L(A_1)$ & $x \in L(A_2)$ $\longrightarrow$ $L(A)$

L^R ~~$= \{x \mid x \in \Sigma^* \& x^R$~~

$L^R = \{x \in \Sigma^* \mid x^R \in L\}$

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Regular language are closed under

- Union
- concatenation
- Kleene closure
- Complement
- Intersection
- Reversal
- Difference $(L_1 - L_2)$
- Right quotient
- Substitution & homomorphism
- Inverse homomorphism
- Cycle(L)

~~Right~~

$L_1, L_2 \subseteq \Sigma^*$

Right quotient of L_1 by L_2

$L_1/L_2 = \{ x \in \Sigma^* \mid \exists y \in L_2 \text{ s.t. } xy \in L_1 \}$

$L_1 = \{a, ab, bab, baba\}$
$L_2 = \{a, ab\}$
$L_1/L_2 = \{\varepsilon, bab, b\}$

$L_1 = 10^*1$
$L_2 = 1$
$L_1/L_2 = 10^*$

$L_1 = 0^*10^*$
$L_1/0^* = 0^*10^*$
$L_1/10^* = 0^*$
$L_1/1 = 0^*$

If L is regular, then so is L/L' for any L'

$A = (Q, \Sigma, \delta, q_0, F)$ s.t. $L(A) = L$

$A' = (Q, \Sigma, \delta, q_0, F')$

$F' = \{q \in Q \mid \hat{\delta}(q, x) \in F \text{ for some } x \in L'\}$

HW - $L(A') = L/L'$

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Homomorphism

$\Sigma_1 \qquad \Sigma_2$

$h : \Sigma_1^* \to \Sigma_2^*$

$h(xy) = h(x)h(y)$

$h(\varepsilon) = \varepsilon$

$x = a_1 a_2 \ldots a_n \in \Sigma_1^*$

$h(x) = h(a_1)h(a_2) \ldots h(a_n)$

Substitution

$\Sigma_1 = \{a, b\}$

$\Sigma_2 = \{0, 1\}$

$h : \Sigma_1 \to p(\Sigma_2^*)$

$h(a) = \{0^n \mid n \geq 0\} - L_1$

$h(b) = \{1^n \mid n \geq 0\} - L_2$

$h(L) = \bigcup_{x \in L} h(x)$

$L = \{a^n b^n \mid n \geq 0\}$

$h(L) = 0^* 1^*$

Thm Σ

$h(a)$ $\forall a \in \Sigma$ is regular

$h(L)$ is regular for every regular language L under Σ

$L = L(r)$

$\forall a \in \Sigma \quad h(a) = r_a \quad L(r_a) = h(a)$

Induction Basis ϕ, ε, a

r_1, r_2

$r_1 + r_2$ $\quad r = \phi \quad r' = \phi \quad h(L(\phi)) = \phi = L(r')$

$r_1 r_2$ $\quad r = \varepsilon \quad r' = \varepsilon \quad h(L(\varepsilon)) = \{\varepsilon\} = L(r')$

r_1^* $\quad r = a \quad r' = r_a \quad h(L(r)) = h(\{a\}) = L(r_a) = L($

IH r_1, r_2 true $< n$ operation.

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$r = r_1 + r_2$

IS – $L(r') = h(L(r))$

$r' = r_1' + r_2'$

$L(r') = \cancel{L(r_1') \cup L(r_2')}\ L(r_1' + r_2')$

$= L(r_1') \cup L(r_2')$

$= h(L(r_1)) \cup h(L(r_2))$

$= h(L(r_1 + r_2))$

$= h(L(r))$

Inverse Homomorphism

$h : \Sigma_1 \longrightarrow \Sigma_2^*$

$L \subseteq \Sigma_2^*$

$h^{-1}(L) = \{ x \in \Sigma_1^* \mid h(x) \in L \}$

only those which have preimages.

