Accelerating Unstructured Mesh Applications using Custom Streaming Architectures

Kyrylo Tkachov

Imperial College London

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Supervisor: Prof. Paul Kelly
2nd marker: Dr. Tony Field

Unstructured meshes

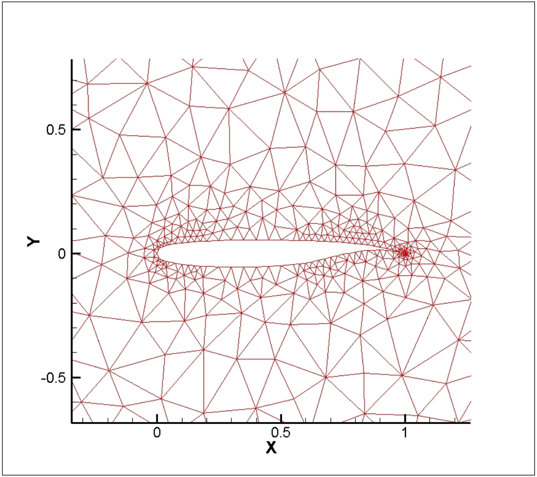


Image from Department of Environmental Engineering, University of Genoa

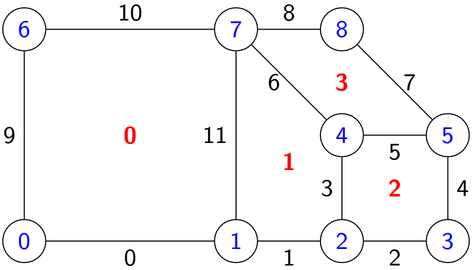
Airfoil: Indirection maps

Elements:

- Nodes
- Cells
- Edges

edge-to-node map = {0,1, 1,2, 2,3, 2,4, 3,5, 4,5, 4,7, 5,8, 7,8, 0,6, 6,7}

cell-to-node map = {0,9,10,11, 1,2,4,7, 2,3,4,5, 4,5,7,8}



Airfoil: Data sets

Data set name	Associated with	Type/Dimension
x	Nodes	$\mathbb{R} \times \mathbb{R}$
q	Cells	$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
q_old	Cells	$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
res	Cells	$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
adt	Cells	$\mathbb{R}$
bound	Edges	$\{0, 1\}$

Airfoil: Kernels

Kernel Name	Iterates over	Reads	Writes
save_soln	Cells	q	q_old
adt_calc	Cells	x, q	adt
res_calc	Edges	x, q, adt	res
bres_calc	(Boundary) Edges	x, q, adt, bound	res
update	Cells	q_old, adt, res	q, res

```
void res_calc(float *x1, float *x2, float *q1, float *q2,
                          float *adt1, float *adt2, float *res1, float *
                                res2) {
  float dx,dy,mu, ri, p1,vol1, p2,vol2, f;
  dx = x1[0] - x2[0];
  dy = x1[1] - x2[1];
  ri = 1.0f/q1[0];
  p1 = gm1*(q1[3]-0.5f*ri*(q1[1]*q1[1]+q1[2]*q1[2]));
  vol1 = ri*(q1[1]*dy - q1[2]*dx);
  ri = 1.0f/q2[0];
  p2 = gm1*(q2[3]-0.5f*ri*(q2[1]*q2[1]+q2[2]*q2[2]));
  vol2 = ri*(q2[1]*dy - q2[2]*dx);
  mu = 0.5f*((*adt1)+(*adt2))*eps;
  f = 0.5f*(vol1* q1[0] + vol2* q2[0] ) + mu*(q1[0]-q2[0]);
  res1[0] += f;
  res2[0] -= f;
  f = 0.5f*(vol1* q1[1] + p1*dy + vol2* q2[1] + p2*dy) + mu*(q1
      [1]-q2[1]);
  res1[1] += f;
  res2[1] -= f;
  f = 0.5f*(vol1* q1[2] - p1*dx + vol2* q2[2] - p2*dx) + mu*(q1
      [2]-q2[2]);
  res1[2] += f;
  res2[2] -= f;
  f = 0.5f*(vol1*(q1[3]+p1) + vol2*(q2[3]+p2) ) + mu*(q1[3]-q2[3])
      ;
  res1[3] += f;
  res2[3] -= f;
}
```

