

Robin Mehta  
AI Homework 3

A	B	C	A and B	A and B $\rightarrow$ C	A $\rightarrow$ C	B $\rightarrow$ C	(A $\rightarrow$ C) $\vee$ (B $\rightarrow$ C)	A $\vee$ B	-(A and B)	-C	-A $\vee$ -B $\vee$ -C	(A $\vee$ B) and (-C $\vee$ -D $\vee$ -E)
1	1	1	1	1	1	1	1	1	0	0	0	0
1	1	0	1	0	0	0	0	1	0	1	1	1
1	0	0	0	1	0	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	0	0	1	1	0
0	0	1	0	1	1	1	1	0	0	0	0	0
0	1	1	0	1	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	1	1	1	0	1	1
0	1	0	0	1	1	0	1	1	1	1	1	1

### Problem 1, Part 1)

- True
- True
- False
- True
- True
- True

### Problem 1, Part 2)

- Prove bi-directionally:
    - If True = true, which means that alpha is valid in every model.
    - If alpha is valid (true in all models), True is still true in every model.
  - Prove alpha entails beta iff (alpha  $\rightarrow$  beta) is valid
    - In every model where alpha is true, beta is also true
    - If (alpha  $\rightarrow$  beta) is valid (true in all models), beta will also be true.
  - Prove alpha  $\equiv$  beta iff (alpha  $\leftrightarrow$  beta)
    - A truth table proves that where (alpha  $\leftrightarrow$  beta), alpha  $\equiv$  beta is true in all models.
    - A truth table proves that where (alpha  $\equiv$  beta), alpha  $\rightarrow$  beta is also true in all models.
  - Prove alpha entails beta iff (alpha  $\wedge$   $\sim$ beta) is unsatisfiable.
    - Let (alpha  $\wedge$   $\sim$ beta) be satisfiable (at least one model exists where alpha is true and beta is false), which does not show entailment.
    - Let (alpha entails beta) be valid. Here, there is never a case where alpha is true and beta is false.
- Overall, if alpha is true and beta is never false, (alpha  $\wedge$   $\sim$ beta) is unsatisfiable.

### Problem 2, Part 1)

A

$\sim C$

$\sim A \vee B$

$C \vee D$

E

$\sim B \vee \sim D \vee \sim E \vee F$

Overall:

$$(A) \wedge (\sim C) \wedge (\sim A \vee B) \wedge (C \vee D) \wedge (E) \wedge (\sim B \vee \sim D \vee \sim E \vee F)$$

### Problem 2, Part 2) Use resolution to prove F

Assume: F is false. Prove using contradiction

$$\sim F \rightarrow (\sim B \vee \sim D \vee \sim E) :$$

$$\sim F \rightarrow \sim B \text{ must be true}$$

true  $\rightarrow \sim(\sim A \vee B)$ , given A, meaning B is true, meaning  $\sim B$  is false. (true  $\rightarrow$  false is false). Contradiction.

OR

$$\sim F \rightarrow \sim D \text{ must be true}$$

true  $\rightarrow (C \vee D)$ , given C, meaning D is true, meaning  $\sim D$  is false. (true  $\rightarrow$  false is false). Contradiction.

OR

$$\sim F \rightarrow \sim E \text{ must be true}$$

true  $\rightarrow \sim E$ , given E is true, meaning  $\sim E$  is false. (true  $\rightarrow$  false is false). Contradiction.

Finally:

$$\sim F \rightarrow \text{false or false or false}$$

$$\text{true} \rightarrow \text{false. Contradiction!}$$

### Problem 3, Part 1: Determine whether the pairs can be unified

1. No. Since B and C are different constants, and y and y are the same variable, these pairs cannot be unified.
2. No. B is not equal to GameOf(x), meaning that B, being a constant, cannot be substituted for GameOf(x).
3. Yes. C(B, FriendOf(B), GameOf(FriendOf(B)))
4. Yes. P(A, FriendOf(B))
5. No. By the unique names assumption, two pairs of different functions cannot be unified.
6. No. By the unique names assumption, A and C are different constants, and so FriendOf(x,x) cannot be substituted with FriendOf(A,C).

### Problem 4

#### Part 1: Translate the knowledge base into first-order-logic (using quantifiers!)

- 1) Student(Pat)
- 2) Restaurant(Ayse)
- 3) GetsScholarship(Pat)
- 4)  $\forall x(\text{Student}(x) \wedge \text{GetScholarship}(x)) \rightarrow \text{SpecialOccasion}(x)$
- 5)  $\text{Ex}(\text{Patron}(\text{Pat}, y))$
- 6)  $\forall x,y,z(\text{Student}(x) \wedge \text{Patron}(x, y) \wedge \text{Restaurant}(y) \wedge \text{SpecialOccasion}(x)) \rightarrow \text{Eats}(x, z=\text{lentil soup})$

#### Part 2: Convert To CNF

- 1)  $\text{Student}(\text{Pat}) \wedge$
- 2)  $\text{Restaurant}(\text{Ayse}) \wedge$
- 3)  $\text{GetsScholarship}(\text{Pat}) \wedge$
- 4)  $\sim \text{Student}(x) \vee \sim \text{GetsScholarship}(x) \vee \text{SpecialOccasion}(x) \wedge$
- 5)  $\text{Patron}(\text{Pat}, y) \wedge$
- 6)  $\sim \text{Student}(x) \vee \sim \text{Patron}(x, y) \vee \sim \text{Restaurant}(y) \vee \sim \text{SpecialOccasion}(x) \vee \text{Eats}(x, z=\text{lentil soup})$

#### Part 3: Prove that Pat eats lentil soup using resolution refutation.

Assume:  $\sim \text{Eats}(x=\text{Pat}, z=\text{lentil soup})$

Given by CNF:

If  $\sim \text{Eats}(x=\text{Pat}, z=\text{lentil soup})$  is true, then:

$\sim \text{SpecialOccasion}(x)$  must be true

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Given: Student(Pat), GetsScholarship(Pat)

$\sim\text{Student(Pat)} \vee \sim\text{GetsScholarship(Pat)} \vee \text{SpecialOccasion(Pat)}$  must be true

false  $\vee$  false  $\vee$  SpecialOccasion(Pat) must be true, so SpecialOccasion(Pat) is true. Contradiction.

