## 5.315 overlap_sboxes

DESCRIPTION LINKS LOGIC

**Origin** Geometry, derived from [338]

**Constraint** overlap_sboxes(K, DIMS, OBJECTS, SBOXES)

**Synonym** overlap.

**Types**

```
VARIABLES  :  collection(v−dvar)
INTEGERS   :  collection(v−int)
POSITIVES  :  collection(v−int)
```

**Arguments**

```
K        :  int
DIMS     :  sint
OBJECTS  :  collection(oid−int, sid−dvar, x − VARIABLES)
SBOXES   :  collection(sid−int, t − INTEGERS, l − POSITIVES)
```

**Restrictions**

$|\text{VARIABLES}| \geq 1$
$|\text{INTEGERS}| \geq 1$
$|\text{POSITIVES}| \geq 1$
required(VARIABLES, v)
$|\text{VARIABLES}| = \text{K}$
required(INTEGERS, v)
$|\text{INTEGERS}| = \text{K}$
required(POSITIVES, v)
$|\text{POSITIVES}| = \text{K}$
$\text{POSITIVES.v} > 0$
$\text{K} > 0$
$\text{DIMS} \geq 0$
$\text{DIMS} < \text{K}$
increasing_seq(OBJECTS, [oid])
required(OBJECTS, [oid, sid, x])
$\text{OBJECTS.oid} \geq 1$
$\text{OBJECTS.oid} \leq |\text{OBJECTS}|$
$\text{OBJECTS.sid} \geq 1$
$\text{OBJECTS.sid} \leq |\text{SBOXES}|$
$|\text{SBOXES}| \geq 1$
required(SBOXES, [sid, t, l])
$\text{SBOXES.sid} \geq 1$
$\text{SBOXES.sid} \leq |\text{SBOXES}|$
do_not_overlap(SBOXES)

**Purpose**

Holds if, for each pair of objects $(O_i, O_j)$, $i < j$, $O_i$ overlaps $O_j$ with respect to a set of dimensions depicted by `DIMS`. $O_i$ and $O_j$ are objects that take a shape among a set of shapes. Each *shape* is defined as a finite set of shifted boxes, where each shifted box is described by a box in a `K`-dimensional space at a given offset (from the origin of the shape) with given sizes. More precisely, a *shifted box* is an entity defined by its shape id `sid`, shift offset `t`, and sizes `l`. Then, a shape is defined as the union of shifted boxes sharing the same shape id. An *object* is an entity defined by its unique object identifier `oid`, shape id `sid` and origin `x`.
An object $O_i$ *overlaps* an object $O_j$ with respect to a set of dimensions depicted by `DIMS` if and only if, there exists a shifted box $s_i$ associated with $O_i$ and there exists a shifted box $s_j$ associated with $O_j$, such that (1) there exists a dimension $d \in$ `DIMS` where the end of $O_i$ in dimension $d$ is strictly greater than the start of $O_j$ in dimension $d$, and (2) the end of $O_j$ in dimension $d$ is strictly greater than the start of $O_i$ in dimension $d$.

**Example**

$$\left\langle \begin{array}{l} 2, \{0,1\}, \\ \left\langle \begin{array}{lll} \texttt{oid}-1 & \texttt{sid}-1 & \texttt{x}-\langle 1,1\rangle, \\ \texttt{oid}-2 & \texttt{sid}-2 & \texttt{x}-\langle 3,2\rangle, \\ \texttt{oid}-3 & \texttt{sid}-3 & \texttt{x}-\langle 2,4\rangle \end{array} \right\rangle, \\ \left\langle \begin{array}{lll} \texttt{sid}-1 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 4,5\rangle, \\ \texttt{sid}-2 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 3,3\rangle, \\ \texttt{sid}-3 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 2,1\rangle \end{array} \right\rangle \end{array} \right\rangle$$

Figure 5.656 shows the objects of the example. Since $O_1$ overlaps both $O_2$ and $O_3$, and since $O_2$ overlaps $O_3$, the `overlap_sboxes` constraint holds.

**Typical**

$|\texttt{OBJECTS}| > 1$

**Symmetries**

- Items of `OBJECTS` are permutable.
- Items of `SBOXES` are permutable.
- Items of `OBJECTS.x`, `SBOXES.t` and `SBOXES.l` are permutable (*same permutation used*).
- `SBOXES.l.v` can be increased.

**Arg. properties**

Suffix-contractible wrt. `OBJECTS`.

