5.105 cycle_or_accessibility

DESCRIPTION LINKS GRAPH

Origin

Inspired by [244].

Constraint

`cycle_or_accessibility(MAXDIST, NCYCLE, NODES)`

Arguments

MAXDIST	:	int
NCYCLE	:	dvar
NODES	:	collection(index−int, succ−dvar, x−int, y−int)

Restrictions

$\mathtt{MAXDIST} \geq 0$
$\mathtt{NCYCLE} \geq 1$
$\mathtt{NCYCLE} \leq |\mathtt{NODES}|$
$\mathtt{required}(\mathtt{NODES}, [\mathtt{index}, \mathtt{succ}, \mathtt{x}, \mathtt{y}])$
$\mathtt{NODES.index} \geq 1$
$\mathtt{NODES.index} \leq |\mathtt{NODES}|$
$\mathtt{distinct}(\mathtt{NODES}, \mathtt{index})$
$\mathtt{NODES.succ} \geq 0$
$\mathtt{NODES.succ} \leq |\mathtt{NODES}|$
$\mathtt{NODES.x} \geq 0$
$\mathtt{NODES.y} \geq 0$

Purpose

Consider a digraph G described by the NODES collection. Cover a subset of the vertices of G by a set of vertex-disjoint circuits in such a way that the following property holds: for each uncovered vertex v_1 of G there exists at least one covered vertex v_2 of G such that the Manhattan distance between v_1 and v_2 is less than or equal to MAXDIST.

Example

$$\left(\begin{array}{l} 3, 2, \left\langle\begin{array}{llll} \mathtt{index}-1 & \mathtt{succ}-6 & \mathtt{x}-4 & \mathtt{y}-5, \\ \mathtt{index}-2 & \mathtt{succ}-0 & \mathtt{x}-9 & \mathtt{y}-1, \\ \mathtt{index}-3 & \mathtt{succ}-0 & \mathtt{x}-2 & \mathtt{y}-4, \\ \mathtt{index}-4 & \mathtt{succ}-1 & \mathtt{x}-2 & \mathtt{y}-6, \\ \mathtt{index}-5 & \mathtt{succ}-5 & \mathtt{x}-7 & \mathtt{y}-2, \\ \mathtt{index}-6 & \mathtt{succ}-4 & \mathtt{x}-4 & \mathtt{y}-7, \\ \mathtt{index}-7 & \mathtt{succ}-0 & \mathtt{x}-6 & \mathtt{y}-4 \end{array}\right\rangle \end{array}\right)$$

Figure 5.249 represents the solution associated with the example. The covered vertices are coloured in blue, while the links starting from the uncovered vertices are dashed. The `cycle_or_accessibility` constraint holds since:

- In the solution we have $\mathtt{NCYCLE} = 2$ disjoint circuits.
- All the 3 uncovered nodes are located at a distance that does not exceed $\mathtt{MAXDIST} = 3$ from at least one covered node.

Typical

$\mathtt{MAXDIST} > 0$
$\mathtt{NCYCLE} < |\mathtt{NODES}|$
$|\mathtt{NODES}| > 2$

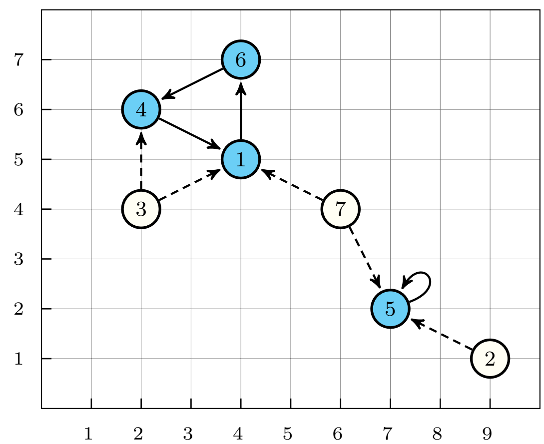


Figure 5.249: Final graph associated with the facilities location problem

Symmetries

- Items of NODES are permutable.
- Attributes of NODES are permutable w.r.t. permutation (index) (succ) (x, y) (*permutation applied to all items*).
- One and the same constant can be added to the x attribute of all items of NODES.
- One and the same constant can be added to the y attribute of all items of NODES.

Arg. properties

Functional dependency: NCYCLE determined by NODES.

Remark

This kind of facilities location problem is described in [244, pages 187–189] pages. In addition to our example they also mention the cost problem that is usually a trade-off between the vertices that are directly covered by circuits and the others.

See also

common keyword: cycle (*graph constraint*).

used in graph description: nvalues_except_0.

