5.51 big_peak

DESCRIPTION LINKS AUTOMATON

Origin

Derived from peak.

Constraint

big_peak(N, VARIABLES, TOLERANCE)

Arguments

N	:	dvar
VARIABLES	:	collection(var−dvar)
TOLERANCE	:	int

Restrictions

$\mathtt{N} \geq 0$
$2 * \mathtt{N} \leq \mathtt{max}(|\mathtt{VARIABLES}| - 1, 0)$
required(VARIABLES, var)
$\mathtt{TOLERANCE} \geq 0$

Purpose

A variable V_p $(1 < p < m)$ of the sequence of variables $\mathtt{VARIABLES} = V_1, \ldots, V_m$ is a *peak* if and only if there exists an i $(1 < i \leq p)$ such that $V_{i-1} < V_i$ and $V_i = V_{i+1} = \cdots = V_p$ and $V_p > V_{p+1}$. Similarly a variable V_v $(1 < k < m)$ is a *valley* if and only if there exists an i $(1 < i \leq v)$ such that $V_{i-1} > V_i$ and $V_i = V_{i+1} = \cdots = V_v$ and $V_v < V_{v+1}$. A peak variable V_p $(1 < p < m)$ is a *potential big peak* wrt a non-negative integer TOLERANCE if and only if:

1. V_p is a peak,
2. $\exists i, j \in [1, m] \mid i < p < j$, V_i is a valley (or $i = 1$ if there is no valley before position p), V_j is a valley (or $i = m$ if there is no valley after position p), $V_p - V_i > \mathtt{TOLERANCE}$, and $V_p - V_j > \mathtt{TOLERANCE}$.

Let i_p and j_p be the largest i and the smallest j satisfying condition 2. Now a potential big peak V_p $(1 < p < m)$ is a *big peak* if and only if the interval $[i, j]$ does not contain any potential big peak that is strictly higher than V_p. The constraint big_peak holds if and only if N is the total number of big peaks of the sequence of variables VARIABLES.

Example

$(7, \langle 4, 2, 2, 4, 3, 8, 6, 7, 7, 9, 5, 6, 3, 12, 12, 6, 6, 8, 4, 5, 1 \rangle, 0)$
$(4, \langle 4, 2, 2, 4, 3, 8, 6, 7, 7, 9, 5, 6, 3, 12, 12, 6, 6, 8, 4, 5, 1 \rangle, 1)$

As shown part Part (A) of Figure 5.133, the first big_peak constraint holds since the sequence 4 2 2 4 3 8 6 7 7 9 5 6 3 12 12 6 6 8 4 5 1 contains seven big peaks wrt a tolerance of 0 (i.e., we consider standard peaks).

As shown part Part (B) of Figure 5.133, the second big_peak constraint holds since the same sequence 4 2 2 4 3 8 6 7 7 9 5 6 3 12 12 6 6 8 4 5 1 contains only four big peaks wrt a tolerance of 1.

Typical

$\mathtt{N} \geq 1$
$|\mathtt{VARIABLES}| > 6$
$\mathtt{range}(\mathtt{VARIABLES.var}) > 1$
$\mathtt{TOLERANCE} > 1$

Symmetries

- Items of VARIABLES can be reversed.
- One and the same constant can be added to the `var` attribute of all items of VARIABLES.

Arg. properties

- Functional dependency: N determined by VARIABLES and TOLERANCE.
- Contractible wrt. VARIABLES when $\mathtt{N} = 0$ and $\mathtt{TOLERANCE} = 0$.

Usage

Useful for constraining the number of *big peaks* of a sequence of domain variables, by ignoring too small valleys that artificially create small peaks wrt TOLERANCE.

See also

specialisation: `peak` *(the tolerance is set to* 0 *and removed)*.

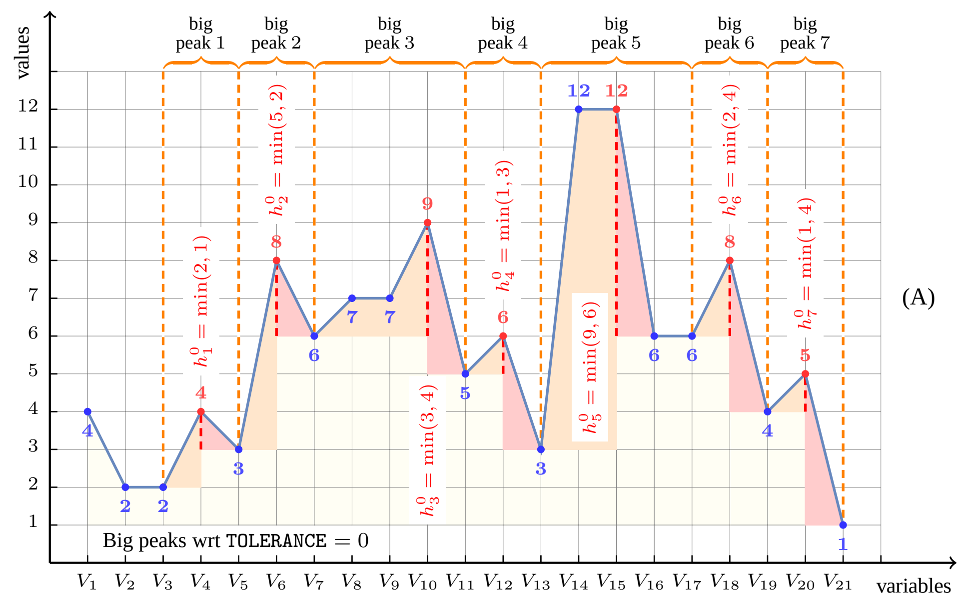
Keywords

characteristic of a constraint: automaton, automaton with counters.

combinatorial object: sequence.

constraint arguments: pure functional dependency.

modelling: functional dependency.



