5.216 length_first_sequence

DESCRIPTION LINKS AUTOMATON

Origin

Inspired by stretch_path

Constraint

length_first_sequence(LEN, VARIABLES)

Arguments

LEN	:	dvar
VARIABLES	:	collection(var−dvar)

Restrictions

$\texttt{LEN} \geq 0$
$\texttt{LEN} \leq |\texttt{VARIABLES}|$
required(VARIABLES, var)

Purpose

LEN is the length of the maximum sequence of variables that take the same value that contains the first variable of the collection VARIABLES (or 0 if the collection is empty).

Example

$(3, \langle 4, 4, 4, 5, 5, 4\rangle)$
$(6, \langle 4, 4, 4, 4, 4, 4\rangle)$
$(5, \langle 4, 4, 4, 4, 4, 1\rangle)$

The first length_first_sequence constraint holds since the sequence associated with the first value of the collection $\texttt{VARIABLES} = \langle 4, 4, 4, 5, 5, 4\rangle$ spans over three consecutive variables.

Typical

$\texttt{LEN} < |\texttt{VARIABLES}|$
$|\texttt{VARIABLES}| > 1$

Symmetry

All occurrences of two distinct values of VARIABLES.var can be swapped; all occurrences of a value of VARIABLES.var can be renamed to any unused value.

Arg. properties

Functional dependency: LEN determined by VARIABLES.

Reformulation

Without loss of generality let assume that the collection $\texttt{VARIABLES} = \langle V_1, V_2, \ldots, V_n\rangle$ has more than one variable. By introducing $2 \cdot n - 1$ 0-1 variables, the length_first_sequence(LEN, VARIABLES) constraint can be expressed in term of $2 \cdot n - 1$ reified constraints and one arithmetic constraint (i.e., a sum_ctr constraint). We first introduce $n - 1$ variables that are respectively set to 1 if and only if two given consecutive variables of the collection VARIABLES are equal:

$B_{1,2} \Leftrightarrow V_1 = V_2,$
$B_{2,3} \Leftrightarrow V_2 = V_3,$
$\ldots\ldots\ldots\ldots\ldots\ldots$
$B_{n-1,n} \Leftrightarrow V_{n-1} = V_n.$

We then introduce n variables $A_1, A_2, \ldots, A_n$ that are respectively associated to the different sliding sequences starting on the first variable of the sequence $V_1\ V_2\ \ldots\ V_n$. Variable A_i is set to 1 if and only if $V_1 = V_2 = \cdots = V_i$:

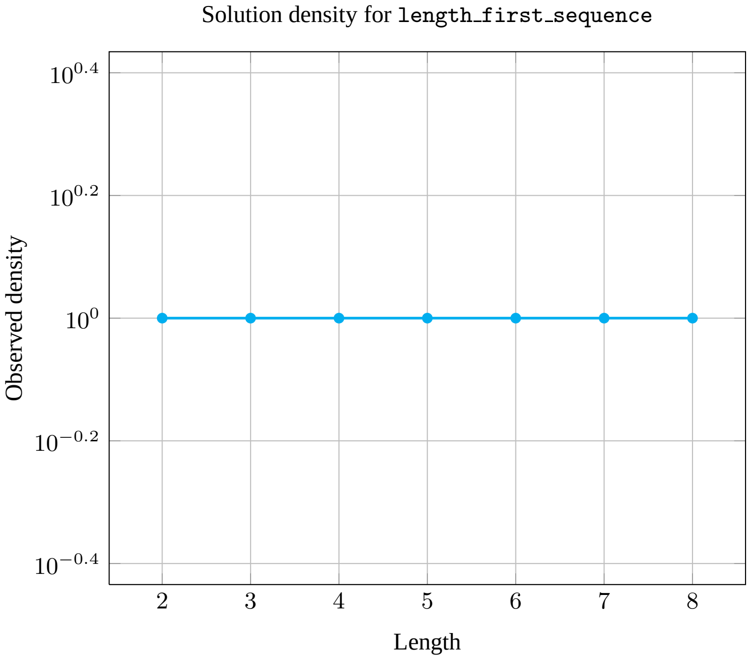
$A_1 = 1,$
$A_2 \Leftrightarrow B_{1,2} \quad \wedge A_1,$
$A_3 \Leftrightarrow B_{2,3} \quad \wedge A_2,$
$\dots\dots\dots\dots\dots\dots$
$A_n \Leftrightarrow B_{n-1,n} \wedge A_{n-1}.$

