5.263 minimum

DESCRIPTION LINKS GRAPH AUTOMATON

Origin CHIP

Constraint minimum(MIN, VARIABLES)

Synonym min.

Arguments

MIN	:	dvar
VARIABLES	:	collection(var−dvar)

Restrictions

$|\text{VARIABLES}| > 0$
required(VARIABLES, var)

Purpose MIN is the minimum value of the collection of domain variables VARIABLES.

Example

$(2, \langle 3, 2, 7, 2, 6\rangle)$
$(7, \langle 8, 8, 7, 8, 7\rangle)$

The first minimum constraint holds since its first argument $\text{MIN} = 2$ is set to the minimum value of the collection $\langle 3, 2, 7, 2, 6\rangle$.

Typical

$|\text{VARIABLES}| > 1$
$\text{range}(\text{VARIABLES.var}) > 1$

Symmetries

- Items of VARIABLES are permutable.
- All occurrences of two distinct values of VARIABLES.var can be swapped.
- One and the same constant can be added to MIN as well as to the var attribute of all items of VARIABLES.

Arg. properties

- Functional dependency: MIN determined by VARIABLES.
- Aggregate: MIN(min), VARIABLES(union).

Usage In some project scheduling problems one has to introduce dummy activities that correspond for instance to the starting time of a given set of activities. In this context one can use the minimum constraint to get the minimum starting time of a set of tasks.

Remark Note that minimum is a constraint and not just a function that computes the minimum value of a collection of variables: potential values of MIN influence the variables of VARIABLES, and reciprocally potential values that can be assigned to variables of VARIABLES influence MIN.

The minimum constraint is called min in **JaCoP** (http://www.jacop.eu/).

Algorithm A filtering algorithm for the `minimum` constraint is described in [27].

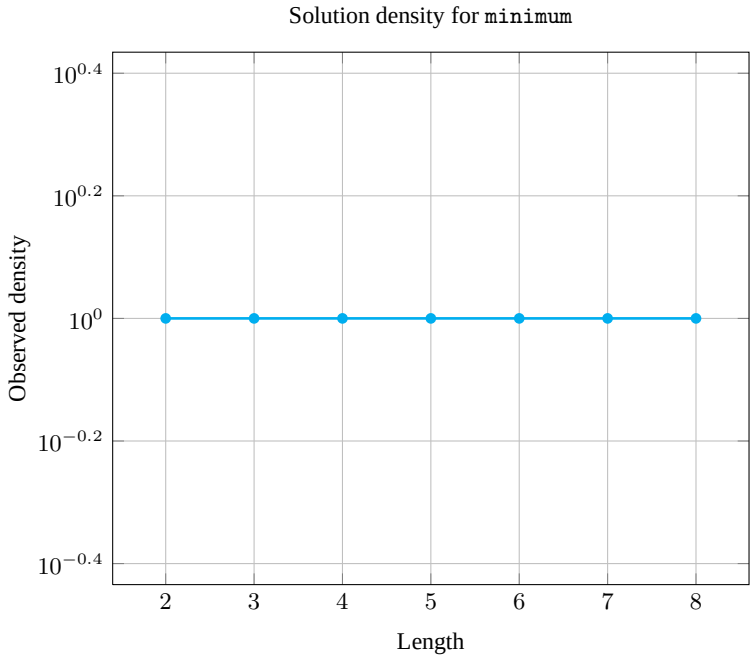
The `minimum` constraint is entailed if all the following conditions hold:

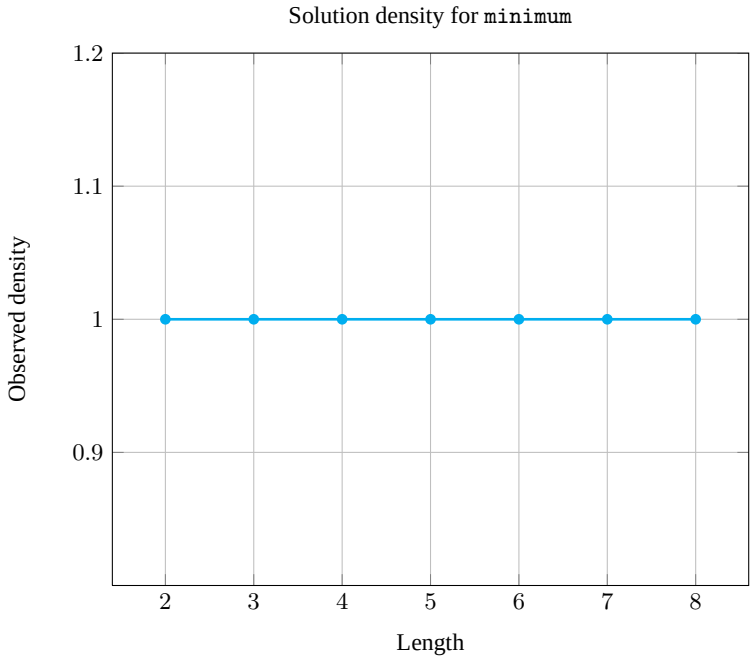
1. `MIN` is fixed.
2. At least one variable of `VARIABLES` is assigned value `MIN`.
3. All variables of `VARIABLES` have their minimum value greater than or equal to value `MIN`.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

