1018 AUTOMATON

5.111 decreasing_peak

DESCRIPTION LINKS AUTOMATON

Origin Derived from peak and decreasing.

Constraint decreasing_peak(VARIABLES)

Argument VARIABLES : collection(var-dvar)

Restrictions |VARIABLES| > 0

Purpose

required(VARIABLES, var)

A variable V_k (1 < k < m) of the sequence of variables VARIABLES $= V_1, \ldots, V_m$ is a peak if and only if there exists an i $(1 < i \le k)$ such that $V_{i-1} < V_i$ and $V_i = V_{i+1} = \cdots = V_k$ and $V_k > V_{k+1}$.

When considering all the peaks of the sequence VARIABLES from left to right enforce all peaks to be decreasing, i.e. the altitude of each peak is less than or equal to the altitude of its preceding peak when it exists.

Example $(\langle 1, 7, 7, 4, 3, 7, 2, 2, 5, 4 \rangle)$

The decreasing_peak constraint holds since the sequence 1 7 7 4 3 7 2 2 5 4 contains three peaks, in bold, that are decreasing.

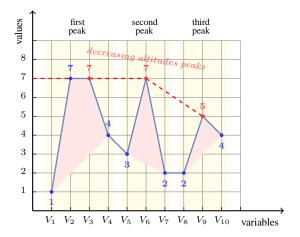


Figure 5.262: Illustration of the **Example** slot: a sequence of ten variables V_1 , V_2 , V_3 , V_4 , V_5 , V_6 , V_7 , V_8 , V_9 , V_{10} respectively fixed to values 1, 7, 7, 4, 3, 7, 2, 2, 5, 4 and its corresponding three peaks, in red, respectively located at altitudes 7, 7 and 5

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Typical

 $\begin{aligned} |\mathtt{VARIABLES}| &\geq 7 \\ \mathtt{range}(\mathtt{VARIABLES.var}) &> 1 \\ \mathtt{peak}(\mathtt{VARIABLES.var}) &\geq 3 \end{aligned}$

Symmetry

One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

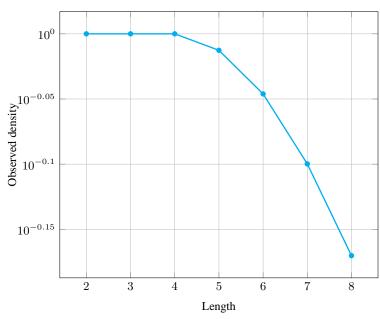
- Prefix-contractible wrt. VARIABLES.
- Suffix-contractible wrt. VARIABLES.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7553	105798	1666878	29090469

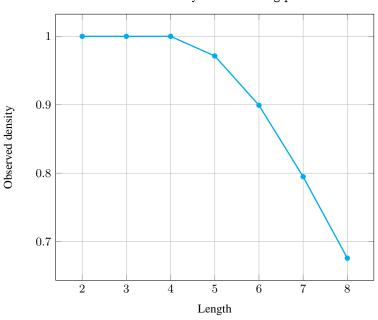
Number of solutions for decreasing peak: domains 0..n

Solution density for decreasing_peak



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Solution density for decreasing_peak



See also implied by: all_equal_peak.

related: increasing_peak, peak.

Keywords characteristic of a constraint: automaton, automaton with counters,

automaton with same input symbol.

combinatorial object: sequence.

constraint network structure: sliding cyclic(1) constraint network(2).

 $\textbf{Cond. implications} \qquad \texttt{decreasing_peak}(\texttt{VARIABLES})$

with peak(VARIABLES.var) > 0 implies not_all_equal(VARIABLES).

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Automaton

Figure 5.263 depicts the automaton associated with the decreasing_peak constraint. To each pair of consecutive variables (VAR $_i$, VAR $_{i+1}$) of the collection VARIABLES corresponds a signature variable S_i . The following signature constraint links VAR $_i$, VAR $_{i+1}$ and S_i : (VAR $_i$ < VAR $_{i+1}$ \Leftrightarrow $S_i = 0) <math>\land$ (VAR $_i$ = VAR $_{i+1}$ \Leftrightarrow $S_i = 1) <math>\land$ (VAR $_i$ > VAR $_{i+1}$ \Leftrightarrow $S_i = 2$).

STATES SEMANTICS : initial stationary or decreasing mode : increasing (before first potential peak) mode : decreasing (after a peak) mode : increasing (after a peak) mode $\{Altitude \leftarrow 0\}$ $\mathtt{VAR}_i < \mathtt{VAR}_{i+1}$ $\mathtt{VAR}_i \leq \mathtt{VAR}_{i+1}$ $\mathtt{VAR}_i > \mathtt{VAR}_{i+1},$ $\mathtt{VAR}_i > \mathtt{VAR}_{i+1},$ $\{Altitude \geq VAR_i,$ $\{Altitude \leftarrow VAR_i\}$ $Altitude \leftarrow VAR_i$ $\mathtt{VAR}_i \leq \mathtt{VAR}_{i+1}$ $\mathtt{VAR}_i \ge \mathtt{VAR}_{i+1}$ $\mathtt{VAR}_i < \mathtt{VAR}_{i+1}$

Figure 5.263: Automaton for the decreasing peak constraint (note the conditional transition from state w to state v testing that the counter Altitude is greater than or equal to VAR $_i$ for enforcing that all peaks from left to right are in decreasing altitude)

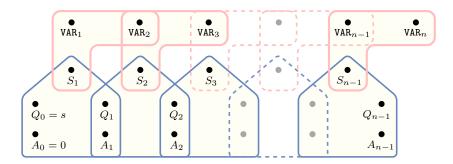


Figure 5.264: Hypergraph of the reformulation corresponding to the automaton of the decreasing peak constraint where A_i stands for the value of the counter Altitude (since all states of the automaton are accepting there is no restriction on the last variable Q_{n-1})