5.141　element_lesseq

Origin	[301]
Constraint	element_lesseq(ITEM, TABLE)
Arguments	ITEM　:　collection(index−dvar, value−dvar) TABLE　:　collection(index−int, value−int)
Restrictions	required(ITEM, [index, value]) ITEM.index $\geq$ 1 ITEM.index $\leq$ |TABLE| |ITEM| = 1 |TABLE| > 0 required(TABLE, [index, value]) TABLE.index $\geq$ 1 TABLE.index $\leq$ |TABLE| distinct(TABLE, index)
Purpose	ITEM[1].value is less than or equal to one of the entries (i.e., the value attribute) of the table TABLE.

Example

$$\left(\begin{array}{l} \langle \text{index} - 3 \; \text{value} - 1 \rangle, \\ \left\langle \begin{array}{ll} \text{index} - 1 & \text{value} - 6, \\ \text{index} - 2 & \text{value} - 9, \\ \text{index} - 3 & \text{value} - 2, \\ \text{index} - 4 & \text{value} - 9 \end{array} \right\rangle \end{array} \right)$$

The element_lesseq constraint holds since ITEM[1].value = 1 is less than or equal to TABLE[ITEM[1].index].value = TABLE[3].value = 2.

Typical	|TABLE| > 1 range(TABLE.value) > 1
Symmetries	• Items of TABLE are permutable. • All occurrences of two distinct values in ITEM.value or TABLE.value can be swapped; all occurrences of a value in ITEM.value or TABLE.value can be renamed to any unused value.
Usage	Used for modelling variable subscripts in linear constraints [301].
Reformulation	By introducing an extra variable VAL, the element_lesseq(⟨index − INDEX value − VALUE⟩, TABLE) constraint can be expressed in term of an elem(⟨index − INDEX value − VAL⟩, TABLE) constraint and of an inequality constraint VALUE $\leq$ VAL.

See also **common keyword:** element, element_greatereq, element_product *(array constraint)*.

implied by: elem.

Keywords **characteristic of a constraint:** automaton, automaton without counters, reified automaton constraint.

constraint arguments: binary constraint.

constraint network structure: centered cyclic(2) constraint network(1).

constraint type: data constraint.

filtering: linear programming, arc-consistency.

modelling: array constraint, table, variable subscript, variable indexing.

Cond. implications element_lesseq(ITEM, TABLE)
  with $minval($ITEM.value$) > 0$
  and   TABLE.value > 0
 implies bin_packing_capa(BINS : TABLE, ITEMS : ITEM).

Arc input(s)	ITEM TABLE
Arc generator	$PRODUCT \mapsto$ collection(item, table)
Arc arity	2
Arc constraint(s)	• item.index $=$ table.index • item.value $\leq$ table.value
Graph property(ies)	**NARC**$= 1$

Graph model

Similar to the element constraint except that the *equality* constraint of the second condition of the arc constraint is replaced by a *less than or equal to* constraint.

Parts (A) and (B) of Figure 5.308 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the unique arc of the final graph is stressed in bold.
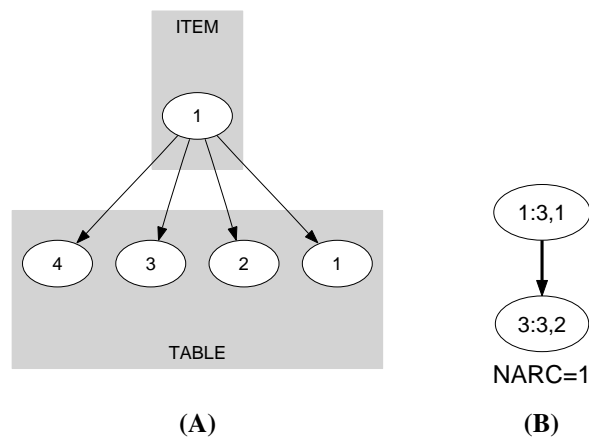

Figure 5.308: Initial and final graph of the element_lesseq constraint

Signature

Since all the index attributes of TABLE are distinct and because of the first arc constraint the final graph cannot have more than one arc. Therefore we can rewrite **NARC** $= 1$ to **NARC** ≥ 1 and simplify $\overline{\overline{\textbf{NARC}}}$ to $\overline{\textbf{NARC}}$.

Automaton
Figure 5.309 depicts the automaton associated with the `element_lesseq` constraint. Let INDEX and VALUE respectively be the `index` and the `value` attributes of the unique item of the ITEM collection. Let $INDEX_i$ and $VALUE_i$ respectively be the `index` and the `value` attributes of the ith item of the TABLE collection. To each quadruple $(INDEX, VALUE, INDEX_i, VALUE_i)$ corresponds a 0-1 signature variable S_i as well as the following signature constraint: $((INDEX = INDEX_i) \wedge (VALUE \le VALUE_i)) \Leftrightarrow S_i$.
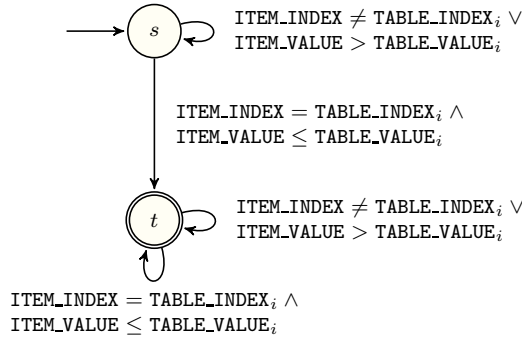
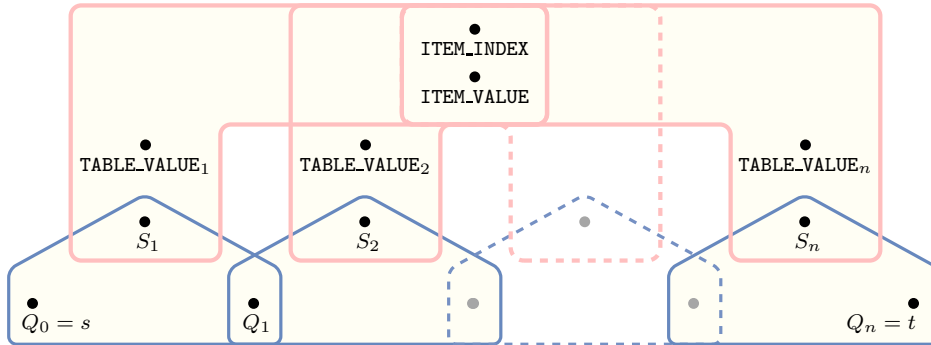Figure 5.309: Automaton of the `element_lesseq` constraint

Figure 5.310: Hypergraph of the reformulation corresponding to the automaton of the `element_lesseq` constraint