## 5.41 atmost_nvalue

DESCRIPTION LINKS GRAPH

**Origin** [62]

**Constraint** atmost_nvalue(NVAL, VARIABLES)

**Synonyms** soft_alldiff_max_var, soft_alldifferent_max_var, soft_alldistinct_max_var.

**Arguments**

| | | |
|---|---|---|
| NVAL | : | dvar |
| VARIABLES | : | collection(var−dvar) |

**Restrictions**

NVAL ≥ min(1, |VARIABLES|)
required(VARIABLES, var)

**Purpose** The number of distinct values taken by the variables of the collection VARIABLES is less than or equal to NVAL.

**Example**

$(4, \langle 3, 1, 3, 1, 6\rangle)$
$(3, \langle 3, 1, 3, 1, 6\rangle)$
$(1, \langle 3, 3, 3, 3, 3\rangle)$

The first atmost_nvalue constraint holds since the collection $\langle 3, 1, 3, 1, 6\rangle$ involves at most 4 distinct values (i.e., in fact 3 distinct values).

**Typical**

NVAL > 1
NVAL < |VARIABLES|
|VARIABLES| > 1

**Symmetries**

- NVAL can be increased.
- Items of VARIABLES are permutable.
- All occurrences of two distinct values of VARIABLES.var can be swapped; all occurrences of a value of VARIABLES.var can be renamed to any unused value.
- An occurrence of a value of VARIABLES.var can be replaced by any value of VARIABLES.var.

**Arg. properties**

Contractible wrt. VARIABLES.

**Remark** This constraint was introduced together with the atleast_nvalue constraint by C. Bessière *et al.* in an article [62] providing filtering algorithms for the nvalue constraint.

It was shown in [69] that, finding out whether a atmost_nvalue constraint has a solution or not is NP-hard. This was achieved by reduction from 3-SAT.

