1528 NARC, PATH

## 5.223 lex\_chain\_greater

DESCRIPTION LINKS GRAPH

Origin Derived from lex\_chain\_less

Constraint lex\_chain\_greater(VECTORS)

Usual name lex\_chain

Argument VECTORS : collection(vec - VECTOR)

**Restrictions**  $|VECTOR| \ge 1$ 

required(VECTOR, var)
required(VECTORS, vec)
same\_size(VECTORS, vec)

For each pair of consecutive vectors  $VECTOR_i$  and  $VECTOR_{i+1}$  of the VECTORS collection we have that  $VECTOR_i$  is lexicographically strictly greater than  $VECTOR_{i+1}$ . Given two vectors,  $\vec{X}$  and  $\vec{Y}$  of n components,  $\langle X_0, \ldots, X_{n-1} \rangle$  and  $\langle Y_0, \ldots, Y_{n-1} \rangle$ ,  $\vec{X}$  is lexicographically strictly greater than  $\vec{Y}$  if and only if  $X_0 > Y_0$  or  $X_0 = Y_0$  and  $\langle X_1, \ldots, X_{n-1} \rangle$  is lexicographically strictly greater than  $\langle Y_1, \ldots, Y_{n-1} \rangle$ .

Example

Purpose

$$\left(\left\langle \mathtt{vec} - \left\langle 5, 2, 6, 3 \right\rangle, \mathtt{vec} - \left\langle 5, 2, 6, 2 \right\rangle, \mathtt{vec} - \left\langle 5, 2, 3, 9 \right\rangle \right)$$

The lex\_chain\_greater constraint holds since:

- The first vector  $\langle 5,2,6,3 \rangle$  of the VECTORS collection is lexicographically strictly greater than the second vector  $\langle 5,2,6,2 \rangle$  of the VECTORS collection.
- The second vector  $\langle 5,2,6,2 \rangle$  of the VECTORS collection is lexicographically strictly greater than the third vector  $\langle 5,2,3,9 \rangle$  of the VECTORS collection.

**Typical** 

```
\begin{aligned} |\mathtt{VECTOR}| &> 1 \\ |\mathtt{VECTORS}| &> 1 \end{aligned}
```

Arg. properties

- Contractible wrt. VECTORS.
- Suffix-extensible wrt. VECTORS.vec (add items at same position).

Usage

This constraint was motivated for breaking symmetry: more precisely when one wants to lexicographically order the consecutive columns of a matrix of decision variables. A further motivation is that using a set of lexicographic ordering constraints between two vectors does usually not allows to come up with a complete pruning.

Algorithm

A filtering algorithm achieving arc-consistency for a chain of lexicographical ordering constraints is presented in [95].

20130730 1529

See also common keyword: lex\_between, lex\_greatereq, lex\_less,

lex\_lesseq(lexicographic order).

implies: lex\_alldifferent, lex\_chain\_greatereq.

part of system of constraints: lex\_greater.
used in graph description: lex\_greater.

**Keywords** application area: floor planning problem.

characteristic of a constraint: vector.

constraint type: decomposition, order constraint, system of constraints.

filtering: arc-consistency.

**heuristics:** heuristics and lexicographical ordering. **modelling:** degree of diversity of a set of solutions.

**modelling exercises:** degree of diversity of a set of solutions. **symmetry:** symmetry, matrix symmetry, lexicographic order.

 $\overline{NARC}$ , PATH

Arc input(s)	VECTORS
Arc generator	$PATH \mapsto collection(vectors1, vectors2)$
Arc arity	2
Arc constraint(s)	<pre>lex_greater(vectors1.vec, vectors2.vec)</pre>
<b>Graph property(ies)</b>	NARC =  VECTORS  - 1

## Graph model

Parts (A) and (B) of Figure 5.478 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. The <code>lex\_chain\_greater</code> constraint holds since all the arc constraints of the initial graph are satisfied.

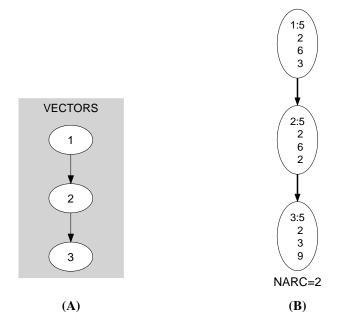


Figure 5.478: Initial and final graph of the lex\_chain\_greater constraint

## Signature

Since we use the PATH arc generator on the VECTORS collection the number of arcs of the initial graph is equal to |VECTORS| - 1. For this reason we can rewrite NARC = |VECTORS| - 1 to  $NARC \ge |VECTORS| - 1$  and simplify  $\overline{NARC}$  to  $\overline{NARC}$ .

20130730 1531