5.242 max_increasing_slope

DESCRIPTION LINKS AUTOMATON

Origin Motivated by time series.

Constraint max_increasing_slope(MAX, VARIABLES)

Arguments

MAX	:	dvar
VARIABLES	:	collection(var−dvar)

Restrictions

MAX ≥ 0
MAX $<$range(VARIABLES.var)
required(VARIABLES, var)
|VARIABLES| > 0

Purpose Given a sequence of variables VARIABLES $= V_1, V_2, \ldots, V_n$, sets MAX to 0 if $\nexists i \in [1, n-1] | V_i < V_{i+1}$, otherwise sets MAX to $\max_{i \in [1,n-1] | V_i < V_{i+1}} (V_{i+1} - V_i)$.

Example

$(4, \langle 1, 1, 5, 8, 6, 2, 2, 1, 2 \rangle)$
$(0, \langle 9, 8, 6, 4, 1, 0 \rangle)$
$(8, \langle 9, 6, 6, 4, 1, 9 \rangle)$

The first max_increasing_slope constraint holds since the sequence 1 1 5 8 6 2 2 1 2 contains two increasing subsequences 1 5 8 and 1 2 and the maximum slope is equal to $\max(5-1, 8-5, 2-1) = 4$ as shown on Figure 5.532.

Typical

MAX > 0
MAX $<$range(VARIABLES.var) $- 1$
|VARIABLES| > 2
range(VARIABLES.var) > 2

Symmetry One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

Functional dependency: MAX determined by VARIABLES.

Usage Getting the maximum slope over the increasing sequences of time series.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for max_increasing_slope: domains $0..n$

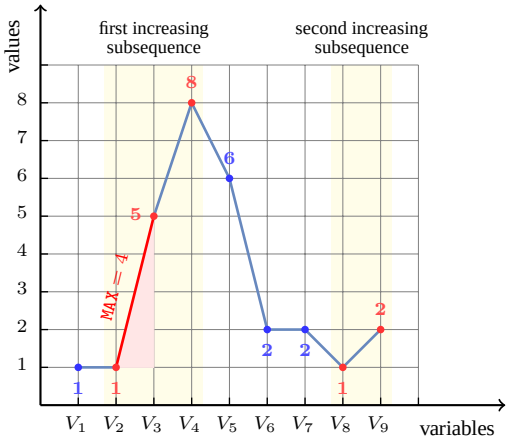
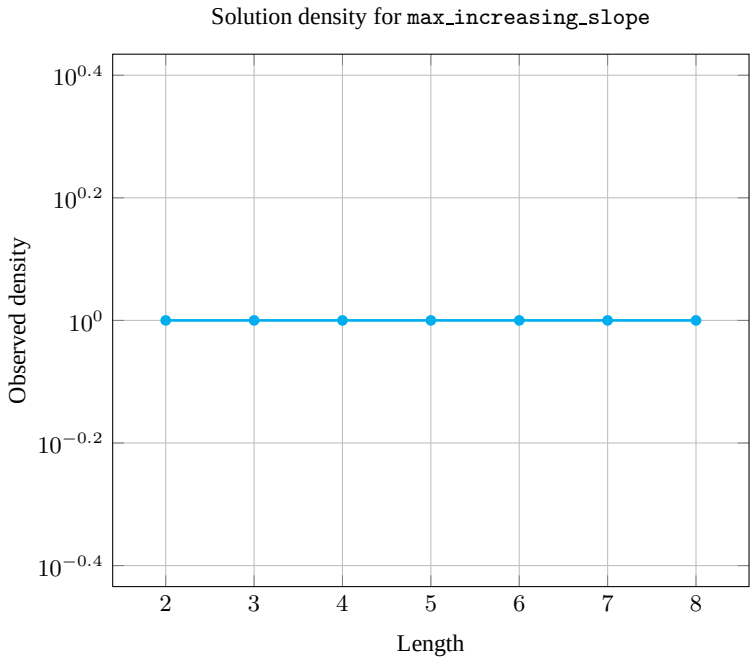
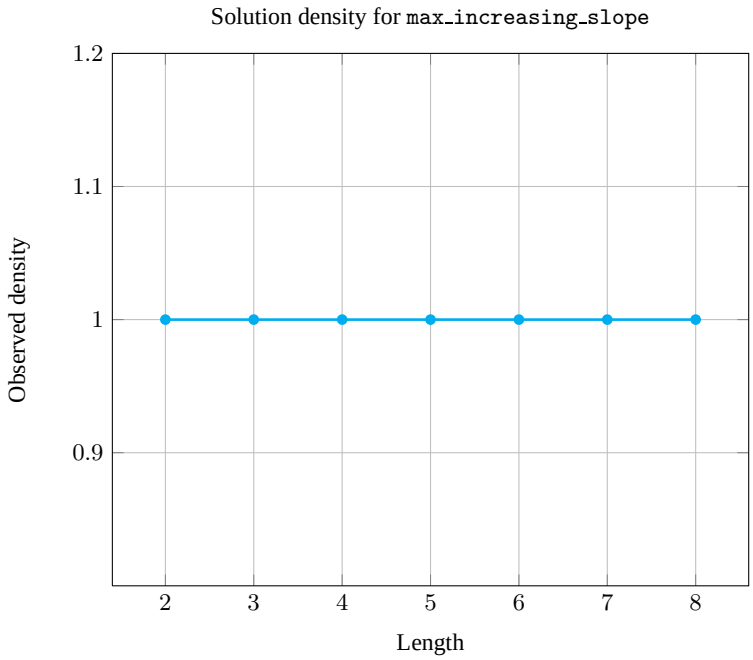


