NARC, PATH

## 5.225 lex\_chain\_greatereq

Origin Derived from lex\_chain\_lesseq

Constraint lex\_chain\_greatereq(VECTORS)

Usual name lex\_chain

Type VECTOR : collection(var-dvar)

Argument VECTORS : collection(vec - VECTOR)

**Restrictions**  $|VECTOR| \ge 1$ 

required(VECTOR, var)
required(VECTORS, vec)
same\_size(VECTORS, vec)

For each pair of consecutive vectors VECTOR<sub>i</sub> and VECTOR<sub>i+1</sub> of the VECTORS collection we have that VECTOR<sub>i</sub> is lexicographically greater than or equal to VECTOR<sub>i+1</sub>. Given two vectors,  $\vec{X}$  and  $\vec{Y}$  of n components,  $\langle X_0, \ldots, X_{n-1} \rangle$  and  $\langle Y_0, \ldots, Y_{n-1} \rangle$ ,  $\vec{X}$  is lexicographically greater than or equal to  $\vec{Y}$  if and only if n = 0 or  $X_0 > Y_0$  or  $X_0 = Y_0$  and  $\langle X_1, \ldots, X_{n-1} \rangle$  is lexicographically greater than or equal to  $\langle Y_1, \ldots, Y_{n-1} \rangle$ .

Example

**Purpose** 

$$(\langle \mathtt{vec} - \langle 5, 2, 6, 2 \rangle, \mathtt{vec} - \langle 5, 2, 6, 2 \rangle, \mathtt{vec} - \langle 5, 2, 3, 9 \rangle))$$

The lex\_chain\_greatereq constraint holds since:

- The first vector  $\langle 5,2,6,2 \rangle$  of the VECTORS collection is lexicographically greater than or equal to the second vector  $\langle 5,2,6,2 \rangle$  of the VECTORS collection.
- The second vector  $\langle 5,2,6,2 \rangle$  of the VECTORS collection is lexicographically greater than or equal to the third vector  $\langle 5,2,3,9 \rangle$  of the VECTORS collection.

**Typical** 

```
\begin{aligned} |\text{VECTOR}| &> 1 \\ |\text{VECTORS}| &> 1 \end{aligned}
```

Arg. properties

- Contractible wrt. VECTORS.
- Suffix-contractible wrt. VECTORS.vec (remove items from same position).

Usage

This constraint was motivated for breaking symmetry: more precisely when one wants to lexicographically order the consecutive columns of a matrix of decision variables. A further motivation is that using a set of lexicographic ordering constraints between two vectors does usually not allows to come up with a complete pruning.

20130730 1543

Algorithm A filtering algorithm achieving arc-consistency for a chain of lexicographical ordering con-

straints is presented in [95].

Six different ways of integrating a chain of lexicographical ordering constraints within non-overlapping constraints like diffn or geost and within their corresponding necessary

condition like the cumulative constraint are shown in [3].

See also common keyword: lex\_between, lex\_greater, lex\_less,

lex\_lesseq(lexicographic order).

implied by: lex\_chain\_greater (non-strict order implied by strict order).

part of system of constraints: lex\_greatereq.
used in graph description: lex\_greatereq.

**Keywords characteristic of a constraint:** vector.

constraint type: system of constraints, decomposition, order constraint.

filtering: arc-consistency.

heuristics: heuristics and lexicographical ordering.

symmetry: symmetry, matrix symmetry, lexicographic order.

 $\overline{NARC}$ , PATH

Arc input(s)	VECTORS
Arc generator	$PATH \mapsto collection(vectors1, vectors2)$
Arc arity	2
Arc constraint(s)	<pre>lex_lesseq(vectors1.vec, vectors2.vec)</pre>
Graph property(ies)	NARC =  VECTORS  - 1

## Graph model

Parts (A) and (B) of Figure 5.498 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. The lex\_chain\_greatereq constraint holds since all the arc constraints of the initial graph are satisfied.

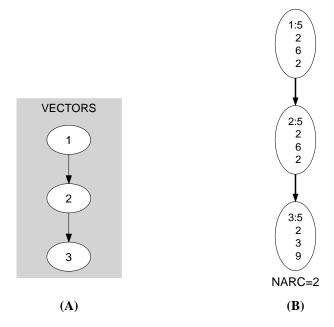


Figure 5.498: Initial and final graph of the lex\_chain\_greatereq constraint

## Signature

Since we use the PATH arc generator on the VECTORS collection the number of arcs of the initial graph is equal to |VECTORS| - 1. For this reason we can rewrite NARC = |VECTORS| - 1 to  $NARC \ge |VECTORS| - 1$  and simplify  $\overline{NARC}$  to  $\overline{NARC}$ .

20130730 1545