

5.253 min_decreasing_slope

	DESCRIPTION	LINKS	AUTOMATON
Origin	Motivated by time series.		
Constraint	min_decreasing_slope(MIN, VARIABLES)		
Arguments	MIN : dvar VARIABLES : collection(var-dvar)		
Restrictions	MIN ≥ 0 MIN <range(VARIABLES.var) required(VARIABLES, var) VARIABLES > 0		
Purpose	Given a sequence of variables $VARIABLES = V_1, V_2, \dots, V_n$, sets MIN to 0 if $\nexists i \in [1, n - 1] V_i > V_{i+1}$, otherwise sets MIN to $\min_{i \in [1, n - 1] V_i > V_{i+1}} (V_i - V_{i+1})$.		
Example	<div>(2, ⟨1, 1, 5, 8, 6, 2, 4, 1, 5⟩)</div> <div>(0, ⟨1, 1, 1, 3, 4, 7, 7, 7, 9⟩)</div> <div>(9, ⟨1, 1, 9, 0, 4, 7, 7, 7, 9⟩)</div> <p>The first min_decreasing_slope constraint holds since the sequence 1 1 5 8 6 2 4 1 5 contains two decreasing subsequences 8 6 2 and 4 1 and the minimum slope is equal to $\min(8 - 6, 6 - 2, 4 - 1) = 2$ as shown on Figure 5.548.</p>		
Typical	MIN > 1 VARIABLES > 2 range(VARIABLES.var) > 2		
Symmetry	One and the same constant can be added to the var attribute of all items of VARIABLES.		
Arg. properties	Functional dependency: MIN determined by VARIABLES.		
Usage	Getting the minimum slope over the decreasing sequences of time series.		
Counting			

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for min_decreasing_slope: domains 0..n

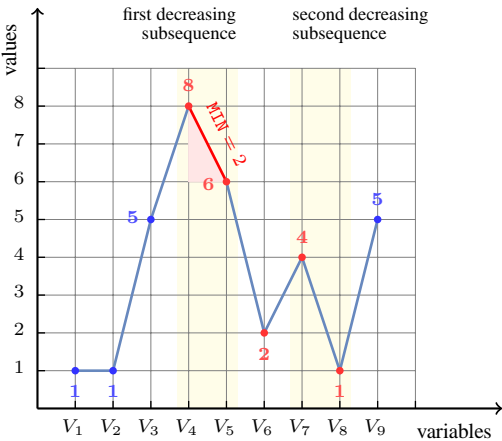
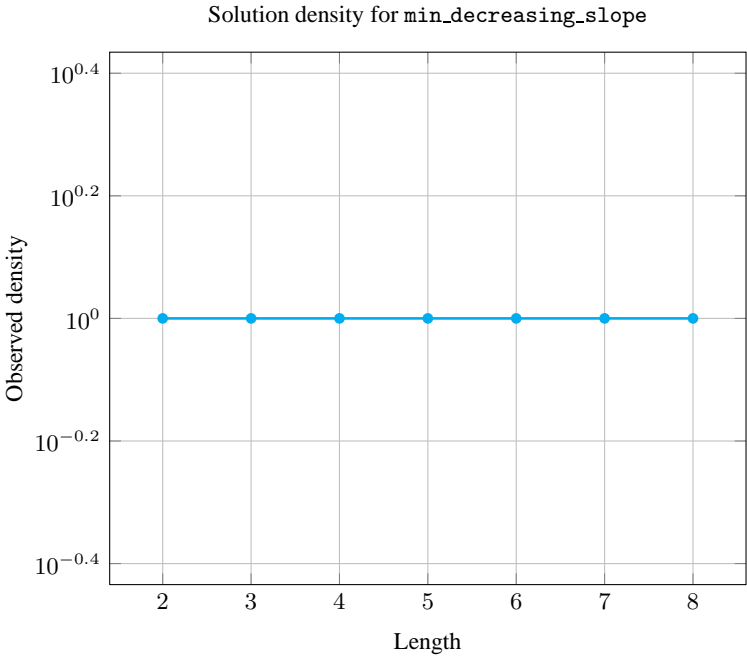
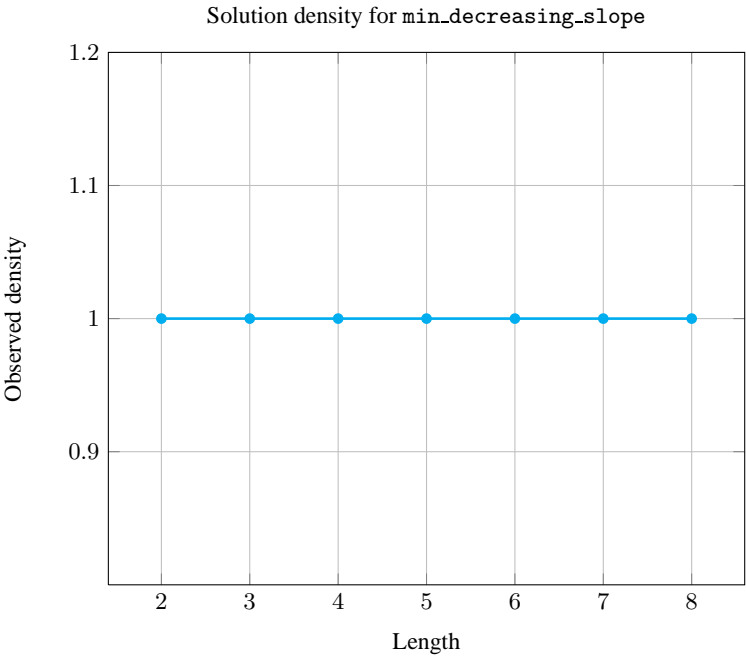


