5.24 among_diff_0

DESCRIPTION LINKS GRAPH AUTOMATON

Origin

Used in the automaton of nvalue.

Constraint

among_diff_0(NVAR, VARIABLES)

Arguments

NVAR	:	dvar
VARIABLES	:	collection(var−dvar)

Restrictions

$\mathtt{NVAR} \geq 0$
$\mathtt{NVAR} \leq |\mathtt{VARIABLES}|$
required(VARIABLES, var)

Purpose

NVAR is the number of variables of the collection VARIABLES that take a value different from 0.

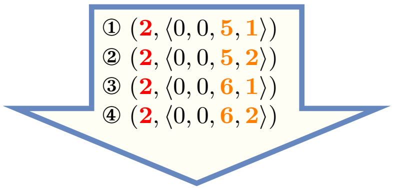
Example

$(3, \langle 0, 5, 5, 0, 1\rangle)$
$(0, \langle 0, 0, 0, 0, 0\rangle)$
$(1, \langle 0, 0, 0, 6, 0\rangle)$

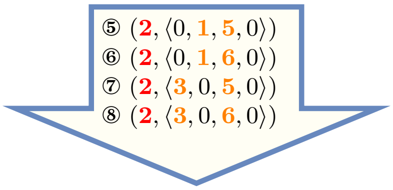
The first among_diff_0 constraint holds since exactly 3 values of the collection of values $\langle 0, 5, 5, 0, 1\rangle$ are different from 0.

All solutions

Figure 5.60 gives all solutions to the following non ground instance of the among_diff_0 constraint: $\mathtt{V}_1 \in \{0, 3\}$, $\mathtt{V}_2 \in [0, 1]$, $\mathtt{V}_3 \in [5, 6]$, $\mathtt{V}_4 \in [0, 2]$, among_diff_0$(\mathbf{2}, \langle \mathtt{V}_1, \mathtt{V}_2, \mathtt{V}_3, \mathtt{V}_4\rangle)$.



① $(\mathbf{2}, \langle 0, 0, \mathbf{5}, \mathbf{1}\rangle)$
② $(\mathbf{2}, \langle 0, 0, \mathbf{5}, \mathbf{2}\rangle)$
③ $(\mathbf{2}, \langle 0, 0, \mathbf{6}, \mathbf{1}\rangle)$
④ $(\mathbf{2}, \langle 0, 0, \mathbf{6}, \mathbf{2}\rangle)$



⑤ $(\mathbf{2}, \langle 0, \mathbf{1}, \mathbf{5}, 0\rangle)$
⑥ $(\mathbf{2}, \langle 0, \mathbf{1}, \mathbf{6}, 0\rangle)$
⑦ $(\mathbf{2}, \langle \mathbf{3}, 0, \mathbf{5}, 0\rangle)$
⑧ $(\mathbf{2}, \langle \mathbf{3}, 0, \mathbf{6}, 0\rangle)$

Figure 5.60: All solutions corresponding to the non ground example of the among_diff_0 constraint of the **All solutions** slot, where the number of variables assigned a value different from zero is equal to $\mathtt{NVAR} = \mathbf{2}$

Typical

$\mathtt{NVAR} > 0$
$\mathtt{NVAR} < |\mathtt{VARIABLES}|$
$|\mathtt{VARIABLES}| > 1$
atleast(1, VARIABLES, 0)
$2 * \mathtt{among\_diff\_0}(\mathtt{VARIABLES.var}) > |\mathtt{VARIABLES}|$

Symmetries

- Items of `VARIABLES` are permutable.
- An occurrence of a value of `VARIABLES.var` that is different from 0 can be replaced by any other value that is also different from 0.

