

5.3 all_differ_from_at_most_k_pos

	DESCRIPTION	LINKS	GRAPH
Origin	Inspired by all_differ_from_at_least_k_pos .		
Constraint	<code>all_differ_from_at_most_k_pos(K, VECTORS)</code>		
Type	VECTOR : <code>collection(var-dvar)</code>		
Arguments	K : <code>int</code> VECTORS : <code>collection(vec - VECTOR)</code>		
Restrictions	<code>required(VECTOR, var)</code> $ \text{VECTOR} \geq 1$ $ \text{VECTOR} \geq K$ $K \geq 0$ <code>required(VECTORS, vec)</code> <code>same_size(VECTORS, vec)</code>		
Purpose	Enforce all pairs of distinct vectors of the VECTORS collection to differ from at most K positions.		
Example	$(2, \langle \text{vec} - \langle 0, 3, 0, 6 \rangle, \text{vec} - \langle 0, 3, 4, 1 \rangle, \text{vec} - \langle 0, 3, 4, 6 \rangle \rangle)$		
	The <code>all_differ_from_at_most_k_pos</code> constraint holds since: <ul style="list-style-type: none"> • The first and second vectors differ from 2 positions, which is less than or equal to $K = 2$. • The first and third vectors differ from 1 position, which is less than or equal to $K = 2$. • The second and third vectors differ from 1 position, which is less than or equal to $K = 2$. 		
Typical	$K > 0$ $K < \text{VECTOR} $ $ \text{VECTORS} > 1$		
Symmetries	<ul style="list-style-type: none"> • Items of VECTORS are permutable. • Items of VECTORS.vec are permutable (<i>same permutation used</i>). 		
Arg. properties	<ul style="list-style-type: none"> • Contractible wrt. VECTORS. • Contractible wrt. VECTORS.vec (<i>remove items from same position</i>). 		
See also	implied by: all_differ_from_exactly_k_pos ($\leq K$ replaced by $= K$). part of system of constraints: differ_from_at_most_k_pos . used in graph description: differ_from_at_most_k_pos .		

Keywords

characteristic of a constraint: disequality, vector.

constraint type: system of constraints, decomposition.

final graph structure: no loop, symmetric.

Arc input(s)	VECTORS
Arc generator	$\text{CLIQUE}(\neq) \mapsto \text{collection}(\text{vectors1}, \text{vectors2})$
Arc arity	2
Arc constraint(s)	$\text{differ_from_at_most_k_pos}(K, \text{vectors1.vec}, \text{vectors2.vec})$
Graph property(ies)	$\text{NARC} = \text{VECTORS} * \text{VECTORS} - \text{VECTORS} $
Graph class	<ul style="list-style-type: none"> • NO_LOOP • SYMMETRIC

Graph model

The **Arc constraint(s)** slot uses the `differ_from_at_most_k_pos` constraint defined in this catalogue.

Parts (A) and (B) of Figure 5.3 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. The previous constraint holds since exactly $3 \cdot (3 - 1) = 6$ arc constraints hold.

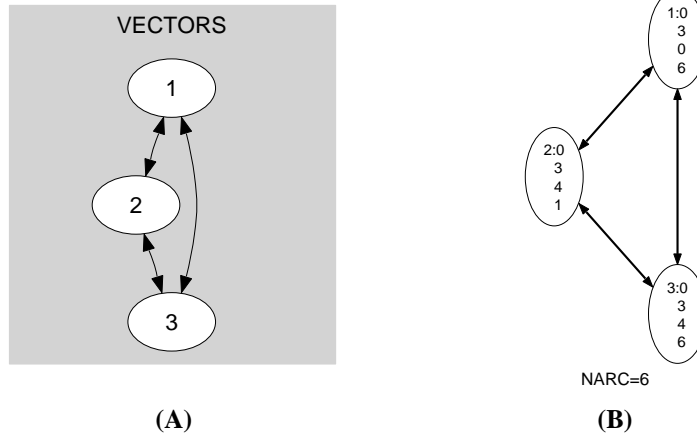


Figure 5.3: Initial and final graph of the `all_differ_from_at_most_k_pos` constraint

Signature

Since we use the $\text{CLIQUE}(\neq)$ arc generator on the items of the **VECTORS** collection, the expression $|\text{VECTORS}| \cdot |\text{VECTORS}| - |\text{VECTORS}|$ corresponds to the maximum number of arcs of the final graph. Therefore we can rewrite the graph property $\text{NARC} = |\text{VECTORS}| \cdot |\text{VECTORS}| - |\text{VECTORS}|$ to $\text{NARC} \geq |\text{VECTORS}| \cdot |\text{VECTORS}| - |\text{VECTORS}|$. This leads to simplify $\overline{\text{NARC}}$ to NARC .

