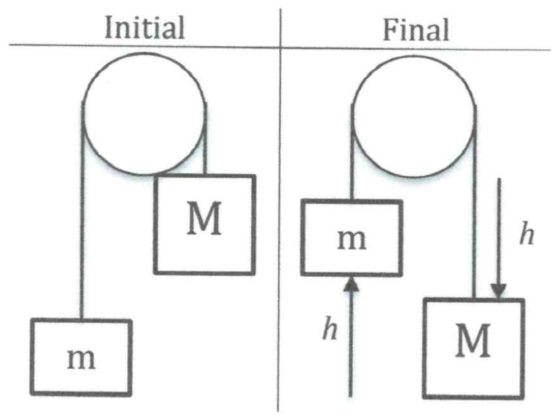
Question 3

For this question, we will guide you through a new way of finding certain information of pulley systems without using kinematics or forces.

Consider the pulley system below. Assume M>m. The system is held at rest initially, and then let go, resulting in mass *m* rising a distance *h* and mass *M* falling a distance *h* (as shown in the diagram). The initial and final forms of energy are derived in the following equations:

$$\Delta E = 0$$
$$= \Delta E_{kinetic} + \Delta U_{gravitational\ potential}$$
$$= \frac{1}{2}mv_f^2 + mgh + \frac{1}{2}Mv_f^2 - Mgh$$
$$= \frac{1}{2}(m+M)v_f^2 + gh(m-M) \qquad \text{(equation 1)}$$



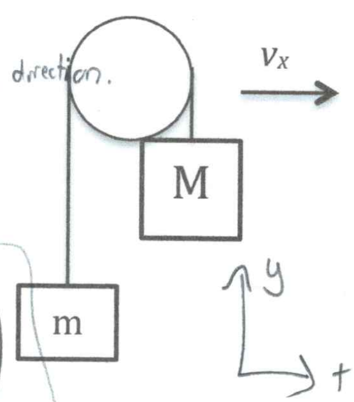
a) Let z = equation 1. Find $\frac{dz}{dt}$ and use it to find the acceleration of both masses. [5]

2 marks $\frac{dz}{dt} = (m+M)va + gv(m-M) = 0$

2 mark $\Rightarrow (m+M)a = g(M-m)$

1 mark $\Rightarrow \boxed{a = \frac{g(M-m)}{m+M}}$

b) Suppose I hold the pulley by its axis hole and hold mass M in place. I now walk and maintain a constant speed v_x as shown in the diagram. While in this constant speed, I let go of mass M and let it fall a distance h. Find the kinetic energy of mass M after it has fallen a distance h. [3]



1 mark From equation 1, $v_{f_y} = \sqrt{\frac{2gh(M-m)}{M+m}} \; (-\hat{y})$ ← This is the ↕y direction.

1 mark $v_{total} = \sqrt{v_{f_y}^2 + v_x^2} = \sqrt{\frac{2gh(M-m)}{M+m} + v_x^2}$

1 marks $\boxed{\Rightarrow E_{K_M} = \frac{1}{2}Mv_{total}^2 = \frac{1}{2}M\left(\frac{2gh(M-m)}{M+m} + v_x^2\right)}$