5.198 interval_and_count

DESCRIPTION LINKS GRAPH AUTOMATON

Origin [126]

Constraint interval_and_count(ATMOST, COLOURS, TASKS, SIZE_INTERVAL)

Arguments

ATMOST	:	int
COLOURS	:	collection(val−int)
TASKS	:	collection(origin−dvar, colour−dvar)
SIZE_INTERVAL	:	int

Restrictions

ATMOST ≥ 0
required(COLOURS, val)
distinct(COLOURS, val)
required(TASKS, [origin, colour])
TASKS.origin ≥ 0
SIZE_INTERVAL > 0

Purpose

First consider the set of tasks of the TASKS collection, where each task has a specific colour that may not be initially fixed. Then consider the intervals of the form $[k \cdot \texttt{SIZE\_INTERVAL}, k \cdot \texttt{SIZE\_INTERVAL} + \texttt{SIZE\_INTERVAL} - 1]$, where k is an integer. The interval_and_count constraint forces that, for each interval I_k previously defined, the total number of tasks, which both are assigned to I_k and take their colour in COLOURS, does not exceed the limit ATMOST.

Example

$$\left(\begin{array}{l} 2, \langle 4\rangle, \\ \left\langle\begin{array}{ll} \texttt{origin}-1 & \texttt{colour}-4, \\ \texttt{origin}-0 & \texttt{colour}-9, \\ \texttt{origin}-10 & \texttt{colour}-4, \\ \texttt{origin}-4 & \texttt{colour}-4 \end{array}\right\rangle, 5 \end{array}\right)$$

Figure 5.459 shows the solution associated with the example. The constraint interval_and_count holds since, for each interval, the number of tasks taking colour 4 does not exceed the limit 2.

Typical

ATMOST > 0
ATMOST $<$ |TASKS|
|COLOURS| > 0
|TASKS| > 1
range(TASKS.origin) > 1
range(TASKS.colour) > 1
SIZE_INTERVAL > 1

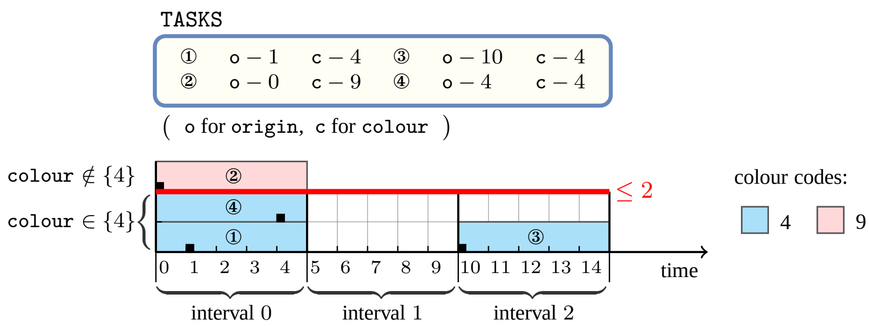


Figure 5.459: The `interval_and_count` solution to the **Example** slot with the use of each interval

Symmetries

- `ATMOST` can be increased.
- Items of `COLOURS` are permutable.
- Items of `TASKS` are permutable.
- One and the same constant can be added to the `origin` attribute of all items of `TASKS`.
- An occurrence of a value of `TASKS.origin` that belongs to the k-th interval, of size `SIZE_INTERVAL`, can be replaced by any other value of the same interval.
- An occurrence of a value of `TASKS.colour` that belongs to `COLOURS.val` (resp. does not belong to `COLOURS.val`) can be replaced by any other value in `COLOURS.val` (resp. not in `COLOURS.val`).

Arg. properties

- Contractible wrt. `COLOURS`.
- Contractible wrt. `TASKS`.

Usage

This constraint was originally proposed for dealing with timetabling problems. In this context the different intervals are interpreted as morning and afternoon periods of different consecutive days. Each colour corresponds to a type of course (i.e., French, mathematics). There is a restriction on the maximum number of courses of a given type each morning as well as each afternoon.

Remark

If we want to only consider intervals that correspond to the morning or to the afternoon we could extend the `interval_and_count` constraint in the following way:

- We introduce two extra parameters `REST` and `QUOTIENT` that correspond to non-negative integers such that `REST` is strictly less than `QUOTIENT`,
- We add the following condition to the arc constraint: $(\texttt{tasks1.origin}/\texttt{SIZE\_INTERVAL}) \equiv \texttt{REST} (\bmod\ \texttt{QUOTIENT})$

Now, if we want to express a constraint on the morning intervals, we set `REST` to 0 and `QUOTIENT` to 2.

Reformulation

Let K denote the index of the last possible interval where the tasks can be assigned: $K = \lfloor \frac{\max_{i\in[1,|\texttt{TASKS}|]}(\texttt{TASKS}[i].\texttt{origin})+\texttt{SIZE\_INTERVAL}-1}{\texttt{SIZE\_INTERVAL}} \rfloor$. The $\texttt{interval\_and\_count}(\texttt{ATMOST}, \texttt{COLOURS}, \texttt{TASKS}, \texttt{SIZE\_INTERVAL})$ constraint can be expressed in term of a set of reified constraints and of K arithmetic constraints (i.e., sum_ctr constraints).

