5.309 ordered_global_cardinality

DESCRIPTION LINKS GRAPH

Origin [312]

Constraint ordered_global_cardinality(VARIABLES, VALUES)

Usual name ordgcc

Synonym ordered_gcc.

Arguments

VARIABLES : collection(var−dvar)
VALUES : collection(val−int, omax−int)

Restrictions

required(VARIABLES, var)
$|\texttt{VALUES}| > 0$
required(VALUES, [val, omax])
increasing_seq(VALUES, [val])
$\texttt{VALUES.omax} \geq 0$
$\texttt{VALUES.omax} \leq |\texttt{VARIABLES}|$

Purpose

For each $i \in [1, |\texttt{VALUES}|]$, the values of the corresponding set of values VALUES$[j]$.val ($i \leq j \leq |\texttt{VALUES}|$) should be taken by at most VALUES$[i]$.omax variables of the VARIABLES collection.
From that previous definition, the omax attributes are decreasing.

Example

$$\left(\begin{array}{l}\langle 2,0,1,0,0\rangle, \\ \langle \texttt{val}-0\ \texttt{omax}-5, \texttt{val}-1\ \texttt{omax}-3, \texttt{val}-2\ \texttt{omax}-1\rangle\end{array}\right)$$

The ordered_global_cardinality constraint holds since the values of the three sets of values $\{0,1,2\}$, $\{1,2\}$ and $\{2\}$ are respectively used no more than 5, 3 and 1 times within the collection $\langle 2,0,1,0,0\rangle$.

Symmetry Items of VARIABLES are permutable.

Arg. properties Contractible wrt. VALUES.

Usage

The ordered_global_cardinality can be used in order to restrict the way we assign the values of the VALUES collection to the variables of the VARIABLES collection. It expresses the fact that, when we use a value v, we implicitly also use all values that are less than or equal to v. As depicted by Figure 5.647 this is for instance the case for a *soft cumulative* constraint where we want to control the shape of cumulative profile by providing for each instant i a variable h_i that gives the height of the cumulative profile at instant i. These variables h_i are passed as the first argument of the ordered_global_cardinality constraint. Then the omax attribute of the j-th item of the VALUES collection gives the maximum number of instants for which the height of the cumulative profile is greater than or equal to value VALUES$[j]$.val. In Figure 5.647 we should have:

- no more than 1 height variable greater than or equal to 2,
- no more than 3 height variables greater than or equal to 1,
- no more than 5 height variables greater than or equal to 0.

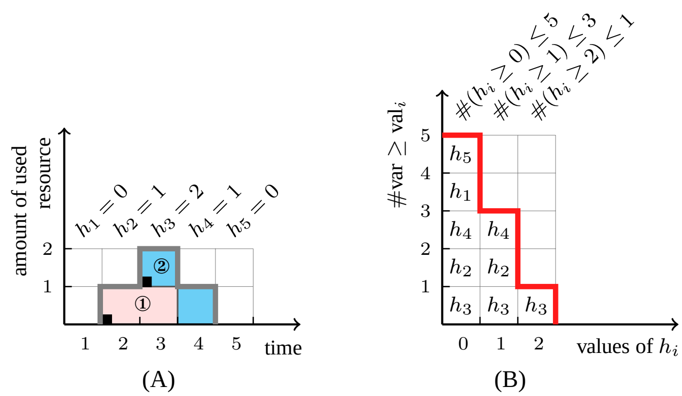


Figure 5.647: (A) A cumulative profile wrt two tasks ① and ②, and its corresponding height variables h_1, $h_2, \ldots, h_5$ giving at each instant how many resource is used (B) profile of value utilisation of the height variables (e.g., value 1 is assigned to variables h_3, h_2, h_4 and therefore used three times)

Remark The original definition of the `ordered_global_cardinality` constraint mentions a third argument, namely the minimum number of occurrences of the smallest value. We omit it since it is redundant.

An other closely related constraint, the `cost_ordered_global_cardinality` constraint was introduced in [312] in order to model the fact that overloads costs may depend of the instant where they occur.