res_calc data requirements

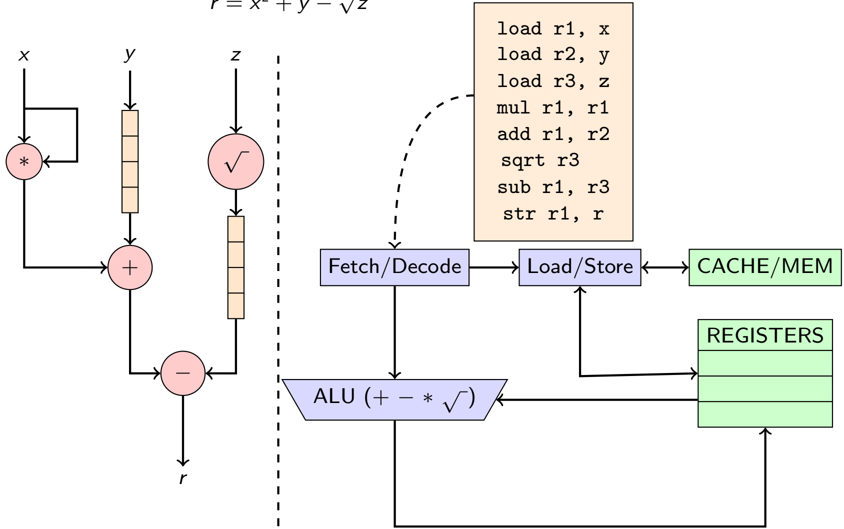
- Iterates over edges
- Processing each edge requires 2 cells, 2 nodes.
- Each edge **increments** two cells (+=).
- Most computationally intensive kernel in Airfoil.

Kernel application and double dereferencing

```
res_calc(
        &x[2*edge[2*i]],
        &x[2*edge[2*i+1]],
        &q[4*ecell[2*i]],
        &q[4*ecell[2*i+1]],
        &adt[ecell[2*i]],
        &adt[ecell[2*i+1]],
        &res[4*ecell[2*i]],
        &res[4*ecell[2*i+1]]
        );
```

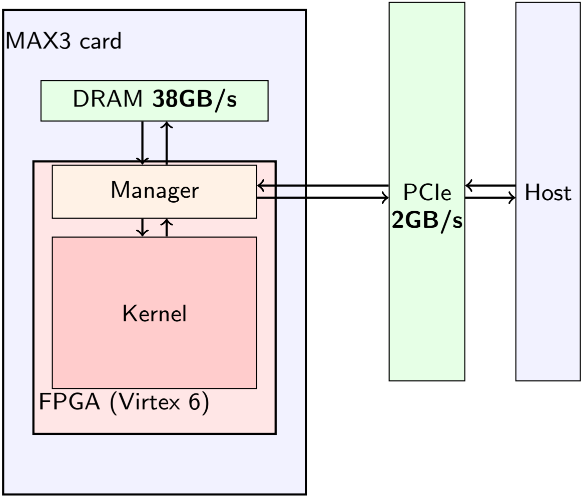
Why custom streaming?

$$r = x^2 + y - \sqrt{z}$$

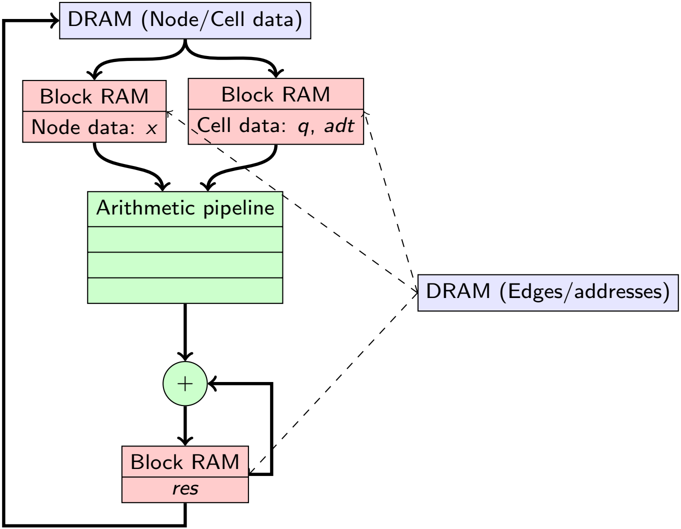


```
load r1, x
load r2, y
load r3, z
mul r1, r1
add r1, r2
 sqrt r3
sub r1, r3
 str r1, r
```