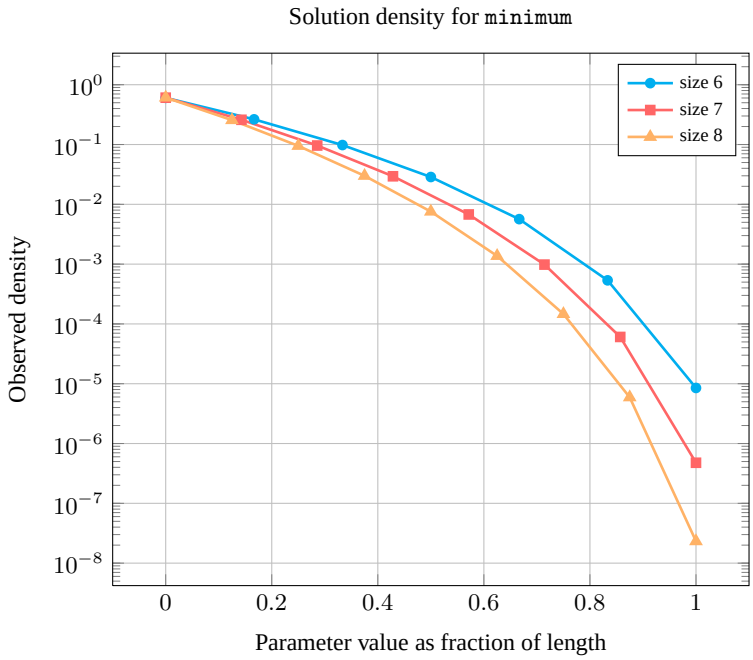
Number of solutions for `minimum`: domains $0..n$

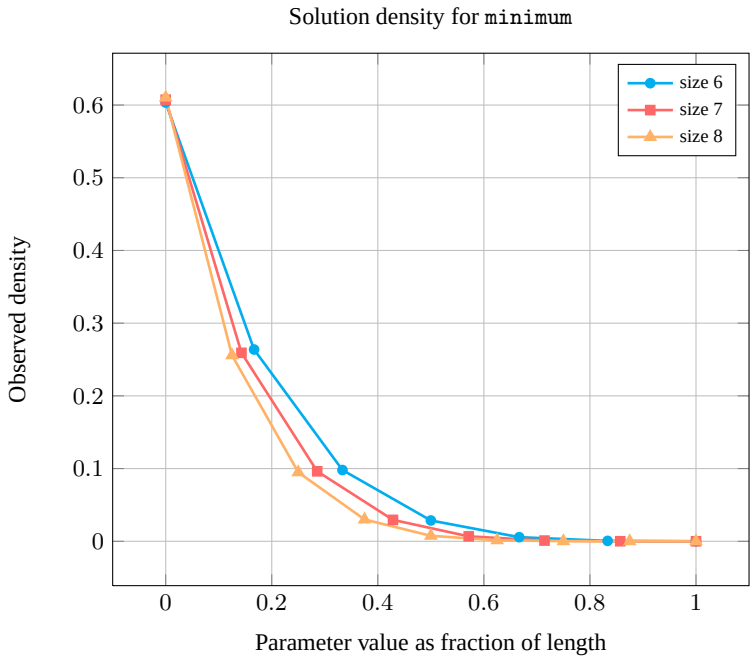




Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	5	37	369	4651	70993	1273609	26269505
	1	3	19	175	2101	31031	543607	11012415
	2	1	7	65	781	11529	201811	4085185
	3	-	1	15	211	3367	61741	1288991
	4	-	-	1	31	665	14197	325089
	5	-	-	-	1	63	2059	58975
	6	-	-	-	-	1	127	6305
	7	-	-	-	-	-	1	255
	8	-	-	-	-	-	-	1

Solution count for `minimum`: domains $0..n$





Systems `min` in **Choco**, `min` in **Gecode**, `min` in **JaCoP**, `minimum` in **MiniZinc**, `minimum` in **SICStus**.

