5.50 between_min_max

DESCRIPTION LINKS GRAPH AUTOMATON

Origin

Used for defining cumulative_convex.

Constraint

between_min_max(VAR, VARIABLES)

Arguments

```
VAR        :  dvar
VARIABLES  :  collection(var−dvar)
```

Restrictions

required(VARIABLES, var)
$|$VARIABLES$| > 0$

Purpose

VAR is greater than or equal to at least one variable of the collection VARIABLES and less than or equal to at least one variable of the collection VARIABLES.

Example

$(3, \langle 1, 1, 4, 8\rangle)$
$(1, \langle 1, 1, 4, 8\rangle)$
$(8, \langle 1, 1, 4, 8\rangle)$

The first between_min_max constraint holds since its first argument 3 is greater than or equal to the minimum value of the values of the collection $\langle 1, 1, 4, 8\rangle$ and less than or equal to the maximum value of $\langle 1, 1, 4, 8\rangle$.

Typical

$|$VARIABLES$| > 1$
range(VARIABLES.var) > 1

Symmetries

- Items of VARIABLES are permutable.
- VAR can be set to any value of VARIABLES.var.

Arg. properties

Extensible wrt. VARIABLES.

Reformulation

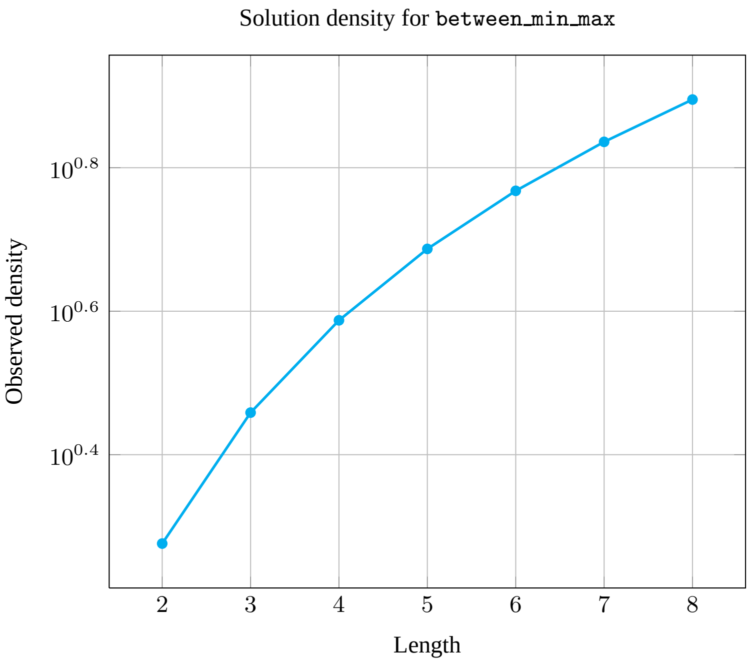
By introducing two extra variables MIN and MAX, the between_min_max(VAR, VARIABLES) constraint can be expressed in term of the following conjunction of constraints:

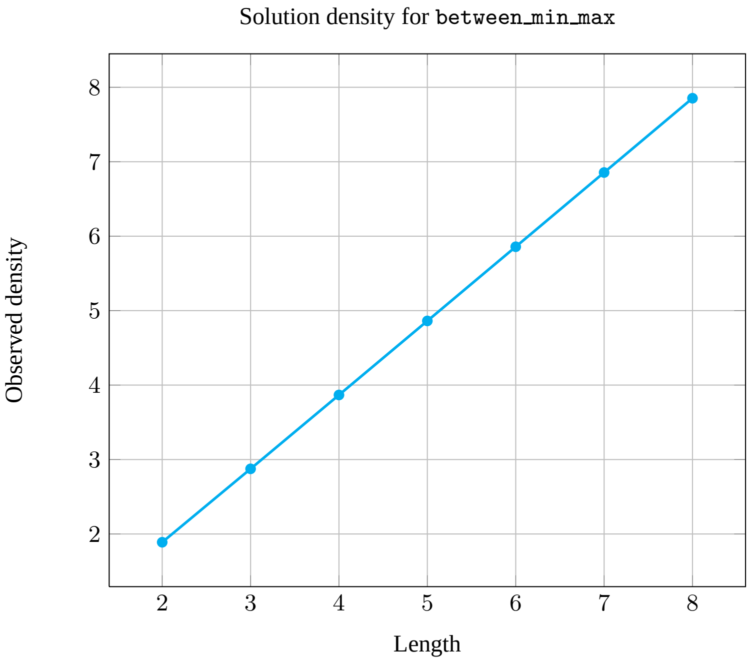
minimum(MIN, VARIABLES),
maximum(MAX, VARIABLES),
VAR $\geq$ MIN,
VAR $\leq$ MAX.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	17	184	2417	37806	689201	14376608	338051265

