5.77 common_modulo

DESCRIPTION LINKS GRAPH

Origin Derived from `common`.

Constraint `common_modulo(NCOMMON1, NCOMMON2, VARIABLES1, VARIABLES2, M)`

Arguments

`NCOMMON1`	:	`dvar`
`NCOMMON2`	:	`dvar`
`VARIABLES1`	:	`collection(var−dvar)`
`VARIABLES2`	:	`collection(var−dvar)`
`M`	:	`int`

Restrictions

`NCOMMON1` ≥ 0
`NCOMMON1` $\leq$ `|VARIABLES1|`
`NCOMMON2` ≥ 0
`NCOMMON2` $\leq$ `|VARIABLES2|`
`required(VARIABLES1, var)`
`required(VARIABLES2, var)`
`M` > 0

Purpose

`NCOMMON1` is the number of variables of the collection of variables `VARIABLES1` taking a value situated in an equivalence class (congruence modulo a fixed number `M`) derived from the values assigned to the variables of `VARIABLES2` and from `M`.
`NCOMMON2` is the number of variables of the collection of variables `VARIABLES2` taking a value situated in an equivalence class (congruence modulo a fixed number `M`) derived from the values assigned to the variables of `VARIABLES1` and from `M`.

Example

$$(3, 4, \langle 0, 4, 0, 8\rangle, \langle 7, 5, 4, 9, 2, 4\rangle, 5)$$

In the example, the last argument `M` $= 5$ defines the equivalence classes $a \equiv 0 \pmod 5$, $a \equiv 1 \pmod 5$, $a \equiv 2 \pmod 5$, $a \equiv 3 \pmod 5$, and $a \equiv 4 \pmod 5$ where a is an integer. As a consequence the items of collection $\langle 0, 4, 0, 8\rangle$ respectively correspond to the equivalence classes $a \equiv 0 \pmod 5$, $a \equiv 4 \pmod 5$, $a \equiv 0 \pmod 5$, and $a \equiv 3 \pmod 5$. Similarly the items of collection $\langle 7, 5, 4, 9, 2, 4\rangle$ respectively correspond to the equivalence classes $a \equiv 2 \pmod 5$, $a \equiv 0 \pmod 5$, $a \equiv 4 \pmod 5$, $a \equiv 4 \pmod 5$, $a \equiv 2 \pmod 5$, and $a \equiv 4 \pmod 5$. The `common_modulo` constraint holds since:

- Its first argument `NCOMMON1` $= 3$ is the number of equivalence classes associated with the items of collection $\langle 0, 4, 0, 8\rangle$ that also correspond to equivalence classes associated with $\langle 7, 5, 4, 9, 2, 4\rangle$.
- Its second argument `NCOMMON2` $= 4$ is the number of equivalence classes associated with the items of collection $\langle 7, 5, 4, 9, 2, 4\rangle$ that also correspond to equivalence classes associated with $\langle 0, 4, 0, 8\rangle$.

Typical

$|\mathtt{VARIABLES1}| > 1$
$\mathtt{range}(\mathtt{VARIABLES1.var}) > 1$
$|\mathtt{VARIABLES2}| > 1$
$\mathtt{range}(\mathtt{VARIABLES2.var}) > 1$
$\mathtt{M} > 1$
$\mathtt{M} < \mathtt{maxval}(\mathtt{VARIABLES1.var})$
$\mathtt{M} < \mathtt{maxval}(\mathtt{VARIABLES2.var})$

Symmetries

- Arguments are permutable w.r.t. permutation $(\mathtt{NCOMMON1}, \mathtt{NCOMMON2})$ $(\mathtt{VARIABLES1}, \mathtt{VARIABLES2})$ $(\mathtt{M})$.
- Items of `VARIABLES1` are permutable.
- Items of `VARIABLES2` are permutable.
- An occurrence of a value u of `VARIABLES1.var` can be replaced by any other value v such that v is congruent to u modulo `M`.
- An occurrence of a value u of `VARIABLES2.var` can be replaced by any other value v such that v is congruent to u modulo `M`.

Arg. properties

- Functional dependency: `NCOMMON1` determined by `VARIABLES1`, `VARIABLES2` and `M`.
- Functional dependency: `NCOMMON2` determined by `VARIABLES1`, `VARIABLES2` and `M`.

See also

specialisation: `common` *(*`variable` mod `constant` *replaced by* `variable`*)*.

Keywords

characteristic of a constraint: modulo.

constraint arguments: constraint between two collections of variables, pure functional dependency.

final graph structure: acyclic, bipartite, no loop.

modelling: functional dependency.

Arc input(s)	VARIABLES1 VARIABLES2
Arc generator	$PRODUCT \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var $\bmod$ M = variables2.var $\bmod$ M
Graph property(ies)	• **NSOURCE**= NCOMMON1 • **NSINK**= NCOMMON2
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Graph model Parts (A) and (B) of Figure 5.200 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NSOURCE** and **NSINK** graph properties, the source and sink vertices of the final graph are stressed with a double circle. Since the graph has only 3 sources and 4 sinks the variables NCOMMON1 and NCOMMON2 are respectively equal to 3 and 4. Note that the vertices corresponding to the variables that take values 8, 7 or 2 were removed from the final graph since there is no arc for which the associated arc constraint holds.

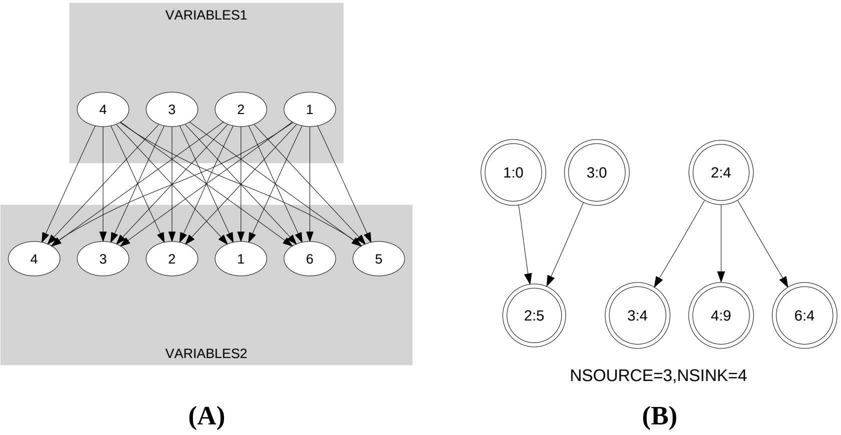

Figure 5.200: Initial and final graph of the common_modulo constraint