

5.221 lex_alldifferent

	DESCRIPTION	LINKS	GRAPH
Origin	J. Pearson		
Constraint	<code>lex_alldifferent(VECTORS)</code>		
Synonyms	<code>lex_alldiff</code> , <code>lex_alldistinct</code> , <code>alldiff_on_tuples</code> , <code>alldifferent_on_tuples</code> , <code>alldistinct_on_tuples</code> .		
Type	VECTOR : <code>collection</code> (<code>var-dvar</code>)		
Argument	VECTORS : <code>collection</code> (<code>vec</code> – VECTOR)		
Restrictions	$ \text{VECTOR} \geq 1$ <code>required</code> (VECTOR, <code>var</code>) <code>required</code> (VECTORS, <code>vec</code>) <code>same_size</code> (VECTORS, <code>vec</code>)		
Purpose	All the vectors of the collection VECTORS are distinct. Two vectors (u_1, u_2, \dots, u_n) and (v_1, v_2, \dots, v_n) are distinct if and only if there exists $i \in [1, n]$ such that $u_i \neq v_i$.		
Example	$((\text{vec} - \langle 5, 2, 3 \rangle, \text{vec} - \langle 5, 2, 6 \rangle, \text{vec} - \langle 5, 3, 3 \rangle))$		
	The <code>lex_alldifferent</code> constraint holds since:		
	<ul style="list-style-type: none"> • The first vector $\langle 5, 2, 3 \rangle$ and the second vector $\langle 5, 2, 6 \rangle$ of the VECTORS collection differ in their third component (i.e., $3 \neq 6$). • The first vector $\langle 5, 2, 3 \rangle$ and the third vector $\langle 5, 3, 3 \rangle$ of the VECTORS collection differ in their second component (i.e., $2 \neq 3$). • The second vector $\langle 5, 2, 6 \rangle$ and the third vector $\langle 5, 3, 3 \rangle$ of the VECTORS collection differ in their second and third components (i.e., $2 \neq 3$ and $6 \neq 3$). 		
Typical	$ \text{VECTOR} > 1$ $ \text{VECTORS} > 1$		
Symmetries	<ul style="list-style-type: none"> • Items of VECTORS are <code>permutable</code>. • Items of VECTORS.<code>vec</code> are <code>permutable</code> (<i>same permutation used</i>). • All occurrences of two distinct tuples of values of VECTORS.<code>vec</code> can be <code>swapped</code>; all occurrences of a tuple of values of VECTORS.<code>vec</code> can be <code>renamed</code> to any unused tuple of values. 		
Arg. properties	<ul style="list-style-type: none"> • <code>Contractible</code> wrt. VECTORS. • <code>Extensible</code> wrt. VECTORS.<code>vec</code> (<i>add items at same position</i>). 		

Usage	When the vectors have two components, the <code>lex_alldifferent</code> constraint allows to directly enforce difference constraints between pairs of variables . Such difference constraints occur for instance in block design problems (e.g., Steiner triples, Kirkman school-girls problem). However, in all these problems a same variable may occur in more than one pair of variables. Consequently, arc-consistency is not achieved any more by the filtering algorithm described in [335] .
Algorithm	A filtering algorithm achieving arc-consistency for the <code>lex_alldifferent</code> constraint is proposed by C.-G. Quimper and T. Walsh in [335] and a longer version is available in [336] and in [337] .
Reformulation	The <code>lex_alldifferent</code> (VECTORS) constraint can be expressed as a clique of lex_different constraints. By associating a n -dimensional box for which all sizes are equal to 1, one can also express the <code>lex_alldifferent</code> (VECTORS) constraint as a diffn or a geost constraint. Enforcing all the n -dimensional boxes to not overlap is equivalent as enforcing all the vectors to be distinct. In the context of the multidimensional sweep algorithm of the geost constraint [38] , it makes more sense to make a complete sweep over the domain of each variable in order not to only restrict the minimum and maximum value of each variable.
See also	<p>generalisation: diffn(vector <i>replaced by</i> orthotope), geost(vector <i>replaced by</i> object).</p> <p>implied by: all_incomparable, lex_chain_greater, lex_chain_less.</p> <p>implies: lex_alldifferent_except_0.</p> <p>part of system of constraints: lex_different.</p> <p>specialisation: alldifferent(vector <i>replaced by</i> variable).</p> <p>used in graph description: lex_different.</p>
Keywords	<p>characteristic of a constraint: vector.</p> <p>constraint type: system of constraints, decomposition.</p> <p>filtering: bipartite matching, arc-consistency.</p> <p>modelling: difference between pairs of variables.</p>

Arc input(s)	VECTORS
Arc generator	$\text{CLIQUE}(<) \mapsto \text{collection}(\text{vectors1}, \text{vectors2})$
Arc arity	2
Arc constraint(s)	$\text{lex_different}(\text{vectors1.vec}, \text{vectors2.vec})$
Graph property(ies)	$\text{NARC} = \text{VECTORS} * (\text{VECTORS} - 1) / 2$

Graph model Parts (A) and (B) of Figure 5.494 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold.

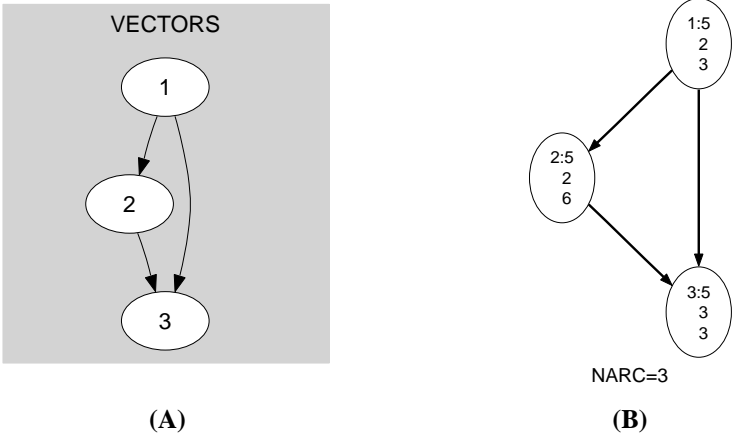


Figure 5.494: Initial and final graph of the `lex_alldifferent` constraint

Signature Since we use the `CLIQUE(<)` arc generator on the `VECTORS` collection the number of arcs of the initial graph is equal to $|\text{VECTORS}| \cdot (|\text{VECTORS}| - 1) / 2$. For this reason we can rewrite $\text{NARC} = |\text{VECTORS}| \cdot (|\text{VECTORS}| - 1) / 2$ to $\text{NARC} \geq |\text{VECTORS}| \cdot (|\text{VECTORS}| - 1) / 2$ and simplify $\overline{\text{NARC}}$ to NARC .

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