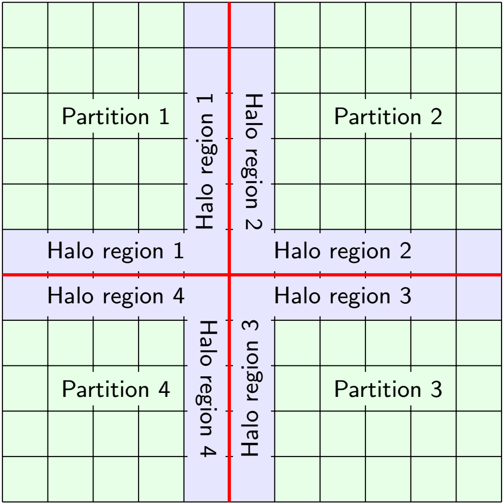
The hardware



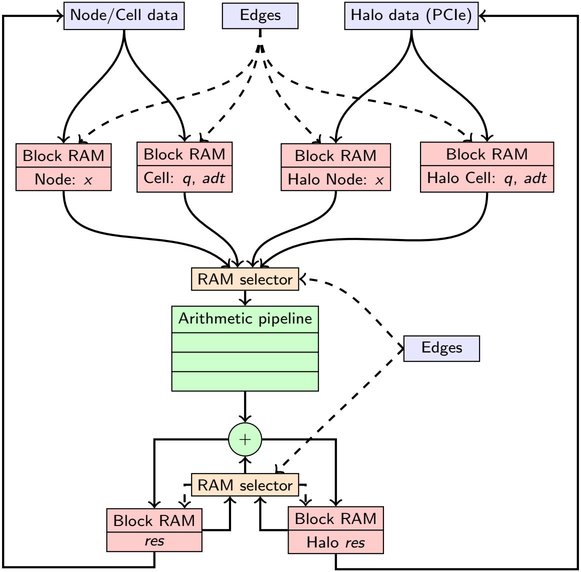
Architecture design: 1st iteration



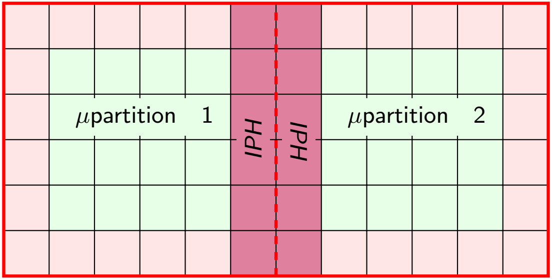
Partitioning and halos



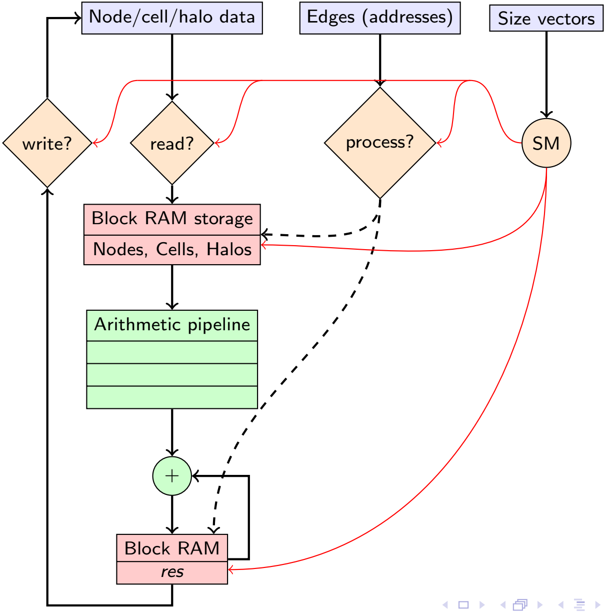
Architecture design: 2nd iteration, Halo Exchange



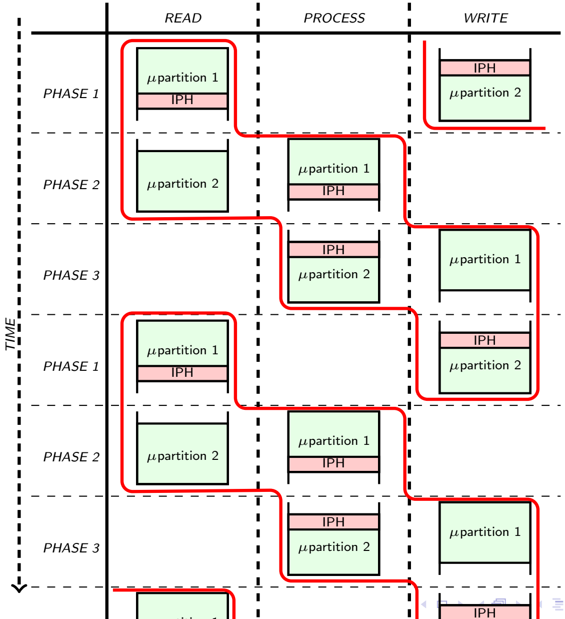
Two-level partitioning: edge processing and I/O interleaving



Architecture design: 3rd iteration



Accelerator phases and execution pattern



Performance Model

We know:

- DRAM bandwidth.
- PCIe bandwidth.
- Clock frequency.
- Partition sizes and therefore the amount of data transferred.

