5.169 golomb

DESCRIPTION LINKS GRAPH

Origin Inspired by [196].

Constraint golomb(VARIABLES)

Argument VARIABLES : collection(var−dvar)

Restrictions
required(VARIABLES, var)
VARIABLES.var ≥ 0
strictly_increasing(VARIABLES)

Purpose Given a strictly increasing sequence $X_1, X_2, \ldots, X_n$, enforce all differences $X_i - X_j$ between two variables X_i and X_j $(i > j)$ to be distinct.

Example $(\langle 0, 1, 4, 6\rangle)$

Figure 5.381 gives a graphical interpretation of the solution given in the example in term of a graph: each vertex corresponds to a value of $\langle 0, 1, 4, 6\rangle$, while each arc depicts a difference between two values. The golomb constraint holds since one can note that these differences $1, 4, 6, 3, 5, 2$ are all-distinct.

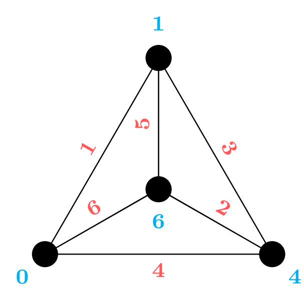


Figure 5.381: Graphical representation of the solution 0, 1, 4, 6 (differences are displayed in light red and are pairwise distinct).

Typical $|\text{VARIABLES}| > 2$

Symmetry One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties Contractible wrt. VARIABLES.

Usage This constraint refers to the Golomb ruler problem. We quote the definition from [380]: "A Golomb ruler is a set of integers (marks) $a_1 < \cdots < a_k$ such that all the differences $a_i - a_j$ $(i > j)$ are distinct".

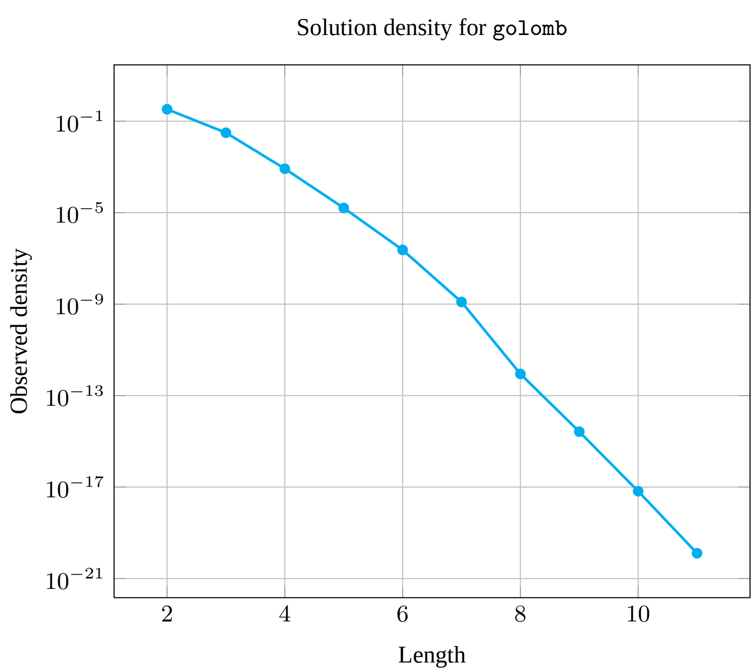
Remark Different constraints models for the Golomb ruler problem were presented in [393].

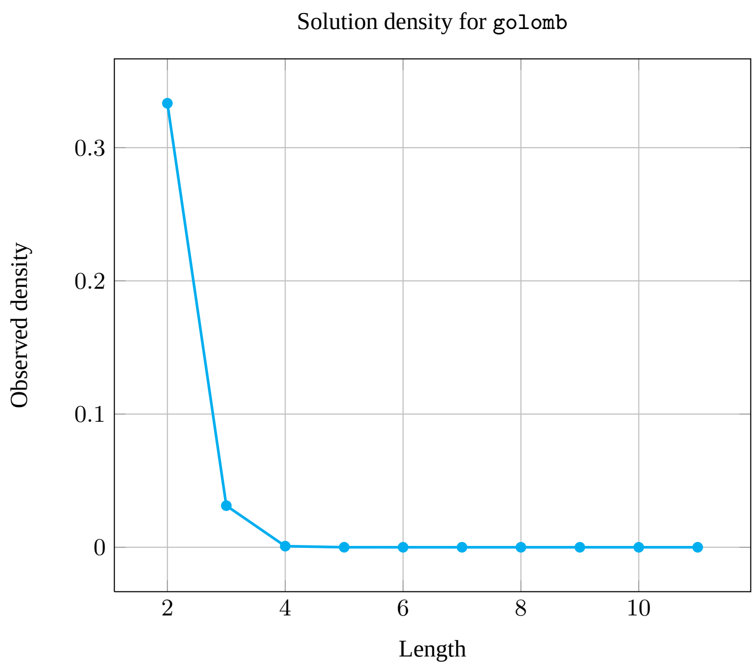
Algorithm At a first glance, one could think that, because it looks so similar to the `alldifferent` constraint, we could have a perfect polynomial filtering algorithm. However this is not true since one retrieves the *same* variable in different vertices of the graph. This leads to the fact that one has incompatible arcs in the bipartite graph (the two classes of vertices correspond to the pair of variables and to the fact that the difference between two pairs of variables takes a specific value). However one can still reuse a similar filtering algorithm as for the `alldifferent` constraint, but this will not lead to perfect pruning.