Number of solutions for between_min_max: domains $0..n$

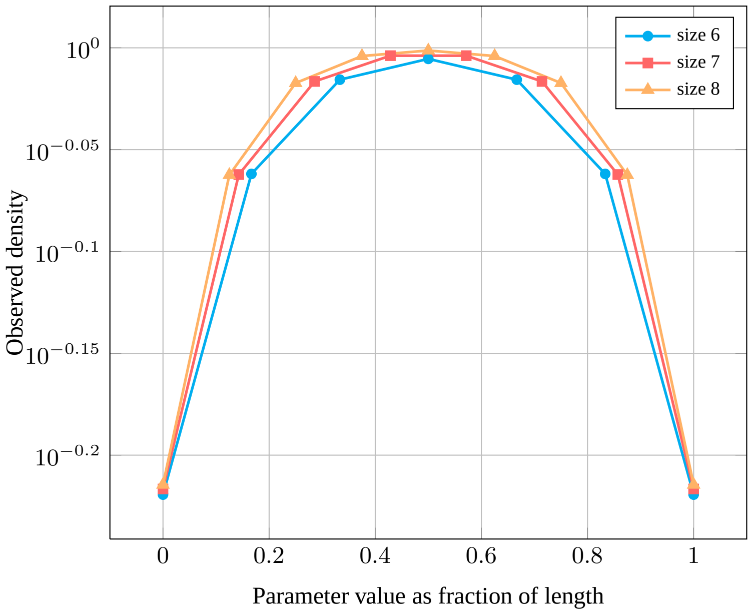

Solution density for between_min_max
Observed density
$10^{0.8}$
$10^{0.6}$
$10^{0.4}$
2
3
4
5
6
7
8
Length

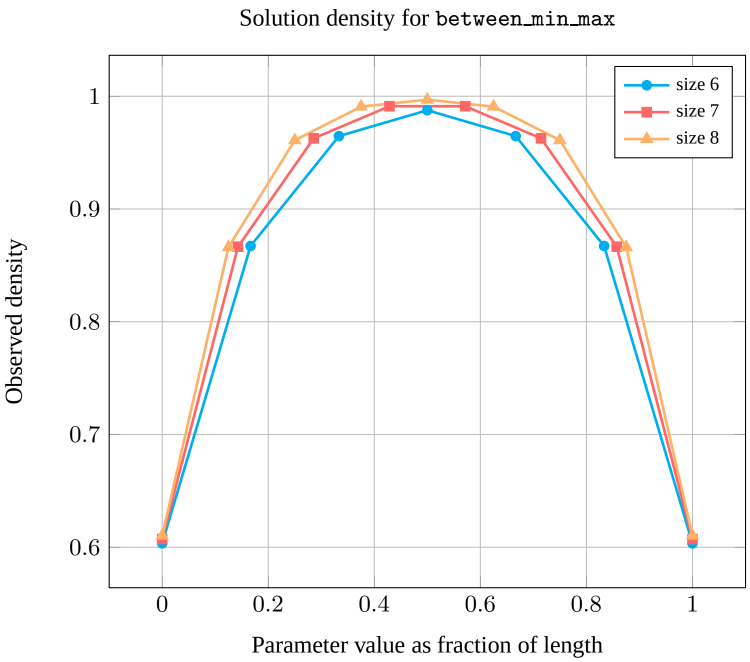

Solution density for between_min_max
Observed density
8
7
6
5
4
3
2
2
3
4
5
6
7
8
Length

Length (n)		2	3	4	5	6	7	8
Total		17	184	2417	37806	689201	14376608	338051265
Parameter value	0	5	37	369	4651	70993	1273609	26269505
	1	7	55	543	6751	102023	1817215	37281919
	2	5	55	593	7501	113489	2018899	41366849
	3	-	37	543	7501	116191	2078581	42649535
	4	-	-	369	6751	113489	2078581	42915649
	5	-	-	-	4651	102023	2018899	42649535
	6	-	-	-	-	70993	1817215	41366849
	7	-	-	-	-	-	1273609	37281919
	8	-	-	-	-	-	-	26269505

Solution count for `between_min_max`: domains $0..n$

Solution density for `between_min_max`





Used in `cumulative_convex`.

