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Spiral patterns in magnets

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Abstract

An autowave model of the spiral pattern formation in magnets located in a rotating magnetic field is proposed. The model is based on the overdamped double sine-Gordon equation. Nucleation of spiral domains is associated with the averaged motion of domain walls in the rotating field. A vortex-type defect (Bloch line) is the core of the spiral. © 1997 Published by Elsevier Science B.V.

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1. Introduction

The formation of spiral domains has been observed in experiments with ferromagnetic films located in a high intensity magnetic field oscillating with a low frequency [1,2]. The patterns exhibited dynamical properties which were very similar to patterns observed in systems driven far from equilibrium or in excitable media [3]. Nevertheless, at present there is no adequate theoretical interpretation of the spiral domains as a kind of autowaves in magnets. Alternative theories which are based on some specific properties of the ferromagnetic films (e.g., boundary magnetostatic effects) are given in Refs. [2,4].

In this paper an autowave model for the formation of spiral domains in bulk ferromagnets under the action of a rotating magnetic field is proposed. The model is based on the overdamped version of the double sine-Gordon equation, which differs from the well-known theories of autowaves in active media [3].

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2. The model

Let us consider a ferromagnet with two anisotropy axes, and an energy density of the form

$$w = \tfrac{1}{2}\alpha[(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2] - \tfrac{1}{2}\beta' M_x^2 + \tfrac{1}{2}\beta M_z^2 - \boldsymbol{M}\cdot\boldsymbol{H}, \qquad (1)$$

where $\boldsymbol{M}$ is the magnetization vector. As a dynamic equation we will use the Landau–Lifshitz equation with the Hilbert relaxation term,

$$\boldsymbol{M}_t = g\boldsymbol{M} \times \delta E/\delta \boldsymbol{M} + \gamma \boldsymbol{M} \times \boldsymbol{M}_t/M,$$
$$E = \int w\,\mathrm{d}V. \qquad (2)$$

Let the anisotropy constant β have a large positive value ($0 < \beta' \ll \beta$); then the vector $\boldsymbol{M}$ tends to align in the (x, y) plane ("easy-plane"). Let us introduce the angle variables θ, φ,

$$\boldsymbol{M}(x,y,t) = (M\cos\theta\cos\varphi, M\cos\theta\sin\varphi, M\sin\theta),$$

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where we suppose that all variables only depend on the x and y coordinates. Let the vector of the external magnetic field $\boldsymbol{H}$ lie in the easy-plane too. One can represent the angles in the form of asymptotic expansions in powers of the small parameter $\epsilon = \beta'/2\beta \ll 1$,

$$\theta = \epsilon\theta_1 + \epsilon^2\theta_2 + \dots,$$

$$\varphi = \varphi_0 + \epsilon\varphi_1 + \epsilon^2\varphi_2 + \dots. \tag{3}$$

Substituting these expansions into Eq. (2) and confining ourselves to the zeroth order approximation in ϵ, we obtain the following equations for the angle variables,

$$-\gamma\varphi_\tau + \Delta\varphi = \sin 2\varphi + h_x(\tau)\sin\varphi - h_y(\tau)\cos\varphi,$$

$$\theta = -\epsilon\varphi_\tau, \tag{4}$$

where we introduce the dimensionless quantities $\tau = \omega_0 t/2$, $\boldsymbol{r}' = \boldsymbol{r}/l_0\sqrt{2}$, $\omega_0 = g\beta' M$ and $l_0^2 = \alpha/\beta'$, $h_x = 2H_x/\beta' M$, $h_y = 2H_y/\beta' M$. In the following we will assume the external magnetic field to be circularly polarized,

$$h_x(\tau) = h\cos\Omega\tau, \quad h_y(\tau) = h\sin\Omega\tau. \tag{5}$$

Deviations of the vector $\boldsymbol{M}$ from the easy-plane will be small for low-frequency processes, where $\Omega \ll 1/\epsilon$.

Let now β have a large negative value. One may put $\beta' = 0$ without loss of generality. This "easy-axis" model is similar to that proposed in Ref. [5]. Now the base plane will be the (x, z) plane and Eq. (4) retains its dimensionless form after the exchange $y \to z$. Thus, Eq. (4) is a rather generic model for the domain pattern formation in bulk ferromagnets.

3. The averaged motion of domain walls

If $h = 0$, Eq. (4) has the well-known stationary soliton solution

$$\varphi_1 = 2\sigma\,\mathrm{arctg}[\exp(-\sqrt{2}y)], \tag{6}$$

which is a 1D domain wall of the Néel type. Here we suppose the wall to be oriented along the x-axis. The wall separates domains with $\varphi(-\infty) = \sigma\pi$ and $\varphi(+\infty) = 0$, where $\sigma = \pm 1$ is the polarity of the wall (it defines the direction of rotation of the vector $\boldsymbol{M}$ in the domain wall).

