## 5.122 disj

**DESCRIPTION** LINKS **GRAPH** Origin [287] Constraint disj(TASKS) start-dvar, duration-dvar, Argument : collection Restrictions required(TASKS, [start, duration, before, position])  ${\tt TASKS.duration} > 1$  ${\tt TASKS.position} \geq 0$  ${\tt TASKS.position} < |{\tt TASKS}|$ All the tasks of the collection TASKS should not overlap. For a given task t the attributes before and position respectively correspond to the set of tasks starting before task t**Purpose** (assuming that the first task is labelled by 1) and to the position of task t (assuming that the first task has position 0).

Example

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\left(\begin{array}{cccc} \text{start}-1 & \text{duration}-3 & \text{before}-\emptyset & \text{position}-0,\\ \text{start}-9 & \text{duration}-1 & \text{before}-\{1,3,4\} & \text{position}-3,\\ \text{start}-7 & \text{duration}-2 & \text{before}-\{1,4\} & \text{position}-2,\\ \text{start}-4 & \text{duration}-1 & \text{before}-\{1\} & \text{position}-1 \end{array}\right)
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Figure 5.273 shows the tasks of the example. Since these tasks do not overlap the disj constraint holds.

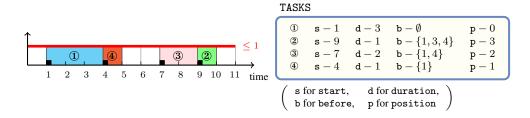


Figure 5.273: Tasks corresponding to the **Example** slot

**Typical** 

 $|\mathtt{TASKS}| > 1$ 

**Symmetries** 

- One and the same constant can be added to the start attribute of all items of TASKS.
- TASKS.duration can be decreased to any value  $\geq 1$ .

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the first task has position 0 we have that, for a given task t, the number of elements of its

Usage The disj constraint was originally applied [287] to solve the *open-shop* problem.

**Remark** This constraint is similar to the disjunctive constraint. In addition to the start and the duration attributes of a task t, the disj constraint introduces a set variable before that represents the set of tasks that end before the start of task t as well as a domain variable position that gives the absolute order of task t in the resource. Since it assumes that

before attribute is equal to the value of its position attribute.

Algorithm The main idea of the algorithm is to apply in a systematic way shaving on the position

attribute of a task. It is implemented in Gecode [374].

See also common keyword: disjunctive (scheduling constraint).

used in graph description: in\_set.

**Keywords** complexity: sequencing with release times and deadlines.

constraint arguments: constraint involving set variables.

constraint type: scheduling constraint, resource constraint, decomposition.

Arc input(s)	TASKS
Arc generator	$CLIQUE(\neq) \mapsto \texttt{collection}(\texttt{tasks1}, \texttt{tasks2})$
Arc arity	2
Arc constraint(s)	<ul> <li>V ( tasks1.start + tasks1.duration ≤ tasks2.start, tasks2.start + tasks2.duration ≤ tasks1.start )</li> <li>tasks1.start + tasks1.duration ≤ tasks2.start ⇔ in_set(tasks1.key, tasks2.before)</li> <li>tasks1.start + tasks1.duration ≤ tasks2.start ⇔ tasks1.position &lt; tasks2.position</li> </ul>
Graph property(ies)	NARC =  TASKS  *  TASKS  -  TASKS

## Graph model

We generate a *clique* with a non-overlapping constraint between each pair of distinct tasks and state that the number of arcs of the final graph should be equal to the number of arcs of the initial graph. For two tasks  $t_1$  and  $t_2$ , the three conditions of the arc constraint respectively correspond to:

- The fact that  $t_1$  ends before the start of  $t_2$  or that  $t_2$  ends before the start of  $t_1$ .
- The equivalence between the fact that  $t_1$  ends before the start of  $t_2$  and the fact that the identifier of task  $t_1$  belongs to the before attribute of task  $t_2$ .
- The equivalence between the fact that  $t_1$  ends before the start of  $t_2$  and the fact that the position attribute of task  $t_1$  is strictly less than the position attribute of task  $t_2$ .

Parts (A) and (B) of Figure 5.274 respectively show the initial and final graph associated with the **Example** slot. The disj constraint holds since all the arcs of the initial graph belong to the final graph: all the non-overlapping constraints holds.

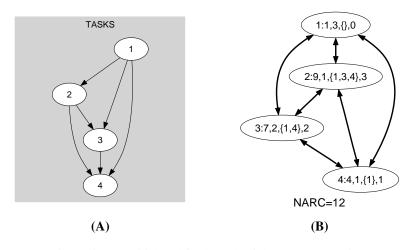


Figure 5.274: Initial and final graph of the disj constraint

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