5.197 intersection_of_intervals

DESCRIPTION LINKS

Origin Inspired by video summarization.

Constraint intersection_of_intervals(INTERSECTION, TASKS, INTERVALS)

Synonyms intersection_between_tasks_and_intervals, ordered_tasks_intersection.

Arguments

INTERSECTION	:	dvar
TASKS	:	collection(origin−dvar, duration−dvar, end−dvar)
INTERVALS	:	collection(low−int, up−int)

Restrictions

INTERSECTION ≥ 0
INTERSECTION $\leq$ sum(TASKS.duration)
require_at_least(2, TASKS, [origin, duration, end])
TASKS.duration ≥ 0
TASKS.origin $\leq$ TASKS.end
required(INTERVALS, [low, up])
INTERVALS.low $\leq$ INTERVALS.up

Purpose

INTERSECTION is the intersection between a collection of ordered tasks TASKS and a collection of ordered fixed intervals INTERVALS:

1. $\forall t \in [1, |\texttt{TASKS}|] : \texttt{TASKS}[t].\texttt{end} = \texttt{TASKS}[t].\texttt{origin} + \texttt{TASKS}[t].\texttt{duration}$,
2. $\forall t \in [1, |\texttt{TASKS}| - 1] : \texttt{TASKS}[t].\texttt{end} \leq \texttt{TASKS}[t+1].\texttt{origin}$,
3. $\forall i \in [1, |\texttt{INTERVALS}| - 1] : \texttt{INTERVALS}[i].\texttt{up} < \texttt{INTERVALS}[i+1].\texttt{low}$,
4. $\texttt{INTERSECTION} = \sum_{\substack{t\in[1,|\texttt{TASKS}|] \\ i\in[1,|\texttt{INTERVALS}|]}} \max(\beta_{t,i} - \alpha_{t,i} + 1, 0)$ with

$$\alpha_{t,i} = \max\begin{pmatrix} \texttt{TASKS}[t].\texttt{origin}, \\ \texttt{INTERVALS}[i].\texttt{low} \end{pmatrix}, \quad \beta_{t,i} = \min\begin{pmatrix} \texttt{TASKS}[t].\texttt{end} - 1, \\ \texttt{INTERVALS}[i].\texttt{up} \end{pmatrix}.$$

Example

$$\left(\begin{array}{l} 3, \left\langle \begin{array}{lll} \texttt{origin}-2 & \texttt{duration}-2 & \texttt{end}-4, \\ \texttt{origin}-7 & \texttt{duration}-2 & \texttt{end}-9, \\ \texttt{origin}-9 & \texttt{duration}-0 & \texttt{end}-9 \end{array} \right\rangle, \\ \langle \texttt{low}-1\ \texttt{up}-3, \texttt{low}-5\ \texttt{up}-5, \texttt{low}-8\ \texttt{up}-9 \rangle \end{array}\right)$$

As illustrated by Figure 5.457, the constraint holds since:

- The first task is included within the first interval and therefore contributes from its total duration 2 to the overall intersection.
- While the second task does not intersect the first and second intervals, it has a non empty intersection of 1 with the third interval.
- The third task does not contribute to the overall intersection since its duration is equal to zero.

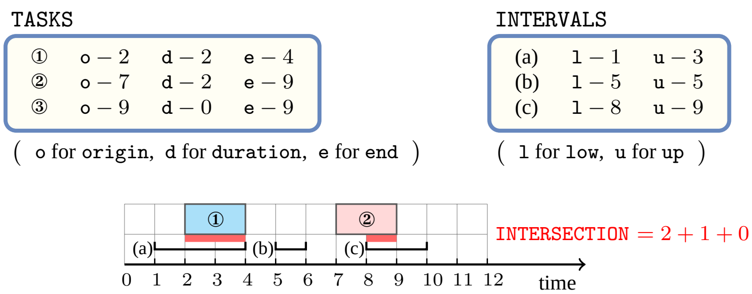

Figure 5.457: The `intersection_of_intervals` solution to the **Example** slot

The overall intersection 3 is equal to $2 + 1 + 0$.

All solutions

Figure 5.458 gives all solutions to the following non ground instance of the `intersection_of_intervals` constraint: $\mathtt{O}_1 \in [0, 1]$, $\mathtt{D}_1 \in [0, 6]$, $\mathtt{E}_1 \in [3, 5]$, $\mathtt{O}_2 \in [0, 6]$, $\mathtt{D}_2 \in [1, 3]$, $\mathtt{E}_2 \in [0, 9]$, `intersection_of_intervals`$(\mathbf{2}, \langle \mathtt{O}_1\ \mathtt{D}_1\ \mathtt{E}_1,\ \mathtt{O}_2\ \mathtt{D}_2\ \mathtt{E}_2\rangle, \langle \mathbf{1\ 3},\ \mathbf{5\ 5},\ \mathbf{8\ 9}\rangle)$.

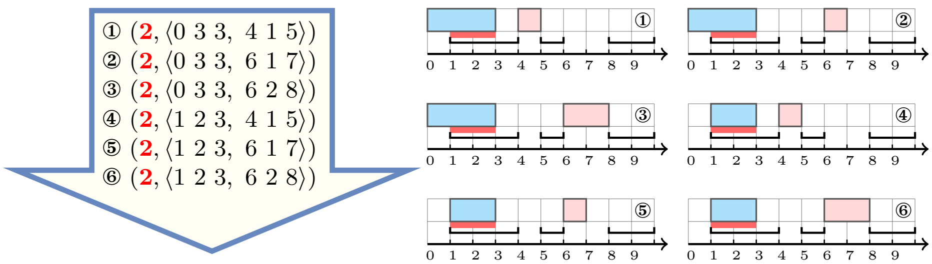

Figure 5.458: All solutions corresponding to the non ground example of the `intersection_of_intervals` constraint of the **All solutions** slot

Typical

$\mathtt{INTERSECTION} > 0$
$|\mathtt{TASKS}| > 1$
$\mathrm{range}(\mathtt{TASKS.duration}) > 1$
$|\mathtt{INTERVALS}| > 1$

Arg. properties

Functional dependency: `INTERSECTION` determined by `TASKS` and `INTERVALS`.

Keywords

constraint type: scheduling constraint.

modelling: zero-duration task.