5.235　link_set_to_booleans

DESCRIPTION　　　　　**LINKS**　　　　　**GRAPH**

Origin　　　Inspired by domain_constraint.

Constraint　　　link_set_to_booleans(SVAR, BOOLEANS)

Arguments
SVAR　　　　: svar
BOOLEANS　: collection(bool−dvar, val−int)

Restrictions
required(BOOLEANS, [bool, val])
BOOLEANS.bool ≥ 0
BOOLEANS.bool ≤ 1
distinct(BOOLEANS, val)

Purpose

Make the link between a set variable SVAR and those 0-1 variables that are associated with each potential value belonging to SVAR: The 0-1 variables, which are associated with a value belonging to the set variable SVAR, are equal to 1, while the remaining 0-1 variables are all equal to 0.

Example

$$\left(\begin{array}{l} \{1, 3, 4\}, \\ \quad \texttt{bool} - 0 \quad \texttt{val} - 0, \\ \quad \texttt{bool} - 1 \quad \texttt{val} - 1, \\ \left\langle \begin{array}{l} \texttt{bool} - 0 \quad \texttt{val} - 2, \\ \texttt{bool} - 1 \quad \texttt{val} - 3, \\ \texttt{bool} - 1 \quad \texttt{val} - 4, \\ \texttt{bool} - 0 \quad \texttt{val} - 5 \end{array} \right\rangle \end{array} \right)$$

In the example, the 0-1 variables associated with the values 1, 3 and 4 are all set to 1, while the other 0-1 variables are set to 0. Consequently, the link_set_to_booleans constraint holds since its first argument SVAR is set to $\{1, 3, 4\}$.

Typical
$|\texttt{BOOLEANS}| > 1$
range(BOOLEANS.bool) > 1

Symmetry　　　Items of BOOLEANS are permutable.

Usage　　　This constraint is used in order to make the link between a formulation using set variables and a formulation based on linear programming.

Systems　　　channel in **Gecode**, link_set_to_booleans in **MiniZinc**.

See also　　　**common keyword:** alldifferent_between_sets, clique (*constraint involving set variables*), domain_constraint (*channelling constraint*), k_cut, path_from_to, roots, strongly_connected, symmetric_cardinality, symmetric_gcc, tour (*constraint involving set variables*).

Keywords

Derived Collection

$$\text{col} \left(\begin{array}{l} \text{SET}-\text{collection}(\text{one}-\text{int}, \text{setvar}-\text{svar}), \\ [\text{item}(\text{one} - 1, \text{setvar} - \text{SVAR})] \end{array} \right)$$

Arc input(s)	SET BOOLEANS
Arc generator	$PRODUCT \mapsto \text{collection}(\text{set}, \text{booleans})$
Arc arity	2
Arc constraint(s)	$\text{booleans.bool} = \text{set.one} \Leftrightarrow \text{in\_set}(\text{booleans.val}, \text{set.setvar})$
Graph property(ies)	$\textbf{NARC} = |\text{BOOLEANS}|$

Graph model

The link_set_to_booleans constraint is modelled with the following bipartite graph. The first set of vertices corresponds to a single vertex containing the set variable. The second class of vertices contains one vertex for each item of the collection BOOLEANS. The arc constraint between the set variable SVAR and one potential value v of the set variable expresses the following:

- If the 0-1 variable associated with v is equal to 1 then v should belong to SVAR.
- Otherwise if the 0-1 variable associated with v is equal to 0 then v should not belong to SVAR.

Since all arc constraints should hold the final graph contains exactly $|\text{BOOLEANS}|$ arcs.

Parts (A) and (B) of Figure 5.519 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. The link_set_to_booleans constraint holds since the final graph contains exactly 6 arcs (one for each 0-1 variable).

Signature

Since the initial graph contains $|\text{BOOLEANS}|$ arcs the maximum number of arcs of the final graph is equal to $|\text{BOOLEANS}|$. Therefore we can rewrite the graph property $\textbf{NARC} = |\text{BOOLEANS}|$ to $\textbf{NARC} \geq |\text{BOOLEANS}|$ and simplify $\overline{\textbf{NARC}}$ to $\overline{\textbf{NARC}}$.
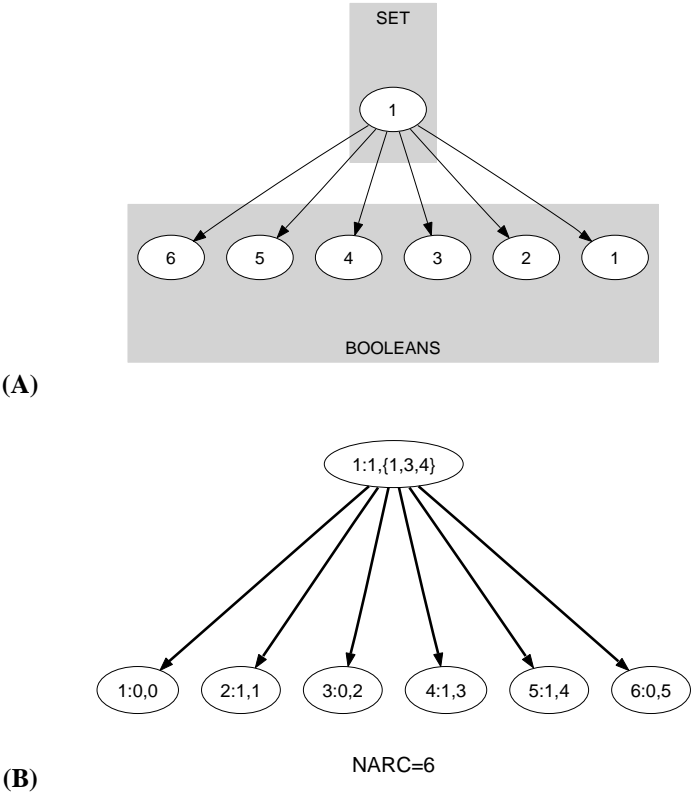
(A)

(B)

NARC=6

Figure 5.519: Initial and final graph of the link_set_to_booleans constraint