

5.32 arith_or

	DESCRIPTION	LINKS	GRAPH	AUTOMATON
Origin	Used in the definition of several automata			
Constraint	<code>arith_or(VARIABLES1, VARIABLES2, RELOP, VALUE)</code>			
Arguments	VARIABLES1 : <code>collection</code> (var-dvar) VARIABLES2 : <code>collection</code> (var-dvar) RELOP : <code>atom</code> VALUE : <code>int</code>			
Restrictions	<code>required</code> (VARIABLES1, var) <code>required</code> (VARIABLES2, var) $ VARIABLES1 = VARIABLES2 $ $RELOP \in [=, \neq, <, \geq, >, \leq]$			
Purpose	Enforce for all pairs of variables $var1_i, var2_i$ of the VARIABLES1 and VARIABLES2 collections to have $var1_i \text{ RELOP VALUE} \vee var2_i \text{ RELOP VALUE}$.			
Example	$((\langle 0, 1, 0, 0, 1 \rangle, \langle 0, 0, 0, 1, 0 \rangle), =, 0)$ <p>The constraint <code>arith_or</code> holds since, for all pairs of variables $var1_i, var2_i$ of the VARIABLES1 and VARIABLES2 collections, there is at least one variable that is equal to 0.</p>			
All solutions	Figure 5.88 gives all solutions to the following non ground instance of the <code>arith_or</code> constraint: $U_1 \in [3, 4], U_2 \in [1, 2], U_3 \in [1, 4], V_1 \in [2, 3], V_2 \in [2, 2], V_3 \in [0, 1]$, <code>arith_or</code> ($\langle U_1, U_2, U_3 \rangle, \langle V_1, V_2, V_3 \rangle, =, 2$)			
	<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid blue; padding: 5px; width: 45%;"> ① $(\langle 3, 1, 2 \rangle, \langle 2, 2, 0 \rangle, =, 2)$ ② $(\langle 3, 1, 2 \rangle, \langle 2, 2, 1 \rangle, =, 2)$ ③ $(\langle 3, 2, 2 \rangle, \langle 2, 2, 0 \rangle, =, 2)$ ④ $(\langle 3, 2, 2 \rangle, \langle 2, 2, 1 \rangle, =, 2)$ </div> <div style="border: 1px solid blue; padding: 5px; width: 45%;"> ⑤ $(\langle 4, 1, 2 \rangle, \langle 2, 2, 0 \rangle, =, 2)$ ⑥ $(\langle 4, 1, 2 \rangle, \langle 2, 2, 1 \rangle, =, 2)$ ⑦ $(\langle 4, 2, 2 \rangle, \langle 2, 2, 0 \rangle, =, 2)$ ⑧ $(\langle 4, 2, 2 \rangle, \langle 2, 2, 1 \rangle, =, 2)$ </div> </div>			
Typical	$ VARIABLES1 > 0$ $RELOP \in [=]$			
Symmetries	<ul style="list-style-type: none"> Arguments are <code>permutable</code> w.r.t. permutation (VARIABLES1, VARIABLES2) (RELOP) (VALUE). Items of VARIABLES1 and VARIABLES2 are <code>permutable</code> (same permutation used). 			

Figure 5.88: All solutions corresponding to the non ground example of the `arith_or` constraint of the **All solutions** slot

Arg. properties

Contractible wrt. VARIABLES1 and VARIABLES2 (*remove items from same position*).

See also

specialisation: **arith** (variable RELOP VALUE \vee variable RELOP VALUE *replaced by* variable RELOP VALUE).

Keywords

characteristic of a constraint: automaton, automaton without counters, reified automaton constraint.

constraint network structure: Berge-acyclic constraint network.

constraint type: decomposition, value constraint.

filtering: arc-consistency.

final graph structure: acyclic, bipartite, no loop.

modelling: disjunction.

Arc input(s)	VARIABLES1 VARIABLES2
Arc generator	<i>PRODUCT</i> (=) \mapsto collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var RELOP VALUE \vee variables2.var RELOP VALUE
Graph property(ies)	NARC = VARIABLES1
Graph class	<ul style="list-style-type: none">• ACYCLIC• BIPARTITE• NO_LOOP

Graph model Parts (A) and (B) of Figure 5.89 respectively show the initial and final graphs associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold.

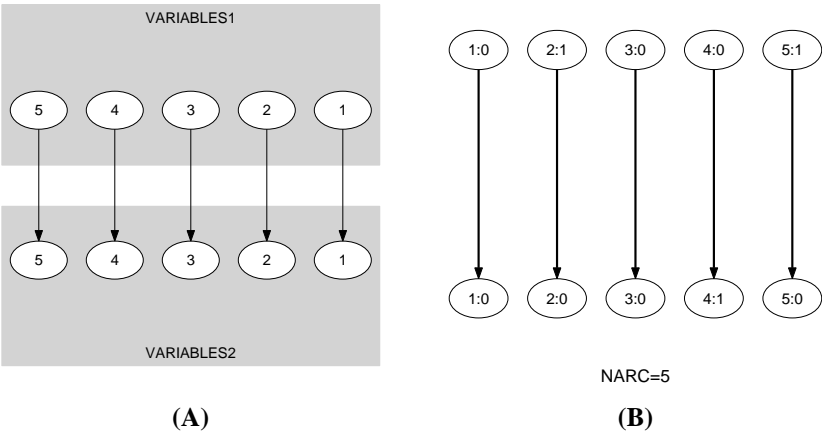
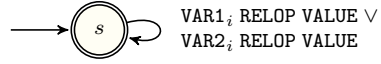
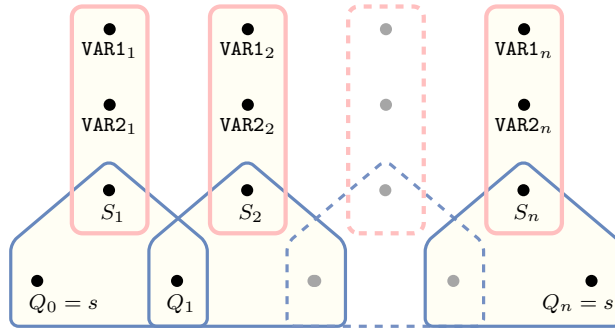


Figure 5.89: Initial and final graph of the **arith_or** constraint

Automaton

Figure 5.90 depicts the automaton associated with the `arith_or` constraint. Let VAR1_i and VAR2_i be the i^{th} variables of the `VARIABLES1` and `VARIABLES2` collections. To each pair of variables $(\text{VAR1}_i, \text{VAR2}_i)$ corresponds a signature variable S_i . The following signature constraint links VAR1_i , VAR2_i and S_i : $\text{VAR1}_i \text{ RELOP VALUE} \vee \text{VAR2}_i \text{ RELOP VALUE} \Leftrightarrow S_i$. The automaton enforces for each pair of variables $\text{VAR1}_i, \text{VAR2}_i$ the condition $\text{VAR1}_i \text{ RELOP VALUE} \vee \text{VAR2}_i \text{ RELOP VALUE}$.

Figure 5.90: Automaton of the `arith_or` constraintFigure 5.91: Hypergraph of the reformulation corresponding to the automaton of the `arith_or` constraint