5.2 all_differ_from_at_least_k_pos

DESCRIPTION LINKS GRAPH

Origin Inspired by [177].

Constraint all_differ_from_at_least_k_pos(K, VECTORS)

Type
VECTOR : collection(var−dvar)

Arguments

K	:	int
VECTORS	:	collection(vec − VECTOR)

Restrictions
required(VECTOR, var)
|VECTOR| ≥ 1
|VECTOR| ≥ K
K ≥ 0
required(VECTORS, vec)
same_size(VECTORS, vec)

Purpose Enforce all pairs of distinct vectors of the VECTORS collection to differ from at least K positions.

Example $\langle 2, \langle \mathtt{vec} - \langle 2,5,2,0\rangle, \mathtt{vec} - \langle 3,6,2,1\rangle, \mathtt{vec} - \langle 3,6,1,0\rangle\rangle\rangle$

The all_differ_from_at_least_k_pos constraint holds since:

- The first and second vectors differ from 3 positions, which is greater than or equal to K = 2.
- The first and third vectors differ from 3 positions, which is greater than or equal to K = 2.
- The second and third vectors differ from 2 positions, which is greater than or equal to K = 2.

Typical
K > 0
|VECTORS| > 1

Symmetries
- Items of VECTORS are permutable.
- Items of VECTORS.vec are permutable (*same permutation used*).

Arg. properties
- Contractible wrt. VECTORS.
- Extensible wrt. VECTORS.vec (*add items at same position*).

See also
implied by: all_differ_from_exactly_k_pos (≥ K *replaced by* = K).
part of system of constraints: differ_from_at_least_k_pos.
used in graph description: differ_from_at_least_k_pos.

Keywords **application area:** bioinformatics.

characteristic of a constraint: disequality, vector.

constraint type: system of constraints, decomposition.

final graph structure: no loop, symmetric.

Cond. implications `all_differ_from_at_least_k_pos`(`K`, `VECTORS`)
with $\texttt{K} \leq |\texttt{VECTORS}|$
implies `atleast_nvector`(`NVEC`, `VECTORS`).

Arc input(s)	VECTORS
Arc generator	$CLIQUE(\neq) \mapsto$ collection(vectors1, vectors2)
Arc arity	2
Arc constraint(s)	differ_from_at_least_k_pos(K, vectors1.vec, vectors2.vec)
Graph property(ies)	NARC$= \lvert$VECTORS$\rvert * \lvert$VECTORS$\rvert - \lvert$VECTORS$\rvert$
Graph class	• NO_LOOP • SYMMETRIC

Graph model

The **Arc constraint(s)** slot uses the differ_from_at_least_k_pos constraint defined in this catalogue.

Parts (A) and (B) of Figure 5.2 respectively show the initial and final graph associated with the **Example** slot. Since we use the NARC graph property, the arcs of the final graph are stressed in bold. The previous constraint holds since exactly $3 \cdot (3-1) = 6$ arc constraints hold.

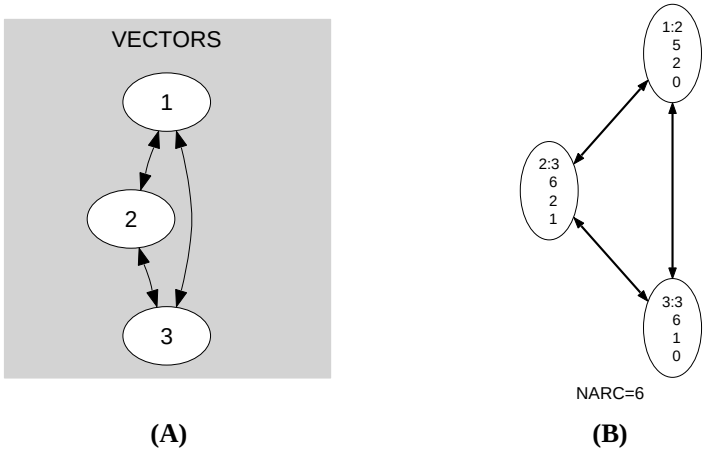

Figure 5.2: Initial and final graph of the all_differ_from_at_least_k_pos constraint

Signature

Since we use the $CLIQUE(\neq)$ arc generator on the items of the VECTORS collection, the expression $\lvert$VECTORS$\rvert \cdot \lvert$VECTORS$\rvert - \lvert$VECTORS$\rvert$ corresponds to the maximum number of arcs of the final graph. Therefore we can rewrite the graph property NARC $= \lvert$VECTORS$\rvert \cdot \lvert$VECTORS$\rvert - \lvert$VECTORS$\rvert$ to NARC $\geq \lvert$VECTORS$\rvert \cdot \lvert$VECTORS$\rvert - \lvert$VECTORS$\rvert$. This leads to simplify NARC to NARC.