5.170 graph_crossing

DESCRIPTION LINKS GRAPH

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Constraint graph_crossing(NCROSS, NODES)

Synonyms crossing, ncross.

Arguments

NCROSS	:	dvar
NODES	:	collection(succ−dvar, x−int, y−int)

Restrictions

$\texttt{NCROSS} \geq 0$
required(NODES, [succ, x, y])
$\texttt{NODES.succ} \geq 1$
$\texttt{NODES.succ} \leq |\texttt{NODES}|$

Purpose NCROSS is the number of proper intersections between line segments, where each line segment is an arc of the directed graph defined by the arc linking a node and its unique successor.

Example

$$\left\langle 2, \left\langle \begin{array}{lll} \texttt{succ}-1 & \texttt{x}-4 & \texttt{y}-7, \\ \texttt{succ}-1 & \texttt{x}-2 & \texttt{y}-5, \\ \texttt{succ}-1 & \texttt{x}-7 & \texttt{y}-6, \\ \texttt{succ}-2 & \texttt{x}-1 & \texttt{y}-2, \\ \texttt{succ}-3 & \texttt{x}-2 & \texttt{y}-2, \\ \texttt{succ}-2 & \texttt{x}-5 & \texttt{y}-3, \\ \texttt{succ}-3 & \texttt{x}-8 & \texttt{y}-2, \\ \texttt{succ}-9 & \texttt{x}-6 & \texttt{y}-2, \\ \texttt{succ}-10 & \texttt{x}-10 & \texttt{y}-6, \\ \texttt{succ}-8 & \texttt{x}-10 & \texttt{y}-1 \end{array} \right\rangle \right\rangle$$

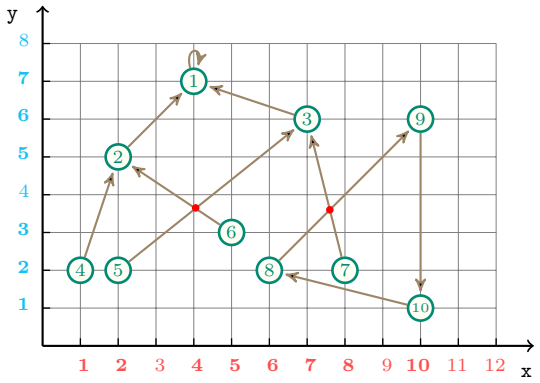
Figure 5.383 shows the line segments associated with the NODES collection. One can note the following line segments intersection:

- Arcs $8 \rightarrow 9$ and $7 \rightarrow 3$ cross,
- Arcs $5 \rightarrow 3$ and $6 \rightarrow 2$ cross also.

Consequently, the graph_crossing constraint holds since its first argument NCROSS is set to 2.

Typical

$|\texttt{NODES}| > 1$
$\texttt{range}(\texttt{NODES.succ}) > 1$
$\texttt{range}(\texttt{NODES.x}) > 1$
$\texttt{range}(\texttt{NODES.y}) > 1$



NODES

1:	succ − 1	x − 4	y − 7
2:	succ − 1	x − 2	y − 5
3:	succ − 1	x − 7	y − 6
4:	succ − 2	x − 1	y − 2
5:	succ − 3	x − 2	y − 2
6:	succ − 2	x − 5	y − 3
7:	succ − 3	x − 8	y − 2
8:	succ − 9	x − 6	y − 2
9:	succ − 10	x − 10	y − 6
10:	succ − 8	x − 10	y − 1

Figure 5.383: Illustration of the **Example** slot: a graph covering with 2 line segments intersections in red (NCROSS = 2)

Symmetries

- Attributes of NODES are permutable w.r.t. permutation (succ) (x, y) (*permutation applied to all items*).
- One and the same constant can be added to the x attribute of all items of NODES.
- One and the same constant can be added to the y attribute of all items of NODES.

Arg. properties

Functional dependency: NCROSS determined by NODES.

Usage

This is a general crossing constraint that can be used in conjunction with one graph covering constraint such as cycle, tree or map. In many practical problems ones want not only to cover a graph with specific patterns but also to avoid too much crossing between the arcs of the final graph.

Remark

We did not give a specific crossing constraint for each graph covering constraint. We feel that it is better to start first with a more general constraint before going in the specificity of the pattern that is used for covering the graph.

See also

common keyword: crossing *(line segments intersection)*, cycle, map, tree *(graph constraint,graph partitioning constraint)*, two_layer_edge_crossing *(line segments intersection)*.

