5.6 all_equal_peak

DESCRIPTION LINKS AUTOMATON

Origin Derived from `peak` and `all_equal`.

Constraint `all_equal_peak(VARIABLES)`

Argument `VARIABLES : collection(var−dvar)`

Restrictions
$|$`VARIABLES`$| > 0$
`required(VARIABLES, var)`

Purpose A variable V_k $(1 < k < m)$ of the sequence of variables `VARIABLES` $= V_1, \dots, V_m$ is a *peak* if and only if there exists an i $(1 < i \leq k)$ such that $V_{i-1} < V_i$ and $V_i = V_{i+1} = \dots = V_k$ and $V_k > V_{k+1}$.
Enforce all the peaks of the sequence `VARIABLES` to be assigned the same value, i.e. to be located at the same altitude.

Example $(\langle 1, 5, 5, 4, 3, 5, 2, 7\rangle)$

The `all_equal_peak` constraint holds since the two peaks, in bold, of the sequence 1 5 **5** 4 3 **5** 2 7 are located at the same altitude 5. Figure 5.7 depicts the solution associated with the example.

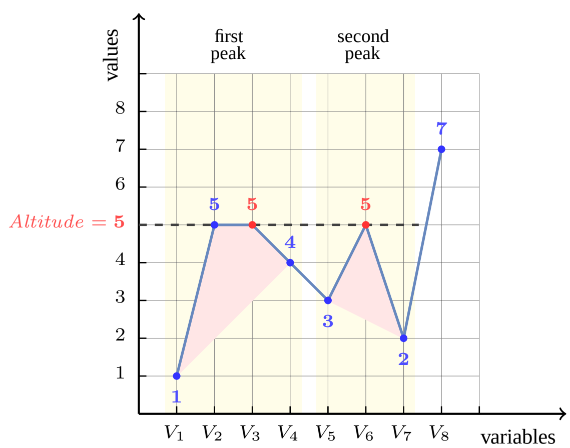


Figure 5.7: Illustration of the **Example** slot: a sequence of eight variables V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8 respectively fixed to values 1, 5, 5, 4, 3, 5, 2, 7 and its corresponding two peaks, in red, both located at altitude 5

Note that the `all_equal_peak` constraint does not enforce that the maximum value of the sequence `VARIABLES` corresponds to the altitude of its peaks since, as shown by the example, the sequence can ends up with an increasing subsequence that go beyond the altitude of its peaks. It also does not enforce that the sequence `VARIABLES` contains at least one peak.

All solutions

Figure 5.8 gives all solutions to the following non ground instance of the `all_equal_peak` constraint: $\mathtt{V}_1 \in \{0,5\}$, $\mathtt{V}_2 \in [2,3]$, $\mathtt{V}_3 = 2$, $\mathtt{V}_4 \in [3,4]$, $\mathtt{V}_5 = 1$, `all_equal_peak`$(\langle \mathtt{V}_1, \mathtt{V}_2, \mathtt{V}_3, \mathtt{V}_4, \mathtt{V}_5 \rangle)$.

① $(\langle 0, 2, 2, \mathbf{3}, 1 \rangle)$
② $(\langle 0, 2, 2, \mathbf{4}, 1 \rangle)$
③ $(\langle 0, \mathbf{3}, 2, \mathbf{3}, 1 \rangle)$
④ $(\langle 5, 2, 2, \mathbf{3}, 1 \rangle)$
⑤ $(\langle 5, 2, 2, \mathbf{4}, 1 \rangle)$
⑥ $(\langle 5, \mathbf{3}, 2, \mathbf{3}, 1 \rangle)$
⑦ $(\langle 5, 3, 2, \mathbf{4}, 1 \rangle)$

Figure 5.8: All solutions corresponding to the non ground example of the `all_equal_peak` constraint of the **All solutions** slot where each peak is coloured in orange

Typical

$|\mathtt{VARIABLES}| \geq 5$
$\mathtt{range}(\mathtt{VARIABLES.var}) > 1$
$\mathtt{peak}(\mathtt{VARIABLES.var}) \geq 2$

Symmetries

- Items of `VARIABLES` can be reversed.
- One and the same constant can be added to the `var` attribute of all items of `VARIABLES`.

