5.193 inflexion

DESCRIPTION LINKS AUTOMATON

Origin

N. Beldiceanu

Constraint

inflexion(N, VARIABLES)

Arguments

N : dvar
VARIABLES : collection(var−dvar)

Restrictions

$\texttt{N} \geq 0$
$\texttt{N} \leq \texttt{max}(0, |\texttt{VARIABLES}| - 2)$
required(VARIABLES, var)

Purpose

N is equal to the number of times that the following conjunctions of constraints hold:

- X_i CTR $X_{i+1} \wedge X_i \neq X_{i+1}$,
- $X_{i+1} = X_{i+2} \wedge \cdots \wedge X_{j-2} = X_{j-1}$,
- $X_{j-1} \neq X_j \wedge X_{j-1}$ ¬CTR X_j.

where X_k is the k^{th} item of the VARIABLES collection and $1 \leq i, i+2 \leq j, j \leq n$ and CTR is $<$ or $>$.

Example

$(3, \langle 1, 1, 4, 8, 8, 2, 7, 1\rangle)$
$(0, \langle 1, 1, 4, 4, 6, 6, 7, 9\rangle)$
$(7, \langle 1, 0, 2, 0, 7, 2, 7, 1, 2\rangle)$

The first inflexion constraint holds since the sequence 1 1 4 8 8 2 7 1 contains three inflexions peaks that respectively correspond to values 8, 2 and 7.

All solutions

Figure 5.446 gives all solutions to the following non ground instance of the inflexion constraint: $\texttt{N} \in \{0, 2\}$, $\texttt{V}_1 = 2$, $\texttt{V}_2 \in [2, 3]$, $\texttt{V}_3 \in [1, 2]$, $\texttt{V}_4 \in [1, 2]$, $\texttt{V}_5 = 3$, $\texttt{inflexion}(\texttt{N}, \langle \texttt{V}_1, \texttt{V}_2, \texttt{V}_3, \texttt{V}_4, \texttt{V}_5\rangle)$.

Typical

$\texttt{N} > 0$
$|\texttt{VARIABLES}| > 2$
range(VARIABLES.var) > 1

Symmetries

- Items of VARIABLES can be reversed.
- One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

Functional dependency: N determined by VARIABLES.

Usage

Useful for constraining the number of *inflexions* of a sequence of domain variables.

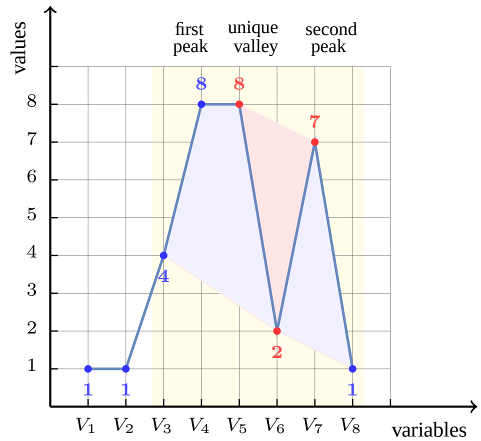


Figure 5.445: Illustration of the first example of the **Example** slot: a sequence of eight variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8$ respectively fixed to values $1, 1, 4, 8, 8, 2, 7, 1$ and its three inflexions in red, two peaks and one valley ($\mathtt{N} = 3$)