Used in minimum_greater_than, next_element, next_greater_element.

See also
common keyword: maximum *(order constraint)*.
comparison swapped: maximum.
generalisation: minimum_modulo *(*variable *replaced by* variable mod constant*)*.
implied by: and.
implies: between_min_max, in.
soft variant: minimum_except_0 *(value* 0 *is ignored)*, open_minimum *(open constraint)*.
specialisation: min_n *(minimum or order* n *replaced by absolute minimum)*.
uses in its reformulation: cycle.

Keywords
characteristic of a constraint: minimum, maxint, automaton, automaton without counters, reified automaton constraint.
constraint arguments: reverse of a constraint, pure functional dependency.
constraint network structure: centered cyclic(1) constraint network(1).
constraint type: order constraint.
filtering: glue matrix, arc-consistency, entailment.
modelling: functional dependency.

Cond. implications
minimum(MIN, VARIABLES)
with first(VARIABLES.var) > MIN
and last(VARIABLES.var) > MIN
implies deepest_valley(DEPTH, VARIABLES).

Arc input(s)	VARIABLES
Arc generator	$CLIQUE \mapsto \texttt{collection}(\texttt{variables1}, \texttt{variables2})$
Arc arity	2
Arc constraint(s)	$\bigvee \left(\begin{array}{l} \texttt{variables1.key} = \texttt{variables2.key}, \\ \texttt{variables1.var} < \texttt{variables2.var} \end{array} \right)$
Graph property(ies)	$\mathbf{ORDER}(0, \texttt{MAXINT}, \texttt{var}) = \texttt{MIN}$

Graph model The condition $\texttt{variables1.key} = \texttt{variables2.key}$ holds if and only if variables1 and variables2 corresponds to the same vertex. It is used in order to enforce to keep all the vertices of the initial graph. $\mathbf{ORDER}(0, \texttt{MAXINT}, \texttt{var})$ refers to the source vertices of the graph, i.e., those vertices that do not have any predecessor.

Parts (A) and (B) of Figure 5.574 respectively show the initial and final graph associated with the first example of the **Example** slot. Since we use the **ORDER** graph property, the vertices of rank 0 (without considering the loops) of the final graph are outlined with a thick circle.

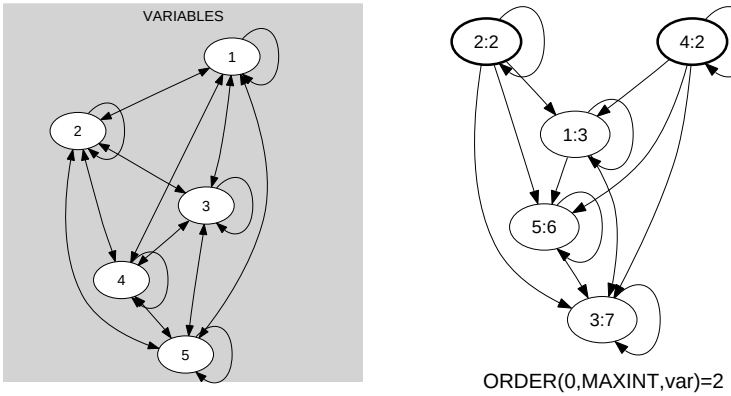


Figure 5.574: Initial and final graph of the minimum constraint

Automaton

Figure 5.575 depicts a first counter free deterministic automaton associated with the `minimum` constraint. Let $\texttt{VAR}_i$ be the i^{th} variable of the `VARIABLES` collection. To each pair $(\texttt{MIN}, \texttt{VAR}_i)$ corresponds a signature variable S_i as well as the following signature constraint: $(\texttt{MIN} < \texttt{VAR}_i \Leftrightarrow S_i = 0) \wedge (\texttt{MIN} = \texttt{VAR}_i \Leftrightarrow S_i = 1) \wedge (\texttt{MIN} > \texttt{VAR}_i \Leftrightarrow S_i = 2)$.

