## 5.240 map

DESCRIPTION LINKS GRAPH

**Origin** Inspired by [376]

**Constraint** map(NBCYCLE, NBTREE, NODES)

**Arguments**

| | | |
|---|---|---|
| NBCYCLE | : | dvar |
| NBTREE | : | dvar |
| NODES | : | collection(index−int, succ−dvar) |

**Restrictions**

NBCYCLE $\geq 0$
NBTREE $\geq 0$
required(NODES, [index, succ])
NODES.index $\geq 1$
NODES.index $\leq |$NODES$|$
distinct(NODES, index)
NODES.succ $\geq 1$
NODES.succ $\leq |$NODES$|$

**Purpose** Number of trees and number of cycles of a map. We take the description of a map from [376, page 459]:

> *"Every map decomposes into a set of connected components, also called connected maps. Each component consists of the set of all points that wind up on the same cycle, with each point on the cycle attached to a tree of all points that enter the cycle at that point."*

**Example**

$$\left\langle \begin{array}{cc} \text{index}-1 & \text{succ}-5, \\ \text{index}-2 & \text{succ}-9, \\ \text{index}-3 & \text{succ}-8, \\ \text{index}-4 & \text{succ}-2, \\ \text{index}-5 & \text{succ}-9, \\ \text{index}-6 & \text{succ}-2, \\ \text{index}-7 & \text{succ}-9, \\ \text{index}-8 & \text{succ}-8, \\ \text{index}-9 & \text{succ}-1 \end{array} \right\rangle$$

The map constraint holds since, as shown by part (B) of Figure 5.529, the graph corresponding to the NODES collection is a map containing NBCYCLE $= 2$ cycles (i.e., a first cycle involving vertices 1, 5 and 9 and a second cycle involving vertex 8) and 3 trees (i.e., two trees respectively involving vertices 7 and 4, 6, 2 and attached to the first cycle, and one tree mentioning vertex 3 linked to the second cycle.)

**Typical**

NBCYCLE $> 0$
NBTREE $> 0$
NBCYCLE $<$ |NODES|
NBCYCLE $<$ NBTREE
|NODES| $> 2$

**Symmetry** Items of NODES are permutable.

**Arg. properties**

- Functional dependency: NBCYCLE determined by NODES.
- Functional dependency: NBTREE determined by NODES.

**See also** **common keyword:** cycle, graph_crossing, tree *(graph partitioning constraint)*.

**Keywords** **constraint arguments:** pure functional dependency.
**constraint type:** graph constraint, graph partitioning constraint.
**filtering:** DFS-bottleneck.
**final graph structure:** connected component.
**modelling:** functional dependency.

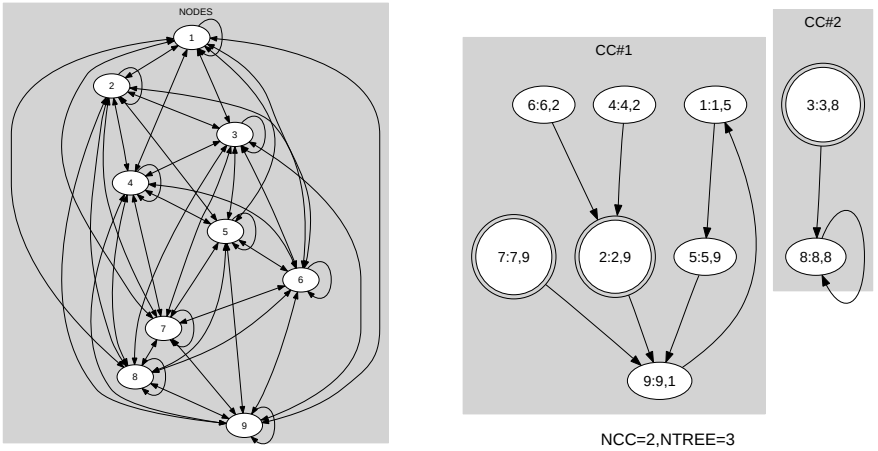
| | |
|---|---|
| **Arc input(s)** | NODES |
| **Arc generator** | $CLIQUE \mapsto$ collection(nodes1, nodes2) |
| **Arc arity** | 2 |
| **Arc constraint(s)** | nodes1.succ = nodes2.index |
| **Graph property(ies)** | • **NCC**= NBCYCLE<br>• **NTREE**= NBTREE |

**Graph model**

Note that, for the argument NBTREE of the map constraint, we consider a definition different from the one used for the argument NTREES of the tree constraint:

- In the map constraint the number of trees NBTREE is equal to the number of vertices of the final graph, which both do not belong to any circuit and have a successor that is located on a circuit. Therefore we count three trees in the context of the **Example** slot.
- In the tree constraint the number of trees NTREES is equal to the number of connected components of the final graph.

Parts (A) and (B) of Figure 5.529 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NCC** graph property, we display the two connected components of the final graph. Each of them corresponds to a connected map. The first connected map is made up from one circuit and two trees, while the second one consists of one circuit and one tree. Since we also use the **NTREE** graph property, we display with a double circle those vertices that do not belong to any circuit but for which at least one successor belongs to a circuit.




**(A)** **(B)**

Figure 5.529: Initial and final graph of the map constraint