Keywords

constraint arguments: pure functional dependency.

constraint type: graph constraint, graph partitioning constraint.

geometry: geometrical constraint, line segments intersection.

modelling: functional dependency.

Arc input(s) NODES

Arc generator $CLIQUE(<) \mapsto \texttt{collection}(\texttt{n1}, \texttt{n2})$

Arc arity 2

Arc constraint(s)

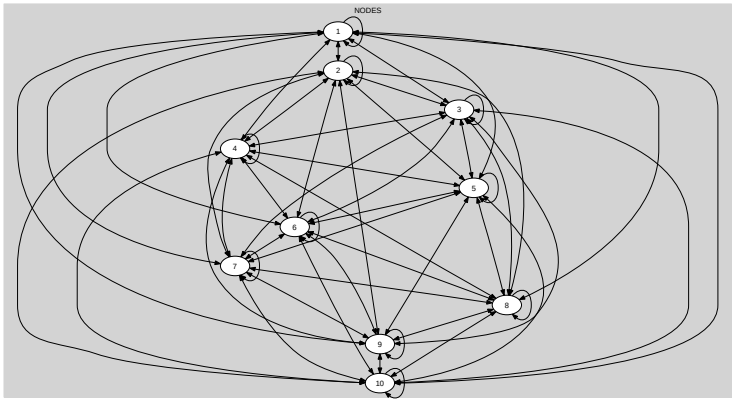
- $\max(\texttt{n1.x}, \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) \geq \min(\texttt{n2.x}, \texttt{NODES}[\texttt{n2.succ}].\texttt{x})$
- $\max(\texttt{n2.x}, \texttt{NODES}[\texttt{n2.succ}].\texttt{x}) \geq \min(\texttt{n1.x}, \texttt{NODES}[\texttt{n1.succ}].\texttt{x})$
- $\max(\texttt{n1.y}, \texttt{NODES}[\texttt{n1.succ}].\texttt{y}) \geq \min(\texttt{n2.y}, \texttt{NODES}[\texttt{n2.succ}].\texttt{y})$
- $\max(\texttt{n2.y}, \texttt{NODES}[\texttt{n2.succ}].\texttt{y}) \geq \min(\texttt{n1.y}, \texttt{NODES}[\texttt{n1.succ}].\texttt{y})$
- $(\texttt{n2.x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) * (\texttt{NODES}[\texttt{n1.succ}].\texttt{y} - \texttt{n1.y}) - (\texttt{NODES}[\texttt{n1.succ}].\texttt{x} - \texttt{n1.x}) * (\texttt{n2.y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y}) \neq 0$
- $(\texttt{NODES}[\texttt{n2.succ}].\texttt{x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) * (\texttt{n2.y} - \texttt{n1.y}) - (\texttt{n2.x} - \texttt{n1.x}) * (\texttt{NODES}[\texttt{n2.succ}].\texttt{y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y}) \neq 0$
- $\texttt{sign}\left(\prod\left(\begin{array}{l}\texttt{n2.x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x},\\ \texttt{NODES}[\texttt{n1.succ}].\texttt{y} - \texttt{n1.y}\end{array}\right) - \prod\left(\begin{array}{l}\texttt{NODES}[\texttt{n1.succ}].\texttt{x} - \texttt{n1.x},\\ \texttt{n2.y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y}\end{array}\right)\right) \neq \texttt{sign}\left(\prod\left(\begin{array}{l}\texttt{NODES}[\texttt{n2.succ}].\texttt{x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x},\\ \texttt{n2.y} - \texttt{n1.y}\end{array}\right) - \prod\left(\begin{array}{l}\texttt{n2.x} - \texttt{n1.x},\\ \texttt{NODES}[\texttt{n2.succ}].\texttt{y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y}\end{array}\right)\right)$

Graph property(ies) **NARC**$=$ NCROSS

Graph model Each node is described by its coordinates x and y, and by its successor succ in the final covering. Note that the co-ordinates are initially fixed. We use the arc generator $CLIQUE(<)$ in order to avoid counting twice the same line segment crossing.

Parts (A) and (B) of Figure 5.384 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. Each arc of the final graph corresponds to a proper intersection between two line segments.

(A)



(B)

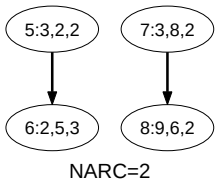


Figure 5.384: Initial and final graph of the `graph_crossing` constraint