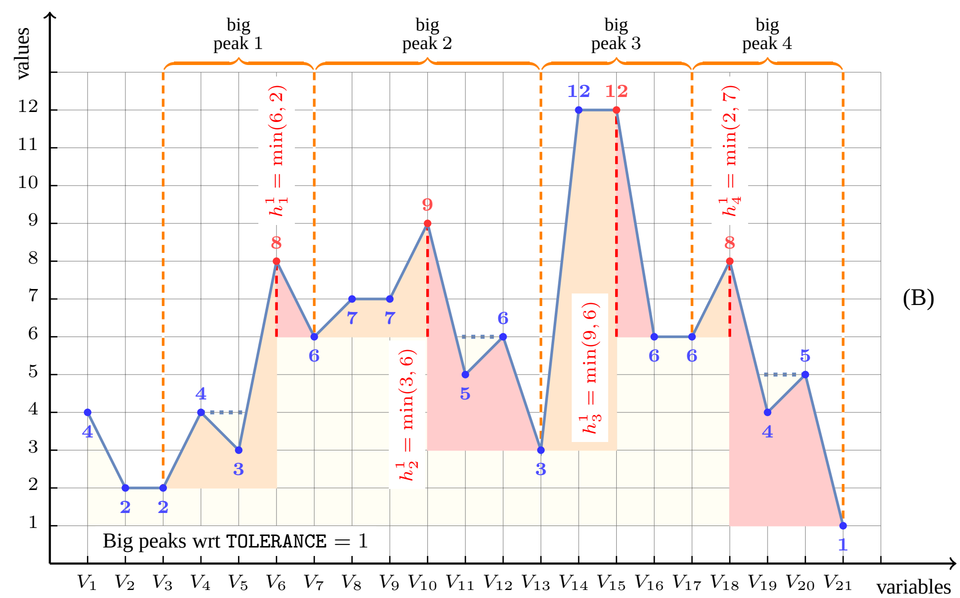


Figure 5.133: Illustration of the **Example** slot: Part (A) a sequence of 21 variables V_1, $V_2, \dots, V_{21}$ respectively fixed to values 4, 2, 2, 4, 3, 8, 6, 7, 7, 9, 5, 6, 3, 12, 12, 6, 6, 8, 4, 5, 1 and its corresponding 7 peaks (TOLERANCE $= 0$ corresponds to standard peaks) with their respective heights $h_1^0 = 1$, $h_2^0 = 2$, $h_3^0 = 3$, $h_4^0 = 1$, $h_5^0 = 6$, $h_6^0 = 2$, $h_7^0 = 1$ (the left and right hand sides of each peak are coloured in light orange and light red) Part (B) the same sequence of variables and its 4 big peaks when TOLERANCE $= 1$ with their respective heights $h_1^1 = 2$, $h_2^1 = 3$, $h_3^1 = 6$, $h_4^1 = 2$

Automaton Figure 5.134 depicts the automaton associated with the big_peak constraint. To each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection VARIABLES corresponds a signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $(\mathtt{VAR}_i < \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\mathtt{VAR}_i = \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 1) \wedge (\mathtt{VAR}_i > \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 2)$.

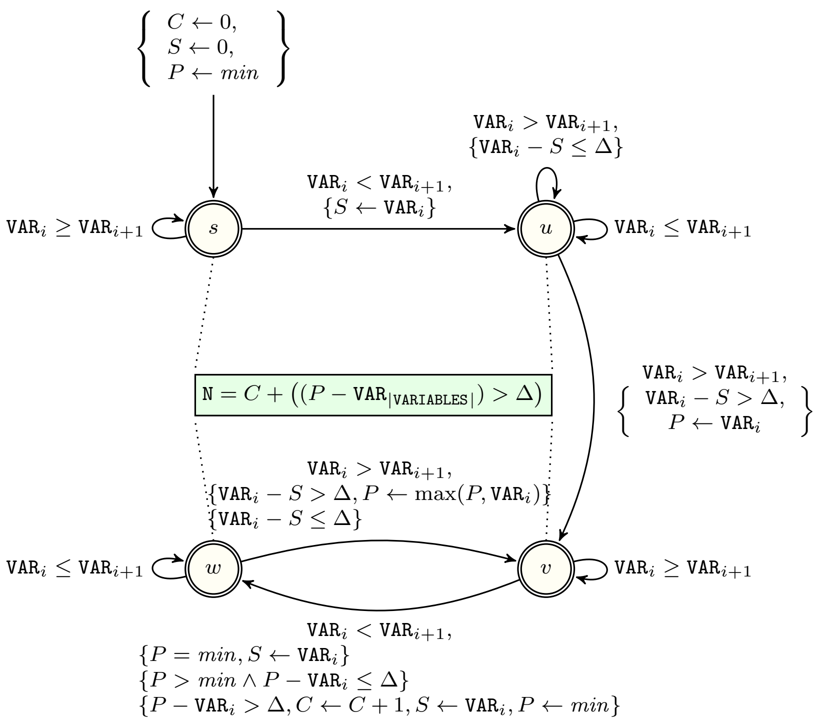


Figure 5.134: Automaton for the big_peak constraint where C, S, P, min and Δ respectively stand for the number of big peaks already encountered, the altitude at the start of the current potential big peak, the altitude of the current potential big peak, the smallest value that can be assigned to a variable of VARIABLES, the TOLERANCE parameter