RK4 Method Comparison

ODE: $dy/dx = x - y$, with initial condition $y(0) = 1$.
Exact solution: $y(x) = x - 1 + 2e^{-x}$.

Explanation:

This exercise compares numerical solutions of the differential equation $dy/dx = x - y$ using RK1 (Euler), RK2 (Midpoint), and RK4 (fourth-order Runge–Kutta).

RK1 shows the largest deviation from the exact curve because it is only first-order accurate, leading to noticeable truncation errors even with moderate step sizes.

RK2 significantly improves accuracy by evaluating the slope at the midpoint of each step, producing a second-order method whose trajectory more closely tracks the exact solution.

RK4 provides excellent accuracy. It evaluates the slope four times per step and averages them in a weighted manner, achieving fourth-order convergence. As expected, the RK4 curve overlaps almost perfectly with the exact analytical solution for this smooth ODE.

The plot below clearly shows that RK4 is the most precise method, RK2 is moderately accurate, and RK1 exhibits the largest errors. This matches the theoretical order of accuracy for each method.