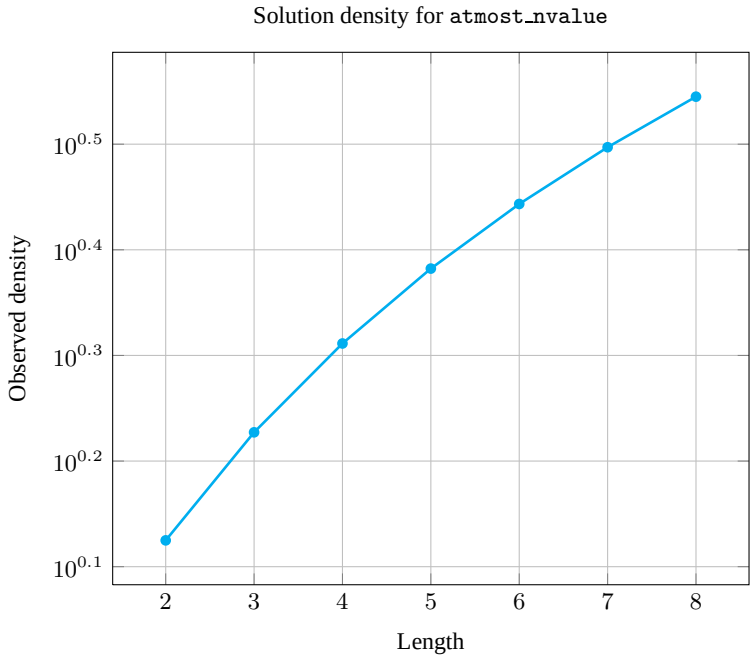
**Algorithm**

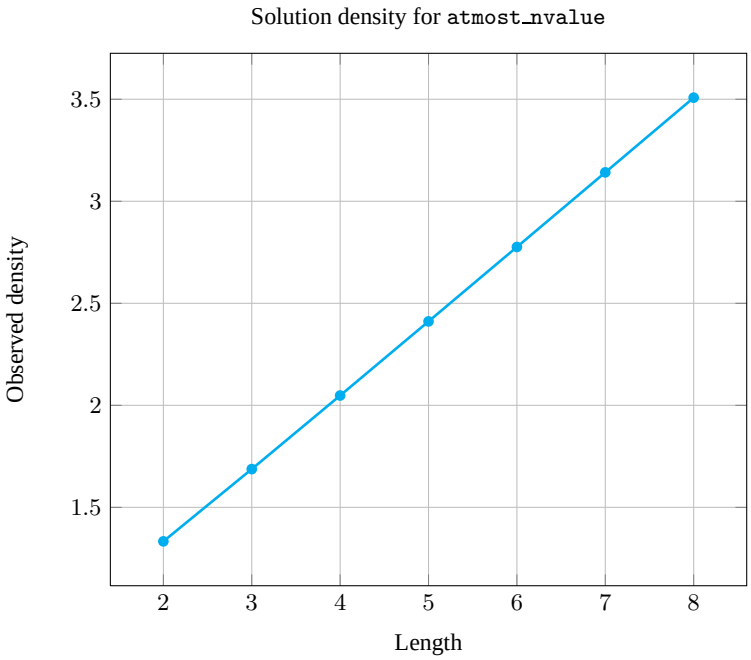
[27] provides an algorithm that achieves bound consistency. [40] provides two filtering algorithms, while [62] provides a greedy algorithm and a graph invariant for evaluating the minimum number of distinct values. [62] also gives a linear relaxation for approximating the minimum number of distinct values.

**Counting**

| Length ($n$) | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Solutions | 12 | 108 | 1280 | 18750 | 326592 | 6588344 | 150994944 |

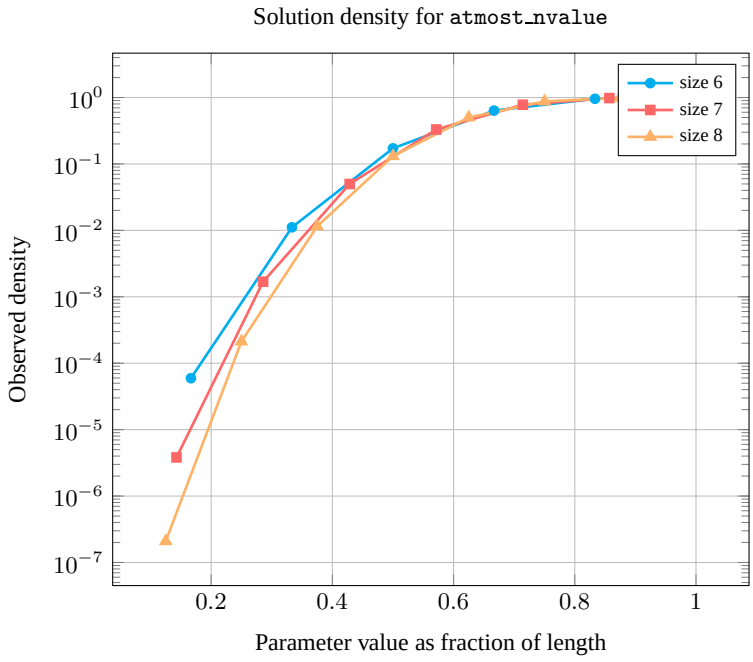
Number of solutions for `atmost_nvalue`: domains $0..n$