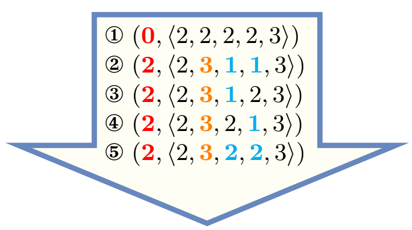


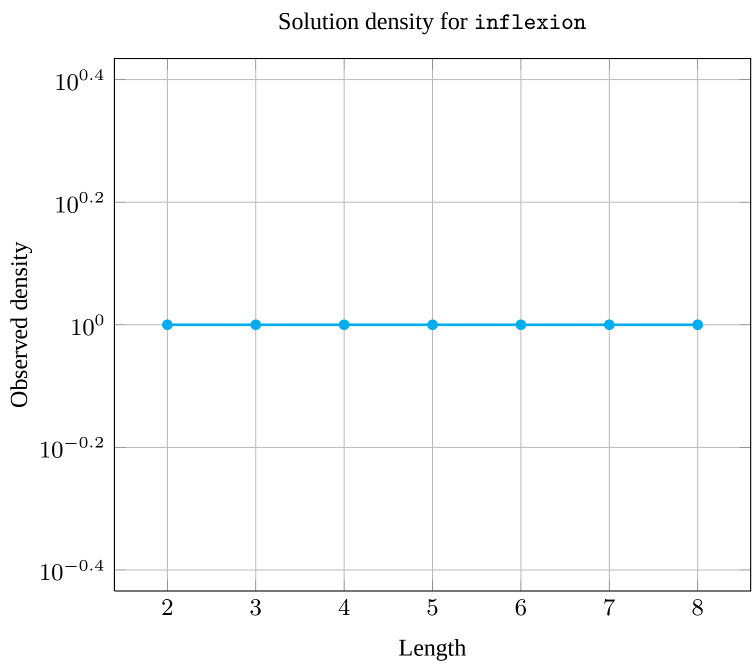
Figure 5.446: All solutions corresponding to the non ground example of the `inflexion` constraint of the **All solutions** slot where each inflexion (i.e. peak or valley) is coloured in orange or cyan

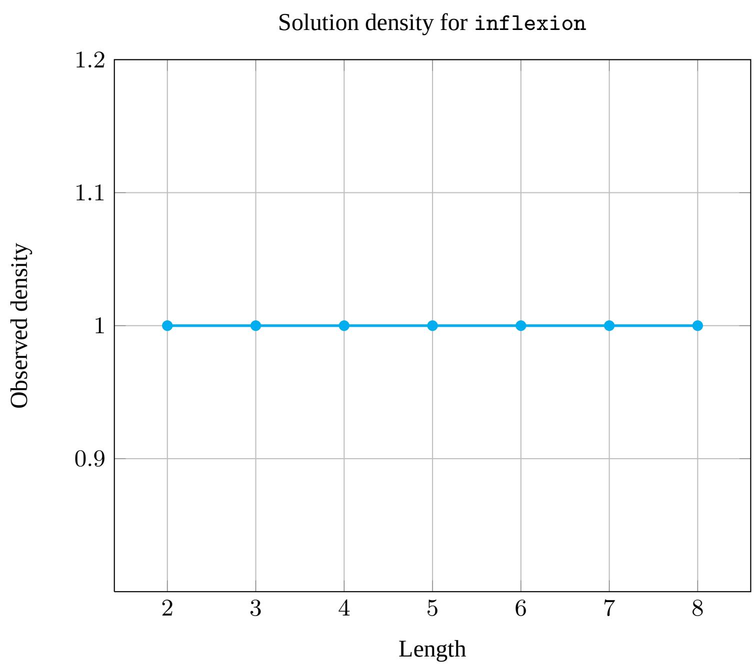
Remark Since the arity of the arc constraint is not fixed, the `inflexion` constraint cannot be currently described with the graph-based representation. However, this would not hold anymore if we were introducing a slot that specifies how to merge adjacent vertices of the final graph.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for `inflexion`: domains $0..n$

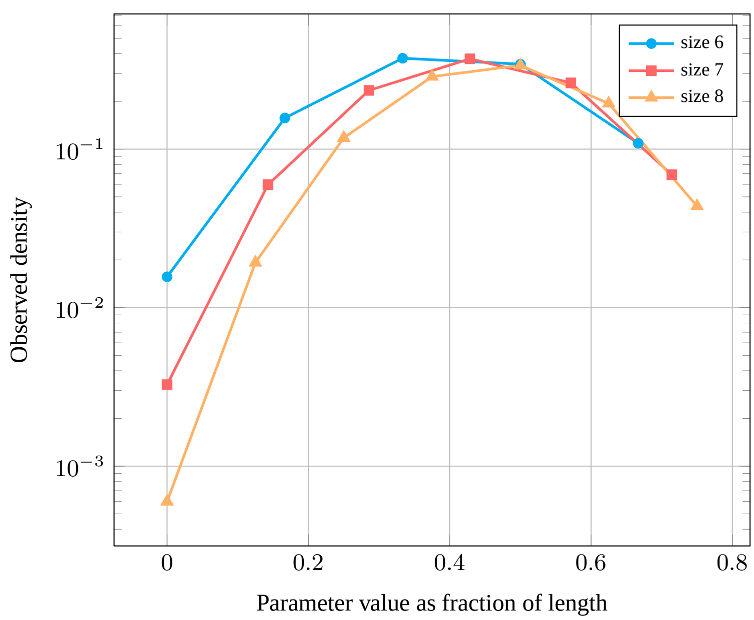

Solution density for inflexion
Observed density
$10^{0.4}$
$10^{0.2}$
10^{0}
$10^{-0.2}$
$10^{-0.4}$
2 3 4 5 6 7 8
Length

