5.147 elements_alldifferent

DESCRIPTION LINKS GRAPH

Origin Derived from elements and alldifferent.

Constraint elements_alldifferent(ITEMS, TABLE)

Synonyms elements_alldiff, elements_alldistinct.

Arguments

ITEMS	:	collection(index−dvar, value−dvar)
TABLE	:	collection(index−int, value−dvar)

Restrictions

required(ITEMS, [index, value])
ITEMS.index ≥ 1
ITEMS.index ≤ |TABLE|
|ITEMS| = |TABLE|
required(TABLE, [index, value])
TABLE.index ≥ 1
TABLE.index ≤ |TABLE|
distinct(TABLE, index)

Purpose All the items of the ITEMS collection should be equal to one of the entries of the table TABLE and all the variables ITEMS.index should take distinct values.

Example

$$\left\langle \begin{array}{llll} \text{index}-2 & \text{value}-9, \\ \text{index}-1 & \text{value}-6, \\ \text{index}-4 & \text{value}-9, \\ \text{index}-3 & \text{value}-2 \end{array} \right\rangle, \left\langle \begin{array}{llll} \text{index}-1 & \text{value}-6, \\ \text{index}-2 & \text{value}-9, \\ \text{index}-3 & \text{value}-2, \\ \text{index}-4 & \text{value}-9 \end{array} \right\rangle$$

The elements_alldifferent constraint holds since, as depicted by Figure 5.338, there is a one to one correspondence between the items of the ITEMS collection and the items of the TABLE collection.

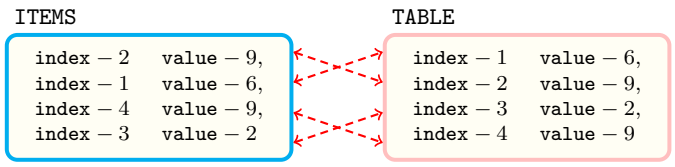

Figure 5.338: Illustration of the one to one correspondence between the items of ITEMS and the items of TABLE

Typical

$|\texttt{ITEMS}| > 1$
$\texttt{range}(\texttt{ITEMS.value}) > 1$
$|\texttt{TABLE}| > 1$
$\texttt{range}(\texttt{TABLE.value}) > 1$

Symmetries

- Arguments are permutable w.r.t. permutation (ITEMS, TABLE).
- Items of ITEMS are permutable.
- Items of TABLE are permutable.
- All occurrences of two distinct values in ITEMS.value or TABLE.value can be swapped; all occurrences of a value in ITEMS.value or TABLE.value can be renamed to any unused value.

Arg. properties

Functional dependency: ITEMS.value determined by ITEMS.index and TABLE.

Usage

Used for replacing by a single elements_alldifferent constraint an alldifferent and a set of element constraints having the following structure:

- The union of the index variables of the element constraints is equal to the set of variables of the alldifferent constraint.
- All the element constraints share exactly the same table.

For instance, the constraint given in the **Example** slot is equivalent to the conjunction of the following set of constraints:

$$\texttt{alldifferent}(\langle \texttt{var}-2, \texttt{var}-1, \texttt{var}-4, \texttt{var}-3 \rangle)$$

$$\texttt{element}\left(\begin{array}{l} \langle\ \texttt{index}-2 \quad \texttt{value}-9\ \rangle, \\ \left\langle \begin{array}{ll} \texttt{index}-1 & \texttt{value}-6, \\ \texttt{index}-2 & \texttt{value}-9, \\ \texttt{index}-3 & \texttt{value}-2, \\ \texttt{index}-4 & \texttt{value}-9 \end{array} \right\rangle \end{array}\right)$$