Performance Model

We can calculate:

- Time to stream micro-partition from DRAM:
  $$t_{DRAM} = \frac{Nonhalo\ node\ and\ cell\ data}{DRAM\ bandwidth}$$
- Time to stream halo data for micro-partition from PCIe:
  $$t_{PCIe} = \frac{Halo\ node\ and\ cell\ data}{PCIe\ bandwidth}$$
- Time to consume edge data during processing: $t_{FPGA} = \frac{Number\ of\ edges}{clock\ frequency \times number\ number\ of\ arithmetic\ pipelines}$

Performance Model

$$\boxed{\text{Total time for each phase: } max(t_{DRAM}, t_{PCIe}, t_{FPGA})}$$

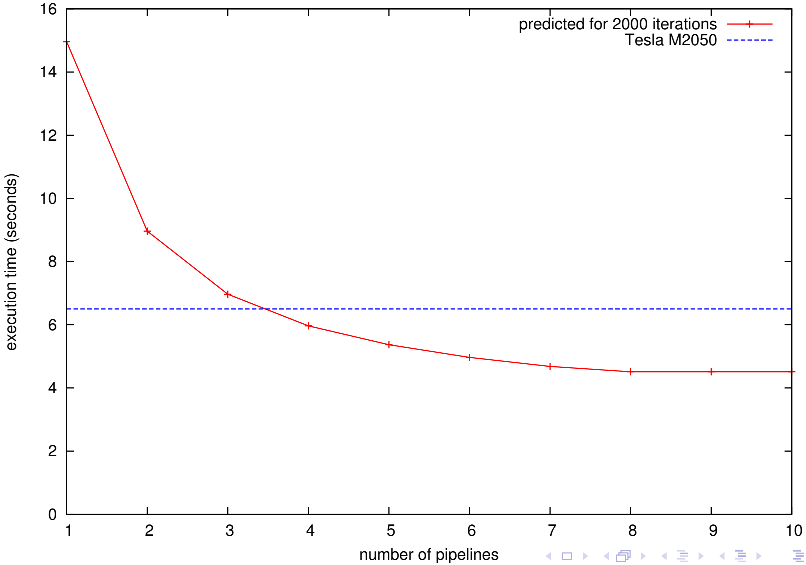
3 phases:

1. Read in data for first micro-partition plus the intra-partition halo. If not first macro-partition, write out second micro-partition and the intra-partition halo.
2. Process first micro-partition, read in the non-IPH data for second micro-partition.
3. Process second micro-partition, write out the non-IPH data for the first micro-partition.

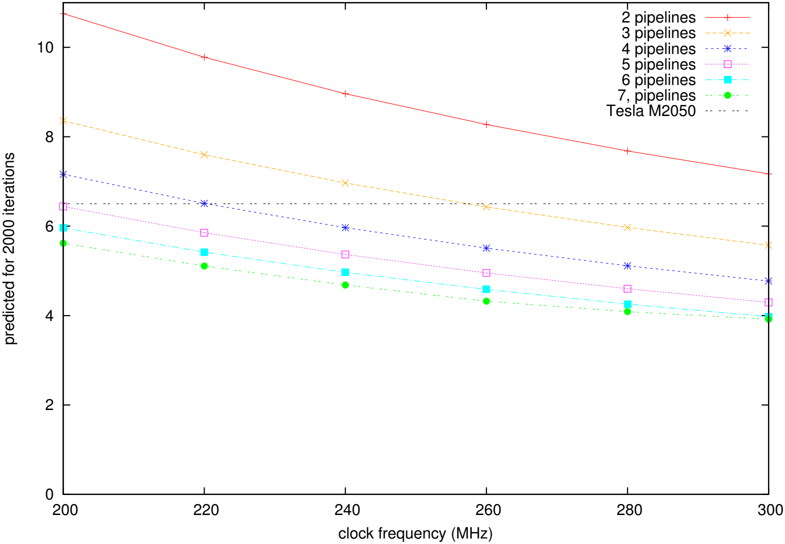
Design space exploration

- We defined a **family** of architectures.
- We can explore the design space using the performance model to find interesting ones.
- We can vary the problem and architecture parameters and predict the effect on performance.

