PHILOS 12A: Introduction to Logic

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WEEK 1
INTRODUCTION TO PROPOSITIONAL LOGIC

1.1 What is Propositional Logic?

1.1.1 Introduction

Propositional Logic is sometimes referred to as "Sentential Logic."

In this course, we use the following definition for a proposition:

Definition 1.1 (Proposition)**.** A proposition refers to a **declarative sentence**.

Example 1.2. The following is a proposition: *"Paris is the capital of France."*

On the other hand, the question *"Is Paris the capital of France?"* is not considered a proposition.

1.1.2 Propositional Connectives

Definition 1.3 (Propositional Connectives)**.** A propositional connective is a word or phrase that we can combine with some other given propositions to build up more complex propositions.

Within the course, we will use p and q to refer to different propositions. With this in mind, we look at the following example:

Example 1.4. Let p denote the proposition *"there is a heatwave today"*, and q to be *"the weather is hot"*. Then, using propositional connectives, we can create the following propositions:

- p and q.
- p or q.
- If p, then q.
- It is not the case that p.
- It is more likely that p than q.
- The people know that p.

Then, we can study reasoning involving these connectives.

Example 1.5 (Reasoning with Or). For example, let p denote *"the infection is viral"* and q be *"the infection is bacterial"*. Then, we have:

1. p or q.
2. It is not the case that p.
3. Therefore, q.

This is an example of reasoning.

! We note that this reasoning is **good** regardless of the propositions substituted in for p or q; we'll be studying patterns of reasoning that is good in virtue of their "form."

Example 1.6 (Reasoning with Implication). Another example can be done using implications as follows:

1. If p, then q.
2. It is not the case that q.
3. Therefore, it is not the case that p.

Again, we note that this example works for any proposition used for p and q.

Example 1.7 (Reasoning with Likeliness). Finally, let us consider the following:

1. It is more likely that p than it is q.
2. Therefore, it is more likely that p than it is (q and r).

Thus far, we've given examples of **good** reasonings. Instead, we will now look at an example of a **bad** reasoning:

Example 1.8 (Bad Reasoning with Implications). Suppose we had the following reasoning:

1. If p, then q.
2. It is not the case that p.
3. Therefore, it is not the case that q.

However, this is **bad**! Even though it isn't the case that p, we don't know whether it is or is not the case that q.

We can use a concrete example to verify this: suppose that p is *"the patient is taking their medicine"* and q is *"the patient is getting better"*.

Then, we suppose that if p, then q. However, it isn't the case that p; i.e., the patient isn't taking their medicine. However, although this is the case, they can still get better, but they could also be getting worse – we can't conclude that it is not the case that q!

Another example that's a bit trickier is given below:

Example 1.9 (Bad Reasoning with Likeliness). Suppose we had:

1. It is more likely that p than it is q.
2. It is more likely that p than it is r.
3. Therefore, it is more likely that p than it is (q or r).

However, this is bad. Again, let us use a concrete example:

- p denotes *"the next card will be a number card"*.
- q denotes *"the next card will be black"*.
- r denotes *"the next card will be red"*.

Then, we see that while each individual premise is true, the next card we pick is guaranteed to be either black or red; thus the conclusion is false.

1.2 Truth-Functional Connectives

In this section, we will examine a special class of propositional connectives: **truth-functional connectives**.
A truth-functional connective contains two parts to them: the "truth" part, and the "functional" part.

1.2.1 Truth

First, we look at the "truth" part.
In this course, we will make the following assumptions for all propositions:

1. All propositions we deal with are either **true** or **false**.
2. Propositions can't be both true *and* false.

We can come up with propositions that don't satisfy these rules.

Example 1.10. The following are propositions that don't satisfy the rules we've established:

1. Hawaiian Pizza is delicious *(propositions of taste)*.
   - This statement doesn't have an objective truth to it.
2. Bob is bald *(vague propositions)*.
   - If we struggle to classify Bob as being bald (due to, say, him having almost no hair), we say that the proposition is neither true or false.
3. This proposition is false *(self-referential propositions)*.
   - Something something set theory, something something Russell's Paradox, something something.

We say that the truth value of a true proposition is `TRUE` (T), and the truth value of a false proposition is `FALSE` (F).

1.2.2 Unary versus Binary Connectives

Now, we can tackle the "functional" part.

We can categorize connectives we've seen thus far into two groups: **unary** and **binary**.

- Unary:
  - "It is not the case that ___".
  - "The police knows that ___".
- Binary:
  - "___ or ___".
  - "___ and ___".
  - "If ___, then ___".
  - "It is more likely that ___ than it is ___".

Definition 1.11 (Arity)**.** The number of propositions a connective takes in is referred to as its **arity**.

! Note that we can consider connectives of arity three or higher, but we'll postpone on that... for now?

1.2.3 Truth-Functionality

For a unary connective $\#$ to be truth-functional, for any proposition p, its truth value $\#p$ must be a function of the truth value of p. In other words, given two propositions p and q with the same truth value, it must be then that $\#p$ and $\#q$ have the same truth value.

Example 1.12 (Unary Truth-Functional)**.** We observe that *"It is not the case that"* is truth-functional:

p	*"It is not the case that p"*
T	F
F	T

On the other hand, *"The police knows that"* isn't:

p	*"The police knows that p"*
T	$?$
F	F

We note that if p is true, the police doesn't necessarily know that that is the case! For example, say that p is *"Ted Bundy is a killer"* and q is *"My neighbour is secretly a killer"*. $\#p$ is true, whereas $\#q$ is false.

Thus, we say that it isn't truth-functional.

For a binary connective $\#$, for it to be truth-functional, we say that for any propositions p and q, the truth value $p\#q$ must be a function of those p and q.

Example 1.13 (Binary Truth-Functional). We see that *"and"* is a truth-functional:

p	q	p and q
T	T	T
T	F	F
F	T	F
F	F	F

Similarly, *"or"* is a truth-functional as well:

p	q	p or q
T	T	T
T	F	T
F	T	T
F	F	F

On the other hand, let us consider the *counterfactual* condition: *"If it had been the case that p, then it would have been the case that q"*. Note that p is referred to as the **antecedent**, and q is the **consequent**.

To see why this isn't a truth-functional, we look at the following:

- Let p be *"I overslept"* (F), and q be *"The speaker gave her lecture"* (T).
  - Substituting this in, we have: *"If it had been that I overslept, it would've been that the speaker gave her lecture"*; this is true, as regardless of whether I overslept or not, the speaker would've still given her lecture.
- Let p be *"I overslept"* (F), and r be *"I arrived on time"* (T).
  - Substituting this in, we have: *"If it had been that I overslept, it would've been that I arrived on time"*; this is false. However, we see that despite q and r having the same truth value, $p\#q$ and $p\#r$ have different truth values! Thus, we conclude that it isn't truth-functional.

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