Counting

Length (n)	2	3	4	5	6	7	8	9	10	11
Solutions	3	2	2	4	8	10	2	2	2	4

Number of solutions for `golomb`: domains $0..k$





See also **common keyword:** `alldifferent` *(all different)*.

implies: `strictly_increasing`.

Keywords **characteristic of a constraint:** disequality, difference, all different, derived collection.

puzzles: Golomb ruler.

Cond. implications

- `golomb`(`VARIABLES`)
  implies `increasing_nvalue`(`NVAL`, `VARIABLES`)
  when `NVAL` =`nval`(`VARIABLES.var`).

- `golomb`(`VARIABLES`)
  implies `soft_alldifferent_ctr`(`C`, `VARIABLES`).

Derived Collection

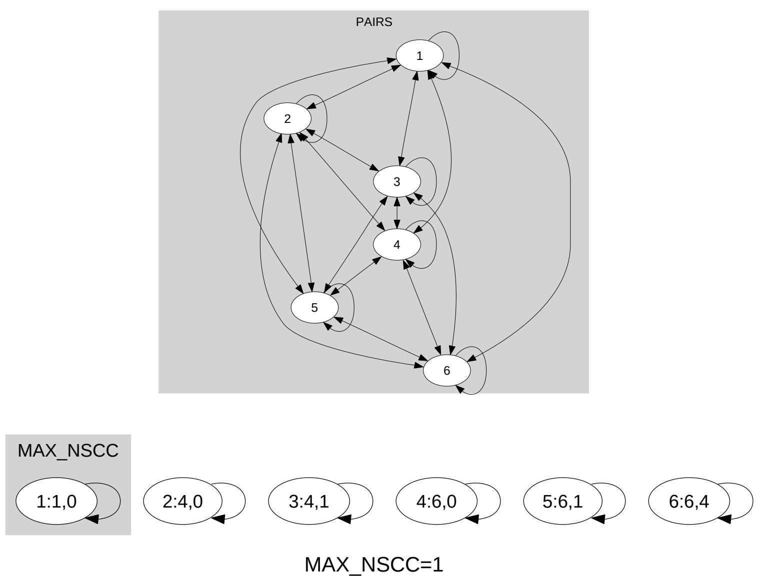
$$\texttt{col}\left(\begin{array}{l}\texttt{PAIRS}-\texttt{collection}(\texttt{x}-\texttt{dvar},\texttt{y}-\texttt{dvar}),\\ [>-\texttt{item}(\texttt{x}-\texttt{VARIABLES.var},\texttt{y}-\texttt{VARIABLES.var})]\end{array}\right)$$

Arc input(s)	PAIRS
Arc generator	$CLIQUE \mapsto$ collection(pairs1, pairs2)
Arc arity	2
Arc constraint(s)	pairs1.y − pairs1.x = pairs2.y − pairs2.x
Graph property(ies)	**MAX_NSCC** ≤ 1

Graph model

When applied on the collection of items ⟨VAR1, VAR2, VAR3, VAR4⟩, the generator of derived collection generates the following collection of items: ⟨VAR2 VAR1, VAR3 VAR1, VAR3 VAR2, VAR4 VAR1, VAR4 VAR2, VAR4 VAR3⟩. Note that we use a binary arc constraint between two vertices and that this binary constraint involves four variables.

Parts (A) and (B) of Figure 5.382 respectively show the initial and final graph associated with the **Example** slot. Since we use the **MAX_NSCC** graph property we show one of the largest strongly connected components of the final graph. The constraint holds since all the strongly connected components have at most one vertex: the differences $1, 2, 3, 4, 5, 6$ that one can construct from the values $0, 1, 4, 6$ assigned to the variables of the VARIABLES collection are all-distinct.



(A)

(B)

Figure 5.382: Initial and final graph of the golomb constraint