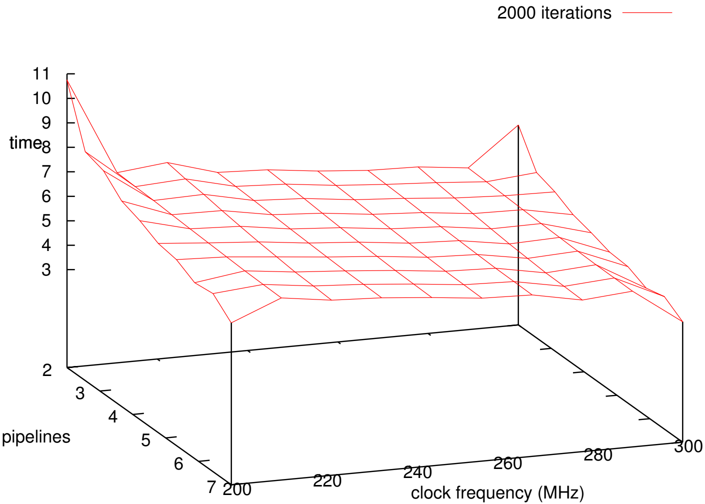
Execution time vs Number of pipelines



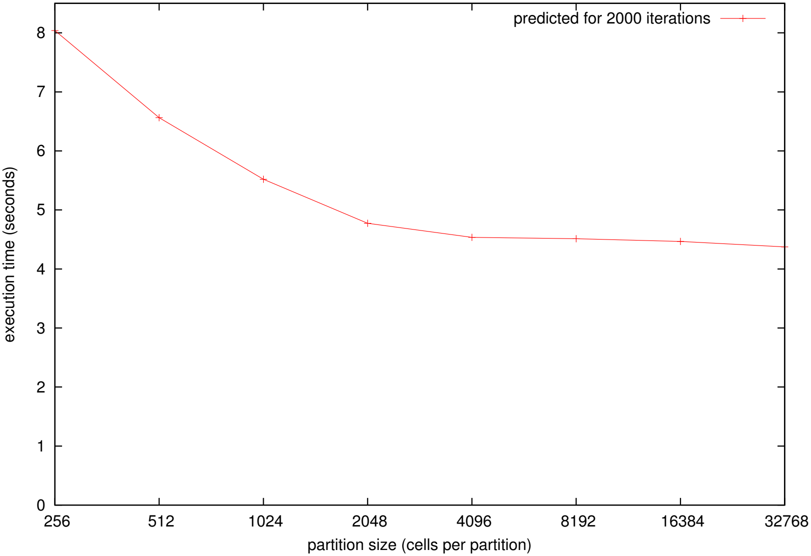
Execution time vs Number of pipelines and clock frequency



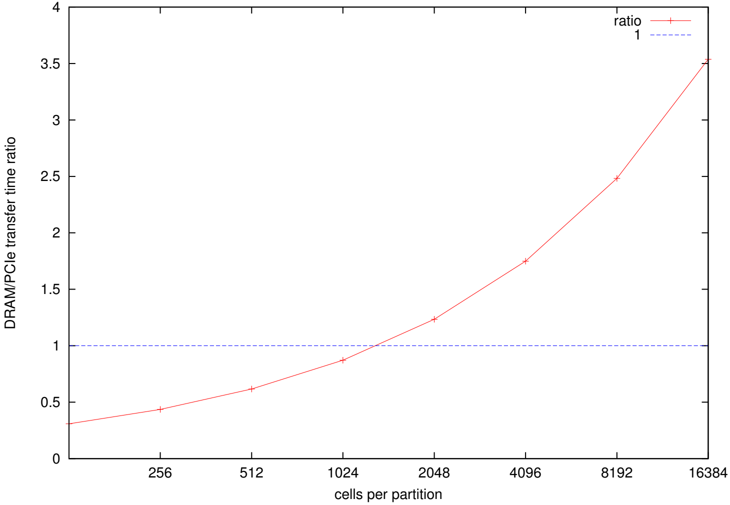
Execution time vs Number of pipelines and clock frequency