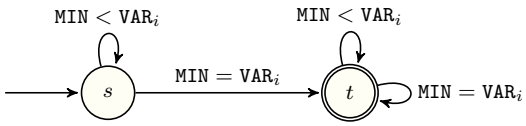


Figure 5.575: Counter free automaton of the `minimum` constraint

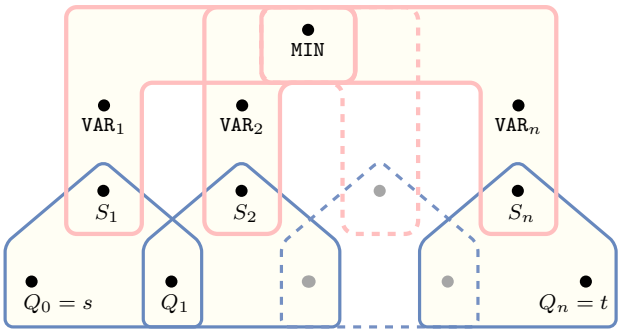


Figure 5.576: Hypergraph of the reformulation corresponding to the automaton of the `minimum` constraint

Figure 5.576 depicts a second counter free non deterministic automaton associated with the `minimum` constraint, where the argument `MIN` is also part of the sequence passed to the automaton.

Figure 5.579 depicts a third deterministic automaton with one counter associated with the `minimum` constraint, where the argument `MIN` is unified to the final value of the counter.

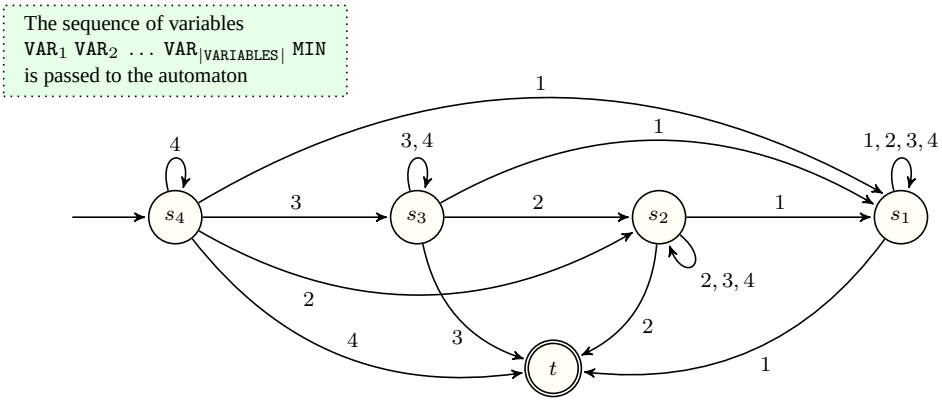


Figure 5.577: Counter free non deterministic automaton of the `minimum`(`MIN`, `VARIABLES`) constraint assuming that the union of the domain of the variables is the set $\{1, 2, 3, 4\}$ and that the elements of `VARIABLES` are first passed to the automaton followed by `MIN` (state s_i means that no value strictly less than value i was found and that value i was already encountered at least once)

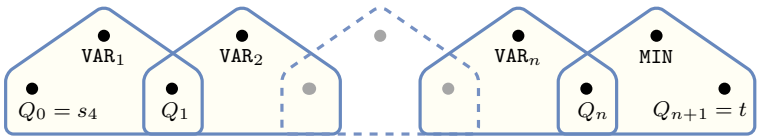


Figure 5.578: Hypergraph of the reformulation corresponding to the counter free non deterministic automaton of the `minimum` constraint

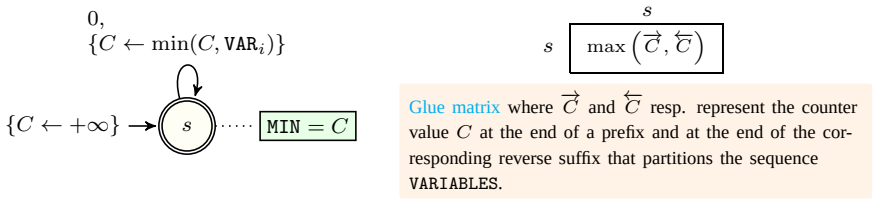


Figure 5.579: Automaton (with one counter) of the `minimum` constraint and its glue matrix

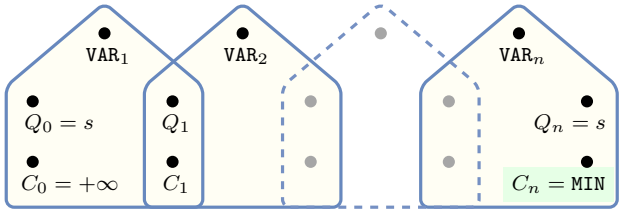


Figure 5.580: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the `minimum` constraint: since all states variables $Q_0, Q_1, \ldots, Q_n$ are fixed to the unique state s of the automaton, the transitions constraints share only the counter variable C and the constraint network is Berge-acyclic