$L(A) = L \qquad A = (Q, \Sigma_2, \delta, q_0, F)$

$A' = (Q, \Sigma_1, \delta', q_0, F)$

$\delta' : Q \times \Sigma_1 \rightarrow Q$

$\delta'(q, a) = \hat{\delta}(q, h(a))$

$L(A') = h^{-1}(L)$

$\forall x \in \Sigma_1^*$

$\hat{\delta}'(q_0, x) = \hat{\delta}(q_0, h(x))$

$|x| = 0$ ✓

$|x| = 1$ ✓

IH $|x| = k$

IS $|x| = k+1$

$\hat{\delta}'(q_0, xa) = \delta'(\hat{\delta}'(q_0, x), a)$

$= \delta'(\hat{\delta}(q_0, h(x)), a)$

$= \hat{\delta}(\hat{\delta}(q_0, h(x)), h(a))$

$= \hat{\delta}(q_0, h(x)h(a))$

$= \hat{\delta}(q_0, h(xa))$

$\text{Cycle}(L) = \{ x_1 x_2 \mid x_2 x_1 \in L \}$

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DFA minimization

$p \equiv q$ indistinguishable ~~or~~ equivalent

if $\forall x \in \Sigma^*$

if $\hat{\delta}(p, x) \subset F$ iff $\hat{\delta}(q, x) \subset F$

→ Equivalence relation – reflexive, transitive, symm.

$a \in \Sigma$

$r = \delta(p, a)$ if $r \not\equiv s$ then $p \not\equiv q$ ∴ of ax

$s = \delta(q, a)$

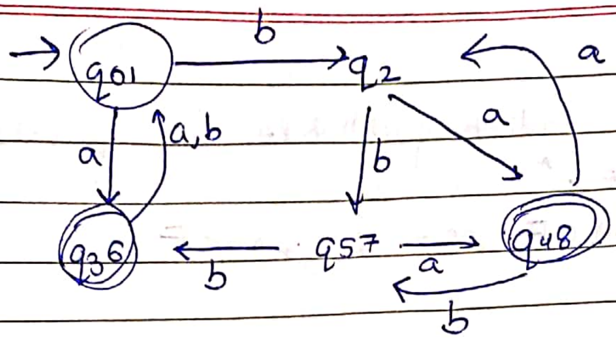
δ	a	b
→ q_0	q_3	q_2
q_1	q_6	q_2
q_2	q_8	q_5
q_3	q_0	q_1
q_4	q_2	q_5
q_5	q_4	q_3
q_6	q_1	q_0
q_7	q_4	q_0
q_8	q_2	q_7
q_9	q_7	q_{10}
q_{10}	q_6	q_7

states not accessible:
$\{q_9, q_{10}\}$

states accessible:
$\{q_0, q_2, q_3, q_1, q_5, q_8, q_6, q_4, q_7\}$

loop
0 1 2

	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
q_1								
q_2	X^2	X^2						
q_3	X^0	X^0	X^0					
q_4	X^0	X^0	X^0	X^1				
q_5	X^1	X^1	X^1	X^0	X^0			
q_6	X^0	X^0	X^0		X^1	X^0		
q_7	X^1	X^1	X^1	X^0	X^0		X^0	
q_8	X^0	X^0	X^0	X^1		X^0	X^1	X^0



An equivalen rel^n $\sim$ on Σ^* is said to be right invariant if for $x, y \in \Sigma^*$

$$x \sim y \Rightarrow \forall z\,(xz \sim yz)$$

Let L be a language over Σ

Define the rel^n $\sim_L$ on Σ^* by

$x, y, z \in \Sigma^*$

$$x \sim_L y \quad \text{iff} \quad \forall z\,(xz \in L \Leftrightarrow yz \in L)$$