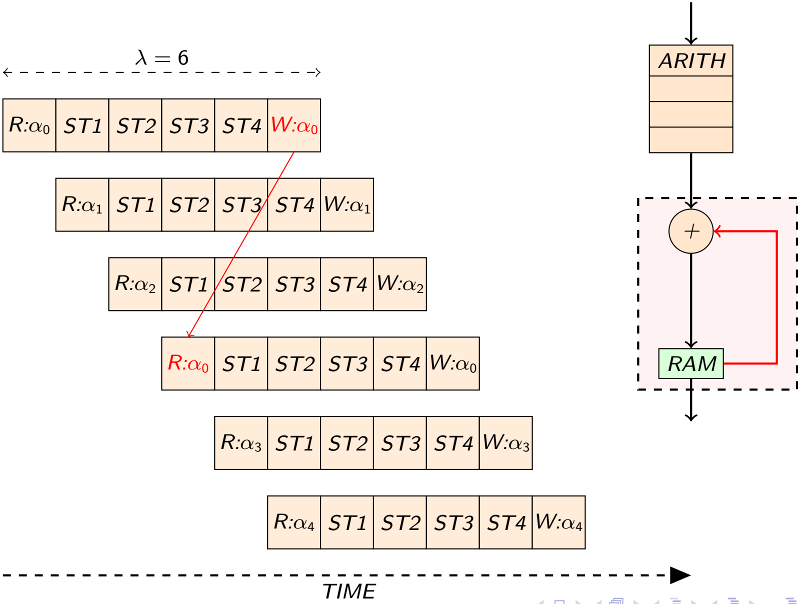
Execution time vs Partition size



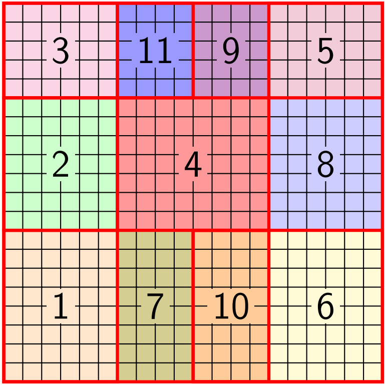
DRAM/PCIe transfer ratio vs Partition size

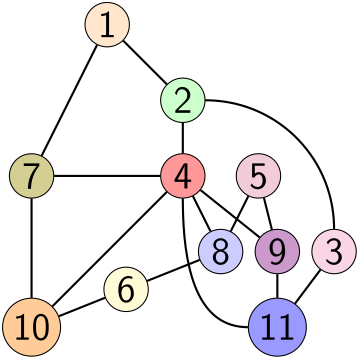


Implementation issues: Edge dependencies in the pipeline


$\lambda = 6$
$R{:}\alpha_0$ | ST1 | ST2 | ST3 | ST4 | $W{:}\alpha_0$
$R{:}\alpha_1$ | ST1 | ST2 | ST3 | ST4 | $W{:}\alpha_1$
$R{:}\alpha_2$ | ST1 | ST2 | ST3 | ST4 | $W{:}\alpha_2$
$R{:}\alpha_0$ | ST1 | ST2 | ST3 | ST4 | $W{:}\alpha_0$
$R{:}\alpha_3$ | ST1 | ST2 | ST3 | ST4 | $W{:}\alpha_3$
$R{:}\alpha_4$ | ST1 | ST2 | ST3 | ST4 | $W{:}\alpha_4$
TIME
ARITH
+
RAM

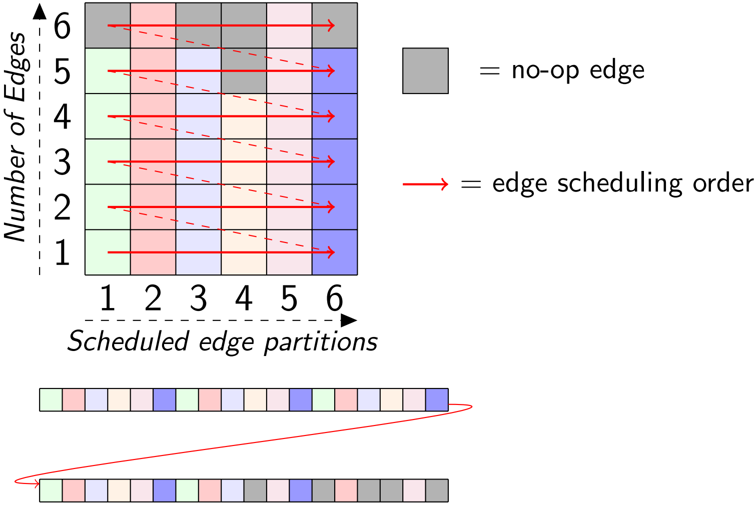
Edge-partitions and adjacency graph scheduling





```
function boolean validSchedule(node[ ] sch, int n, int $\lambda$,
Graph $g$)
    for $i$ in $[0..n-1]$ do
        for $j := 1$ ; $j < \lambda$ ; $j := j + 1$ do
            if $sch[i]$ adjacent to $sch[(i + j)\%n]$ in $g$ then
                return FALSE
            end if
        end for
    end for
    return TRUE
end function
```