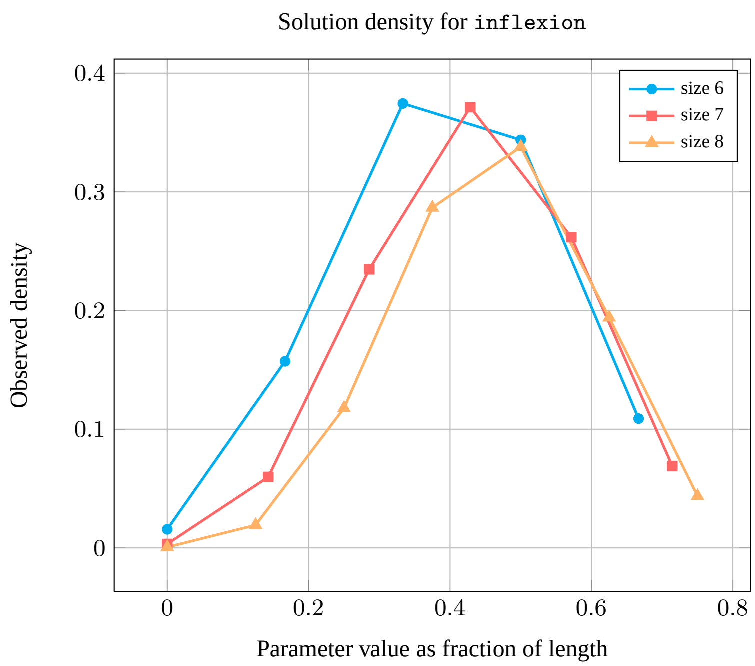

Solution density for inflexion
Observed density
1.2
1.1
1
0.9
2 3 4 5 6 7 8
Length

Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	9	36	135	498	1841	6856	25731
	1	-	28	320	2588	18494	125284	828120
	2	-	-	170	3348	44058	492320	5069970
	3	-	-	-	1342	40446	778936	12341184
	4	-	-	-	-	12810	549152	14547186
	5	-	-	-	-	-	144604	8354520
	6	-	-	-	-	-	-	1880010

Solution count for `inflexion`: domains $0..n$

Solution density for `inflexion`





See also **common keyword:** `global_contiguity`, `min_dist_between_inflexion`, `peak`, `valley` *(sequence)*.

Keywords **characteristic of a constraint:** automaton, automaton with counters, automaton with same input symbol.

combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

constraint network structure: sliding cyclic(1) constraint network(2).

filtering: glue matrix.

modelling: functional dependency.

Cond. implications

- `inflexion`(N, VARIABLES)
  with $\texttt{N} > 0$
  implies `atleast_nvalue`(NVAL, VARIABLES)
  when $\texttt{NVAL} = 2$.

- `inflexion`(N, VARIABLES)
  with `valley`(VARIABLES.var) $= 0$
  implies `peak`(N, VARIABLES).

- `inflexion`(N, VARIABLES)
  with `peak`(VARIABLES.var) $= 0$
  implies `valley`(N, VARIABLES).

Automaton Figure 5.447 depicts the automaton associated with the `inflexion` constraint. To each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $(\mathtt{VAR}_i < \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\mathtt{VAR}_i = \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 1) \wedge (\mathtt{VAR}_i > \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 2)$.