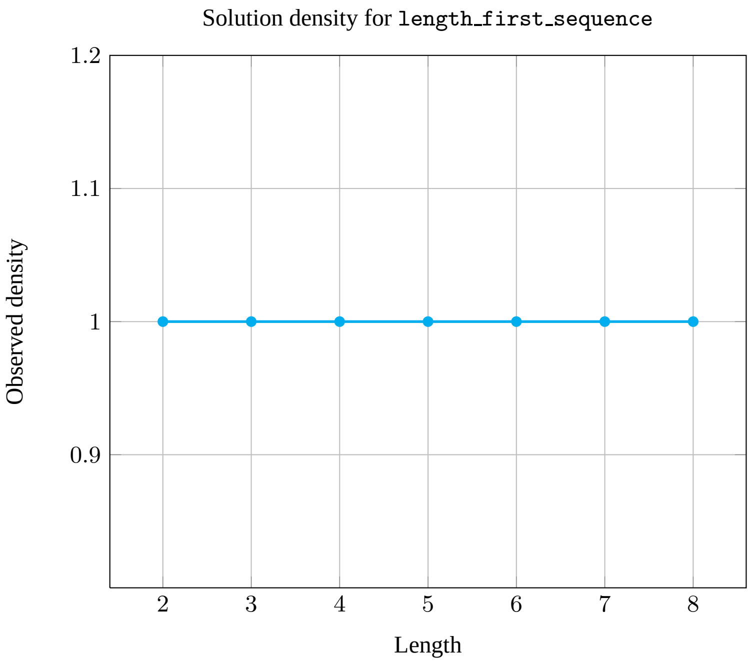
Finally we state the following arithmetic constraint:
$\texttt{LEN} = A_1 + A_2 + \cdots + A_n.$

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for `length_first_sequence`: domains $0..n$

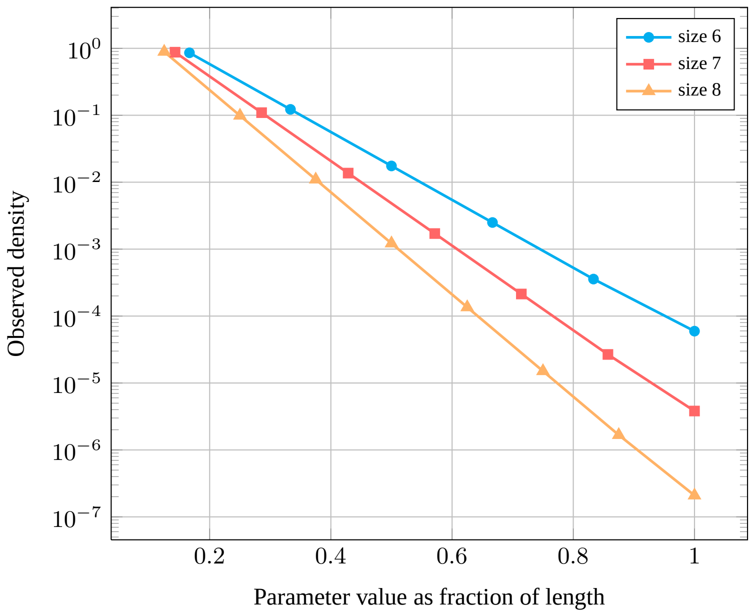




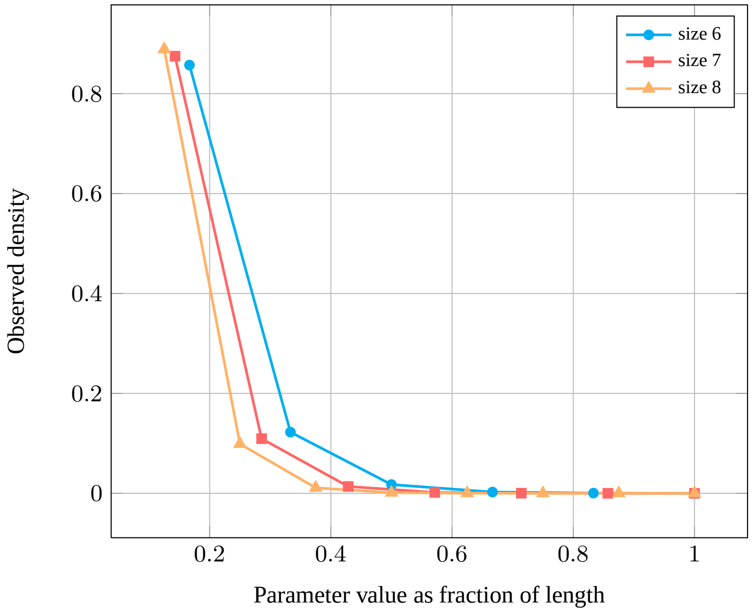
Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	1	6	48	500	6480	100842	1835008	38263752
	2	3	12	100	1080	14406	229376	4251528
	3	-	4	20	180	2058	28672	472392
	4	-	-	5	30	294	3584	52488
	5	-	-	-	6	42	448	5832
	6	-	-	-	-	7	56	648
	7	-	-	-	-	-	8	72
	8	-	-	-	-	-	-	9