$x \sim_L y$

$z \in \Sigma^*$

$$\forall w\,(x\underbrace{zw}_{u} \in L \Leftrightarrow y\underbrace{zw}_{u} \in L)$$

since $x \sim_L y$

so

$x \sim_L y$ is right invariant

$$A = (Q, \Sigma, \delta, q_0, F)$$

$\sim_A$ on Σ^*

$x, y \in \Sigma^*$

$$x \sim_A y \quad \text{iff} \quad \hat{\delta}(q_0, x) = \hat{\delta}(q_0, y)$$

$\sim_A$ is equivalence relation and right invariant

$$\hat{\delta}(q_0, xz) = \hat{\delta}(\hat{\delta}(q_0, y), z)$$
$$= \hat{\delta}(q_0, yz)$$

Myhill Nerode Theorem - if and only if condition

Let L be a language over Σ
The following ~~eq~~ statements are equivalent

1- L is accepted by a DFA
2- There exists a right invariant equivalence relation $\sim$ of finite index on Σ^* st L is the union of some of the equivalence classes of $\sim$
3- The equivalence rel^n $\sim_L$ is of finite order.

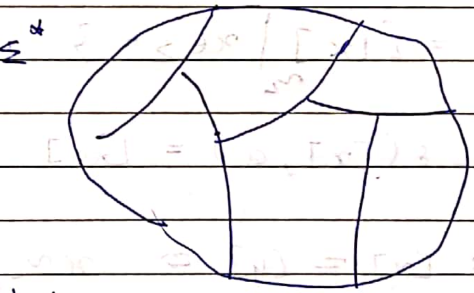
index = no. of equiv. classes

① ⇒ ②

$$A = (Q, \Sigma, \delta, q_0, F)$$
$$L = L(A)$$

$\underline{\sim_A}$

for $x \in \Sigma^*$ if $\hat{\delta}(q_0, x) = p$



$$[x]_{\sim_A} = \{ y \in \Sigma^* \mid \hat{\delta}(q_0, y) = p \}$$

$q \in Q$ $\quad C_q = \{ x \in \Sigma^* \mid \hat{\delta}(q_0, x) = q \}$

$$L = \{ x \in \Sigma^* \mid \hat{\delta}(q_0, x) \in F \}$$
$$= \bigcup_{p \in F} \{ x \in \Sigma^* \mid \hat{\delta}(q_0, x) = p \}$$
$$= \bigcup_{p \in F} C_p$$

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(2) $\Rightarrow$ (3)

$\sim$

$\sim_L$ is a refinement of $\sim$

if $x, y \in \Sigma^*$ $x \sim y$ the $x \sim_L y$

$x \sim y \Rightarrow \forall z (xz \sim yz)$

$\forall (xz \in L \longleftrightarrow yz \in L)$

xz and yz belong to same eqv. class

so either both accepted or

both rejected.

(3) $\Rightarrow$ (1)

Assume $\sim_L$ is of finite index.

$A_L = (Q, \Sigma, \delta, q_0, F)$

$Q = \{ [x]_{\sim_L} \mid x \in \Sigma^* \}$

δ : $\delta([x], a) = [xa]$ $\forall [x] \in Q, a \in \Sigma$

Well defined : $[x] = [y] \Rightarrow x \sim_L y \Rightarrow xa \sim_L ya$

$\Downarrow$

$[xa] = [ya]$

$q_0 = [\varepsilon]$

$F = \{ [x] \in Q \mid x \in L \}$

$L(A_L) = L$

To prove : $\hat{\delta}(q_0, w) = [w]$ $\forall w \in \Sigma^*$

$w \in L \Longleftrightarrow [w] \in F$

Base $|w| = 0 \quad w = \varepsilon$

$\hat{\delta}(q_0, \varepsilon) = q_0 = [\varepsilon]$

$\hat{\delta}(q_0, a) = \hat{\delta}([\varepsilon], a) = [\varepsilon a] = [a]$

IS – $\hat{\delta}(q_0, xa) = \delta(\hat{\delta}(q_0, x), a)$

$= \delta([x], a)$ — IH

$= \delta\, [xa]$ by def.

