

## 5.92 counts

|                        | DESCRIPTION   | LINKS | GRAPH | AUTOMATON |
|------------------------|---|-------|-------|-----------|
| <b>Origin</b>          | Derived from <code>count</code> .   |       |       |           |
| <b>Constraint</b>      | <code>counts(VARIABLES, VARIABLES, RELOP, LIMIT)</code>   |       |       |           |
| <b>Arguments</b>       | VALUES : <code>collection(val-int)</code><br>VARIABLES : <code>collection(var-dvar)</code><br>RELOP : <code>atom</code><br>LIMIT : <code>dvar</code>  |       |       |           |
| <b>Restrictions</b>    | <code>required(VARIABLES, val)</code><br><code>distinct(VARIABLES, val)</code><br><code>required(VARIABLES, var)</code><br>RELOP $\in [=, \neq, <, \geq, >, \leq]$  |       |       |           |
| <b>Purpose</b>         | Let $N$ be the number of variables of the <code>VARIABLES</code> collection assigned to a value of the <code>VALUES</code> collection. Enforce condition $N$ RELOP <code>LIMIT</code> to hold.  |       |       |           |
| <b>Example</b>         | $(\langle 1, 3, 4, 9 \rangle, \langle 4, 5, 5, 4, 1, 5 \rangle, =, 3)$ <p>Values 1, 3, 4 and 9 of the <code>VALUES</code> collection are assigned to 3 items of the <code>VARIABLES = <math>\langle 4, 5, 5, 4, 1, 5 \rangle</math></code> collection. The <code>counts</code> constraint holds since this number is in fact equal (RELOP is set to <code>=</code>) to the last argument of the <code>counts</code> constraint.</p>                             |       |       |           |
| <b>Typical</b>         | $ \text{VALUES}  > 1$<br>$ \text{VARIABLES}  > 1$<br><code>range(VARIABLES.var) &gt; 1</code><br>$ \text{VARIABLES}  >  \text{VALUES} $<br>RELOP $\in [=, <, \geq, >, \leq]$<br>LIMIT $> 0$<br>LIMIT $<  \text{VARIABLES} $   |       |       |           |
| <b>Symmetries</b>      | <ul style="list-style-type: none"> <li>Items of <code>VALUES</code> are <code>permutable</code>.</li> <li>Items of <code>VARIABLES</code> are <code>permutable</code>.</li> <li>An occurrence of a value of <code>VARIABLES.var</code> that belongs to <code>VALUES.val</code> (resp. does not belong to <code>VALUES.val</code>) can be <code>replaced</code> by any other value in <code>VALUES.val</code> (resp. not in <code>VALUES.val</code>).</li> </ul> |       |       |           |
| <b>Arg. properties</b> | <ul style="list-style-type: none"> <li><code>Contractible</code> wrt. <code>VARIABLES</code> when RELOP <math>\in [&lt;, \leq]</math>.</li> <li><code>Extensible</code> wrt. <code>VARIABLES</code> when RELOP <math>\in [\geq, &gt;]</math>.</li> <li><code>Aggregate</code>: <code>VALUES(sunion)</code>, <code>VARIABLES(union)</code>, <code>RELOP(id)</code>, <code>LIMIT(+)</code> when RELOP <math>\in [&lt;, \leq, \geq, &gt;]</math>.</li> </ul>       |       |       |           |

|                      |  |
|----------------------|--|
| <b>Usage</b>         | Used in the <b>Constraint(s) on sets</b> slot for defining some constraints like <a href="#">assign_and_counts</a> .   |
| <b>Reformulation</b> | The <code>count(VALUE, VARIABLES, RELOP, LIMIT)</code> constraint can be expressed in term of the conjunction <a href="#">among</a> ( $N$ , VARIABLES, VALUES) $\wedge$ $N$ RELOP LIMIT.   |
| <b>Systems</b>       | <a href="#">count</a> in <b>Gecode</b> .   |
| <b>Used in</b>       | <a href="#">assign_and_counts</a> .  |
| <b>See also</b>      | <b>assignment dimension added:</b> <a href="#">assign_and_counts</a> ( <i>assignment dimension introduced</i> ).<br><b>common keyword:</b> <a href="#">among</a> ( <i>value constraint, counting constraint</i> ).<br><b>specialisation:</b> <a href="#">count</a> (variable $\in$ VALUES <i>replaced by</i> variable=VALUE).  |
| <b>Keywords</b>      | <b>characteristic of a constraint:</b> <a href="#">automaton</a> , <a href="#">automaton with counters</a> .<br><b>constraint network structure:</b> <a href="#">alpha-acyclic constraint network(2)</a> .<br><b>constraint type:</b> <a href="#">value constraint</a> , <a href="#">counting constraint</a> .<br><b>filtering:</b> <a href="#">arc-consistency</a> .<br><b>final graph structure:</b> <a href="#">acyclic</a> , <a href="#">bipartite</a> , <a href="#">no loop</a> . |