Solution count for `length_first_sequence`: domains $0..n$

Solution density for `length_first_sequence`



Solution density for `length_first_sequence`



See also **common keyword:** `length_last_sequence` (*counting constraint*,*sequence*).

Keywords **characteristic of a constraint:** automaton, automaton with counters.

combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

constraint network structure: sliding cyclic(1) constraint network(2).

constraint type: value constraint, counting constraint.

filtering: glue matrix.

modelling: functional dependency.

Automaton Figure 5.488 depicts the automaton associated with the length_first_sequence constraint. To each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection VARIABLES corresponds a signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $\mathtt{VAR}_i = \mathtt{VAR}_{i+1} \Leftrightarrow S_i$.

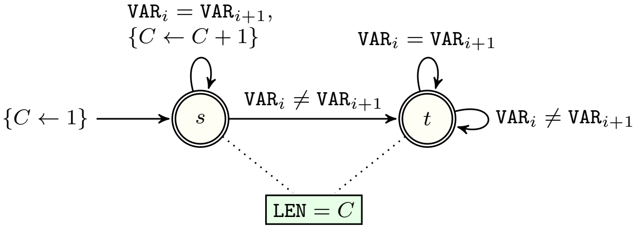

$\mathtt{VAR}_i = \mathtt{VAR}_{i+1}$,
$\{C \leftarrow C+1\}$
$\mathtt{VAR}_i = \mathtt{VAR}_{i+1}$
$\{C \leftarrow 1\}$
s
$\mathtt{VAR}_i \neq \mathtt{VAR}_{i+1}$
t
$\mathtt{VAR}_i \neq \mathtt{VAR}_{i+1}$
$\mathtt{LEN} = C$

Figure 5.488: Automaton of the length_first_sequence constraint when $|\mathtt{VARIABLES}| \geq 2$

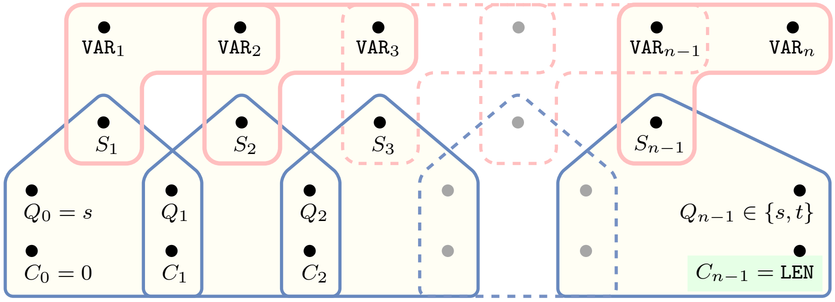
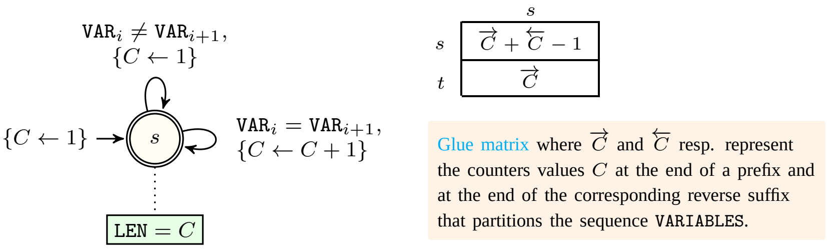


Figure 5.489: Hypergraph of the reformulation corresponding to the automaton of the length_first_sequence constraint



	s
s	$\overrightarrow{C} + \overleftarrow{C} - 1$
t	$\overrightarrow{C}$

Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counters values C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

Figure 5.490: Automaton of the reverse of the length_first_sequence constraint (i.e., the length_last_sequence constraint) when $|\mathtt{VARIABLES}| \geq 2$ and corresponding glue matrix between length_first_sequence and its reverse length_last_sequence