1. For each task $\texttt{TASKS}[i]$ $(i \in [1, |\texttt{TASKS}|])$ of the TASKS collection we create a 0-1 variable B_i that will be set to 1 if and only if task $\texttt{TASKS}[i]$ takes a colour within the set of colours COLOURS:
   $$\begin{aligned} B_i \Leftrightarrow\ & \texttt{TASKS}[i].\texttt{colour} = \texttt{COLOURS}[1].\texttt{val} \vee \\ & \texttt{TASKS}[i].\texttt{colour} = \texttt{COLOURS}[2].\texttt{val} \vee \\ & \dots\dots\dots\dots\dots\dots\dots\dots \\ & \texttt{TASKS}[i].\texttt{colour} = \texttt{COLOURS}[|\texttt{COLOURS}|].\texttt{val}. \end{aligned}$$
2. For each task $\texttt{TASKS}[i]$ $(i \in [1, |\texttt{TASKS}|])$ and for each interval $[k \cdot \texttt{SIZE\_INTERVAL}, k \cdot \texttt{SIZE\_INTERVAL} + \texttt{SIZE\_INTERVAL} - 1]$ $(k \in [0, K])$ we create a 0-1 variable B_{ik} that will be set to 1 if and only if, both task $\texttt{TASKS}[i]$ takes a colour within the set of colours COLOURS, and the origin of task $\texttt{TASKS}[i]$ is assigned within interval $[k \cdot \texttt{SIZE\_INTERVAL}, k \cdot \texttt{SIZE\_INTERVAL} + \texttt{SIZE\_INTERVAL} - 1]$:
   $$\begin{aligned} B_{ik} \Leftrightarrow\ & B_i \wedge \\ & \texttt{TASKS}[i].\texttt{origin} \geq k \cdot \texttt{SIZE\_INTERVAL} \wedge \\ & \texttt{TASKS}[i].\texttt{origin} \leq k \cdot \texttt{SIZE\_INTERVAL} + \texttt{SIZE\_INTERVAL} - 1 \end{aligned}$$
3. Finally, for each interval $[k \cdot \texttt{SIZE\_INTERVAL}, k \cdot \texttt{SIZE\_INTERVAL} + \texttt{SIZE\_INTERVAL} - 1]$ $(k \in [0, K])$, we impose the sum $B_{1k} + B_{2k} + \cdots + B_{|\texttt{TASKS}|k}$ to not exceed the maximum allowed capacity ATMOST.

See also

assignment dimension removed: among_low_up *(assignment dimension corresponding to intervals is removed).*

related: interval_and_sum *(*among_low_up *constraint replaced by* sum_ctr*).*

used in graph description: among_low_up.

Keywords

application area: assignment.

characteristic of a constraint: coloured, automaton, automaton with array of counters.

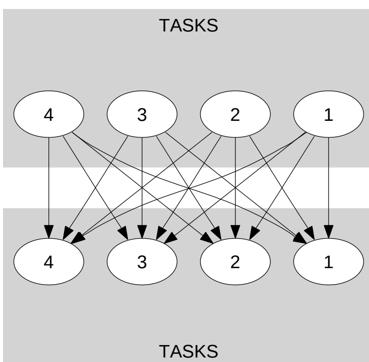
constraint type: timetabling constraint, resource constraint, temporal constraint.

modelling: assignment dimension, interval.

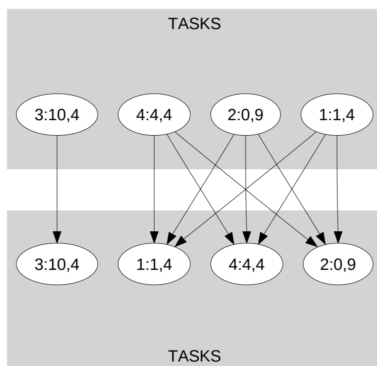
Arc input(s)	`TASKS TASKS`
Arc generator	$PRODUCT \mapsto$ `collection(tasks1, tasks2)`
Arc arity	2
Arc constraint(s)	`tasks1.origin/SIZE_INTERVAL = tasks2.origin/SIZE_INTERVAL`
Sets	SUCC $\mapsto$ [`source`, `variables` − `col` (`VARIABLES`−`collection`(`var`−`dvar`), [`item`(`var` − `TASKS.colour`)])]
Constraint(s) on sets	`among_low_up`(0, `ATMOST`, `variables`, `COLOURS`)

Graph model We use a bipartite graph where each class of vertices corresponds to the different tasks of the `TASKS` collection. There is an arc between two tasks if their origins belong to the same interval. Finally we enforce an `among_low_up` constraint on each set $\mathcal{S}$ of successors of the different vertices of the final graph. This put a restriction on the maximum number of tasks of $\mathcal{S}$ for which the colour attribute takes its value in `COLOURS`.

Parts (A) and (B) of Figure 5.460 respectively show the initial and final graph associated with the **Example** slot. Each connected component of the final graph corresponds to items that are all assigned to the same interval.



(A)



(B)

Figure 5.460: Initial and final graph of the `interval_and_count` constraint

Automaton Figure 5.461 depicts the automaton associated with the `interval_and_count` constraint. Let $\texttt{COLOUR}_i$ be the `colour` attribute of the i^{th} item of the `TASKS` collection. To each pair $(\texttt{COLOURS}, \texttt{COLOUR}_i)$ corresponds a signature variable S_i as well as the following signature constraint: $\texttt{COLOUR}_i \in \texttt{COLOURS} \Leftrightarrow S_i$.

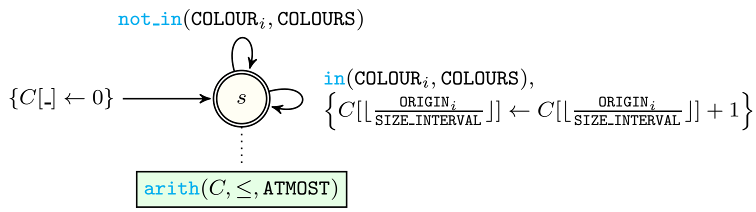


Figure 5.461: Automaton of the `interval_and_count` constraint