**Remark**

One of the eight relations of the *Region Connection Calculus* [338].

**See also**

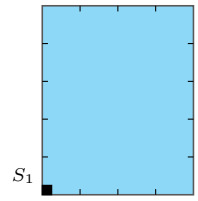
**common keyword:** `contains_sboxes`, `coveredby_sboxes`, `covers_sboxes`, `disjoint_sboxes`, `equal_sboxes`, `inside_sboxes`, `meet_sboxes` *(rcc8)*, `non_overlap_sboxes` *(geometrical constraint,logic)*.

**Keywords**

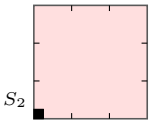
**constraint type:** logic.

**geometry:** geometrical constraint, rcc8.

**miscellaneous:** obscure.

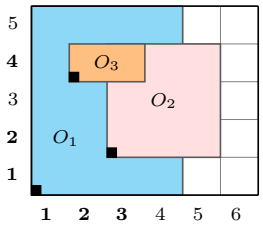



(A) Shape of the first object




(B) Shape of the second object


(C) Shape of the third object




OBJECTS

| | | | |
|---|---|---|---|
| $O_1$: | oid $-$ 1 | sid $-$ 1 | x $-$ $\langle 1, 1\rangle$ |
| $O_2$: | oid $-$ 2 | sid $-$ 2 | x $-$ $\langle 3, 2\rangle$ |
| $O_3$: | oid $-$ 3 | sid $-$ 3 | x $-$ $\langle 2, 4\rangle$ |

(D) Three objects where $O_1$ both overlaps $O_2$ and $O_3$ and where $O_2$ overlaps $O_3$

Figure 5.656: (D) the three mutually overlapping objects $O_1$, $O_2$, $O_3$ of the **Example** slot respectively assigned shapes $S_1$, $S_2$, $S_3$; (A), (B), (C) shapes $S_1$, $S_2$ and $S_3$ are made up from a single shifted box.

**Logic**

- $\texttt{origin}(\texttt{O1}, \texttt{S1}, \texttt{D}) \stackrel{\text{def}}{=} \texttt{O1.x}(\texttt{D}) + \texttt{S1.t}(\texttt{D})$
- $\texttt{end}(\texttt{O1}, \texttt{S1}, \texttt{D}) \stackrel{\text{def}}{=} \texttt{O1.x}(\texttt{D}) + \texttt{S1.t}(\texttt{D}) + \texttt{S1.l}(\texttt{D})$
- $\texttt{overlap\_sboxes}(\texttt{Dims}, \texttt{O1}, \texttt{S1}, \texttt{O2}, \texttt{S2}) \stackrel{\text{def}}{=}$
  $\forall \texttt{D} \in \texttt{Dims}$
  $$\bigwedge \left( \begin{array}{l} \texttt{end}(\texttt{O1}, \texttt{S1}, \texttt{D}) > \\ \texttt{origin}(\texttt{O2}, \texttt{S2}, \texttt{D}) \\ \texttt{end}(\texttt{O2}, \texttt{S2}, \texttt{D}) > \\ \texttt{origin}(\texttt{O1}, \texttt{S1}, \texttt{D}) \end{array} \right)'$$
- $\texttt{overlap\_objects}(\texttt{Dims}, \texttt{O1}, \texttt{O2}) \stackrel{\text{def}}{=}$
  $\forall \texttt{S1} \in \texttt{sboxes}([\texttt{O1.sid}])$
  $\exists \texttt{S2} \in \texttt{sboxes}\left( \left[ \texttt{O2.sid} \right] \right)$
  $$\texttt{overlap\_sboxes} \left( \begin{array}{l} \texttt{Dims}, \\ \texttt{O1}, \\ \texttt{S1}, \\ \texttt{O2}, \\ \texttt{S2} \end{array} \right)$$
- $\texttt{all\_overlap}(\texttt{Dims}, \texttt{OIDS}) \stackrel{\text{def}}{=}$
  $\forall \texttt{O1} \in \texttt{objects}(\texttt{OIDS})$
  $\forall \texttt{O2} \in \texttt{objects}(\texttt{OIDS})$
  $\texttt{O1.oid} < \Rightarrow$
  $\texttt{O2.oid}$
  $$\texttt{overlap\_objects} \left( \begin{array}{l} \texttt{Dims}, \\ \texttt{O1}, \\ \texttt{O2} \end{array} \right)$$
- $\texttt{all\_overlap}(\texttt{DIMENSIONS}, \texttt{OIDS})$