Question 1: Proofs

Prove the following statements true or false. Do not use the amount of space as an indication to the length of the answers. [3 marks each, no marks for completely wrong reasoning]

a) Suppose x and y are integers. If $xy > 0$, then $x + y > 0$.

False. Suppose $x = -1$ & $y = -2$.

Notice $xy = (-1)(-2) = 2 > 0$

but $(-1) + (-2) = -3 \not> 0$

Thus, false.

Note: Suppose a and b are integers. If a | b, then b = ka for some integer k. The converse is also true: If b = ka for some integers a,b, and k, then obviously a | b.

b) Suppose u, w and z are integers. If w | u and w | z, then w | (u + z).

True.

If $w|u$, then $\exists k \in \mathbb{Z}$ such that $kw = u$ ①

Also $w|z$, so $\exists m \in \mathbb{Z}$ such that $mw = z$ ②

Add ① + ② $= kw + mw = u + z$

$\Rightarrow w(k+m) = u + z$

Since $k+m$ is an integer, $w | (u+z)$

1 mark

1 mark

1 mark