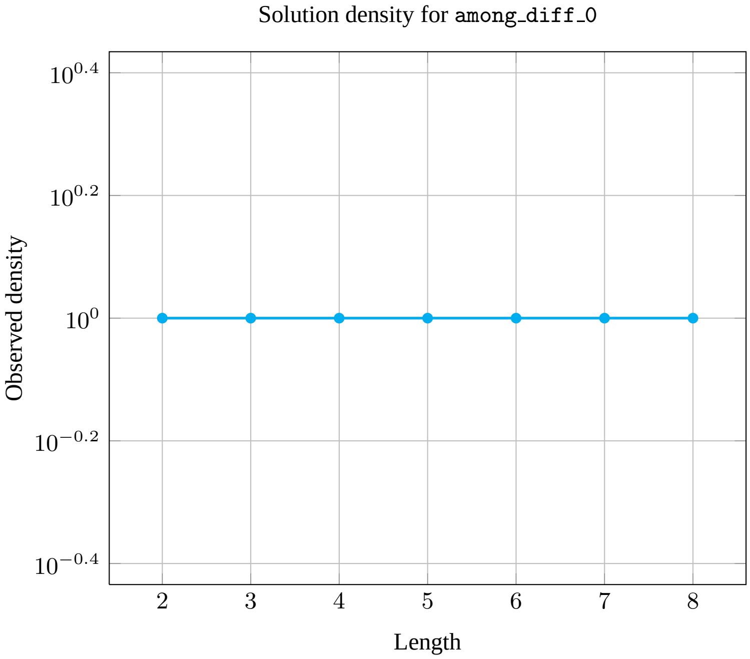
Arg. properties

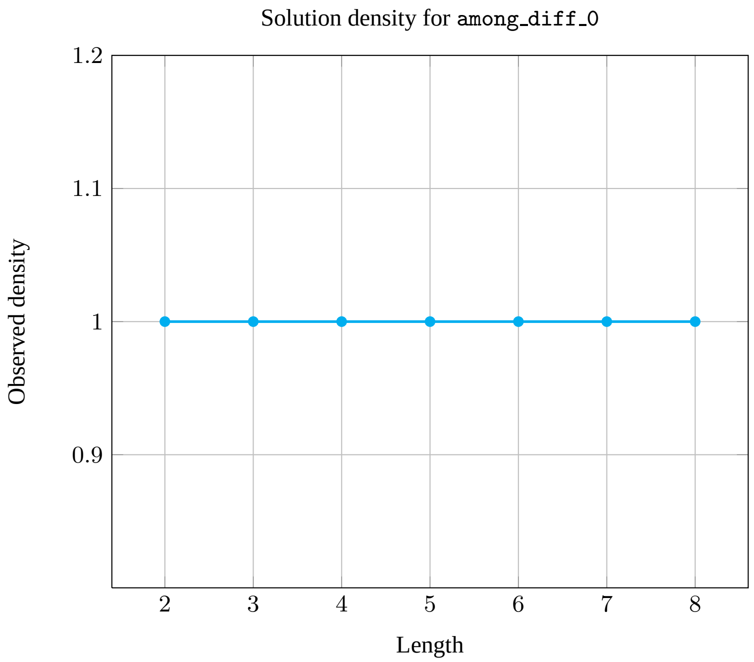
- Functional dependency: `NVAR` determined by `VARIABLES`.
- Contractible wrt. `VARIABLES` when `NVAR` $= 0$.
- Contractible wrt. `VARIABLES` when `NVAR` $= |$`VARIABLES`$|$.
- Aggregate: `NVAR`$(+)$, `VARIABLES`(`union`).

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

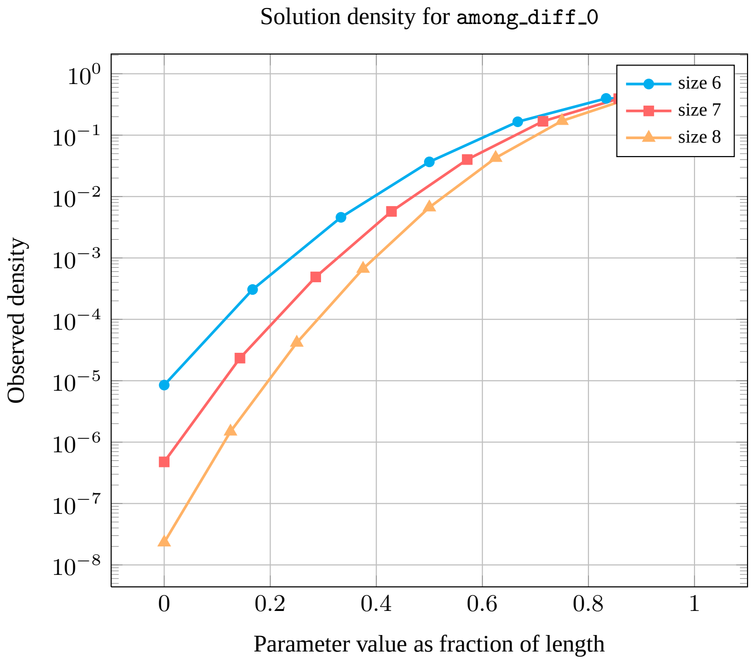
Number of solutions for `among_diff_0`: domains $0..n$

