5.40 atmost1

DESCRIPTION LINKS

Origin [365]

Constraint atmost1(SETS)

Synonym pair_atmost1.

Argument SETS : collection(s−svar, c−int)

Restrictions required(SETS, [s, c])
SETS.c ≥ 1

Purpose Given a collection of set variables $s_1, s_2, \ldots, s_n$ and their respective cardinality $c_1, c_2, \ldots, c_n$, the atmost1 constraint forces the following two conditions:

- $\forall i \in [1, n] : |s_i| = c_i$,
- $\forall i, j \in [1, n]\ (i < j) : |s_i \bigcap s_j| \leq 1$.

Example

$$\left\{ \left\langle \begin{array}{ll} \mathtt{s} - \{5,8\} & \mathtt{c} - 2, \\ \mathtt{s} - \{5\} & \mathtt{c} - 1, \\ \mathtt{s} - \{5,6,7\} & \mathtt{c} - 3, \\ \mathtt{s} - \{1,4\} & \mathtt{c} - 2 \end{array} \right\rangle \right\}$$

The atmost1 constraint holds since:

- $|\{5,8\}| = 2, |\{5\}| = 1, |\{5,6,7\}| = 3, |\{1,4\}| = 2$.
- $|\{5,8\} \bigcap \{5\}| \leq 1, |\{5,8\} \bigcap \{5,6,7\}| \leq 1, |\{5,8\} \bigcap \{1,4\}| \leq 1,$
  $|\{5\} \bigcap \{5,6,7\}| \leq 1, |\{5\} \bigcap \{1,4\}| \leq 1,$
  $|\{5,6,7\} \bigcap \{1,4\}| \leq 1$.

Typical |SETS| > 1

Symmetries

- Items of SETS are permutable.
- All occurrences of two distinct values of SETS.s can be swapped; all occurrences of a value of SETS.s can be renamed to any unused value.

Arg. properties Contractible wrt. SETS.

Remark When we have only two set variables the atmost1 constraint was called pair_atmost1 in [428].

Algorithm C. Bessière *et al.* have shown in [68] that it is NP-hard to enforce bound consistency for the `atmost1` constraint. Consequently, following the first filtering algorithm from A. Sadler and C. Gervet [365], W.-J. van Hoeve and A. Sabharwal have proposed an algorithm that enforces bound-consistency when the `atmost1` constraint involves only two sets variables [428].

Systems `at_most1` in **MiniZinc**.

Keywords **constraint arguments:** constraint involving set variables.

constraint type: predefined constraint.

filtering: bound-consistency.