Keywords

constraint type: graph constraint.

final graph structure: strongly connected component.

geometry: geometrical constraint.

modelling: functional dependency.

problems: facilities location problem.

Arc input(s)	NODES
Arc generator	$CLIQUE \mapsto$ `collection`(nodes1, nodes2)
Arc arity	2
Arc constraint(s)	nodes1.succ = nodes2.index
Graph property(ies)	• **NTREE** $= 0$ • **NCC** $=$ NCYCLE

Arc input(s)	NODES
Arc generator	$CLIQUE \mapsto$ `collection`(nodes1, nodes2)
Arc arity	2
Arc constraint(s)	$\bigvee\left(\begin{array}{l}\texttt{nodes1.succ} = \texttt{nodes2.index},\\ \bigwedge\left(\begin{array}{l}\texttt{nodes1.succ} = 0,\\ \texttt{nodes2.succ} \neq 0,\\ \texttt{abs}(\texttt{nodes1.x} - \texttt{nodes2.x}) + \texttt{abs}(\texttt{nodes1.y} - \texttt{nodes2.y}) \leq \texttt{MAXDIST}\end{array}\right)\end{array}\right)$
Graph property(ies)	**NVERTEX** $= \lvert\texttt{NODES}\rvert$
Sets	$\text{PRED} \mapsto \left[\begin{array}{l}\texttt{variables} - \texttt{col}\left(\begin{array}{l}\texttt{VARIABLES}-\texttt{collection}(\texttt{var}-\texttt{dvar}),\\ [\texttt{item}(\texttt{var} - \texttt{NODES.succ})]\end{array}\right),\\ \texttt{destination}\end{array}\right]$
Constraint(s) on sets	nvalues_except_0(variables, $=$, 1)

Graph model For each vertex v we have introduced the following attributes:

- `index`: the label associated with v,
- `succ`: if v is not covered by a circuit then 0; If v is covered by a circuit then index of the successor of v.
- `x`: the `x`-coordinate of v,
- `y`: the `y`-coordinate of v.

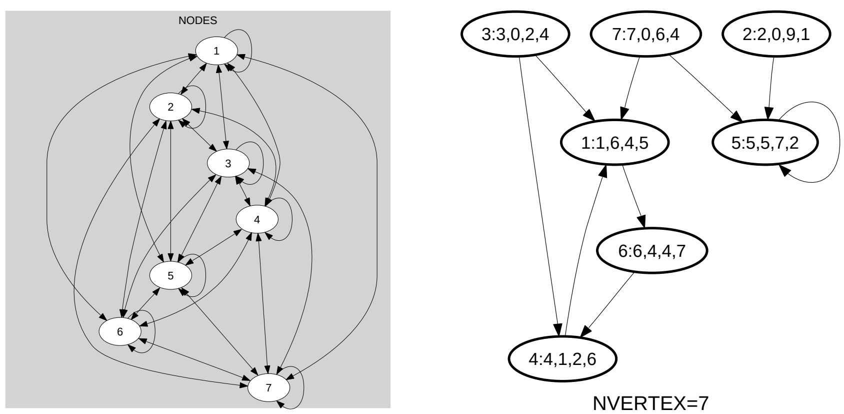
The first graph constraint forces all vertices, which have a non-zero successor, to form a set of `NCYCLE` vertex-disjoint circuits.

The final graph associated with the second graph constraint contains two types of arcs:

- The arcs belonging to one circuit (i.e., `nodes1.succ` = `nodes2.index`),
- The arcs between one vertex v_1 that does not belong to any circuit (i.e., `nodes1.succ` $= 0$) and one vertex v_2 located on a circuit (i.e., `nodes2.succ` $\neq 0$) such that the Manhattan distance between v_1 and v_2 is less than or equal to `MAXDIST`.

In order to specify the fact that each vertex is involved in at least one arc we use the graph property **NVERTEX** $= \lvert\texttt{NODES}\rvert$. Finally the dynamic constraint `nvalues_except_0`(`variables`, $=$, 1) expresses the fact that, for each vertex v, there is exactly one predecessor of v that belongs to a circuit.

Parts (A) and (B) of Figure 5.250 respectively show the initial and final graph associated with the second graph constraint of the **Example** slot.



(A) **(B)**

Figure 5.250: Initial and final graph of the `cycle_or_accessibility` constraint

Signature Since $|\texttt{NODES}|$ is the maximum number of vertices of the final graph associated with the second graph constraint we can rewrite $\textbf{NVERTEX} = |\texttt{NODES}|$ to $\textbf{NVERTEX} \geq |\texttt{NODES}|$. This leads to simplify $\overline{\underline{\textbf{NVERTEX}}}$ to $\overline{\textbf{NVERTEX}}$.