5.123 disjoint

DESCRIPTION LINKS GRAPH AUTOMATON

Origin

Derived from `alldifferent`.

Constraint

`disjoint(VARIABLES1, VARIABLES2)`

Arguments

VARIABLES1	:	collection(var−dvar)
VARIABLES2	:	collection(var−dvar)

Restrictions

required(VARIABLES1, var)
required(VARIABLES2, var)

Purpose

Each variable of the collection `VARIABLES1` should take a value that is distinct from all the values assigned to the variables of the collection `VARIABLES2`.

Example

$(\langle 1, 9, 1, 5\rangle, \langle 2, 7, 7, 0, 6, 8\rangle)$

In this example, values $1, 5, 9$ are used by the variables of `VARIABLES1` and values $0, 2, 6, 7, 8$ by the variables of `VARIABLES2`. Since there is no intersection between the two previous sets of values the `disjoint` constraint holds.

All solutions

Figure 5.292 gives all solutions to the following non ground instance of the `disjoint` constraint: $\mathtt{U}_1 \in [0, 2]$, $\mathtt{U}_2 \in [1, 2]$, $\mathtt{U}_3 \in [1, 2]$, $\mathtt{V}_1 \in [0, 1]$, $\mathtt{V}_2 \in [1, 2]$, `disjoint`$(\langle \mathtt{U}_1, \mathtt{U}_2, \mathtt{U}_3\rangle, \langle \mathtt{V}_1, \mathtt{V}_2\rangle)$.

① $(\langle 0, 2, 2\rangle, \langle 1, 1\rangle)$
② $(\langle 1, 1, 1\rangle, \langle 0, 2\rangle)$
③ $(\langle 2, 2, 2\rangle, \langle 0, 1\rangle)$
④ $(\langle 2, 2, 2\rangle, \langle 1, 1\rangle)$

Figure 5.292: All solutions corresponding to the non ground example of the `disjoint` constraint of the **All solutions** slot

Typical

$|\mathtt{VARIABLES1}| > 1$
$|\mathtt{VARIABLES2}| > 1$

Symmetries

- Arguments are permutable w.r.t. permutation (VARIABLES1, VARIABLES2).
- Items of VARIABLES1 are permutable.
- Items of VARIABLES2 are permutable.
- An occurrence of a value of VARIABLES1.var can be replaced by any value of VARIABLES1.var.
- An occurrence of a value of VARIABLES2.var can be replaced by any value of VARIABLES2.var.
- All occurrences of two distinct values in VARIABLES1.var or VARIABLES2.var can be swapped; all occurrences of a value in VARIABLES1.var or VARIABLES2.var can be renamed to any unused value.

Arg. properties

- Contractible wrt. VARIABLES1.
- Contractible wrt. VARIABLES2.

Remark

Despite the fact that this is not an uncommon constraint, it can not be modelled in a compact way neither with a *disequality* constraint (i.e., two given variables have to take distinct values) nor with the `alldifferent` constraint. The `disjoint` constraint can bee seen as a special case of the `common`(NCOMMON1, NCOMMON2, VARIABLES1, VARIABLES2) constraint where NCOMMON1 and NCOMMON2 are both set to 0.

MiniZinc (`http://www.minizinc.org/`) has a `disjoint` constraint between two set variables rather than between two collections of variables.

Algorithm

Let us note:

- n_1 the minimum number of distinct values taken by the variables of the collection VARIABLES1.
- n_2 the minimum number of distinct values taken by the variables of the collection VARIABLES2.
- n_{12} the maximum number of distinct values taken by the union of the variables of VARIABLES1 and VARIABLES2.

One invariant to maintain for the `disjoint` constraint is $n_1 + n_2 \leq n_{12}$. A lower bound of n_1 and n_2 can be obtained by using the algorithms provided in [27, 40]. An exact upper bound of n_{12} can be computed by using a bipartite matching algorithm.

Used in

`k_disjoint`.

See also

generalisation: `disjoint_tasks` *(*`variable` *replaced by* `task`*)*.

implies: `alldifferent_on_intersection`, `lex_different`.

system of constraints: `k_disjoint`.

Keywords

characteristic of a constraint: disequality, automaton, automaton with array of counters.

constraint type: value constraint.

filtering: bipartite matching.

modelling: empty intersection.

Arc input(s) `VARIABLES1` `VARIABLES2`

Arc generator $PRODUCT \mapsto$ `collection`(`variables1`, `variables2`)

Arc arity 2

Arc constraint(s) `variables1.var` $=$ `variables2.var`

Graph property(ies) **NARC** $= 0$

Graph model *PRODUCT* is used in order to generate the arcs of the graph between all variables of `VARIABLES1` and all variables of `VARIABLES2`. Since we use the graph property **NARC** $= 0$ the final graph will be empty. Figure 5.293 shows the initial graph associated with the **Example** slot. Since we use the **NARC** $= 0$ graph property the final graph is empty.

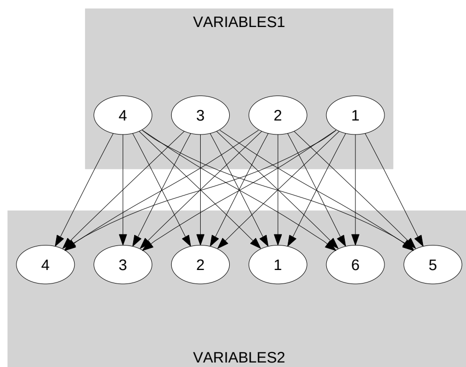

Figure 5.293: Initial graph of the `disjoint` constraint (the final graph is empty)

Signature Since 0 is the smallest number of arcs of the final graph we can rewrite **NARC** $= 0$ to **NARC** ≤ 0. This leads to simplify $\underline{\overline{\mathbf{NARC}}}$ to $\underline{\mathbf{NARC}}$.

Automaton Figure 5.294 depicts the automaton associated with the `disjoint` constraint. To each variable $\texttt{VAR1}_i$ of the collection `VARIABLES1` corresponds a signature variable S_i that is equal to 0. To each variable $\texttt{VAR2}_i$ of the collection `VARIABLES2` corresponds a signature variable $S_{i+|\texttt{VARIABLES1}|}$ that is equal to 1.

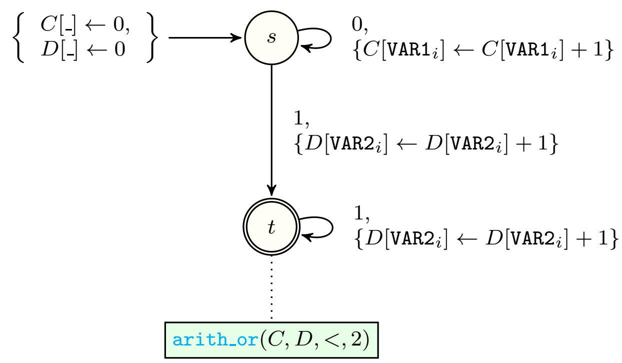

Figure 5.294: Automaton of the $\texttt{disjoint}(\texttt{VARIABLES1}, \texttt{VARIABLES2})$ constraint, where state s handles variables of the collection `VARIABLES1` and state t handles variables of the collection `VARIABLES2`