5.10 all_incomparable

DESCRIPTION LINKS GRAPH

Origin Inspired by incomparable rectangles.

Constraint all_incomparable(VECTORS)

Synonym all_incomparables.

Type VECTOR : collection(var−dvar)

Argument VECTORS : collection(vec − VECTOR)

Restrictions
required(VECTOR, var)
|VECTOR| ≥ 1
required(VECTORS, vec)
|VECTORS| ≥ 1
same_size(VECTORS, vec)

Purpose Enforce for each pair of distinct vectors of the VECTORS collection the fact that they are incomparable. Two vectors VECTOR1 and VECTOR2 are incomparable if and only, when the components of both vectors are ordered, and respectively denoted by SVECTOR1 and SVECTOR2, we neither have SVECTOR1$[i]$.var $\leq$ SVECTOR2$[i]$.var (for all $i \in [1, |\text{SVECTOR1}|]$) nor have SVECTOR2$[i]$.var $\leq$ SVECTOR1$[i]$.var (for all $i \in [1, |\text{SVECTOR1}|]$).

Example

$$\left(\left\langle\begin{array}{l}\text{vec} - \langle 1, 18\rangle,\\ \text{vec} - \langle 2, 16\rangle,\\ \text{vec} - \langle 3, 13\rangle,\\ \text{vec} - \langle 4, 11\rangle,\\ \text{vec} - \langle 5, 10\rangle,\\ \text{vec} - \langle 6, 9\rangle,\\ \text{vec} - \langle 7, 7\rangle\end{array}\right\rangle\right)$$

The all_incomparable constraint holds since all distinct pairs of vectors are incomparable as illustrated by Figure 5.21.

All solutions Figure 5.22 gives all solutions to the following non ground instance of the all_incomparable constraint: $U_1 \in [1, 2]$, $V_1 \in [0, 5]$, $U_2 \in [3, 5]$, $V_2 \in [2, 3]$, $U_3 \in [0, 6]$, $V_3 \in [2, 5]$, all_incomparable$(\langle\langle U_1, V_1\rangle, \langle U_2, V_2\rangle, \langle U_3, V_3\rangle\rangle)$.

Typical
|VECTOR| > 1
|VECTORS| > 1
|VECTORS| > |VECTOR|

Symmetry Items of VECTORS are permutable.

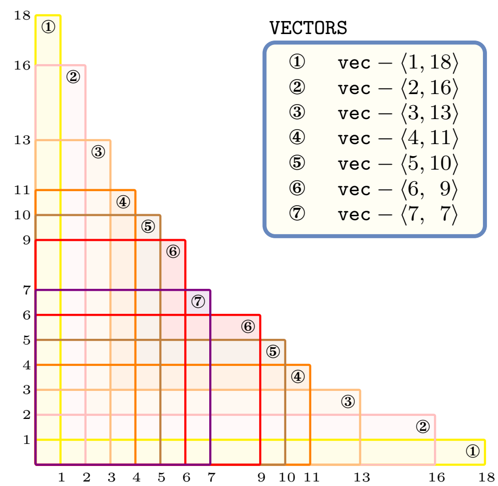


Figure 5.21: Illustrating the incomparability of vectors $\langle 1,18\rangle$, $\langle 2,16\rangle$, $\langle 3,13\rangle$, $\langle 4,11\rangle$, $\langle 5,10\rangle$, $\langle 6,9\rangle$, $\langle 7,7\rangle$: first to each vector we associate a rectangle whose sizes are the components of the vector; second no matter whether we rotate a rectangle from $90°$ or not, one rectangle can not be included in another rectangle.

① $(\langle\langle 1,4\rangle,\langle 3,2\rangle,\langle 0,5\rangle\rangle)$
② $(\langle\langle 1,4\rangle,\langle 3,3\rangle,\langle 0,5\rangle\rangle)$
③ $(\langle\langle 1,5\rangle,\langle 3,3\rangle,\langle 2,4\rangle\rangle)$
④ $(\langle\langle 1,5\rangle,\langle 3,3\rangle,\langle 4,2\rangle\rangle)$
⑤ $(\langle\langle 1,5\rangle,\langle 4,2\rangle,\langle 3,3\rangle\rangle)$
⑥ $(\langle\langle 2,4\rangle,\langle 3,3\rangle,\langle 0,5\rangle\rangle)$
⑦ $(\langle\langle 2,4\rangle,\langle 3,3\rangle,\langle 1,5\rangle\rangle)$

Figure 5.22: All solutions corresponding to the non ground example of the `all_incomparable` constraint of the **All solutions** slot

Arg. properties

Contractible wrt. `VECTORS`.

See also

implies: `lex_alldifferent`.

part of system of constraints: `incomparable`.

used in graph description: `incomparable`.

Keywords

characteristic of a constraint: vector.

constraint type: system of constraints, decomposition.

final graph structure: no loop, symmetric.

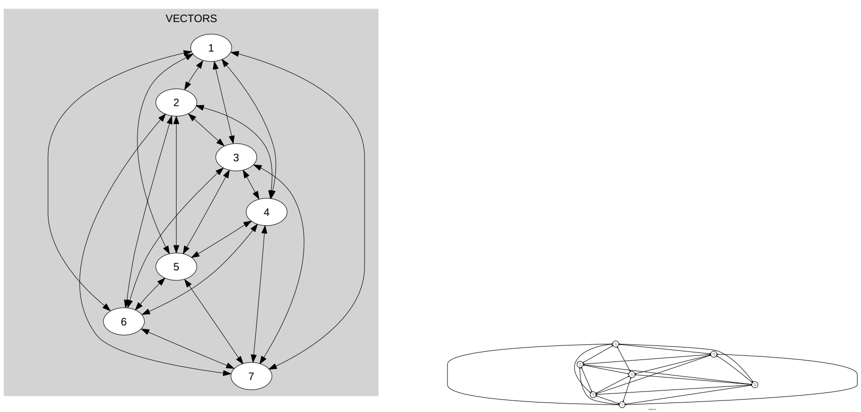
Cond. implications

- `all_incomparable`(`VECTORS`)
  with $|$`VECTOR`$| = 2$
  implies `k_disjoint`(`SETS` : `VECTORS`).
- `all_incomparable`(`VECTORS`)
  with $|$`VECTOR`$| = 2$
  implies `twin`(`PAIRS` : `VECTORS`).

Arc input(s)	`VECTORS`
Arc generator	$CLIQUE(\neq) \mapsto$ `collection`(`vectors1`, `vectors2`)
Arc arity	2
Arc constraint(s)	`incomparable`(`vectors1.vec`, `vectors2.vec`)
Graph property(ies)	**NARC** $=$ |`VECTORS`| $*$ |`VECTORS`| $-$ |`VECTORS`|
Graph class	• `NO_LOOP` • `SYMMETRIC`

Graph model The **Arc constraint(s)** slot uses the `incomparable` constraint defined in this catalogue.

Parts (A) and (B) of Figure 5.23 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. The previous constraint holds since exactly $3 \cdot (3-1) = 6$ arc constraints hold.



(A) **(B)**

Figure 5.23: Initial and final graph of the `all_incomparable` constraint