~~let L be a language over Σ~~

$\text{index}(\sim_L) \leq \text{index}(\sim_A) \leq$ no. of states

$L = \{x \in \{a,b\}^* \mid ab \text{ is a substring of } x\}$

$\sim_L \begin{pmatrix} 1. & [\varepsilon] \\ 2. & [a] \\ 3. & [ab] \end{pmatrix}$

$x = \varepsilon \quad z = b$

$x = a \quad \varepsilon b \notin L$ but $ab \in L$

$x \in \{a, b\}^*$

ab is substring — $[ab]$

ab ✗ substring — a^n, b^n, $b^m a^n$

$L = \{a^n b^n \mid n \geq 0\}$

diff. $\left(\begin{matrix} [a^n] \\ [a^m] \end{matrix} \right.$ $\quad b^n$

$m \neq n$

Thm. For every DFA A, There is eq. min. state DFA A'

Proof
$A = (Q, \Sigma, \delta, q_0, F)$
$A' = (Q', \Sigma, \delta', q_0', F')$
$L(A) = L(A')$

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$Q' = \{ [q] \mid q$ is accessible from $q_0 \}$
~~These~~ The eq. classes of Q wrt $\equiv$
that contain states accessible from q_0.

$q' = [q_0]$
$F' = \{ [q] \in Q \mid q \in F \}$
$\delta' : Q' \times \Sigma \to Q'$

$\delta'([q], a) = [\delta(q,a)]$

well defined $[p] = [q]$

$\hat{\delta}(\delta(p,a), x) = \hat{\delta}(\delta(p,q), x)$
$\hat{\delta}(p, ax) = \hat{\delta}(q, ax)$

$\delta(p,a) \equiv \delta(q,a)$

claim - $L(A) = L(A')$

$\hat{\delta'}([q_0], x) = [\hat{\delta}(q_0, x)] \quad \forall x \in \Sigma^*$

Base : ε
$[q_0] = [q_0]$

IS - $\hat{\delta'}([q_0], xa) = \delta'(\hat{\delta'}([q_0], x), a)$
$= \delta'([\hat{\delta}(q_0, x)], a)$ By def
$[\delta(\hat{\delta}(q_0,x), a)]$
$= [\hat{\delta}(q_0, xa)]$

claim 2 A' is a minimal state DFA accepting L

To prove : The index of $(\sim_L)$ = The index of $\sim_{A'}$
index of $\sim_L \le$ The index of $\sim_{A'}$

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Contradiction : suppose $\sim_{A'}$ has a greater index

then $\exists$ $x \in \Sigma^*$ $y \in \Sigma^*$

$x \sim_L y$ but not $x \sim_{A'} y$

$\hat{\delta}'([q_0], x) \neq \hat{\delta}'([q_0], y)$

$p = [\hat{\delta}([q_0], x)] \neq [\hat{\delta}([q_0], y)] = q$

contradiction How?

FA with output

Moore machine

$M = (Q, \Sigma, \Delta, \lambda, \delta, q_0)$

$\lambda : Q \rightarrow \Delta$

$x = a_1 a_2 \ldots a_n, \; n \geq 0$

The output $\lambda(q_0)\, \lambda(q_1) \ldots \lambda(q_n)$

$q_1 = \delta(q_0, a_1)$

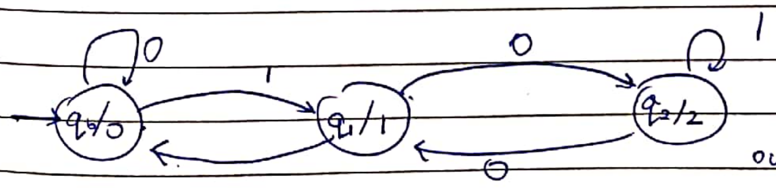
$q_2 = \delta(q_1, a_2)$

$\vdots$

$\delta(q_{i-1}, a_i) = q_i$

For DFA $\Delta = \{0, 1\}$

$\lambda(q) = 1 \leftarrow$ Final state



00101

output: 0 0 0 1 2 2

residue % 3

treated as

binary integer.