|                     |   |
|---------------------|---|
| Arc input(s)        | VARIABLES VALUES  |
| Arc generator       | <i>PRODUCT</i> $\mapsto$ collection(variables, values)  |
| Arc arity           | 2   |
| Arc constraint(s)   | variables.var = values.val  |
| Graph property(ies) | <b>NARC</b> RELOP LIMIT   |
| Graph class         | <ul style="list-style-type: none"><li>• ACYCLIC</li><li>• BIPARTITE</li><li>• NO_LOOP</li></ul> |

**Graph model**

Because of the arc constraint `variables.var = values.val` and since each domain variable can take at most one value, **NARC** is the number of variables taking a value in the `VALUES` collection.

Parts (A) and (B) of Figure 5.223 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold.

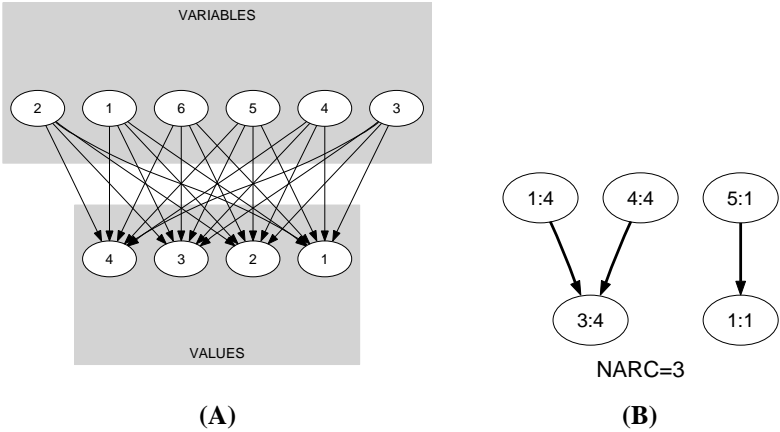


Figure 5.223: Initial and final graph of the counts constraint

**Automaton**

Figure 5.224 depicts the automaton associated with the `counts` constraint. To each variable  $\text{VAR}_i$  of the collection `VARIABLES` corresponds a 0-1 signature variable  $S_i$ . The following signature constraint links  $\text{VAR}_i$  and  $S_i$ :  $\text{VAR}_i \in \text{VALUES} \Leftrightarrow S_i$ .

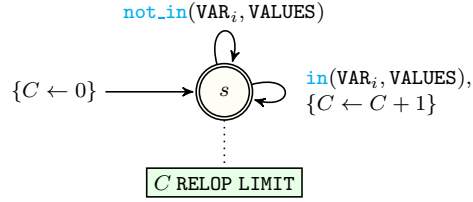


Figure 5.224: Automaton of the `counts` constraint

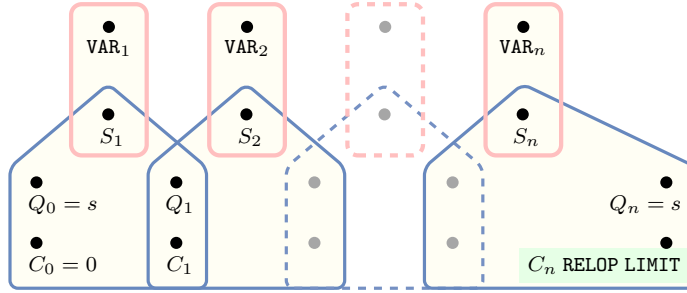


Figure 5.225: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the `counts` constraint: since all states variables  $Q_0, Q_1, \dots, Q_n$  are fixed to the unique state  $s$  of the automaton, the transitions constraints share only the counter variable  $C$  and the constraint network is Berge-acyclic