5.281 non_overlap_sboxes

DESCRIPTION LINKS LOGIC

Origin Geometry, derived from [38]

Constraint non_overlap_sboxes(K, DIMS, OBJECTS, SBOXES)

Synonyms non_overlap, non_overlapping.

Types

VARIABLES : collection(v−dvar)
INTEGERS : collection(v−int)
POSITIVES : collection(v−int)

Arguments

K : int
DIMS : sint
OBJECTS : collection(oid−int, sid−dvar, x − VARIABLES)
SBOXES : collection(sid−int, t − INTEGERS, l − POSITIVES)

Restrictions

$|\mathtt{VARIABLES}| \geq 1$
$|\mathtt{INTEGERS}| \geq 1$
$|\mathtt{POSITIVES}| \geq 1$
required(VARIABLES, v)
$|\mathtt{VARIABLES}| = \mathtt{K}$
required(INTEGERS, v)
$|\mathtt{INTEGERS}| = \mathtt{K}$
required(POSITIVES, v)
$|\mathtt{POSITIVES}| = \mathtt{K}$
$\mathtt{POSITIVES.v} > 0$
$\mathtt{K} > 0$
$\mathtt{DIMS} \geq 0$
$\mathtt{DIMS} < \mathtt{K}$
increasing_seq(OBJECTS, [oid])
required(OBJECTS, [oid, sid, x])
$\mathtt{OBJECTS.oid} \geq 1$
$\mathtt{OBJECTS.oid} \leq |\mathtt{OBJECTS}|$
$\mathtt{OBJECTS.sid} \geq 1$
$\mathtt{OBJECTS.sid} \leq |\mathtt{SBOXES}|$
required(SBOXES, [sid, t, l])
$\mathtt{SBOXES.sid} \geq 1$
$\mathtt{SBOXES.sid} \leq |\mathtt{SBOXES}|$

Purpose

Holds if, for each pair of objects (O_i, O_j), $i < j$, O_i and O_j do not overlap with respect to a set of dimensions depicted by `DIMS`. O_i and O_j are objects that take a shape among a set of shapes. Each *shape* is defined as a finite set of shifted boxes, where each shifted box is described by a box in a `K`-dimensional space at a given offset (from the origin of the shape) with given sizes. More precisely, a *shifted box* is an entity defined by its shape id `sid`, shift offset `t`, and sizes `l`. Then, a shape is defined as the union of shifted boxes sharing the same shape id. An *object* is an entity defined by its unique object identifier `oid`, shape id `sid` and origin `x`.
An object O_i *does not overlap* an object O_j with respect to a set of dimensions depicted by `DIMS` if and only if, for all shifted box s_i associated with O_i and for all shifted box s_j associated with O_j, there exists a dimension $d \in$ `DIMS` such that the start of s_i in dimension d is greater than or equal to the end of s_j in dimension d, or the start of s_j in dimension d is greater than or equal to the end of s_i in dimension d.

Example

$$\left(\begin{array}{l}
2, \{0,1\}, \\
\left\langle\begin{array}{lll}
\texttt{oid}-1 & \texttt{sid}-1 & \texttt{x}-\langle 4,1\rangle, \\
\texttt{oid}-2 & \texttt{sid}-3 & \texttt{x}-\langle 2,2\rangle, \\
\texttt{oid}-3 & \texttt{sid}-4 & \texttt{x}-\langle 5,4\rangle
\end{array}\right\rangle, \\
\left\langle\begin{array}{lll}
\texttt{sid}-1 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 1,1\rangle, \\
\texttt{sid}-1 & \texttt{t}-\langle 1,0\rangle & \texttt{l}-\langle 1,3\rangle, \\
\texttt{sid}-1 & \texttt{t}-\langle 0,2\rangle & \texttt{l}-\langle 1,1\rangle, \\
\texttt{sid}-2 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 3,1\rangle, \\
\texttt{sid}-2 & \texttt{t}-\langle 0,1\rangle & \texttt{l}-\langle 1,1\rangle, \\
\texttt{sid}-2 & \texttt{t}-\langle 2,1\rangle & \texttt{l}-\langle 1,1\rangle, \\
\texttt{sid}-3 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 1,2\rangle, \\
\texttt{sid}-4 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 1,1\rangle
\end{array}\right\rangle
\end{array}\right)$$

Figure 5.605 shows the objects of the example. Since O_1 and O_2 do not overlap, since O_1 and O_3 do not overlap, and since O_2 and O_3 also do not overlap, the `non_overlap_sboxes` constraint holds.

Typical

$|\texttt{OBJECTS}| > 1$

Symmetries

- Items of `OBJECTS` are permutable.
- Items of `SBOXES` are permutable.
- Items of `OBJECTS.x`, `SBOXES.t` and `SBOXES.l` are permutable (*same permutation used*).
- `SBOXES.l.v` can be decreased to any value ≥ 1.