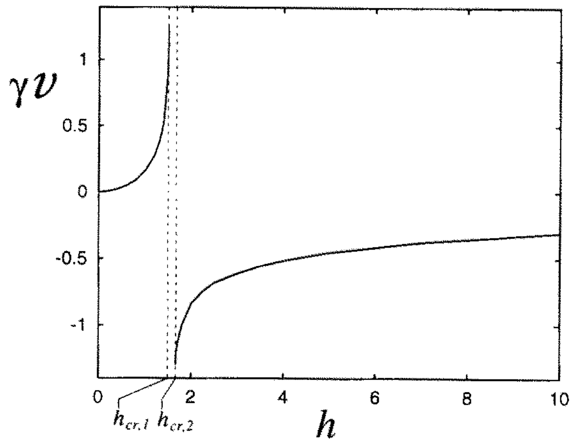

Fig. 1. Dependence of the drift velocity of domain walls on the amplitude h of the rotating magnetic field (5). $\gamma\Omega = 1.3$.

Under the action of the rotating field (5), the domain wall begins a complex motion which consists of oscillations of the wall location and width. The oscillations are accompanied by the slow averaged drift of the wall in the normal direction (i.e., along the y-axis for the case (6)). For h small, the averaged motion has been investigated analytically (see, e.g., Ref. [6]). The moving wall has the asymptotic form

$$\varphi = \varphi_1(y - v\tau) + \mathrm{O}(h), \tag{7}$$

where the velocity of the drift motion is $v \sim \sigma h^2/\gamma$. Computer simulations with Eq. (4) confirm the analytic results for low fields, but with increasing h the dependence $v(h)$ becomes more rapid and, for some $h = h_{\mathrm{cr},1}$, a break occurs (see Fig. 1). In the range $h_{\mathrm{cr},1} < h < h_{\mathrm{cr},2}$, no stable domain structures were observed. The domain walls appeared again for large fields, $h > h_{\mathrm{cr},2}$, but now they will have another shape. To explain this we consider fields $h \gg 1$ and use an asymptotic expansion in powers of $1/h$. It follows from Eq. (4) that the principal term of the expansion for the domain wall will take the form

$$\varphi = \varphi_2(y - v\tau) + \Omega\tau + \mathrm{O}(1/h),$$

$$\varphi_2 = 4\sigma\,\mathrm{arctg}(\sqrt{h}y), \tag{8}$$

with a drift velocity $v \sim -\sigma/\gamma h$. One can see that the magnetic moment $\boldsymbol{M}$ rotates locally with the frequency Ω. Then the domain wall should be interpreted as a localized phase shift of this rotation from $\varphi = \Omega\tau$ for

$y \to -\infty$ to $\varphi = 2\pi + \Omega\tau$ for $y \to +\infty$. The width of this "phase" wall is about $1/\sqrt{h}$.

In the whole range $h > h_{\mathrm{cr},2}$ the wall dynamics, like in the case $h < h_{\mathrm{cr},1}$, is more complicated than the asymptotic behavior (8), but qualitively, the wall remains a 2π-"phase" domain wall having a drift velocity; this is shown in Fig. 1.

The critical fields $h_{\mathrm{cr},1}$ and $h_{\mathrm{cr},2}$ depend on the frequency Ω. If $\Omega \to 0$, then $h_{\mathrm{cr},1} \to 1$ and $h_{\mathrm{cr},2} \to 1$, but for $\gamma\Omega \gg 1$, the linear dependences $h_{\mathrm{cr},1} \approx 0.22\Omega$ and $h_{\mathrm{cr},2} \approx \Omega$ are realized.

4. Spiral patterns

Besides trivial solutions like π-domain walls (6), Eq. (4) has stationary solutions with vortex-type singularities,

$$\varphi \sim k \operatorname{arctg}(x/y), \quad x^2 + y^2 \to 0, \tag{9}$$

where $k = \pm1, \pm2, \ldots$ is the topological charge. The singularities are localized in the domain wall and may be interpreted as horizontal Bloch lines (BL) in the Néel wall [5,7].

Let us consider the simplest structure, which consists of a quasi-one-dimensional wall of the type (6) with only one BL ($k = \pm1$), localized in the point (0,0). The distribution of the magnetization in this singular configuration is shown in Fig. 2. The segments of the domain wall on the right and left hand sides from BL have different polarities. Computer simulations with Eq. (4) show that the stationary configuration described above turns out to be unstable under the action of the oscillating magnetic field (5), which produces a spiral structure.

For low fields $h < h_{\mathrm{cr},1}$, the process of spiral nucleation is shown in Fig. 3. In this case the guiding action of the easy-plane anisotropy on the magnetic moment is the dominant effect and the vector $\boldsymbol{M}$ in homogeneous regions has only small oscillations around some mean values $\langle\varphi\rangle \approx 0$ (dark regions in Fig. 3) and $\langle\varphi\rangle \approx \pi$ (light regions). Here $\langle\ldots\rangle$ is the average over the field period $T = 2\pi/\Omega$. In the final stage the spiral domain has appeared; this covers the whole simulation region and rotates in anti-clockwise direction.

