5.93 coveredby_sboxes

DESCRIPTION LINKS LOGIC

Origin Geometry, derived from [338]

Constraint `coveredby_sboxes(K, DIMS, OBJECTS, SBOXES)`

Synonym `coveredby`.

Types

```
VARIABLES  : collection(v-dvar)
INTEGERS   : collection(v-int)
POSITIVES  : collection(v-int)
```

Arguments

```
K        : int
DIMS     : sint
OBJECTS  : collection(oid-int, sid-dvar, x - VARIABLES)
SBOXES   : collection(sid-int, t - INTEGERS, l - POSITIVES)
```

Restrictions

$|\mathtt{VARIABLES}| \geq 1$
$|\mathtt{INTEGERS}| \geq 1$
$|\mathtt{POSITIVES}| \geq 1$
$\mathtt{required}(\mathtt{VARIABLES}, \mathtt{v})$
$|\mathtt{VARIABLES}| = \mathtt{K}$
$\mathtt{required}(\mathtt{INTEGERS}, \mathtt{v})$
$|\mathtt{INTEGERS}| = \mathtt{K}$
$\mathtt{required}(\mathtt{POSITIVES}, \mathtt{v})$
$|\mathtt{POSITIVES}| = \mathtt{K}$
$\mathtt{POSITIVES.v} > 0$
$\mathtt{K} > 0$
$\mathtt{DIMS} \geq 0$
$\mathtt{DIMS} < \mathtt{K}$
$\mathtt{increasing\_seq}(\mathtt{OBJECTS}, [\mathtt{oid}])$
$\mathtt{required}(\mathtt{OBJECTS}, [\mathtt{oid}, \mathtt{sid}, \mathtt{x}])$
$\mathtt{OBJECTS.oid} \geq 1$
$\mathtt{OBJECTS.oid} \leq |\mathtt{OBJECTS}|$
$\mathtt{OBJECTS.sid} \geq 1$
$\mathtt{OBJECTS.sid} \leq |\mathtt{SBOXES}|$
$\mathtt{required}(\mathtt{SBOXES}, [\mathtt{sid}, \mathtt{t}, \mathtt{l}])$
$|\mathtt{SBOXES}| \geq 1$
$\mathtt{SBOXES.sid} \geq 1$
$\mathtt{SBOXES.sid} \leq |\mathtt{SBOXES}|$
$\mathtt{do\_not\_overlap}(\mathtt{SBOXES})$

Purpose

Holds if, for each pair of objects (O_i, O_j), $i < j$, O_i is covered by O_j with respect to a set of dimensions depicted by `DIMS`. O_i and O_j are objects that take a shape among a set of shapes. Each *shape* is defined as a finite set of shifted boxes, where each shifted box is described by a box in a `K`-dimensional space at a given offset (from the origin of the shape) with given sizes. More precisely, a *shifted box* is an entity defined by its shape id `sid`, shift offset `t`, and sizes `l`. Then, a shape is defined as the union of shifted boxes sharing the same shape id. An *object* is an entity defined by its unique object identifier `oid`, shape id `sid` and origin `x`.
An object O_i *is covered by* an object O_j with respect to a set of dimensions depicted by `DIMS` if and only if, for all shifted box s_i of O_i, there exists a shifted box s_j of O_j such that:

- For all dimensions $d \in$ `DIMS`, (1) the start of s_j in dimension d is less than or equal to the start of s_i in dimension d, and (2) the end of s_i in dimension d is less than or equal to the end of s_j in dimension d.
- There exists a dimension d where, (1) the start of s_j in dimension d coincide with the start of s_i in dimension d, or (2) the end of s_j in dimension d coincide with the end of s_i in dimension d.

Example

$$\left(\begin{array}{l}2,\{0,1\},\\ \left\langle\begin{array}{lll}\texttt{oid}-1 & \texttt{sid}-4 & \texttt{x}-\langle 2,3\rangle,\\ \texttt{oid}-2 & \texttt{sid}-2 & \texttt{x}-\langle 2,2\rangle,\\ \texttt{oid}-3 & \texttt{sid}-1 & \texttt{x}-\langle 1,1\rangle\end{array}\right\rangle,\\ \left\langle\begin{array}{lll}\texttt{sid}-1 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 3,3\rangle,\\ \texttt{sid}-1 & \texttt{t}-\langle 3,0\rangle & \texttt{l}-\langle 2,2\rangle,\\ \texttt{sid}-2 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 2,2\rangle,\\ \texttt{sid}-2 & \texttt{t}-\langle 2,0\rangle & \texttt{l}-\langle 1,1\rangle,\\ \texttt{sid}-3 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 2,2\rangle,\\ \texttt{sid}-3 & \texttt{t}-\langle 2,1\rangle & \texttt{l}-\langle 1,1\rangle,\\ \texttt{sid}-4 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 1,1\rangle\end{array}\right\rangle\end{array}\right)$$

Figure 5.226 shows the objects of the example. Since O_1 is covered by both O_2 and O_3, and since O_2 is covered by O_3, the `coveredby_sboxes` constraint holds.

Typical

$|\texttt{OBJECTS}| > 1$

Symmetries

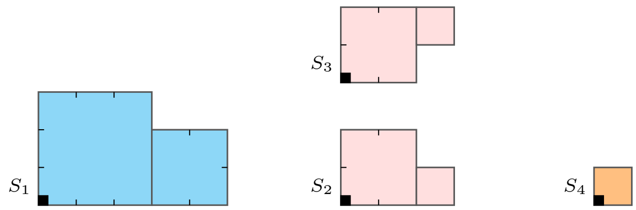
- Items of `SBOXES` are permutable.
- Items of `OBJECTS.x`, `SBOXES.t` and `SBOXES.l` are permutable (*same permutation used*).