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FA with output

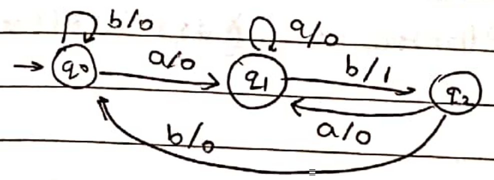
Moore m/c
Mealy m/c

Mealy

$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$

$\lambda : Q \times \Sigma \rightarrow \Delta$

$x = a_1 a_2 \ldots a_n$



δ	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_0

λ	a	b
q_0	0	0
q_1	0	1
q_2	0	0

2-DFA $\quad \delta : Q \times \Sigma \rightarrow (Q \times \{L, R\})$

input : wx

$w q x$

$L(M) = \{ w \in \Sigma^* \mid q_0 w \vdash^* w p \quad$ for som $p \in F \}$

Equivalent to DFA

Two DFA ← language same or not?

DFA language infinite, empty, finite?

if L(M) is not empty then it must accept a ~~language~~ string of length < n(states)

Proof:- $x = uvw$ Pumping lemma

infinite if it accepts

$n \leq |x| < 2n$

Context Free Grammar

$$G = (N, \Sigma, P, S)$$

$A \to \alpha$ $\alpha \in (N \cup \Sigma)^*$

Simplification of CFG

Useless Symbols: Let G be a grammar.
A symbol $X \in N \cup \Sigma$ is useful if there is a derivation
$S \overset{*}{\Rightarrow} \alpha X \beta \overset{*}{\Rightarrow} w$ for some $\alpha, \beta \in (N \cup \Sigma)^*$
and $w \in \Sigma^*$ otherwise X is useless

1. $S \overset{*}{\Rightarrow} \alpha X \beta$
2. $X \overset{*}{\Rightarrow} x$

Necessary but not sufficient

$G: S \to AB$
$A \to a$

$L(G) = \phi$

GENERATING

A is useless (B is also useless)

Proof:
For every $G = (N, \Sigma, P, S)$ L(G) must be non empty
there exist $G' = ((N', \Sigma, P', S)$
$A \in N'$
$A \overset{*}{\Rightarrow} x$

1. old N $\leftarrow \phi$
2. New N $\leftarrow \{A \mid A \to x \in P$ for some $x \in \Sigma^*\}$
3. while old N $\neq$ New N do
4. old N $\leftarrow$ New N
5. New N $\leftarrow$ old N $\cup \{A \mid A \to \alpha \in P$ for some α in $(\Sigma \cup \text{old N})^*\}$
6. $N' \leftarrow$ New N

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REACH ABLE

For every $G = (N, \Sigma, P, S)$

$\exists$ $G' = (N', \Sigma, P', S)$ s.t

$X \in (N' \cup \Sigma')$

$S \overset{*}{\Rightarrow} \alpha X \beta \qquad \alpha, \beta \in (N' \cup \Sigma)^*$

$N' \cup \Sigma' = \{ S, \ldots \}$

$A \rightarrow X_1 X_2 \ldots X_K$

Readable → generating

$S \rightarrow \cancel{AB} \mid a$ B is non generating

$\cancel{A \rightarrow a}$

Readable $\{S, a\}$

$S \rightarrow \cancel{AB} \mid a$ Readable then generating

$A \rightarrow a$

Eg:

$S \rightarrow aS \mid A \mid \epsilon$

$A \rightarrow a$

$\cancel{B \rightarrow aa}$

$\cancel{C \rightarrow aCB}$

genset $= \{A, B, S\}$

Readable $= \{S, A\}$

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Thm For any CFG $G = (N, \Sigma, P, S)$. There is a CFG G' with no ε-production or unit production such that $L(G') = L(G) - \{\varepsilon\}$

Proof - $\hat{P}$ contains all production in P and closed under

i) If $A \to \alpha B \beta$ and $B \to \varepsilon$ are in $\hat{P}$ then Then $A \to \alpha\beta$ is in $\hat{P}$, and

ii) If $A \to B$ and $B \to \gamma$ are in $\hat{P}$, then $A \to \gamma$ is in $\hat{P}$.