Algorithm A filtering algorithm achieving arc-consistency in $O(|\texttt{VARIABLES}|+|\texttt{VALUES}|)$ is described in [312]. It is based on the equivalence between the following two statements:

1. the `ordered_global_cardinality` constraint has a solution,
2. all variables of the `VARIABLES` collection assigned to their respective minimum value correspond to a solution to the `ordered_global_cardinality` constraint.

Reformulation The `ordered_global_cardinality`$(\langle \texttt{var} - V_1, \texttt{var} - V_2, \ldots, \texttt{var} - V_{|\texttt{VARIABLES}|}\rangle,$ $\langle \texttt{val} - v_1\ \texttt{omax} - o_1, \texttt{val} - v_2\ \texttt{omax} - o_2, \ldots, \texttt{val} - v_{|\texttt{VALUES}|}\ \texttt{omax} - o_{|\texttt{VALUES}|}\rangle)$ constraint can be reformulated into a `global_cardinality`$(\langle \texttt{var} - V_1, \texttt{var} - V_2, \ldots, \texttt{var} - V_{|\texttt{VARIABLES}|}\rangle, \langle \texttt{val} - v_1\ \texttt{noccurrence} - N_1, \texttt{val} - v_2\ \texttt{noccurrence} - N_2, \ldots, \texttt{val} - v_{|\texttt{VALUES}|}\ \texttt{noccurrence} - N_{|\texttt{VALUES}|}\rangle)$ and $|\texttt{VALUES}|$ sliding linear inequalities constraints of the form:

$$\begin{aligned} N_1 + N_2 + \cdots + N_{|\texttt{VALUES}|} &\leq o_1, \\ N_2 + \cdots + N_{|\texttt{VALUES}|} &\leq o_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots &, \\ N_{|\texttt{VALUES}|} &\leq o_{|\texttt{VALUES}|}. \end{aligned}$$

However, with the next example, T. Petit and J.-C. Régin have shown that this reformulation hinders propagation:

1. $V_1 \in \{0,1\}$, $V_2 \in \{0,1\}$, $V_3 \in \{0,1,2\}$, $V_4 \in \{2,3\}$, $V_5 \in \{2,3\}$.
2. global_cardinality($\langle V_1, V_2, V_3, V_4, V_5 \rangle$, $\langle$val $-$ 1 noccurrence $-$ N_1, val $-$ 2 noccurrence $-$ N_2, val $-$ 3 noccurrence $-$ $N_3\rangle$),
3. $N_1 + N_2 + N_3 \leq 3 \wedge N_2 + N_3 \leq 2 \wedge N_3 \leq 2$.

The previous reformulation does not remove value 2 from the domain of variable V_3.

See also

related: cumulative *(controlling the shape of the cumulative profile for breaking symmetry)*, global_cardinality_low_up, increasing_global_cardinality *(the order is imposed on the main variables, and not on the count variables)*.

root concept: global_cardinality.

Keywords

application area: assignment.

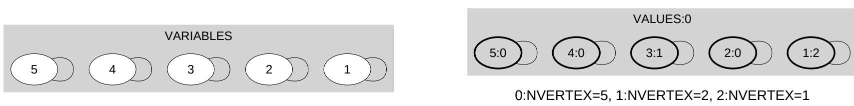
constraint type: value constraint, order constraint.

filtering: arc-consistency.

For all items of `VALUES`:

Arc input(s)	`VARIABLES`
Arc generator	$SELF \mapsto$ `collection`(`variables`)
Arc arity	1
Arc constraint(s)	`variables.var` $\geq$ `VALUES.val`
Graph property(ies)	**NVERTEX** $\leq$ `VALUES.omax`

Graph model Since we want to express one unary constraint for each value we use the "For all items of `VALUES`" iterator. Part (A) of Figure 5.648 shows the initial graphs associated with each value 0, 1 and 2 of the `VALUES` collection of the **Example** slot. Part (B) of Figure 5.648 shows the corresponding final graph associated with value 0. Since we use the **NVERTEX** graph property, the vertices of the final graph is stressed in bold.



(A) **(B)**

Figure 5.648: Initial and final graph of the `ordered_global_cardinality` constraint