1628 ORDER, CLIQUE

## 5.244 max\_n

DESCRIPTION LINKS GRAPH

Origin [27]

Constraint max\_n(MAX, RANK, VARIABLES)

Arguments MAX : dvar RANK : int

VARIABLES : collection(var-dvar)

**Restrictions**  $RANK \ge 0$ 

RANK < |VARIABLES| |VARIABLES| > 0

required(VARIABLES, var)

Purpose

MAX is the maximum value of rank RANK (i.e., the RANK  $^{th}$  largest distinct value, identical values are merged) of the collection of domain variables VARIABLES. The maximum value has rank 0.

**Example** 

```
(6,1,\langle 3,1,7,1,6\rangle)
```

The max\_n constraint holds since its first argument MAX = 6 is fixed to the second (i.e., RANK + 1) largest distinct value of the collection (3, 1, 7, 1, 6).

**Typical** 

```
\begin{aligned} & \text{RANK} > 0 \\ & \text{RANK} < 3 \\ & | \text{VARIABLES}| > 1 \\ & \text{range}(\text{VARIABLES.var}) > 1 \end{aligned}
```

**Symmetries** 

- Items of VARIABLES are permutable.
- One and the same constant can be added to MAX as well as to the var attribute of all items of VARIABLES.

Arg. properties

Functional dependency: MAX determined by RANK and VARIABLES.

Algorithm

[27].

Reformulation

The constraint <code>among\_var(1, \langle MAX \rangle, VARIABLES)</code> enforces MAX to be assigned one of the values of VARIABLES. The constraint <code>nvalue(NVAL, VARIABLES)</code> provides a hand on the number of distinct values assigned to the variables of VARIABLES. By associating to each variable  $V_i$  ( $i \in [1, |VARIABLES|]$ ) of the VARIABLES collection a <code>rank</code> variable  $R_i \in [0, |VARIABLES| - 1]$  with the reified constraint  $R_i = RANK \Leftrightarrow V_i = MAX$ , the inequality  $R_i < NVAL$ , and by creating for each pair of variables  $V_i, V_j$  ( $i, j < i \in [1, |VARIABLES|]$ ) the reified constraints

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$$\begin{split} V_i > V_j &\Leftrightarrow R_i < R_j, \\ V_i = V_j &\Leftrightarrow R_i = R_j, \\ V_i < V_j &\Leftrightarrow R_i > R_j, \\ \text{one can reformulate the max\_n constraint in term of } 3 \cdot \frac{|\text{VARIABLES}| \cdot (|\text{VARIABLES}| - 1)}{2} + 1 \text{ reified} \end{split}$$
constraints.

See also

comparison swapped: min\_n.

generalisation: maximum (absolute maximum replaced by maximum or order n).

Keywords

characteristic of a constraint: rank, maximum.

constraint arguments: pure functional dependency.

constraint type: order constraint. modelling: functional dependency.

Arc input(s)	VARIABLES
Arc generator	$CLIQUE \mapsto \texttt{collection}(\texttt{variables1}, \texttt{variables2})$
Arc arity	2
Arc constraint(s)	$\bigvee \left(egin{array}{c} { t variables1.key} = { t variables2.key}, \ { t variables1.var} > { t variables2.var} \end{array} ight)$
Graph property(ies)	$\frac{\mathbf{ORDER}(\mathtt{RANK},\mathtt{MININT},\mathtt{var}) = \mathtt{MAX}}{}$

## Graph model

Parts (A) and (B) of Figure 5.535 respectively show the initial and final graph associated with the **Example** slot. Since we use the **ORDER** graph property, the vertex of rank 1 (without considering the loops) of the final graph is outlined with a thick circle.

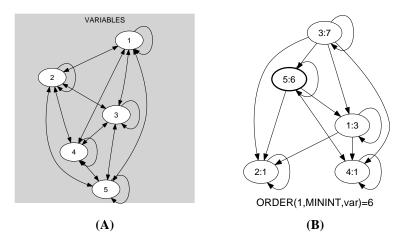


Figure 5.535: Initial and final graph of the max\_n constraint

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