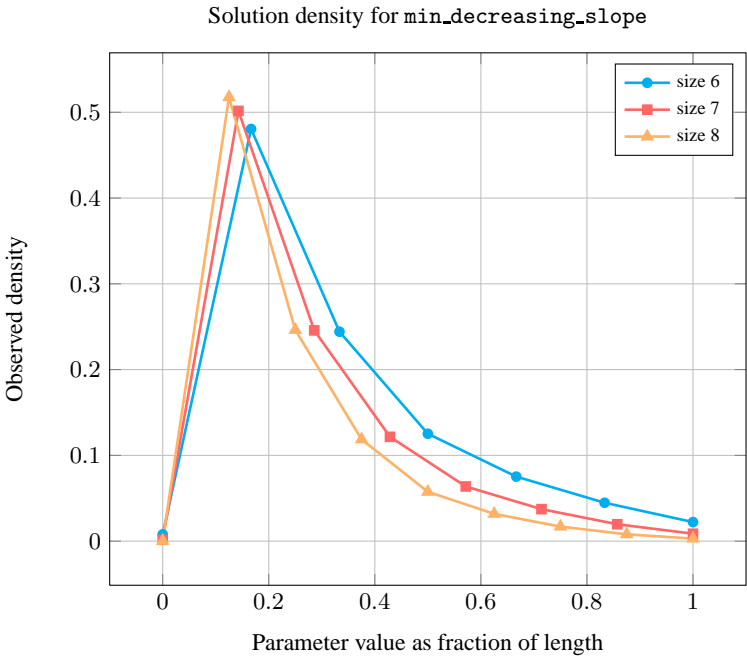
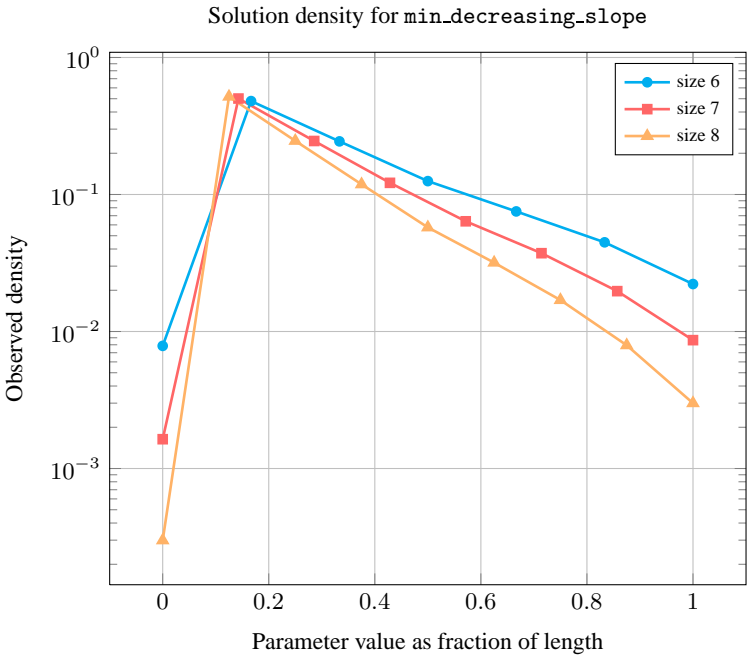
Figure 5.548: Illustration of the first example of the **Example** slot: a sequence of nine variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$ respectively fixed to values 1, 1, 5, 8, 6, 2, 4, 1, 5 and the corresponding minimum slope on the strictly decreasing subsequences 8 6 2 and 4 1 (MIN = 2)