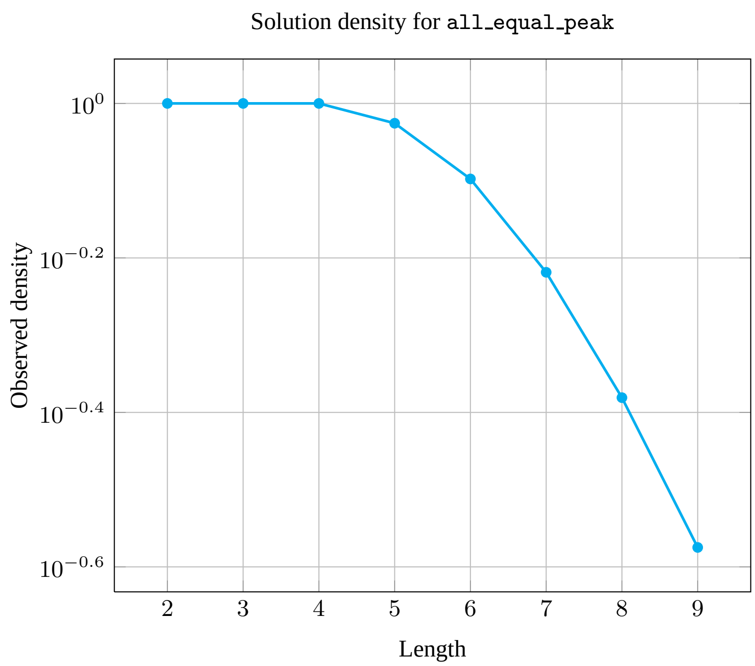
Arg. properties

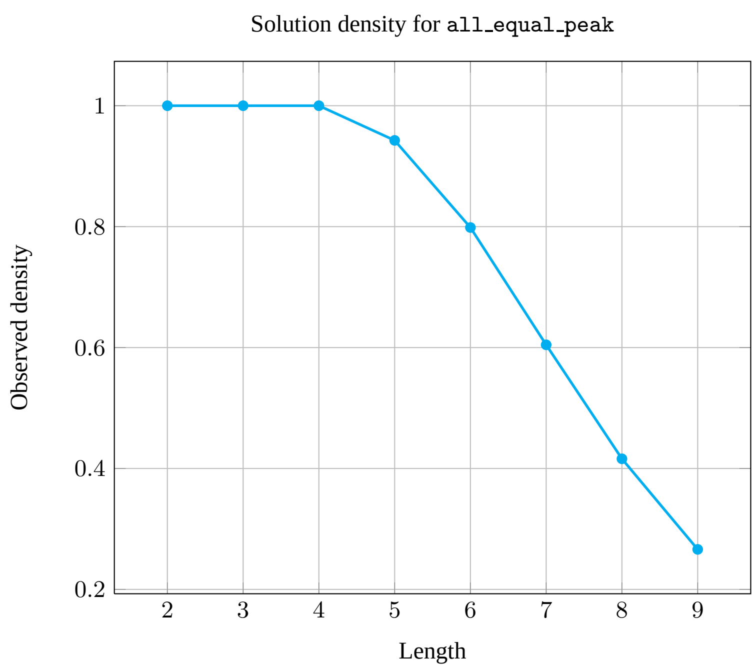
- Prefix-contractible wrt. `VARIABLES`.
- Suffix-contractible wrt. `VARIABLES`.

Counting

Length (n)	2	3	4	5	6	7	8	9
Solutions	9	64	625	7330	93947	1267790	17908059	266201992

Number of solutions for `all_equal_peak`: domains $0..n$





See also **implied by:** all_equal_peak_max.

implies: decreasing_peak, increasing_peak.

related: `all_equal_valley`, `peak`.

Keywords

characteristic of a constraint: automaton, automaton with counters, automaton with same input symbol.

combinatorial object: sequence.

constraint network structure: sliding cyclic(1) constraint network(2).

Cond. implications

- `all_equal_peak`(`VARIABLES`)
  with `peak`(`VARIABLES.var`) > 1
  implies `some_equal`(`VARIABLES`).

- `all_equal_peak`(`VARIABLES`)
  with `peak`(`VARIABLES.var`) > 0
  implies `not_all_equal`(`VARIABLES`).

Automaton

Figure 5.9 depicts the automaton associated with the `all_equal_peak` constraint. To each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $(\mathtt{VAR}_i < \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\mathtt{VAR}_i = \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 1) \wedge (\mathtt{VAR}_i > \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 2)$.

STATES SEMANTICS

s	: initial stationary or decreasing mode	$(\{= \mid >\}^*)$
i	: increasing (before first potential peak) mode	$(< \{< \mid =\}^*)$
j	: decreasing (after a peak) mode	$(> \{> \mid =\}^*)$
k	: increasing (after a peak) mode	$(< \{< \mid =\}^*)$

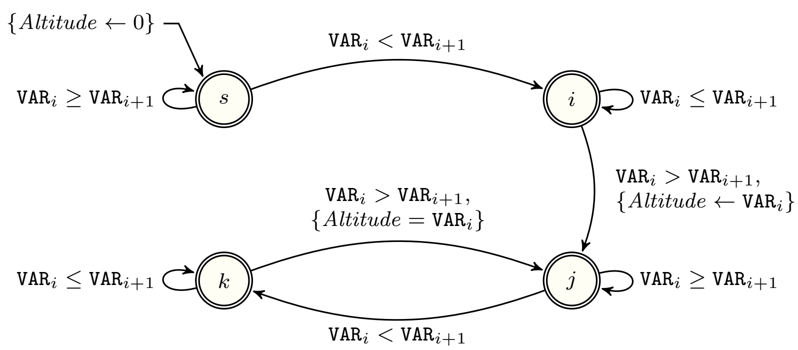


Figure 5.9: Automaton for the `all_equal_peak` constraint (note the conditional transition from state k to state j testing that the counter $Altitude$ is equal to $\mathtt{VAR}_i$ for enforcing that all peaks are located at the same altitude)

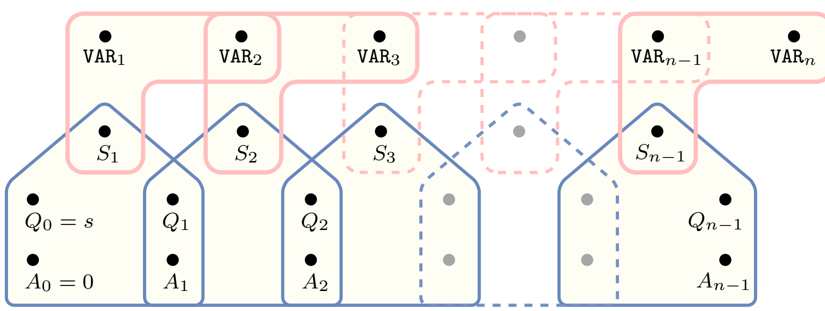


Figure 5.10: Hypergraph of the reformulation corresponding to the automaton of the `all_equal_peak` constraint where A_i stands for the value of the counter $Altitude$ (since all states of the automaton are accepting there is no restriction on the last variable Q_{n-1})