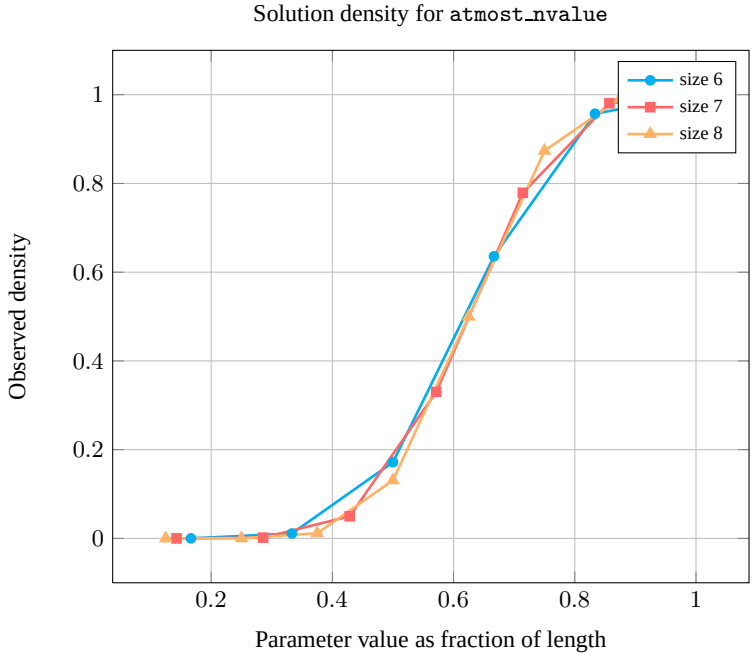





| Length ($n$) | | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Total | | 12 | 108 | 1280 | 18750 | 326592 | 6588344 | 150994944 |
| Parameter value | 1 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 2 | 9 | 40 | 145 | 456 | 1309 | 3536 | 9153 |
| | 3 | - | 64 | 505 | 3456 | 20209 | 104672 | 496017 |
| | 4 | - | - | 625 | 7056 | 74809 | 692672 | 5639841 |
| | 5 | - | - | - | 7776 | 112609 | 1633472 | 21515841 |
| | 6 | - | - | - | - | 117649 | 2056832 | 37603521 |
| | 7 | - | - | - | - | - | 2097152 | 42683841 |
| | 8 | - | - | - | - | - | - | 43046721 |

Solution count for `atmost_nvalue`: domains $0..n$




**Systems** `atMostNValue` in **Choco**.

**See also** **comparison swapped:** `atleast_nvalue`.

**implied by:** nvalue *(*$\leq$ NVAL *replaced by* $=$ NVAL*)*.

**related:** soft_all_equal_max_var, soft_all_equal_min_ctr, soft_all_equal_min_var, soft_alldifferent_ctr, soft_alldifferent_var.

**Keywords**

**complexity:** 3-SAT.

**constraint type:** counting constraint, value partitioning constraint.

**filtering:** bound-consistency.

**final graph structure:** strongly connected component, equivalence.

**modelling:** number of distinct equivalence classes, number of distinct values.

| | |
|---|---|
| **Arc input(s)** | VARIABLES |
| **Arc generator** | $CLIQUE \mapsto$ collection(variables1, variables2) |
| **Arc arity** | 2 |
| **Arc constraint(s)** | variables1.var = variables2.var |
| **Graph property(ies)** | **NSCC** $\leq$ NVAL |
| **Graph class** | EQUIVALENCE |

**Graph model** Parts (A) and (B) of Figure 5.112 respectively show the initial and final graph associated with the first example of the **Example** slot. Since we use the **NSCC** graph property we show the different strongly connected components of the final graph. Each strongly connected component corresponds to a specific value that is assigned to some variables of the VARIABLES collection. The 3 following values 1, 3 and 6 are used by the variables of the VARIABLES collection.

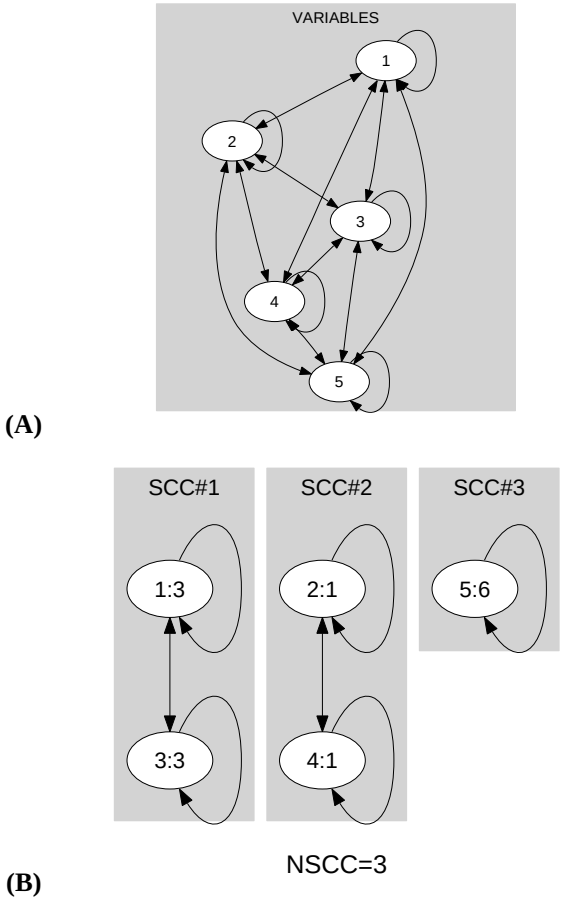



Figure 5.112: Initial and final graph of the `atmost_nvalue` constraint