No-op edges


Number of Edges
6
5
4
3
2
1
1 2 3 4 5 6
Scheduled edge partitions
= no-op edge
= edge scheduling order

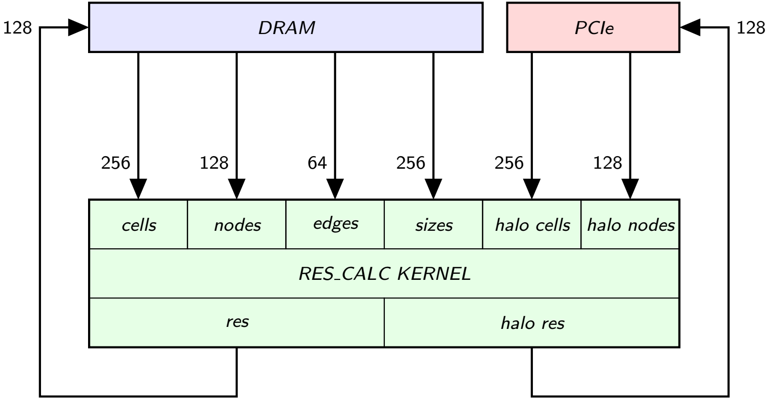
Complexity of edge scheduling

- Take dual adjacency graph: nodes connected in dual graph if they are **not** connected in the original.
- Graph scheduling problem transforms into Hamiltonian path problem with extra adjacency constraint.
- **NP-complete!**.
- Best we can do is search through the schedule space.
- $O(n!)$ (n number of nodes in the adjacency graph).

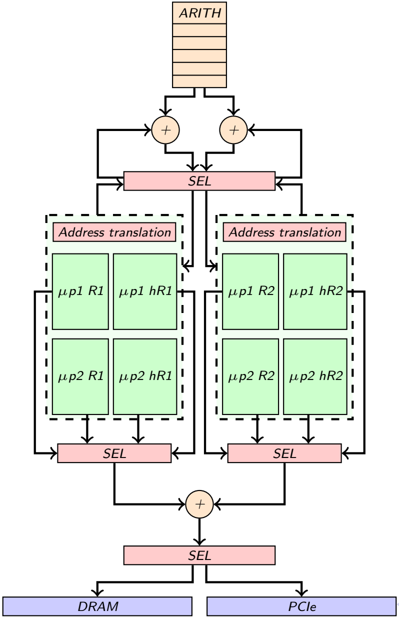
Graph colouring and no-op edge-partitions

- Can group together partitions with same colour.
- To produce schedule with window-width λ add λ no-op edge-partitions after each colour group. Add $\lambda \times c$ no-op partitions (c-number of colours used to colour graph).
- Optimal colouring still **NP**-complete, but we can efficiently find sub-optimal but adequate colouring.
- We use greedy graph colouring algorithm. For each node assign lowest colour not assigned to its neighbours. Worst-case time complexity is $O(n^3)$