Length (<i>n</i>)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	6	20	70	252	924	3432	12870
	1	2	22	256	3512	56537	1051936	22280084
	2	1	14	145	1864	28728	515372	10601773
	3	-	8	98	1062	14729	255076	5106480
	4	-	-	56	704	8853	133672	2475484
	5	-	-	-	382	5266	78198	1369232
	6	-	-	-	-	2612	41330	730161
	7	-	-	-	-	-	18136	341618
	8	-	-	-	-	-	-	129019

Solution count for min_decreasing_slope: domains 0..*n*



Keywords characteristic of a constraint: automaton, automaton with counters.
combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

filtering: glue matrix.

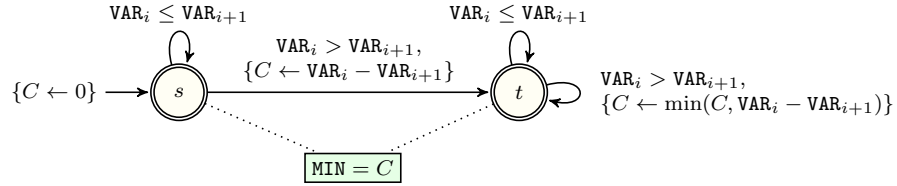
modelling: functional dependency.

Cond. implications

```
min_decreasing_slope(MIN, VARIABLES)
  with range(VARIABLES.var) = MIN + 1
implies max_decreasing_slope(MAX, VARIABLES)
  when range(VARIABLES.var) = MAX + 1.
```

Automaton

Figure 5.549 depicts the automaton associated with the `min_decreasing_slope` constraint. To each pair of consecutive variables $(\text{VAR}_i, \text{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i . The following signature constraint links VAR_i , VAR_{i+1} and S_i : $(\text{VAR}_i \leq \text{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\text{VAR}_i > \text{VAR}_{i+1} \Leftrightarrow S_i = 1)$.



Glue matrix where \vec{C} and \overleftarrow{C} resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

	s	t
s	0	\overleftarrow{C}
t	\vec{C}	$\min(\vec{C}, \overleftarrow{C})$

Figure 5.549: Automaton for the `min_decreasing_slope` constraint and its glue matrix (note that the reverse of `min_decreasing_slope` is `min_increasing_slope`)