OR

$\sim\text{Restaurant(y)}$  must be true

Given: Restaurant(y) is true. Contradiction.

OR

$\sim\text{Patron(x, y)}$  must be true

Given: Patron(Pat, y) is true. Contradiction.

OR

$\sim\text{Student(x)}$  must be true

Given: Student(Pat) is true. Contradiction.

Finally:

$\sim\text{Student(x)} \vee \sim\text{Patron(x, y)} \vee \sim\text{Restaurant(y)} \vee \sim\text{SpecialOccasion(x)} \vee \text{Eats(x, z=lentil soup)}$

False  $\vee$  False  $\vee$  False  $\vee$  False  $\vee$  False. Contradiction!

## Problem 5:

**Part 1:** List unary constraints, if any:

- 1) None
- 2) Constraints i, ii, iii, iv, v, vi

**Part 2:**

1) Possible ordered list of the deliverables:

- a) BDFHJCK violates constraint iii.

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- b) CJBHDKF violates constraint i.
- c) HBDFJCK does not violate any constraints!
- d) HBDFKJC violates constraint v.
- e) HJDBFKC violates constraint iv.

1) Domains after applying arc consistency:

B: 1,2,3,4,5,6

C: 1,2,3,4,5,6,7

D: 2,3,4,5,6,7

E: 1,2,3,4,5,6,7

F: 1,2,3,4,5,6,7

H: 1,2,3,4,5,6

J: 1,2,3,4,5,6,7

K: 2,3,4,5,6,7

3) Backtracking

	B	C	D	F	H	J	K
Initial	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7
B=1	1	3, 4, 5, 6, 7	2	3, 4, 5, 6, 7	3, 4, 5, 6, 7	3, 4, 5, 6, 7	3, 4, 5, 6, 7
D=2	1	3, 4, 5, 6, 7	2	3, 4, 5, 6, 7	Backtrack		
B=2	2	1, 3, 4, 5, 6, 7	1, 3	1, 3, 4, 5, 6, 7	1, 3, 4, 5, 6, 7	1, 3, 4, 5, 6, 7	3, 4, 5, 6, 7

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		5, 6, 7		5, 6, 7	6, 7	5, 6, 7	7
D=1	2	3, 4, 5, 6, 7	1	3, 4, 5, 6, 7	Backtrack		
D=3	2	1, 4, 5, 6, 7	3	1, 3, 4, 5, 6, 7	1	1, 4, 5, 6, 7	1, 4, 5, 6, 7
H=1	2	4, 5, 6, 7	3	4	1	4, 5, 6, 7	4, 5, 6, 7
F=4	2	5, 6, 7	3	4	1	5, 6, 7	5, 6, 7
C=5	2	5	3	4	1	6, 7	6, 7
J=6	2	5	3	4	1	6	Backtrack
C=6	2	6	3	4	1	5, 7	5, 7
J=5	2	6	3	4	1	5	7
K=7	2	6	3	4	1	5	7

Problem 6, Part 1:

	4	5	6
A	1, 4, 7, or 9	2	1, 4, 7, or 9
B	3	1, 4, 7, or 9	5
C	8	1, 4, 7, or 9	6

2) Reduce the domain for variables: List the new domains.

	4	5	6
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A	4, or 9	2	1, 4, or 7
B	3	4, or 7	5
C	8	7, or 9	6

3) Minimum-remaining value heuristic:

We would consider A4, B5, or C5 since these have 2 values left to consider each.

Part 2: Code

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```
robin_mehta_solver.py x
91 def ac3(sudoku):
92     numQueue = list(sudoku)
93     box = sudoku[0][0]
94     for (x,y), value in np.ndenumerate(sudoku):
95         if (value == box):
96             solved = True
97     solved_sudoku = np.copy(sudoku)
98     return False, solved_sudoku
99
100 '''
101 Backtracking search Algorithm
102 Input: 2D numpy matrix
103 Output: return True if a solution is found, with solved sudoku, False
104 '''
105 def check(sudoku, box, x, y):
106     for (a,b), value in np.ndenumerate(sudoku):
107         if (value != box and x == a and y == b):
108             if (x == a or y == b):
109                 return False
110     return True
111
112 def bts(sudoku):
113     solved = False
114     for (x,y), value in np.ndenumerate(sudoku):
115         if (value == "*"):
116             solved = False
117     solved_sudoku = np.cdp(sudoku)
118     return solved, solved_sudoku
119
120 '''
121 Main function
122 '''
123 def main():
124     sudoku_list = read_sudoku(sys.argv[1])
125     solved_sudokus = []
126     for sudoku in sudoku_list:
127         print_sudoku(sudoku)
128         if sys.argv[2] == 'ac3':
129             print 'Using AC-3'
```