5.257 min_n

DESCRIPTION LINKS GRAPH AUTOMATON

Origin [27]

Constraint `min_n`(`MIN`, `RANK`, `VARIABLES`)

Arguments

`MIN`	:	`dvar`
`RANK`	:	`int`
`VARIABLES`	:	`collection`(`var`−`dvar`)

Restrictions

$|\texttt{VARIABLES}| > 0$
$\texttt{RANK} \geq 0$
$\texttt{RANK} < |\texttt{VARIABLES}|$
`required`(`VARIABLES`, `var`)

Purpose

`MIN` is the minimum value of rank `RANK` (i.e., the $\texttt{RANK}^{th}$ smallest distinct value, identical values are merged) of the collection of domain variables `VARIABLES`. The minimum value has rank 0.

Example

$(3, 1, \langle 3, 1, 7, 1, 6\rangle)$

The `min_n` constraint holds since its first argument $\texttt{MIN} = 3$ is fixed to the second (i.e., $\texttt{RANK} + 1$) smallest distinct value of the collection $\langle 3, 1, 7, 1, 6\rangle$. Note that identical values are only counted once: this is why the minimum of order 1 is 3 instead of 1.

Typical

$\texttt{RANK} > 0$
$\texttt{RANK} < 3$
$|\texttt{VARIABLES}| > 1$
`range`(`VARIABLES.var`) > 1

Symmetries

- Items of `VARIABLES` are permutable.
- One and the same constant can be added to `MIN` as well as to the `var` attribute of all items of `VARIABLES`.

Arg. properties

Functional dependency: `MIN` determined by `RANK` and `VARIABLES`.

Algorithm [27].

Reformulation

The constraint `among_var`(1, ⟨`MIN`⟩, `VARIABLES`) enforces `MIN` to be assigned one of the values of `VARIABLES`. The constraint `nvalue`(`NVAL`, `VARIABLES`) provides a hand on the number of distinct values assigned to the variables of `VARIABLES`. By associating to each variable V_i ($i \in [1, |\texttt{VARIABLES}|]$) of the `VARIABLES` collection a *rank* variable $R_i \in [0, |\texttt{VARIABLES}| - 1]$ with the reified constraint $R_i = \texttt{RANK} \Leftrightarrow V_i = \texttt{MIN}$, the inequality $R_i < \texttt{NVAL}$, and by creating for each pair of variables V_i, V_j ($i, j < i \in [1, |\texttt{VARIABLES}|]$)

the reified constraints
$V_i < V_j \Leftrightarrow R_i < R_j$,
$V_i = V_j \Leftrightarrow R_i = R_j$,
$V_i > V_j \Leftrightarrow R_i > R_j$,
one can reformulate the `min_n` constraint in term of $3 \cdot \frac{|\mathtt{VARIABLES}|\cdot(|\mathtt{VARIABLES}|-1)}{2} + 1$ reified constraints.

See also

comparison swapped: `max_n`.

generalisation: `minimum` *(absolute minimum replaced by minimum or order* `n`*).*

used in reformulation: `among_var`, `nvalue`.

Keywords

characteristic of a constraint: rank, minimum, maxint, automaton, automaton with array of counters.

constraint arguments: pure functional dependency.

constraint type: order constraint.

modelling: functional dependency.

Cond. implications

- $\mathtt{min\_n}(\mathtt{MIN}, \mathtt{RANK}, \mathtt{VARIABLES})$
  implies $\mathtt{atleast}(\mathtt{N}, \mathtt{VARIABLES}, \mathtt{MIN})$
  when $\mathtt{N} = 1$.
- $\mathtt{min\_n}(\mathtt{MIN}, \mathtt{RANK}, \mathtt{VARIABLES})$
  with $\mathtt{RANK} = 1$
  and $\mathtt{minval}(\mathtt{VARIABLES.var}) = 1$
  implies $\mathtt{minimum\_greater\_than}(\mathtt{VAR1}, \mathtt{VAR2}, \mathtt{VARIABLES})$.

Arc input(s) `VARIABLES`

Arc generator $CLIQUE \mapsto \texttt{collection}(\texttt{variables1}, \texttt{variables2})$

Arc arity 2

Arc constraint(s) $\bigvee\left(\begin{array}{l}\texttt{variables1.key} = \texttt{variables2.key},\\ \texttt{variables1.var} < \texttt{variables2.var}\end{array}\right)$

Graph property(ies) **ORDER**$(\texttt{RANK}, \texttt{MAXINT}, \texttt{var}) = \texttt{MIN}$

Graph model Parts (A) and (B) of Figure 5.556 respectively show the initial and final graph associated with the **Example** slot. Since we use the **ORDER** graph property, the vertex of rank 1 (without considering the loops) of the final graph is shown in grey.

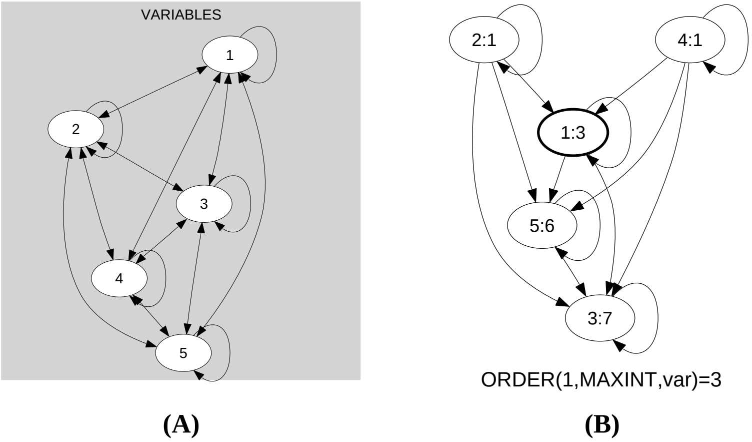

Figure 5.556: Initial and final graph of the `min_n` constraint

Automaton

Figure 5.557 depicts the automaton associated with the min_n constraint. Figure 5.557 depicts the automaton associated with the min_n constraint. To each item of the collection VARIABLES corresponds a signature variable S_i that is equal to 1.

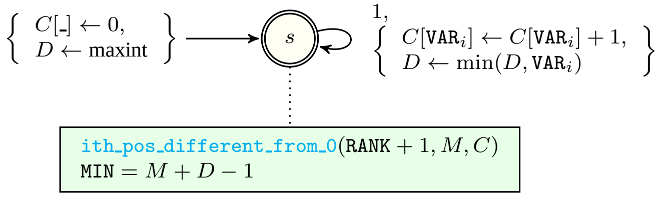
$\{ C[\_] \leftarrow 0, D \leftarrow \text{maxint} \}$
s
1,
$\{ C[\texttt{VAR}_i] \leftarrow C[\texttt{VAR}_i] + 1, D \leftarrow \min(D, \texttt{VAR}_i) \}$
ith_pos_different_from_0($\texttt{RANK} + 1, M, C$)
$\texttt{MIN} = M + D - 1$

Figure 5.557: Automaton of the min_n constraint