The simulations indicate that the nucleation of the spiral domains is induced by the averaged motion of

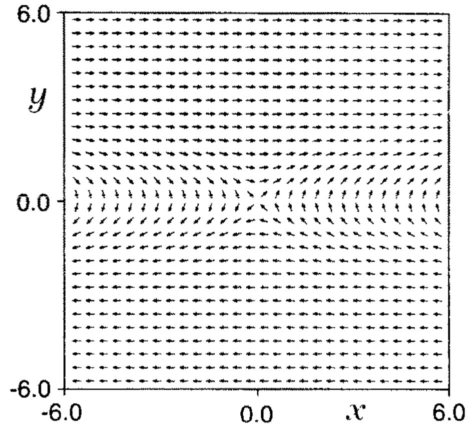

Fig. 2. Distribution of the magnetization in the Néel domain wall with the horizontal Bloch line localized in the point (0,0).

domain walls under the action of the rotating magnetic field (5). It is important to note that the direction of the averaged motion depends on the polarity of the wall. In the initial stage of the process the segments of the wall at opposite sides of the BL have different polarities and this induces opposite directions for their motions (see the intermediate stage in Fig. 2b) followed by a twisting into a spiral.

The influence of the easy-plain anisotropy is insignificant for large fields $h > h_{\mathrm{cr},2}$. In homogeneous regions, the vector $\boldsymbol{M}$ rotates together with the field $\boldsymbol{H}(\tau)$ with the same frequency Ω and lags behind the field for an averaged phase shift $\langle\Delta\varphi\rangle \approx \arcsin(\Omega/h)$. The spiral pattern is also nucleated in this case. The final pattern is shown in Fig. 4 where dark regions correspond to $\langle\varphi - \Omega\tau\rangle - \langle\Delta\varphi\rangle \approx 0$ and light ones to $\langle\varphi - \Omega\tau\rangle - \langle\Delta\varphi\rangle \approx \pi$. The differences with the low field case are that now the domain has an opposite twisting owing to the opposite direction of the averaged motion of the walls for $h > h_{\mathrm{cr},2}$ and the dark regions have a small width of the order of the domain wall thickness.

In both cases the spiral domains exhibited good stability in spite of perturbations which were introduced by the boundaries of the calculation region (boundary conditions $(\nabla\varphi)_n = 0$ were imposed).

The characteristic patterns are obtained after switching off the external field $\boldsymbol{H}$. The spirals were preserved for a long period of time, untwisting slowly in opposite direction. For example, the spiral domain shown in Fig. 3c was untwisted to a one step spiral in a time

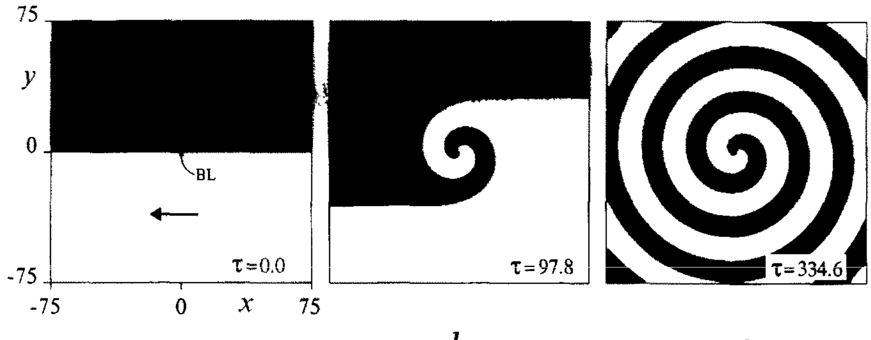

Fig. 3. Nucleation of the spiral domain in the rotating magnetic field. $\gamma\Omega = 1.3, h = 1.4 < h_{cr,1} \approx 1.5$. Dark domains: $\langle\phi\rangle \approx 0$, light domains: $\langle\phi\rangle \approx \pi$.

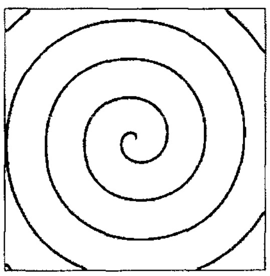

Fig. 4. Spiral domain for $h = 2.5 > h_{cr,2}$. $\gamma\Omega = 1.3$.

of $\tau = 1000$.

In the intermediate region $h_{cr,1} < h < h_{cr,2}$ the vector $\boldsymbol{M}$ executes nonregular rotations with a frequency unequal to Ω. Nonregular processes of nucleation of the spirals were observed, followed by their destruction. The spiral domains are unstable in this region.

5. Conclusion

Two factors are crucial for nucleation of spiral domains in the double sine-Gordon model (4). The first one is the averaged motion of the domain walls which depends on the polarity of the wall. The second factor is the existence of topological defects (Bloch lines in our case) in the domain wall. The defect becomes the core of the spiral. Such a structure of spiral patterns has a qualitative analogy with "reverberators" in active media [8], where the defect may be treated as a "point of phase change".

The influence of the crystal anisotropy in the easy-plane is not a significant factor for spiral nucleation. The spirals also are nucleated for $\beta' = 0$ (i.e., in the limit $h \gg h_{cr,2}$). This case is similar to spiral formation in nematics located in a rotating magnetic field [9–11].

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