Implementation issues: FPGA accelerator, manager configuration



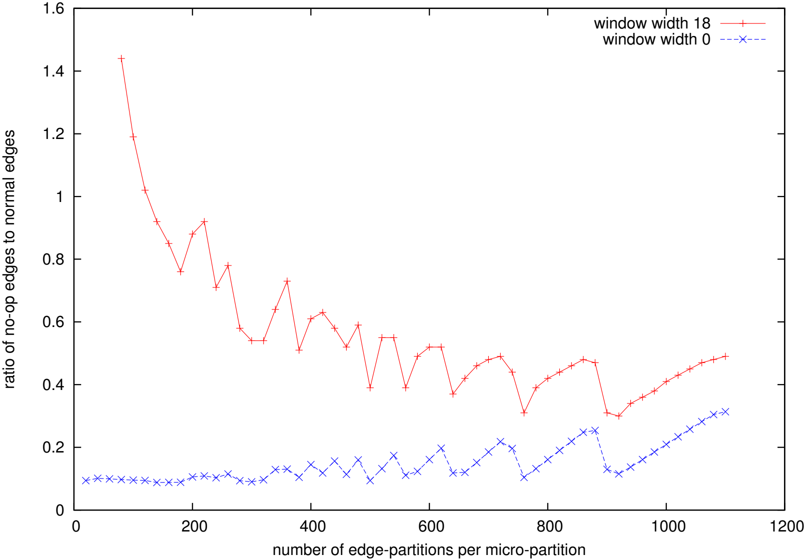
Two-port limitation on RAMs



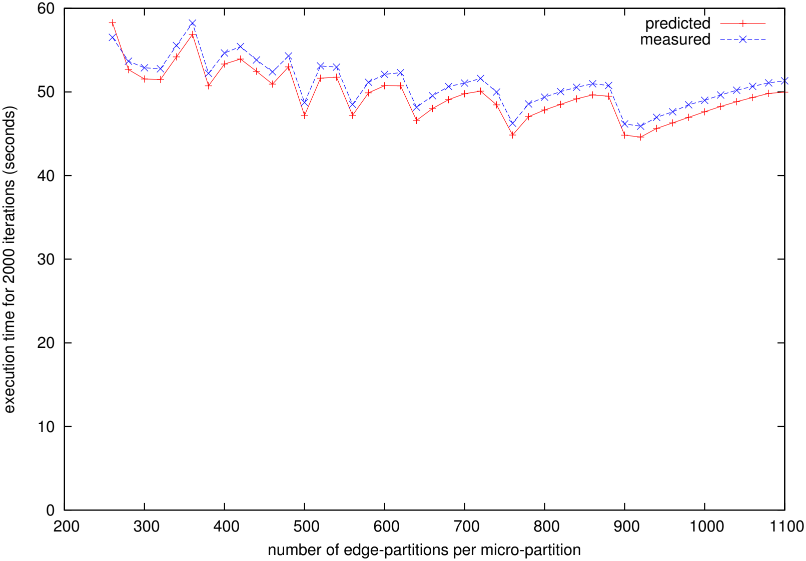
A note on correctness

- Sample implementation gives wrong arithmetic results.
- Through simulations and debugging tracked down to result commiting part of accelerator design.
- *NaN*s from no-op edges commited to result RAMs.
- Kernel consumes and produces correct amount of data in the correct order. Processes correct number of edges.
- *Can still trust the performance results.*

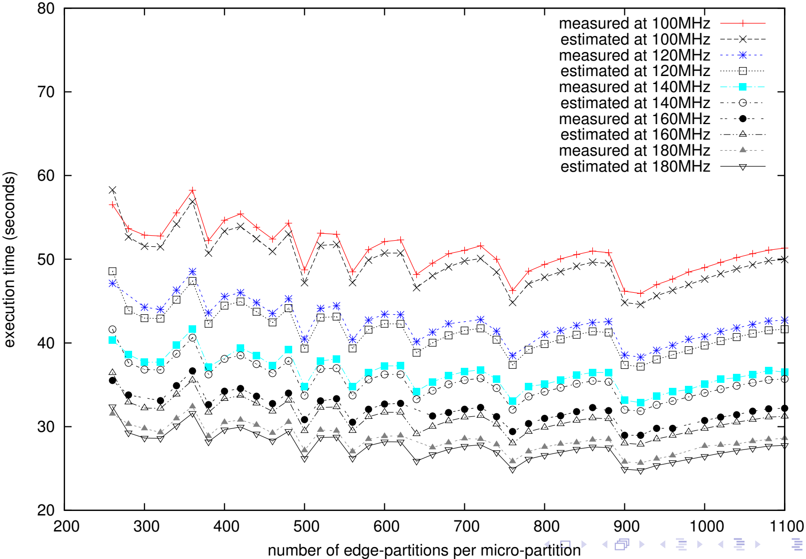
Evaluation: No-op edges



Evaluation: Performance model validation



Evaluation: Performance model validation, various frequencies



Conclusions

- Performance model is validated!
- We can accurately predict the performance of a design space of architectures.
- Simple memory hierarchy provides high predictability. No cache-misses, no non-deterministic thread scheduling by OS.
- Unstructured memory accesses transformed into highly predictable and easily modelled streaming model!
- We showed that an interesting speedup can be achieved.
- Performance rivaling 448-core GPU implementation with only 4-5 pipelines running at a fraction of the clock frequency!

Further work

- Accelerate other kernels: different element iteration requires different data layout!
- Compilation system: plug in architecture parameters and generate host code and accelerator.
- Data formatting: reduce padding, increase bandwidth utilisation.
- Build multi-pipe designs: model predicts they offer the most performance benefits.