5.95   crossing

<div align="center">DESCRIPTION   LINKS   GRAPH</div>

Origin   Inspired by [122].

Constraint   `crossing(NCROSS, SEGMENTS)`

Arguments   `NCROSS   :  dvar`
                          `SEGMENTS :  collection(ox−dvar, oy−dvar, ex−dvar, ey−dvar)`

Restrictions   `NCROSS ≥ 0`
                          `NCROSS ≤ (|SEGMENTS| ∗ |SEGMENTS| − |SEGMENTS|)/2`
                          `required(SEGMENTS, [ox, oy, ex, ey])`

Purpose   NCROSS is the number of line segments intersections between the line segments defined by the SEGMENTS collection. Each line segment is defined by the coordinates $(\texttt{ox}, \texttt{oy})$ and $(\texttt{ex}, \texttt{ey})$ of its two extremities.

Example
$$\left(3, \left\langle \begin{array}{llll} \texttt{ox} - 1 & \texttt{oy} - 4 & \texttt{ex} - 9 & \texttt{ey} - 2, \\ \texttt{ox} - 1 & \texttt{oy} - 1 & \texttt{ex} - 3 & \texttt{ey} - 5, \\ \texttt{ox} - 3 & \texttt{oy} - 2 & \texttt{ex} - 7 & \texttt{ey} - 4, \\ \texttt{ox} - 9 & \texttt{oy} - 1 & \texttt{ex} - 9 & \texttt{ey} - 4 \end{array} \right\rangle \right)$$

Figure 5.228 provides a picture of the example with the corresponding four line segments of the SEGMENTS collection. The `crossing` constraint holds since its first argument NCROSS is set to 3, which is actually the number of line segments intersections.
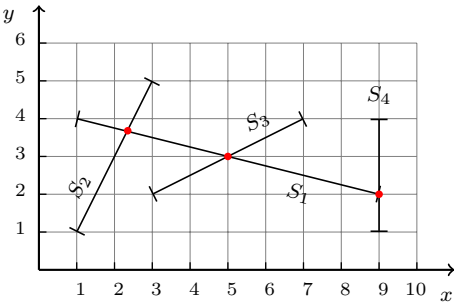

Figure 5.228: Illustration of the **Example** slot: intersection, in red, between the four line segments S_1, S_2, S_3 and S_4 (NCROSS = 3)

Typical $|\texttt{SEGMENTS}| > 1$

Symmetries

- Items of `SEGMENTS` are permutable.
- Attributes of `SEGMENTS` are permutable w.r.t. permutation (ox, oy) (ex, ey) (*permutation applied to all items*).
- One and the same constant can be added to the `ox` and `ex` attributes of all items of `SEGMENTS`.
- One and the same constant can be added to the `oy` and `ey` attributes of all items of `SEGMENTS`.

Arg. properties

Functional dependency: `NCROSS` determined by `SEGMENTS`.

See also

common keyword: `graph_crossing`, `two_layer_edge_crossing` *(line segments intersection)*.

Keywords

constraint arguments: pure functional dependency.

final graph structure: acyclic, no loop.

geometry: geometrical constraint, line segments intersection.

modelling: functional dependency.

Arc input(s)	SEGMENTS
Arc generator	$CLIQUE(<) \mapsto \texttt{collection}(\texttt{s1}, \texttt{s2})$
Arc arity	2
Arc constraint(s)	• $\max(\texttt{s1.ox}, \texttt{s1.ex}) \geq \min(\texttt{s2.ox}, \texttt{s2.ex})$ • $\max(\texttt{s2.ox}, \texttt{s2.ex}) \geq \min(\texttt{s1.ox}, \texttt{s1.ex})$ • $\max(\texttt{s1.oy}, \texttt{s1.ey}) \geq \min(\texttt{s2.oy}, \texttt{s2.ey})$ • $\max(\texttt{s2.oy}, \texttt{s2.ey}) \geq \min(\texttt{s1.oy}, \texttt{s1.ey})$ • $\bigvee \left(\begin{array}{l} \dfrac{(\texttt{s2.ox} - \texttt{s1.ex}) * (\texttt{s1.ey} - \texttt{s1.oy}) -}{(\texttt{s1.ex} - \texttt{s1.ox}) * (\texttt{s2.oy} - \texttt{s1.ey})} = 0, \\ \dfrac{(\texttt{s2.ex} - \texttt{s1.ex}) * (\texttt{s2.oy} - \texttt{s1.oy}) -}{(\texttt{s2.ox} - \texttt{s1.ox}) * (\texttt{s2.ey} - \texttt{s1.ey})} = 0, \\ \text{sign} \left(\begin{array}{l} (\texttt{s2.ox} - \texttt{s1.ex}) * (\texttt{s1.ey} - \texttt{s1.oy}) - \\ (\texttt{s1.ex} - \texttt{s1.ox}) * (\texttt{s2.oy} - \texttt{s1.ey}) \end{array} \right) \neq \\ \text{sign} \left(\begin{array}{l} (\texttt{s2.ex} - \texttt{s1.ex}) * (\texttt{s2.oy} - \texttt{s1.oy}) - \\ (\texttt{s2.ox} - \texttt{s1.ox}) * (\texttt{s2.ey} - \texttt{s1.ey}) \end{array} \right) \end{array} \right)$
Graph property(ies)	$\textbf{NARC}=$ NCROSS
Graph class	• ACYCLIC • NO_LOOP

Graph model
Each line segment is described by the x and y coordinates of its two extremities. In the arc generator we use the restriction $<$ in order to generate a single arc for each pair of segments. This is required, since otherwise we would count more than once a given line segments intersection.

Parts (A) and (B) of Figure 5.229 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. An arc constraint expresses the fact the two line segments intersect. It is taken from [122, page 889]. Each arc of the final graph corresponds to a line segments intersection.
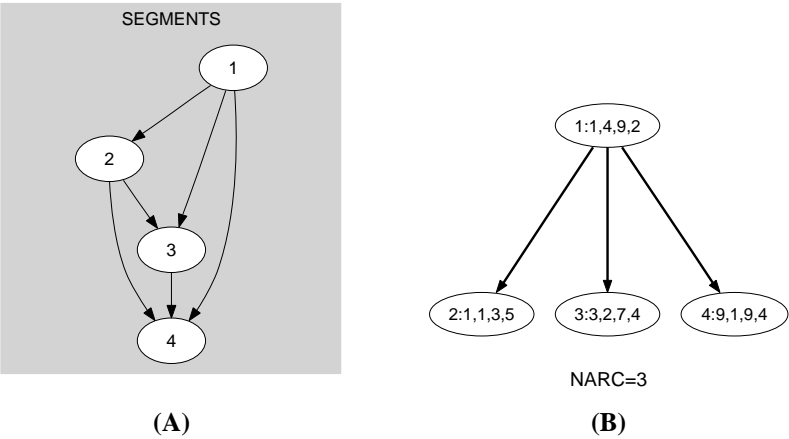
Figure 5.229: Initial and final graph of the crossing constraint