5.305 orchard

DESCRIPTION LINKS GRAPH

Origin [224]

Constraint orchard(NROW, TREES)

Arguments

NROW	:	dvar
TREES	:	collection(index−int, x−dvar, y−dvar)

Restrictions

NROW ≥ 0
TREES.index ≥ 1
TREES.index $\leq |$TREES$|$
required(TREES, [index, x, y])
distinct(TREES, index)
TREES.x ≥ 0
TREES.y ≥ 0

Purpose

Orchard problem [224]:

> *"Your aid I want, Nine trees to plant, In rows just half a score, And let there be, In each row, three—Solve this: I ask no more!"*

Example

$$\left\langle 10, \left\langle \begin{array}{lll} \texttt{index}-1 & \texttt{x}-0 & \texttt{y}-0, \\ \texttt{index}-2 & \texttt{x}-4 & \texttt{y}-0, \\ \texttt{index}-3 & \texttt{x}-8 & \texttt{y}-0, \\ \texttt{index}-4 & \texttt{x}-2 & \texttt{y}-4, \\ \texttt{index}-5 & \texttt{x}-4 & \texttt{y}-4, \\ \texttt{index}-6 & \texttt{x}-6 & \texttt{y}-4, \\ \texttt{index}-7 & \texttt{x}-0 & \texttt{y}-8, \\ \texttt{index}-8 & \texttt{x}-4 & \texttt{y}-8, \\ \texttt{index}-9 & \texttt{x}-8 & \texttt{y}-8 \end{array} \right\rangle \right\rangle$$

The 10 alignments of 3 trees correspond to the following triples of trees: $(1,2,3)$, $(1,4,8)$, $(1,5,9)$, $(2,4,7)$, $(2,5,8)$, $(2,6,9)$, $(3,5,7)$, $(3,6,8)$, $(4,5,6)$, $(7,8,9)$. Figure 5.644 shows the 9 trees and the 10 alignments corresponding to the example.

Typical

NROW > 0
$|$TREES$| > 3$

Symmetries

- Items of TREES are permutable.
- Attributes of TREES are permutable w.r.t. permutation (index) (x, y) (*permutation applied to all items*).
- One and the same constant can be added to the x attribute of all items of TREES.
- One and the same constant can be added to the y attribute of all items of TREES.

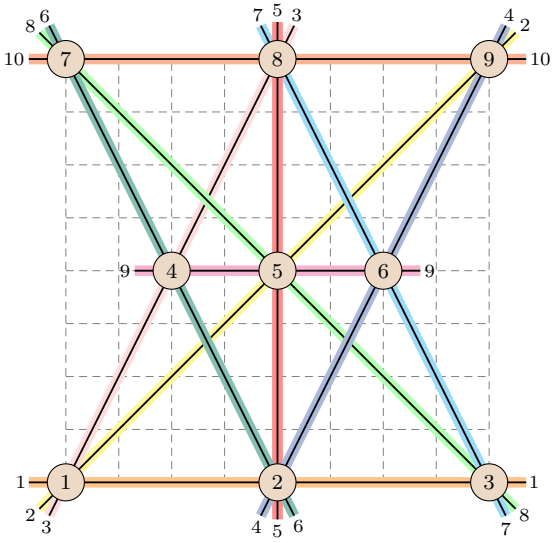

Figure 5.644: Nine trees with 10 alignments of 3 trees

Arg. properties

Functional dependency: `NROW` determined by `TREES`.

Keywords

characteristic of a constraint: hypergraph.

constraint arguments: pure functional dependency.

geometry: geometrical constraint, alignment.

modelling: functional dependency.

Arc input(s)	TREES
Arc generator	$CLIQUE(<) \mapsto$ collection(trees1, trees2, trees3)
Arc arity	3
Arc constraint(s)	$\sum \begin{pmatrix} \texttt{trees1.x} * \texttt{trees2.y} - \texttt{trees1.x} * \texttt{trees3.y}, \\ \texttt{trees1.y} * \texttt{trees3.x} - \texttt{trees1.y} * \texttt{trees2.x}, \\ \texttt{trees2.x} * \texttt{trees3.y} - \texttt{trees2.y} * \texttt{trees3.x} \end{pmatrix} = 0$
Graph property(ies)	**NARC**= NROW

Graph model The arc generator $CLIQUE(<)$ with an arity of three is used in order to generate all the arcs of the directed hypergraph. Each arc is an ordered triple of trees. We use the restriction $<$ in order to generate a single arc for each set of three trees. This is required, since otherwise we would count more than once a given alignment of three trees. The formula used within the arc constraint expresses the fact that the three points of respective coordinates $(\texttt{trees}_1\texttt{.x}, \texttt{trees}_1\texttt{.y})$, $(\texttt{trees}_2\texttt{.x}, \texttt{trees}_2\texttt{.y})$ and $(\texttt{trees}_3\texttt{.x}, \texttt{trees}_3\texttt{.y})$ are aligned. It corresponds to the development of the expression:

$$\begin{vmatrix} \texttt{trees}_1\texttt{.x} & \texttt{trees}_2\texttt{.y} & 1 \\ \texttt{trees}_2\texttt{.x} & \texttt{trees}_2\texttt{.y} & 1 \\ \texttt{trees}_3\texttt{.x} & \texttt{trees}_3\texttt{.y} & 1 \end{vmatrix} = 0$$