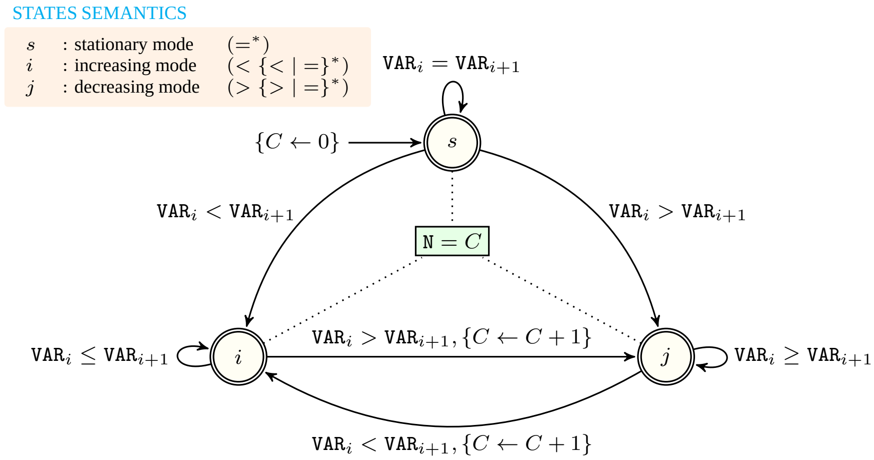


Figure 5.447: Automaton of the `inflexion` constraint (state s means that we are in *stationary* mode, state i means that we are in *increasing* mode, state j means that we are in *decreasing* mode, a new inflexion is detected each time we switch from increasing to decreasing mode – or conversely from decreasing to increasing mode – and the counter C is incremented accordingly)

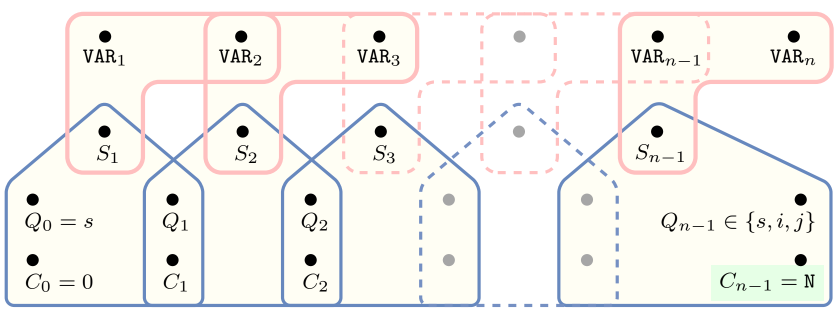


Figure 5.448: Hypergraph of the reformulation corresponding to the automaton of the `inflexion` constraint

Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

	$s\ (=^*)$	$i\ (<\{<\mid=\}^*)$	$j\ (>\{>\mid=\}^*)$
$s\ (=^*)$	0	$\overleftarrow{C}$	$\overleftarrow{C}$
$i\ (<\{<\mid=\}^*)$	$\overrightarrow{C}$	$\overrightarrow{C}+1+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$
$j\ (>\{>\mid=\}^*)$	$\overrightarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+1+\overleftarrow{C}$

Figure 5.449: Glue matrix associated with the automaton of the `inflexion` constraint

`inflexion`($\mathtt{N}=3, \langle 1,1,4,8,8,2,7,1\rangle$)

	1	1	4	8	8	2	2	7	1	
	=	<	<	=	>		>	<		
i	0	1	2	3	4	5	2	1	0	i
$\overrightarrow{Q_i}$	s	s	i	i	i	j	j	i	s	$\overleftarrow{Q_i}$
$\overrightarrow{C_i}$	0	0	0	0	0	1	1	0	0	$\overleftarrow{C_i}$
$\overrightarrow{\mathtt{N}_i}$	0	0	0	0	0	1	1	0	0	$\overleftarrow{\mathtt{N}_i}$

`inflexion`$\left(\overrightarrow{\mathtt{N}_5}=1, \langle 1,1,4,8,8,2\rangle\right)$ `inflexion`$\left(\overleftarrow{\mathtt{N}_2}=1, \langle 1,7,2\rangle\right)$

glue matrix entry associated with the state pair (j, j):

$$\mathtt{N}=\overrightarrow{\mathtt{C}_5}+1+\overleftarrow{\mathtt{C}_2}=1+1+1=3$$

Figure 5.450: Illustrating the use of the state pair (j, j) of the glue matrix for linking `N` with the counters variables obtained after reading the prefix $1, 1, 4, 8, 8, 2$ and corresponding suffix $2, 7, 1$ of the sequence $1, 1, 4, 8, 8, 2, 7, 1$; note that the suffix $2, 7, 1$ (in pink) is proceed in reverse order; the left (resp. right) table shows the initialisation (for $i = 0$) and the evolution (for $i > 0$) of the state of the automaton and its counter C upon reading the prefix $1, 1, 4, 8, 8, 2$ (resp. the reverse suffix $1, 7, 2$).