$L(G) \subseteq L(\hat{G})$

$L(\hat{G}) \supseteq L(G)$ $\qquad$ $A \to \alpha\beta$ consists of $A \to \alpha B \beta$ and $B \to \varepsilon$

$L(G) = L(\hat{G})$

Suppose a derivation of smallest length with ε production

$$S \xRightarrow{*} \alpha_1 B \alpha_2 \xRightarrow{1} \alpha_1 \alpha_2 \xRightarrow{*} x$$

Assume we applied $A \to \alpha B \beta$ earlier

$$S \xRightarrow[m]{*} \gamma A \delta \xRightarrow{1} \gamma \alpha B \beta \delta \xRightarrow{n} \alpha_1 B \alpha_2 \xRightarrow{1} \alpha_1 \alpha_2 \xRightarrow{k} x$$

$m+n+k+2$

$$S \xRightarrow[m]{*} \gamma A \delta \xRightarrow{1} \gamma \alpha \beta \delta \xRightarrow{n} \alpha_1 \alpha_2 \xRightarrow{k} x$$

$m+n+k+1$

$G' = (N, \Sigma, P', S)$

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Standard form.

CNF : Chomsky Normal Form

$A \to BC$

$A \to a$

$G = (N, \Sigma, P, S)$ No ε production terminal. $G' = (N', \Sigma, P', S)$

$N' = N \cup \{A_a, \ldots\}$

$A \to \alpha = (\underbrace{x_1, x_2}_{\text{Nonterm}} \ldots x_i \ldots x_k)$ $x_i \in (N \cup \Sigma)$

$x_i = a$ $P' = P \cup \{A_a \to a, \ldots\}$

$A_a \to a$

GNF (Greibach Normal Form)

$A \to a B_1 B_2 \ldots B_K$ $k \geq 0$

Lemma: Let $G = (N, \Sigma, P, S)$ be a CFG

Let $A \to \alpha_1 B \alpha_2$ be a production in P and $B \to \beta_1 | \beta_2 | \ldots | \beta_r$ be the set of all B-production. Let $G_1 = (N_1, \Sigma, P_1, S)$ be obtained from G by deleting the production $A \to \alpha_1 B \alpha_2$ from P and adding the productions

$A \to \alpha_1 \beta_1 \alpha_2 | \alpha_1 \beta_2 \alpha_2 | \ldots | \alpha_1 \beta_r \alpha_2$

$L(G) = L(G_1)$

Lemma: Let $G = (N, \Sigma, P, S)$ be a CFG

Let $A \to A\alpha_1 | A\alpha_2 | \ldots | A\alpha_r$ be the set of all A-productions for which A is the leftmost symbol in the rhs. Let $A \to \beta_1 | \beta_2 \ldots | \beta_s$ be the remaining A-productions

$G_2 = (N \cup \{B\}, \Sigma, P_2, S)$ be the CFG formed by adding the non-terminal B to N and replacing all A-prod^n by the production

1) $A \to \beta_i$, $A \to \beta_i B$ } $1 \le i \le s$

2. $B \to \alpha_i$, $B \to \alpha_i B$ | $1 \le i \le r$

① $A \underset{a}{\Rightarrow} A\alpha_{i_1} \Rightarrow A\alpha_{i_2}\alpha_{i_1} \cdots \Rightarrow A\alpha_{ip}\alpha_{ip-1} \cdots \alpha_{i_1}$

$\Rightarrow \beta_j \alpha_{ip}\alpha_{ip-1} \cdots \alpha_{i_1}$

② $A \underset{a_2}{\Rightarrow} \beta_j B \Rightarrow \beta_j \alpha_{ip} B \cdots \Rightarrow \beta_j \alpha_{ip}\alpha_{ip-1} \ldots \alpha_{i_1}$

HW - Reverse.