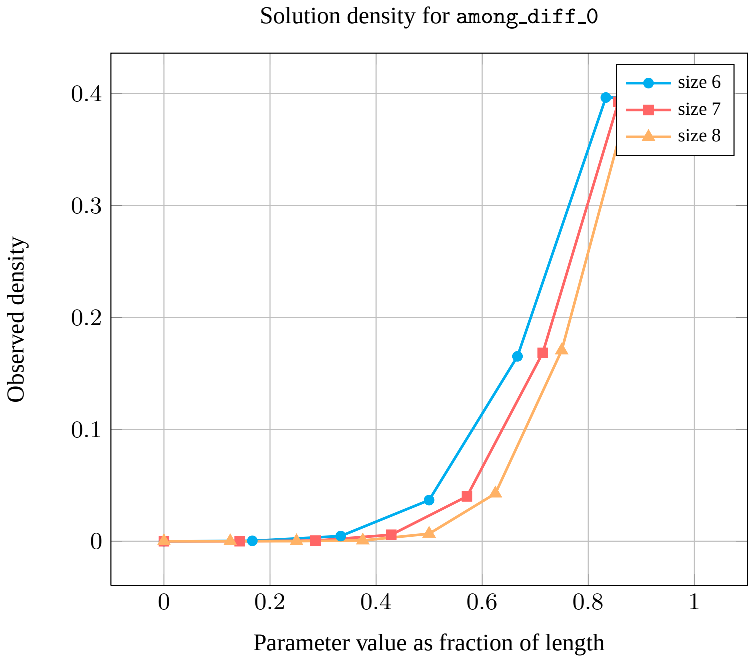




Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	1	1	1	1	1	1	1
	1	4	9	16	25	36	49	64
	2	4	27	96	250	540	1029	1792
	3	-	27	256	1250	4320	12005	28672
	4	-	-	256	3125	19440	84035	286720
	5	-	-	-	3125	46656	352947	1835008
	6	-	-	-	-	46656	823543	7340032
	7	-	-	-	-	-	823543	16777216
	8	-	-	-	-	-	-	16777216

Solution count for among_diff_0: domains $0..n$





See also

common keyword: `nvalue` *(counting constraint)*.

generalisation: `among` *(*`variable` $\neq 0$ *replaced by* `variable` $\in$ `values`*)*.

Keywords

characteristic of a constraint: joker value, automaton, automaton with counters.

constraint arguments: pure functional dependency.

constraint network structure: alpha-acyclic constraint network(2).

constraint type: value constraint, counting constraint.

filtering: arc-consistency.

modelling: functional dependency.

Arc input(s)	`VARIABLES`
Arc generator	$SELF \mapsto$ `collection(variables)`
Arc arity	1
Arc constraint(s)	`variables.var` $\neq 0$
Graph property(ies)	**NARC**= `NVAR`

Graph model Since this is a unary constraint we employ the $SELF$ arc generator in order to produce an initial graph with a single loop on each vertex.

Parts (A) and (B) of Figure 5.61 respectively show the initial and final graph associated with first example of the **Example** slot. Since we use the **NARC** graph property, the loops of the final graph are stressed in bold.

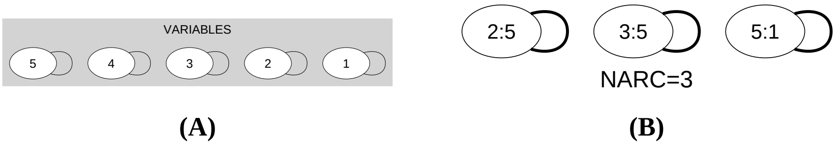


Figure 5.61: Initial and final graph of the `among_diff_0` constraint

Automaton Figure 5.62 depicts the automaton associated with the among_diff_0 constraint. To each variable $\texttt{VAR}_i$ of the collection VARIABLES corresponds a 0-1 signature variable S_i. The following signature constraint links $\texttt{VAR}_i$ and S_i: $\texttt{VAR}_i \neq 0 \Leftrightarrow S_i$. The automaton counts the number of variables of the VARIABLES collection that take a value different from 0 and finally assigns this number to NVAR.

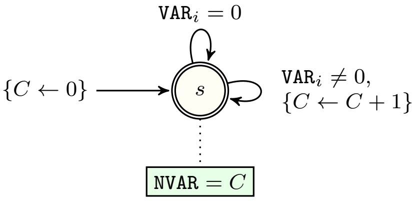


Figure 5.62: Automaton of the among_diff_0 constraint

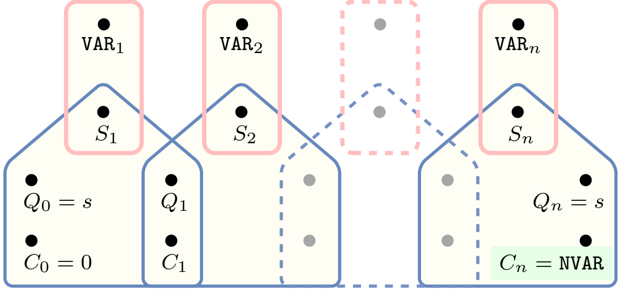


Figure 5.63: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the among_diff_0 constraint: since all states variables $Q_0, Q_1, \ldots, Q_n$ are fixed to the unique state s of the automaton, the transitions constraints share only the counter variable C and the constraint network is Berge-acyclic