$$\texttt{element}\left(\begin{array}{l} \langle\ \texttt{index}-1 \quad \texttt{value}-6\ \rangle, \\ \left\langle \begin{array}{ll} \texttt{index}-1 & \texttt{value}-6, \\ \texttt{index}-2 & \texttt{value}-9, \\ \texttt{index}-3 & \texttt{value}-2, \\ \texttt{index}-4 & \texttt{value}-9 \end{array} \right\rangle \end{array}\right)$$

$$\texttt{element}\left(\begin{array}{l} \langle\ \texttt{index}-3 \quad \texttt{value}-2\ \rangle, \\ \left\langle \begin{array}{ll} \texttt{index}-1 & \texttt{value}-6, \\ \texttt{index}-2 & \texttt{value}-9, \\ \texttt{index}-3 & \texttt{value}-2, \\ \texttt{index}-4 & \texttt{value}-9 \end{array} \right\rangle \end{array}\right)$$

$$\texttt{element}\left(\begin{array}{l} \langle\ \texttt{index}-4 \quad \texttt{value}-9\ \rangle, \\ \left\langle \begin{array}{ll} \texttt{index}-1 & \texttt{value}-6, \\ \texttt{index}-2 & \texttt{value}-9, \\ \texttt{index}-3 & \texttt{value}-2, \\ \texttt{index}-4 & \texttt{value}-9 \end{array} \right\rangle \end{array}\right)$$

As a practical example of utilisation of the `elements_alldifferent` constraint we show how to model the link between a permutation consisting of a single cycle and its expanded form. For instance, to the permutation $3, 6, 5, 2, 4, 1$ corresponds the sequence 3 5 4 2 6 1. Let us note $S_1, S_2, S_3, S_4, S_5, S_6$ the permutation and $V_1V_2V_3V_4V_5V_6$ its expanded form (see Figure 5.339).

The constraint:

$$\texttt{elements\_alldifferent}\left(\left\langle \begin{array}{ll} \texttt{index} - V_1 & \texttt{value} - V_2, \\ \texttt{index} - V_2 & \texttt{value} - V_3, \\ \texttt{index} - V_3 & \texttt{value} - V_4, \\ \texttt{index} - V_4 & \texttt{value} - V_5, \\ \texttt{index} - V_5 & \texttt{value} - V_6, \\ \texttt{index} - V_6 & \texttt{value} - V_1 \end{array} \right\rangle, \left\langle \begin{array}{ll} \texttt{index} - 1 & \texttt{value} - S_1, \\ \texttt{index} - 2 & \texttt{value} - S_2, \\ \texttt{index} - 3 & \texttt{value} - S_3, \\ \texttt{index} - 4 & \texttt{value} - S_4, \\ \texttt{index} - 5 & \texttt{value} - S_5, \\ \texttt{index} - 6 & \texttt{value} - S_6 \end{array} \right\rangle\right)$$

models the fact that $S_1, S_2, S_3, S_4, S_5, S_6$ corresponds to a permutation with a single cycle. It also expresses the link between the variables $S_1, S_2, S_3, S_4, S_5, S_6$ and $V_1, V_2, V_3, V_4, V_5, V_6$.

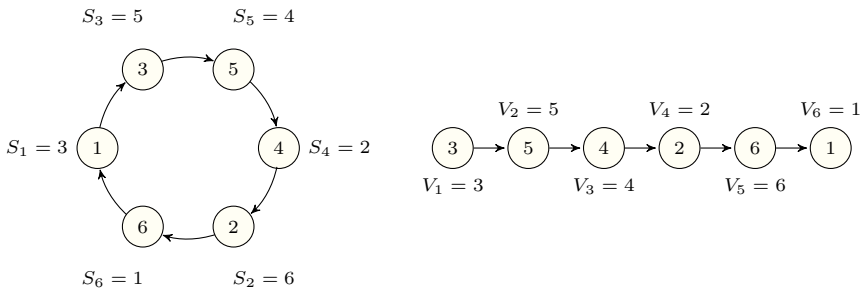

Figure 5.339: Two representations of a permutation containing a single cycle

Reformulation

The `elements_alldifferent`$(\langle \texttt{index} - I_1\ \texttt{value} - V_1, \texttt{index} - I_2\ \texttt{value} - V_2, \ldots, \texttt{index} - I_{|\texttt{ITEMS}|}\ \texttt{value} - V_{|\texttt{ITEMS}|}\rangle, \texttt{TABLE})$ constraint can be expressed in term of a conjunction of $|\texttt{ITEMS}|$ `elem` constraints and of one `alldifferent` constraint of the form:

`elem`$(\langle \texttt{index} - I_1\ \texttt{value} - V_1\rangle, \texttt{TABLE})$,
`elem`$(\langle \texttt{index} - I_2\ \texttt{value} - V_2\rangle, \texttt{TABLE})$,
...
`elem`$(\langle \texttt{index} - I_{|\texttt{ITEMS}|}\ \texttt{value} - V_{|\texttt{ITEMS}|}\rangle, \texttt{TABLE})$,
`alldifferent`$(\langle I_1, I_2, \ldots, I_{|\texttt{ITEMS}|}\rangle)$.

See also

implies: elements, indexed_sum.

used in reformulation: alldifferent, elem, element.

Keywords

characteristic of a constraint: disequality.

combinatorial object: permutation.

constraint type: data constraint.

modelling: array constraint, table, functional dependency.

Cond. implications

elements_alldifferent(ITEMS, TABLE)
with TABLE.value ≥ 0
implies bin_packing_capa(TABLE, ITEMS).

Arc input(s)	ITEMS TABLE
Arc generator	$PRODUCT \mapsto$ collection(items, table)
Arc arity	2
Arc constraint(s)	• items.index = table.index • items.value = table.value
Graph property(ies)	NVERTEX= |ITEMS| + |TABLE|

Graph model

The fact that all variables ITEMS.index are pairwise different is derived from the conjunctions of the following facts:

- From the graph property $\mathbf{NVERTEX} = |\texttt{ITEMS}| + |\texttt{TABLE}|$ it follows that all vertices of the initial graph belong also to the final graph,
- A vertex v belongs to the final graph if there is at least one constraint involving v that holds,
- From the first condition items.index = table.index of the arc constraint, and from the restriction distinct(TABLE.index) it follows: for all vertices v generated from the collection ITEMS at most one constraint involving v holds.

Parts (A) and (B) of Figure 5.340 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NVERTEX** graph property, the vertices of the final graph are stressed in bold.

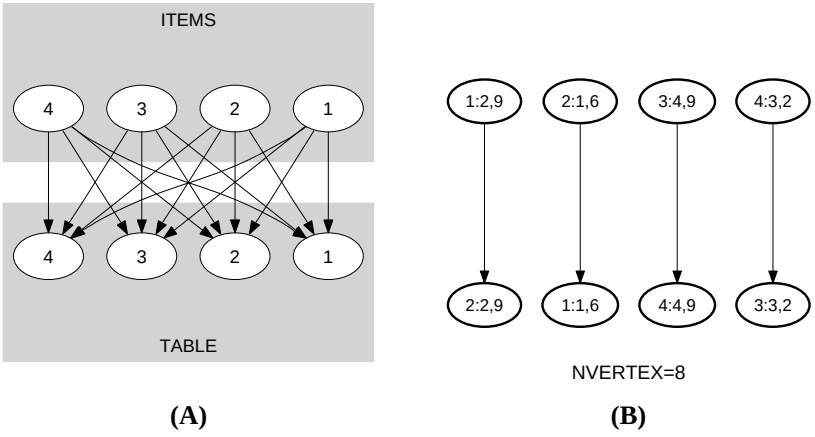

Figure 5.340: Initial and final graph of the elements_alldifferent constraint

Signature

Since the final graph cannot have more than |ITEMS| + |TABLE| vertices one can simplify NVERTEX to NVERTEX.