

## 5.59 cardinality\_atmost

	DESCRIPTION	LINKS	GRAPH	AUTOMATON
Origin	Derived from <a href="#">global_cardinality</a> .			
Constraint	<code>cardinality_atmost(ATMOST, VARIABLES, VALUES)</code>			
Arguments	ATMOST : <code>dvar</code> VARIABLES : <code>collection(var-dvar)</code> VALUES : <code>collection(val-int)</code>			
Restrictions	$ATMOST \geq 0$ $ATMOST \leq  VARIABLES $ <a href="#">required</a> (VARIABLES, var) <a href="#">required</a> (VALUES, val) <a href="#">distinct</a> (VALUES, val)			
Purpose	ATMOST is the maximum number of occurrences of each value of VALUES within the variables of the collection VARIABLES.			
Example	$(2, \langle 2, 1, 7, 1, 2 \rangle, \langle 5, 7, 2, 9 \rangle)$ <p>In this example, values 5, 7, 2 and 9 occur respectively 0, 1, 2 and 0 times within the collection <math>\langle 2, 1, 7, 1, 2 \rangle</math>. As a consequence, the <code>cardinality_atmost</code> constraint holds since its first argument ATMOST is assigned to the maximum number of occurrences 2.</p>			
Typical	$ATMOST > 0$ $ATMOST <  VARIABLES $ $ VARIABLES  > 1$ $ VALUES  > 0$ $ VARIABLES  >  VALUES $			
Symmetries	<ul style="list-style-type: none"> <li>Items of VARIABLES are <a href="#">permutable</a>.</li> <li>Items of VALUES are <a href="#">permutable</a>.</li> <li>An occurrence of a value of VARIABLES.var that does not belong to VALUES.val can be <a href="#">replaced</a> by any other value that also does not belong to VALUES.val.</li> <li>All occurrences of two distinct values in VARIABLES.var or VALUES.val can be <a href="#">swapped</a>; all occurrences of a value in VARIABLES.var or VALUES.val can be <a href="#">renamed</a> to any unused value.</li> </ul>			
Arg. properties	<b>Functional dependency:</b> ATMOST determined by VARIABLES and VALUES.			
Usage	An application of the <code>cardinality_atmost</code> constraint is to enforce a maximum use of values.			

<b>Remark</b>	This is a restricted form of a variant of the <a href="#">among</a> constraint and of the <a href="#">global_cardinality</a> constraint. In the original <a href="#">global_cardinality</a> constraint, one specifies for each value its minimum and maximum number of occurrences.
<b>Algorithm</b>	See <a href="#">global_cardinality</a> [342].
<b>See also</b>	<b>generalisation:</b> <a href="#">global_cardinality</a> ( <i>single count variable replaced by an individual count variable for each value</i> ), <a href="#">multi_inter_distance</a> ( <i>window of size 1 replaced by window of DIST consecutive values</i> ). <b>implied by:</b> <a href="#">among</a> .
<b>Keywords</b>	<b>application area:</b> assignment. <b>characteristic of a constraint:</b> automaton, automaton with array of counters. <b>constraint arguments:</b> pure functional dependency. <b>constraint type:</b> value constraint. <b>filtering:</b> arc-consistency. <b>final graph structure:</b> acyclic, bipartite, no loop. <b>modelling:</b> at most, functional dependency.

Arc input(s)	VARIABLES VALUES
Arc generator	<i>PRODUCT</i> $\mapsto$ collection(variables, values)
Arc arity	2
Arc constraint(s)	variables.var = values.val
Graph property(ies)	<i>MAX_ID</i> = ATMOST
Graph class	<ul style="list-style-type: none"><li>• ACYCLIC</li><li>• BIPARTITE</li><li>• NO_LOOP</li></ul>

**Graph model** Parts (A) and (B) of Figure 5.150 respectively show the initial and final graph associated with the **Example** slot. Since we use the *MAX\_ID* graph property, the vertex that has the maximum number of predecessor is stressed with a double circle.

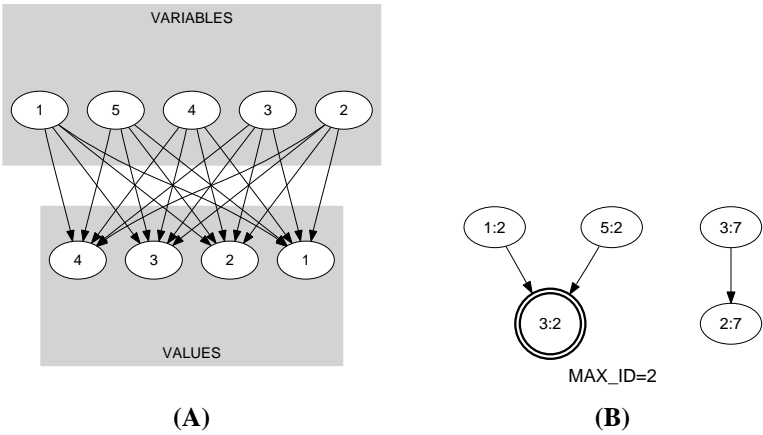


Figure 5.150: Initial and final graph of the cardinality\_atmost constraint

**Automaton**

Figure 5.151 depicts the automaton associated with the `cardinality_atmost` constraint. To each variable  $\text{VAR}_i$  of the collection `VARIABLES` corresponds a 0-1 signature variable  $S_i$ . The following signature constraint links  $\text{VAR}_i$  and  $S_i$ :  $\text{VAR}_i \in \text{VALUES} \Leftrightarrow S_i$ .

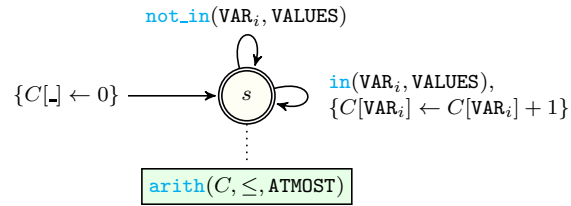


Figure 5.151: Automaton of the `cardinality_atmost` constraint