## 5.45 balance\_interval

DESCRIPTION LINKS GRAPH AUTOMATON

Origin Derived from balance.

Constraint balance\_interval(BALANCE, VARIABLES, SIZE\_INTERVAL)

Arguments BALANCE : dvar

VARIABLES : collection(var-dvar)

SIZE\_INTERVAL : int

**Restrictions** BALANCE > 0

 $BALANCE \le max(0, |VARIABLES| - 2)$ 

required(VARIABLES, var)

 ${\tt SIZE\_INTERVAL}>0$ 

Consider the largest set  $S_1$  (respectively the smallest set  $S_2$ ) of variables of the collection VARIABLES that take their value in a same interval [SIZE\_INTERVAL·k, SIZE\_INTERVAL·k + SIZE\_INTERVAL - 1], where k is an integer. BALANCE is equal to the difference between the cardinality of  $S_2$  and the cardinality of  $S_1$ .

Example

**Purpose** 

 $(3, \langle 6, 4, 3, 3, 4 \rangle, 3)$ 

In the example, the third argument SIZE\_INTERVAL = 3 defines the following family of intervals  $[3 \cdot k, 3 \cdot k + 2]$ , where k is an integer. Values 6,4,3,3 and 4 are respectively located within intervals [6,8], [3,5], [3,5], [3,5] and [3,5]. Therefore intervals [6,8] and [3,5] are respectively used 1 and 4 times. The balance\_interval constraint holds since its first argument BALANCE is assigned to the difference between the maximum and minimum number of the previous occurrences (i.e., 4-1).

**Typical** 

```
\begin{split} |{\tt VARIABLES}| &> 2 \\ {\tt SIZE\_INTERVAL} &> 1 \\ {\tt SIZE\_INTERVAL} &< {\tt range}({\tt VARIABLES.var}) \end{split}
```

**Symmetries** 

- Items of VARIABLES are permutable.
- An occurrence of a value of VARIABLES.var that belongs to the k-th interval, of size SIZE\_INTERVAL, can be replaced by any other value of the same interval.

Arg. properties

Functional dependency: BALANCE determined by VARIABLES and SIZE\_INTERVAL.

Usage

An application of the balance\_interval constraint is to enforce a *balanced assignment* of interval of values, no matter how many distinct interval of values will be used. In this case one will *push down* the maximum value of the first argument of the balance\_interval constraint.

See also

specialisation: balance (variable/constant replaced by variable).

20030820 695

**Keywords** application area: assignment.

characteristic of a constraint: automaton, automaton with array of counters.

constraint arguments: pure functional dependency.

constraint type: value constraint.
final graph structure: equivalence.

modelling: interval, balanced assignment, functional dependency.

Arc input(s)	VARIABLES
Arc generator	$\textcolor{red}{CLIQUE} {\mapsto} \texttt{collection}(\texttt{variables1}, \texttt{variables2})$
Arc arity	2
Arc constraint(s)	<pre>variables1.var/SIZE_INTERVAL = variables2.var/SIZE_INTERVAL</pre>
Graph property(ies)	RANGE_NSCC= BALANCE
Graph class	EQUIVALENCE

## Graph model

The graph property **RANGE\_NSCC** constraints the difference between the sizes of the largest and smallest strongly connected components.

Parts (A) and (B) of Figure 5.120 respectively show the initial and final graph associated with the **Example** slot. Since we use the **RANGE\_NSCC** graph property, we show the largest and smallest strongly connected components of the final graph.

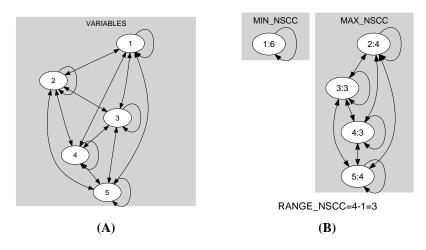


Figure 5.120: Initial and final graph of the balance\_interval constraint

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Automaton

Figure 5.121 depicts the automaton associated with the balance\_interval constraint. To each item of the collection VARIABLES corresponds a signature variable  $S_i$  that is equal to 1.

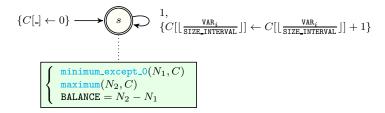


Figure 5.121: Automaton of the balance\_interval constraint