Homework 3

Michael Pham

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Contents

1 Polynomial Basis

2   Linear Independence

> **Problem 2.1.** Suppose $v_1, \ldots, v_m$ are linearly independent in V and $w \in V$. Prove that
>
> $$\dim \operatorname{Span}(v_1 - w, v_2 - w, \ldots, v_m - w) \geq m - 1$$

Solution. We first observe that since $v_1, \ldots, v_m$ are linearly independent, then that means that $\dim \operatorname{Span}(v_1, v_2, \ldots, v_m) = m$. Furthermore, since $\operatorname{Span}(w)$ is the span of only one vector, it follows then that $\dim \operatorname{Span}(w) \leq 1$.

Now, let us define

Now, with this in mind, let us define the space $V' := \operatorname{Span}(v_1 - w, v_2 - w, \ldots, v_m - w)$. ∎