See also **implied by:** `and`, `deepest_valley`, `first_value_diff_0`, `highest_peak`, `in`, `maximum`, `minimum`.

Keywords **characteristic of a constraint:** automaton, automaton without counters, reified automaton constraint.

constraint network structure: centered cyclic(1) constraint network(1).

Derived Collection	col(ITEM−collection(var−dvar), [item(var − VAR)])

Arc input(s)	ITEM VARIABLES
Arc generator	$PRODUCT$ ↦ collection(item, variables)
Arc arity	2
Arc constraint(s)	item.var $\geq$ variables.var
Graph property(ies)	**NARC** ≥ 1
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Arc input(s)	ITEM VARIABLES
Arc generator	$PRODUCT$ ↦ collection(item, variables)
Arc arity	2
Arc constraint(s)	item.var $\leq$ variables.var
Graph property(ies)	**NARC** ≥ 1
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Graph model Parts (A) and (B) of Figure 5.130 respectively show the initial and final graph associated with the second graph constraint of the first example of the **Example** slot. Since we use the **NARC** graph property, the two arcs of the final graph are stressed in bold. The constraint holds since 3 is greater than 1 and since 3 is less than 8.

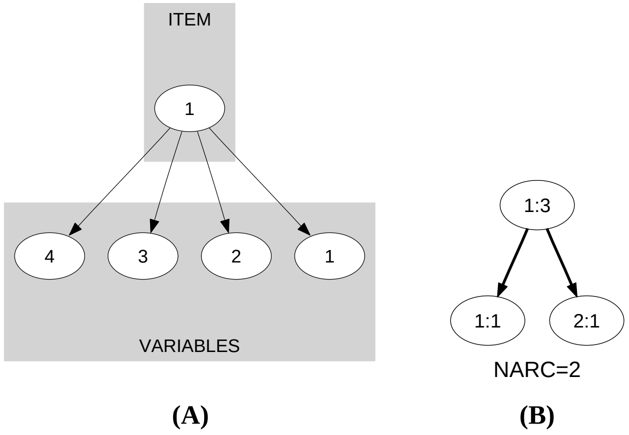


Figure 5.130: Initial and final graph of the between_min_max constraint

Automaton Figure 5.131 depicts the automaton associated with the between_min_max constraint. To each pair $(\texttt{VAR}, \texttt{VAR}_i)$, where $\texttt{VAR}_i$ is a variable of the collection VARIABLES corresponds a signature variable S_i. The following signature constraint links VAR, $\texttt{VAR}_i$ and S_i: $(\texttt{VAR} < \texttt{VAR}_i \Leftrightarrow S_i = 0) \wedge (\texttt{VAR} = \texttt{VAR}_i \Leftrightarrow S_i = 1) \wedge (\texttt{VAR} > \texttt{VAR}_i \Leftrightarrow S_i = 2)$.

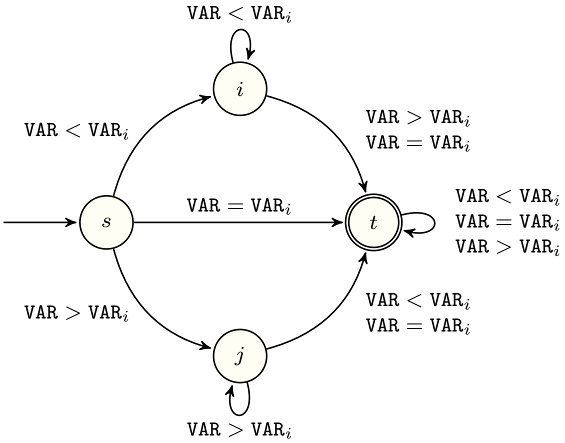


Figure 5.131: Automaton of the between_min_max constraint

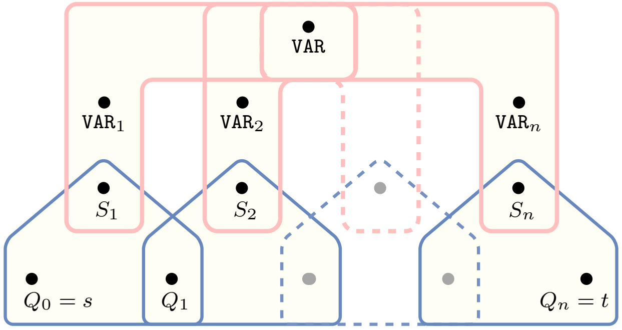


Figure 5.132: Hypergraph of the reformulation corresponding to the automaton of the between_min_max constraint