Figure 5.532: Illustration of the first example of the **Example** slot: a sequence of nine variables V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9 respectively fixed to values 1, 1, 5, 8, 6, 2, 2, 1, 2 and the corresponding maximum slope on the strictly increasing subsequences 1 5 8 and 1 2 (`MAX` $= 4$)

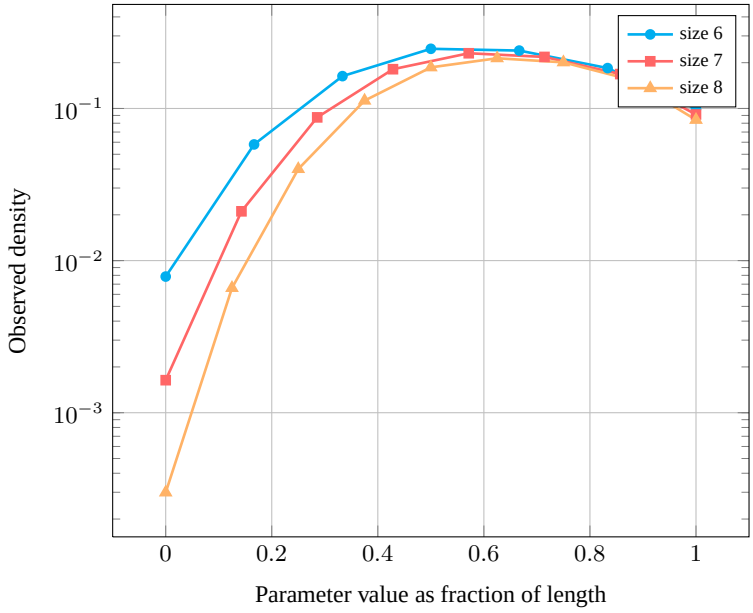




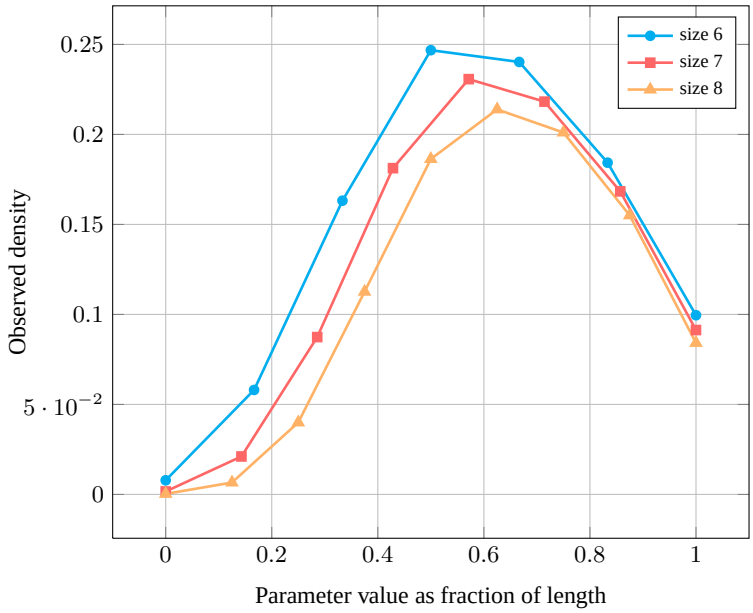
Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	6	20	70	252	924	3432	12870
	1	2	20	151	1036	6828	44220	284405
	2	1	16	188	1952	19200	183304	1721425
	3	-	8	142	2106	29035	380116	4847301
	4	-	-	74	1584	28266	483840	8021350
	5	-	-	-	846	21684	457632	9208124
	6	-	-	-	-	11712	353088	8654931
	7	-	-	-	-	-	191520	6673834
	8	-	-	-	-	-	-	3622481

Solution count for `max_increasing_slope`: domains $0..n$

Solution density for `max_increasing_slope`



Solution density for `max_increasing_slope`



Keywords **characteristic of a constraint:** automaton, automaton with counters.
combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

filtering: glue matrix.

modelling: functional dependency.

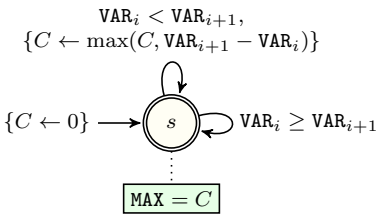
Cond. implications

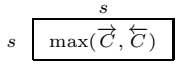
- `max_increasing_slope`(`MAX`, `VARIABLES`)
  with `range`(`VARIABLES.var`) = `MAX` + 1
  implies `longest_increasing_sequence`(`L`, `VARIABLES`)
  when `range`(`VARIABLES.var`) = `L` + 1.

- `max_increasing_slope`(`MAX`, `VARIABLES`)
  with `MAX` = 1
  implies `min_increasing_slope`(`MIN`, `VARIABLES`)
  when `MIN` = 1.

Automaton

Figure 5.533 depicts the automaton associated with the `max_increasing_slope` constraint. To each pair of consecutive variables $(\texttt{VAR}_i, \texttt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links $\texttt{VAR}_i$, $\texttt{VAR}_{i+1}$ and S_i: $(\texttt{VAR}_i \geq \texttt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\texttt{VAR}_i < \texttt{VAR}_{i+1} \Leftrightarrow S_i = 1)$.





Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

Figure 5.533: Automaton for the `max_increasing_slope` constraint and its glue matrix (note that the reverse of `max_increasing_slope` is `max_decreasing_slope`)