

How to prove false/true.

- If true: use math / step-by-step reasoning to prove it.
- If false: give one case where the hypothesis is satisfied, but the conclusion is false.

Ex. 1 If I drink a liquid, then it is apple juice.

False. Suppose I drink milk. Milk is a liquid. But it is not apple juice.

Ex. 2 $a, b \in \mathbb{Z}$, If $a \mid 2b$, then $a \mid b$.

False. $a=6$ & $b=3$, $6 \mid 2(3)$
but $6 \nmid 3$.

Ex. 3 If $a \mid 3$ and $a \mid b$, then $a \mid (3+b)$

True. If $a \mid 3$, then $\exists k$ such that $ka=3$. ①

Also since $a \mid b$, then $\exists m$ such that $ma=b$ ②

$$\textcircled{1} + \textcircled{2} = ka + ma = (3+b)$$

$$a(k+m) = (3+b)$$

Since $k+m$ is an int, thus $a \mid (3+b)$

□/QED

Ex. 4 There is a block (mass M) on a table. Nothing is moving. If I push down on the block with a force $F \geq 0$ Newtons, then the normal force on the block is $\geq Mg$.

True.
+ $\uparrow \hat{y}$



Newton's 1st law $\Rightarrow F_{\text{net},y} = 0 \Rightarrow 0 = N - F - Mg$

So $N = F + Mg$

Since $F \geq 0$, thus $N \geq Mg$.

□.