Remark

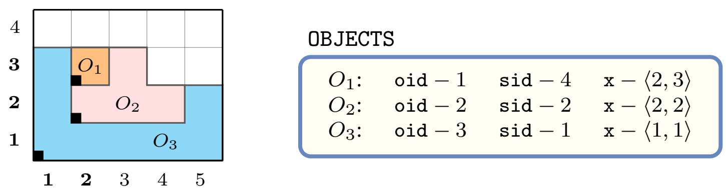
One of the eight relations of the *Region Connection Calculus* [338]. The constraint `coveredby_sboxes` is a restriction of the original relation since it requires that each shifted box of an object is covered by one shifted box of the other object.

See also

common keyword: `contains_sboxes`, `covers_sboxes`, `disjoint_sboxes`, `equal_sboxes`, `inside_sboxes`, `meet_sboxes` (*rcc8*), `non_overlap_sboxes` (*geometrical constraint*,*logic*), `overlap_sboxes` (*rcc8*).



(A) Shape of the third object (B) Shapes of the second object (C) Shape of the first object



(D) Three objects O_1, O_2, O_3, where O_1 is covered by both O_2 and O_3 and where O_2 is covered by O_3

Figure 5.226: (D) the three objects O_1, O_2, O_3 of the **Example** slot respectively assigned shapes S_4, S_2, S_1; (A), (B), (C) shapes S_1, S_2, S_3 and S_4 are respectively made up from 2, 2, 2 and 1 single shifted box.

Keywords

constraint type: logic.

geometry: geometrical constraint, rcc8.

miscellaneous: obscure.

Logic

- $\mathtt{origin}(\mathtt{O1},\mathtt{S1},\mathtt{D}) \stackrel{\text{def}}{=} \mathtt{O1.x}(\mathtt{D}) + \mathtt{S1.t}(\mathtt{D})$
- $\mathtt{end}(\mathtt{O1},\mathtt{S1},\mathtt{D}) \stackrel{\text{def}}{=} \mathtt{O1.x}(\mathtt{D}) + \mathtt{S1.t}(\mathtt{D}) + \mathtt{S1.l}(\mathtt{D})$
- $\mathtt{coveredby\_sboxes}(\mathtt{Dims},\mathtt{O1},\mathtt{S1},\mathtt{O2},\mathtt{S2}) \stackrel{\text{def}}{=}$
$$\bigwedge\left(\begin{array}{l} \forall \mathtt{D} \in \mathtt{Dims} \\ \bigwedge\left(\begin{array}{l} \mathtt{origin}\left(\begin{array}{l}\mathtt{O2},\\ \mathtt{S2},\\ \mathtt{D}\end{array}\right) \leq \\ \mathtt{origin}\left(\begin{array}{l}\mathtt{O1},\\ \mathtt{S1},\\ \mathtt{D}\end{array}\right) \end{array}, \begin{array}{l} \mathtt{end}(\mathtt{O1},\mathtt{S1},\mathtt{D}) \leq \\ \mathtt{end}(\mathtt{O2},\mathtt{S2},\mathtt{D}) \end{array}\right), \\ \exists \mathtt{D} \in \mathtt{Dims} \\ \bigvee\left(\begin{array}{l} \mathtt{origin}\left(\begin{array}{l}\mathtt{O2},\\ \mathtt{S2},\\ \mathtt{D}\end{array}\right) = \\ \mathtt{origin}\left(\begin{array}{l}\mathtt{O1},\\ \mathtt{S1},\\ \mathtt{D}\end{array}\right) \end{array}, \begin{array}{l} \mathtt{end}(\mathtt{O1},\mathtt{S1},\mathtt{D}) = \\ \mathtt{end}(\mathtt{O2},\mathtt{S2},\mathtt{D}) \end{array}\right) \end{array}\right)$$
- $\mathtt{coveredby\_objects}(\mathtt{Dims},\mathtt{O1},\mathtt{O2}) \stackrel{\text{def}}{=}$
$$\begin{array}{l} \forall \mathtt{S1} \in \mathtt{sboxes}([\mathtt{O1.sid}]) \\ \quad \exists \mathtt{S2} \in \mathtt{sboxes}\left(\left[\ \mathtt{O2.sid}\ \right]\right) \\ \quad \mathtt{coveredby\_sboxes}\left(\begin{array}{l}\mathtt{Dims},\\ \mathtt{O1},\\ \mathtt{S1},\\ \mathtt{O2},\\ \mathtt{S2}\end{array}\right) \end{array}$$
- $\mathtt{all\_coveredby}(\mathtt{Dims},\mathtt{OIDS}) \stackrel{\text{def}}{=}$
$$\begin{array}{l} \forall \mathtt{O1} \in \mathtt{objects}(\mathtt{OIDS}) \\ \quad \forall \mathtt{O2} \in \mathtt{objects}(\mathtt{OIDS}) \\ \quad\quad \begin{array}{l}\mathtt{O1.oid} < \\ \mathtt{O2.oid}\end{array} \Rightarrow \\ \quad\quad \mathtt{coveredby\_objects}\left(\begin{array}{l}\mathtt{Dims},\\ \mathtt{O1},\\ \mathtt{O2}\end{array}\right) \end{array}$$
- $\mathtt{all\_coveredby}(\mathtt{DIMENSIONS},\mathtt{OIDS})$