Thm - Every CFL L ~~w/o~~ w/o ε can be generated by a grammar for which every prod^n is of the form $A \to a B_1 B_2 \ldots B_k$, $k \ge 0$.

Proof - All prod^n are of form

$A \to BC$

$A \to a$ i.e CNF form

$G = (N, \Sigma, P, S)$

$N = \{A_1, A_2 \ldots A_m\}$

Step 1 - $A_i \to A_j \gamma$ $j > i$

Suppose $1 \le i < k$ already GNF

A_k production

$A_k \to A_j \gamma$ $\quad j < k$

$A_k \to A_l \gamma$ $\quad l \geq k$

$j = k$

$A_k \to A_k \gamma$

$A_i \to A_j \gamma$ $\quad j > i$

$A_i \to a \gamma$ $\quad a \in \Sigma$

$B_i \to \gamma$ $\quad \gamma \in (N \cup \{B_1, B_2, \ldots B_k\})^*$

Step 2 $\quad A_m \to a\gamma$

$A_{m-1} \to A_m \gamma$

Ex. $A \to BB$

$B \to AC \mid a$

$C \to AB \mid BA \mid a$

} CNF

$A_1 \to A_2 A_2$

$A_2 \to A_1 A_3 \mid a$

$A_3 \to A_1 A_2 \mid A_2 A_1 \mid a$

$\boxed{A_1 \to A_2 A_2}$

$A_2 \to A_2 A_2 A_3 \mid a$ $\longrightarrow$ Lemma 2

$A_3 \to A_2 A_2 A_2 \mid A_2 A_1 \mid a$

$\boxed{A_2 \to a B_2 \mid a}$

$B_2 \to A_2 A_3 \mid A_2 A_3 B_2$

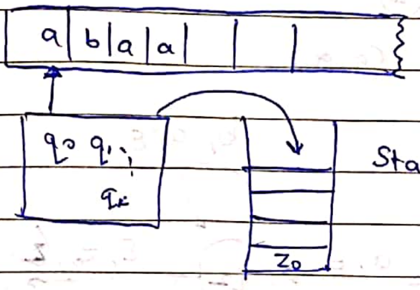
$A_3 \to A_2 A_2 A_2 \mid A_2 A_1 \mid a$

$\boxed{A_3 \to a B_2 A_2 A_2 \mid a B_2 A_2 A_1 \mid a B_2 A_1 \mid a}$

$\boxed{B_2 \to a B_2 A_3 \mid a A_3 \mid a B_2 A_3 B_2 \mid a A_3 B_2}$

CS-205

Pushdown automata (Non deterministic) (PDA)
↳ accepts CFL



Σ

Γ (alphabet in stack)

Stack

Formally, a PDA is a 7-tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \longrightarrow Q \times \Gamma^*$$

finite sub sets of $Q \times \Gamma^*$

$$\delta(q, a, z) = \{(p_1, \gamma_1)\ (p_2, \gamma_2) \ldots (p_k, \gamma_k)\}$$

$\gamma_i \in \Gamma^*$

$\gamma_i = x_1 x_2 \ldots x_m$

Conf $\quad (q, aw, z\gamma) \longmapsto (p, w, \alpha\gamma)$

$\delta(q, a, z)$ contains (p, α)

Notion of accepting by final state

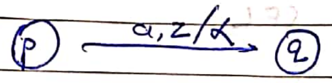
$L(M) = \{x \in \Sigma^* \mid (q_0, x, Z_0) \overset{*}{\longmapsto} (p, \varepsilon, \alpha)$ for some $p \in F$ and $\alpha \in \Gamma^*\}$.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \phi)$$