Arg. properties

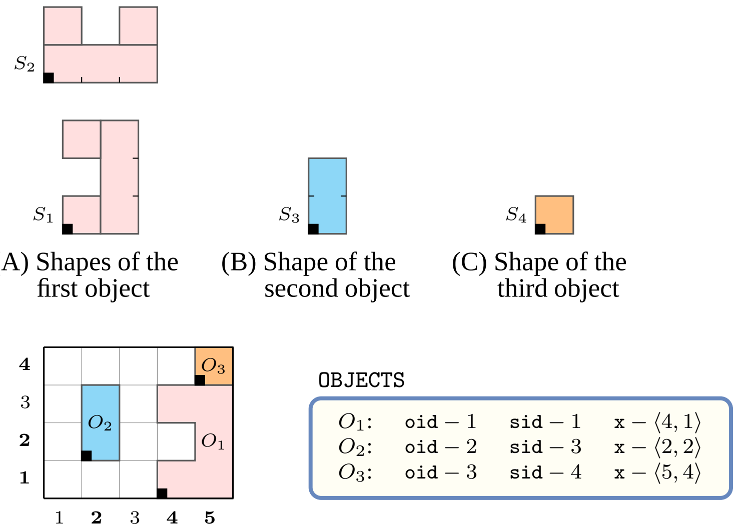
Suffix-contractible wrt. `OBJECTS`.

Remark

In addition from preventing objects to overlap, the `disjoint_sboxes` constraint also enforces that borders and corners of objects are not directly in contact.

See also

common keyword: `contains_sboxes`, `coveredby_sboxes`, `covers_sboxes` (*geometrical constraint between shifted boxes*), `diffn` (*geometrical constraint,non-overlapping*), `disjoint_sboxes`,


S_2
S_1
S_3
S_4
4
3
2
1
1
2
3
4
5
O_3
O_2
O_1
OBJECTS
O_1: oid − 1 sid − 1 x − ⟨4, 1⟩
O_2: oid − 2 sid − 3 x − ⟨2, 2⟩
O_3: oid − 3 sid − 4 x − ⟨5, 4⟩

(A) Shapes of the first object

(B) Shape of the second object

(C) Shape of the third object

(D) Three objects where O_1 does neither overlap O_2 nor O_3 and where O_2 and O_3 also do not overlap

Figure 5.605: (D) the three pairwise non-overlapping objects O_1, O_2, O_3 of the **Example** slot respectively assigned shapes S_1, S_3, S_4; (A), (B), (C) shapes S_1, S_2, S_3 and S_4 are respectively made up from 3, 3, 1 and 1 disjoint shifted box.

`equal_sboxes` *(geometrical constraint between shifted boxes)*, `geost`, `geost_time` *(geometrical constraint,non-overlapping)*, `inside_sboxes`, `meet_sboxes`, `overlap_sboxes` *(geometrical constraint between shifted boxes)*, `visible` *(geometrical constraint)*.

implied by: `disjoint_sboxes`.

Keywords

constraint type: logic.

geometry: geometrical constraint, non-overlapping.

Logic

- $\mathtt{origin}(\mathtt{O1},\mathtt{S1},\mathtt{D}) \overset{\text{def}}{=} \mathtt{O1.x}(\mathtt{D}) + \mathtt{S1.t}(\mathtt{D})$
- $\mathtt{end}(\mathtt{O1},\mathtt{S1},\mathtt{D}) \overset{\text{def}}{=} \mathtt{O1.x}(\mathtt{D}) + \mathtt{S1.t}(\mathtt{D}) + \mathtt{S1.l}(\mathtt{D})$
- $$\begin{array}{l}
\mathtt{non\_overlap\_sboxes}(\mathtt{Dims},\mathtt{O1},\mathtt{S1},\mathtt{O2},\mathtt{S2}) \overset{\text{def}}{=} \\
\quad \exists \mathtt{D} \in \mathtt{Dims} \\
\quad \bigvee \left(\begin{array}{l} \quad \mathtt{end}(\mathtt{O1},\mathtt{S1},\mathtt{D}) \leq \\ \quad \mathtt{origin}\left(\begin{array}{l}\mathtt{O2},\\ \mathtt{S2},\\ \mathtt{D}\end{array}\right) , \\ \quad \mathtt{end}(\mathtt{O2},\mathtt{S2},\mathtt{D}) \leq \\ \quad \mathtt{origin}\left(\begin{array}{l}\mathtt{O1},\\ \mathtt{S1},\\ \mathtt{D}\end{array}\right) \end{array} \right)
\end{array}$$
- $$\begin{array}{l}
\mathtt{non\_overlap\_objects}(\mathtt{Dims},\mathtt{O1},\mathtt{O2}) \overset{\text{def}}{=} \\
\quad \forall \mathtt{S1} \in \mathtt{sboxes}([\mathtt{O1.sid}]) \\
\quad\quad \forall \mathtt{S2} \in \mathtt{sboxes}\left(\left[\ \mathtt{O2.sid} \ \right] \right) \\
\quad\quad \mathtt{non\_overlap\_sboxes}\left(\begin{array}{l}\mathtt{Dims},\\ \mathtt{O1},\\ \mathtt{S1},\\ \mathtt{O2},\\ \mathtt{S2}\end{array}\right)
\end{array}$$
- $$\begin{array}{l}
\mathtt{all\_non\_overlap}(\mathtt{Dims},\mathtt{OIDS}) \overset{\text{def}}{=} \\
\quad \forall \mathtt{O1} \in \mathtt{objects}(\mathtt{OIDS}) \\
\quad\quad \forall \mathtt{O2} \in \mathtt{objects}\left(\ \mathtt{OIDS} \ \right) \\
\quad\quad\quad \begin{array}{l}\mathtt{O1.oid} < \\ \mathtt{O2.oid}\end{array} \Rightarrow \\
\quad\quad\quad \mathtt{non\_overlap\_objects}\left(\begin{array}{l}\mathtt{Dims},\\ \mathtt{O1},\\ \mathtt{O2}\end{array}\right)
\end{array}$$
- $\mathtt{all\_non\_overlap}(\mathtt{DIMENSIONS},\mathtt{OIDS})$