5.75   common

DESCRIPTION   **LINKS**   **GRAPH**

Origin   N. Beldiceanu

Constraint   common(NCOMMON1, NCOMMON2, VARIABLES1, VARIABLES2)

Arguments
```
NCOMMON1    :  dvar
NCOMMON2    :  dvar
VARIABLES1  :  collection(var−dvar)
VARIABLES2  :  collection(var−dvar)
```

Restrictions
```
NCOMMON1 ≥ 0
NCOMMON1 ≤ |VARIABLES1|
NCOMMON2 ≥ 0
NCOMMON2 ≤ |VARIABLES2|
required(VARIABLES1, var)
required(VARIABLES2, var)
```

Purpose

NCOMMON1 is the number of variables of the collection of variables VARIABLES1 taking a value in VARIABLES2.
NCOMMON2 is the number of variables of the collection of variables VARIABLES2 taking a value in VARIABLES1.

Example $(3, 4, \langle 1, 9, 1, 5 \rangle, \langle 2, 1, 9, 9, 6, 9 \rangle)$

The common constraint holds since:

- Its first argument NCOMMON1 $= 3$ corresponds to the number of values of the collection $\langle 1, 9, 1, 5 \rangle$ that occur within $\langle 2, 1, 9, 9, 6, 9 \rangle$.

- Its second argument NCOMMON2 $= 4$ corresponds to the number of values of the collection $\langle 2, 1, 9, 9, 6, 9 \rangle$ that occur within $\langle 1, 9, 1, 5 \rangle$.

All solutions   Figure 5.197 gives all solutions to the following non ground instance of the common constraint: NCOMMON1 $\in$ $[0, 1]$, NCOMMON2 $\in$ $[2, 3]$, U_1 $\in$ $[1, 2]$, U_2 $\in$ $[1, 2]$, U_3 $\in$ $[0, 1]$, U_4 $\in$ $[5, 6]$, V_1 $\in$ $[5, 6]$, V_2 $\in$ $[1, 2]$, V_3 $\in$ $[0, 1]$, common(NCOMMON1, NCOMMON2, $\langle U_1, U_2, U_3, U_4 \rangle, \langle V_1, V_2, V_3 \rangle)$.

Typical
```
|VARIABLES1| > 1
range(VARIABLES1.var) > 1
|VARIABLES2| > 1
range(VARIABLES2.var) > 1
```

$$
\begin{array}{l}
① \ (1, 2, \langle 1, 2, 0, 5 \rangle, \langle 6, 1, 1 \rangle) \\
② \ (1, 2, \langle 1, 2, 0, 6 \rangle, \langle 5, 1, 1 \rangle) \\
③ \ (1, 2, \langle 2, 1, 0, 5 \rangle, \langle 6, 1, 1 \rangle) \\
④ \ (1, 2, \langle 2, 1, 0, 6 \rangle, \langle 5, 1, 1 \rangle) \\
⑤ \ (1, 2, \langle 2, 2, 1, 5 \rangle, \langle 6, 1, 1 \rangle) \\
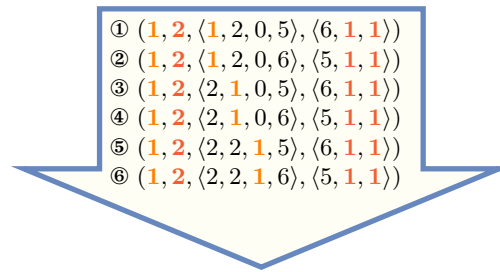⑥ \ (1, 2, \langle 2, 2, 1, 6 \rangle, \langle 5, 1, 1 \rangle)
\end{array}
$$

Figure 5.197: All solutions corresponding to the non ground example of the `common` constraint of the **All solutions** slot

Symmetries

- Arguments are permutable w.r.t. permutation (NCOMMON1, NCOMMON2) (VARIABLES1, VARIABLES2).
- Items of VARIABLES1 are permutable.
- Items of VARIABLES2 are permutable.
- All occurrences of two distinct values in VARIABLES1.var or VARIABLES2.var can be swapped; all occurrences of a value in VARIABLES1.var or VARIABLES2.var can be renamed to any unused value.

Arg. properties

- Functional dependency: NCOMMON1 determined by VARIABLES1 and VARIABLES2.
- Functional dependency: NCOMMON2 determined by VARIABLES1 and VARIABLES2.

Remark

It was shown in [70] that, finding out whether the `common` constraint has a solution or not is NP-hard. This was achieved by reduction from 3-SAT.

See also

common keyword: `alldifferent_on_intersection`, `nvalue_on_intersection`, `same_intersection` *(constraint on the intersection)*.

generalisation: `common_interval` *(variable replaced by variable/constant)*, `common_modulo` *(variable replaced by variable mod constant)*, `common_partition` *(variable replaced by variable ∈ partition)*.

related: `among_var`, `roots`.

root concept: `among`.

specialisation: `uses` *(NCOMMON2=|VARIABLES2|)*.

Keywords

complexity: 3-SAT.

constraint arguments: constraint between two collections of variables, pure functional dependency.

constraint type: constraint on the intersection.

final graph structure: acyclic, bipartite, no loop.

modelling: functional dependency.

Arc input(s)	VARIABLES1 VARIABLES2
Arc generator	$PRODUCT \mapsto \text{collection}(\texttt{variables1}, \texttt{variables2})$
Arc arity	2
Arc constraint(s)	variables1.var = variables2.var
Graph property(ies)	• **NSOURCE**= NCOMMON1 • **NSINK**= NCOMMON2
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Graph model

Parts (A) and (B) of Figure 5.198 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NSOURCE** and **NSINK** graph properties, the source and sink vertices of the final graph are stressed with a double circle. Since the final graph has only 3 sources and 4 sinks the variables NCOMMON1 and NCOMMON2 are respectively equal to 3 and 4. Note that all the vertices corresponding to the variables that take values 5, 2 or 6 were removed from the final graph since there is no arc for which the associated equality constraint holds.
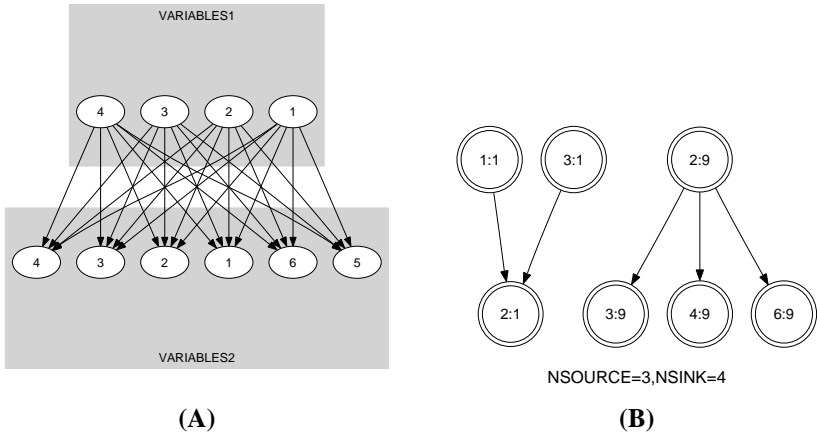

(A)   **(B)**

Figure 5.198: Initial and final graph of the common constraint