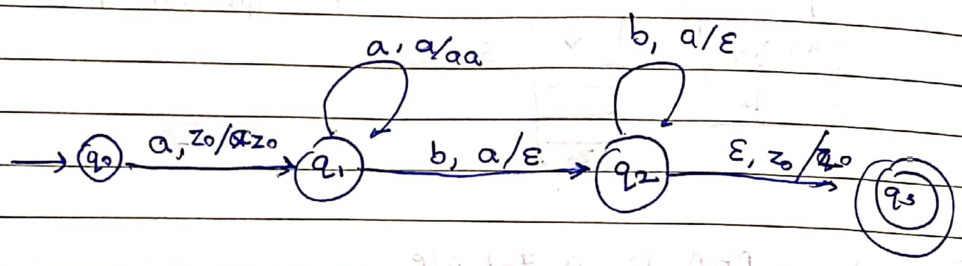
acceptance by empty stack

$L(M) = \{x \in \Sigma^* \mid (q_0, x, Z_0) \overset{*}{\longmapsto} (p, \varepsilon, \varepsilon)$ for some $p \in Q\}$

$\{a^n b^n \mid n \geq 1\}$



(p, a, z) contain (q, α)



$\delta(q_0, a, z_0) = (q_1, a z_0)$

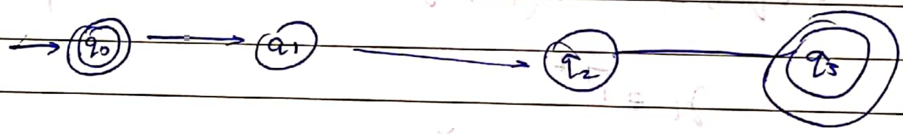
$\delta(q_1, a, a) = (q_1, aa)$

$\delta(q_1, b, a) = (q_2, \varepsilon)$

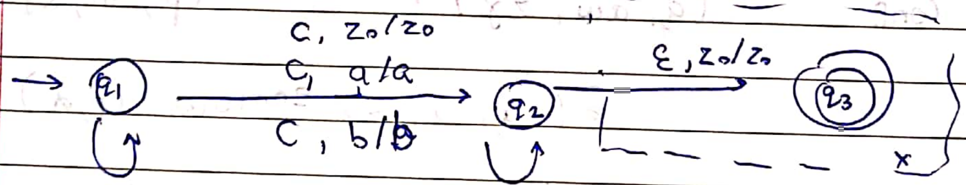
$\delta(q_2, b, a) = (q_2, \varepsilon)$

$\delta(q_2, \varepsilon, z_0) = (q_3, z_0)$

$\{a^n, b^n \mid n \geq 0\}$



$\{wcw^R \mid w \in \{a,b\}^*\}$



a, z_0/az_0 a, a, ε

b, z_0/bz_0 b, b, ε

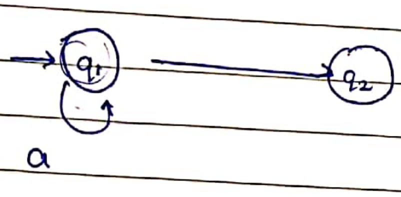
a, a/aa

a, b/ab   ε, z_0/ε ← PDA by empty stack

b, a/ba

b, b/bb

$\{ww^R \mid w \in \{a,b\}^*\}$ ← Replace a by ε



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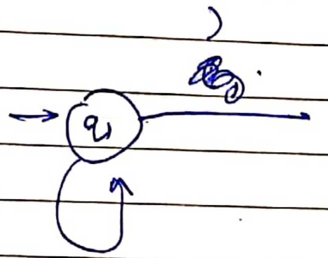

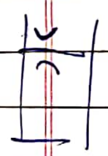

$($, z_0 / $(z_0$

$($, $($ / $(($

$)$, $($ / ε

ε, z_0 / ε