5.201 inverse_offset

DESCRIPTION LINKS GRAPH

Origin Gecode

Constraint inverse_offset(SOFFSET, POFFSET, NODES)

Synonym channel.

Arguments

SOFFSET : int
POFFSET : int
NODES : collection(index−int, succ−dvar, pred−dvar)

Restrictions

required(NODES, [index, succ, pred])
NODES.index ≥ 1
NODES.index $\leq$ |NODES|
distinct(NODES, index)
NODES.succ $\geq 1 +$ SOFFSET
NODES.succ $\leq$ |NODES| + SOFFSET
NODES.pred $\geq 1 +$ POFFSET
NODES.pred $\leq$ |NODES| + POFFSET

Purpose

Enforce each vertex of a digraph to have exactly one predecessor and one successor. In addition the following two statements are equivalent:

1. The successor of the i^{th} node minus **SOFFSET** is equal to j.
2. The predecessor of the j^{th} node minus **POFFSET** is equal to i.

I.e., $\texttt{NODES}[i].\texttt{succ} - \textbf{SOFFSET} = j \Leftrightarrow \texttt{NODES}[j].\texttt{pred} - \textbf{POFFSET} = i$.

Example

$$\left(\begin{array}{l} -1, 0, \left\langle \begin{array}{lll} \texttt{index}-1 & \texttt{succ}-4 & \texttt{pred}-3, \\ \texttt{index}-2 & \texttt{succ}-2 & \texttt{pred}-5, \\ \texttt{index}-3 & \texttt{succ}-0 & \texttt{pred}-2, \\ \texttt{index}-4 & \texttt{succ}-6 & \texttt{pred}-8, \\ \texttt{index}-5 & \texttt{succ}-1 & \texttt{pred}-1, \\ \texttt{index}-6 & \texttt{succ}-7 & \texttt{pred}-7, \\ \texttt{index}-7 & \texttt{succ}-5 & \texttt{pred}-4, \\ \texttt{index}-8 & \texttt{succ}-3 & \texttt{pred}-6 \end{array} \right\rangle \end{array}\right)$$

The inverse_offset constraint holds since:

- $\texttt{NODES}[1].\texttt{succ} - (-1) = 5 \Leftrightarrow \texttt{NODES}[5].\texttt{pred} - 0 = 1$,
- $\texttt{NODES}[2].\texttt{succ} - (-1) = 3 \Leftrightarrow \texttt{NODES}[3].\texttt{pred} - 0 = 2$,
- $\texttt{NODES}[3].\texttt{succ} - (-1) = 1 \Leftrightarrow \texttt{NODES}[1].\texttt{pred} - 0 = 3$,
- $\texttt{NODES}[4].\texttt{succ} - (-1) = 7 \Leftrightarrow \texttt{NODES}[7].\texttt{pred} - 0 = 4$,
- $\texttt{NODES}[5].\texttt{succ} - (-1) = 2 \Leftrightarrow \texttt{NODES}[2].\texttt{pred} - 0 = 5$.

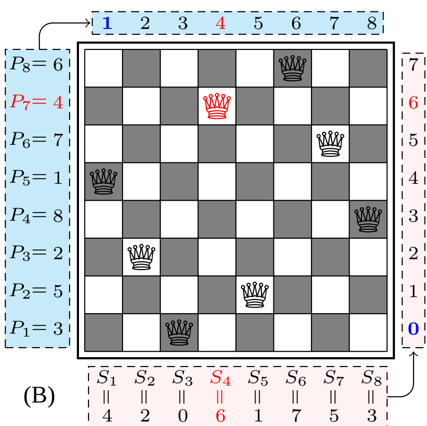
- $\texttt{NODES}[6].\texttt{succ} - (-1) = 8 \Leftrightarrow \texttt{NODES}[8].\texttt{pred} - 0 = 6$.
- $\texttt{NODES}[7].\texttt{succ} - (-1) = 6 \Leftrightarrow \texttt{NODES}[6].\texttt{pred} - 0 = 7$.
- $\texttt{NODES}[8].\texttt{succ} - (-1) = 4 \Leftrightarrow \texttt{NODES}[4].\texttt{pred} - 0 = 8$.

Figure 5.467 shows the board that can be associated with this example.

(A)

NODES

①	index − 1	succ − 4	pred − 3
②	index − 2	succ − 2	pred − 5
③	index − 3	succ − 0	pred − 2
④	index − 4	succ − 6	pred − 8
⑤	index − 5	succ − 1	pred − 1
⑥	index − 6	succ − 7	pred − 7
⑦	index − 7	succ − 5	pred − 4
⑧	index − 8	succ − 3	pred − 6



(B)

$\texttt{SOFFSET} = 1 \quad \texttt{POFFSET} = 0$

$S_1 - 1 = 5 \Leftrightarrow P_5 - 0 = 1$

$S_2 - 1 = 3 \Leftrightarrow P_3 - 0 = 2$

$S_3 - 1 = 1 \Leftrightarrow P_1 - 0 = 3$

$S_4 - 1 = 7 \Leftrightarrow P_7 - 0 = 4$

$S_5 - 1 = 2 \Leftrightarrow P_2 - 0 = 5$

$S_6 - 1 = 8 \Leftrightarrow P_8 - 0 = 6$

$S_7 - 1 = 6 \Leftrightarrow P_6 - 0 = 7$

$S_8 - 1 = 4 \Leftrightarrow P_4 - 0 = 8$

(C)

Figure 5.467: **Example** slot where we highlight the fourth item in red showing the relation between S_4 and P_7, where S_i and P_i (with $1 \leq i \leq 8$) respectively stands for the successor and predecessor attributes of the $i^{\texttt{th}}$ item of the NODES collection (A) Collection of nodes passed to the inverse_offset constraint, (B) Corresponding board, (C) Conditions linking the successor and the predecessor attributes via the offsets $\texttt{SOFFSET} = 1$ and $\texttt{POFFSET} = 0$.

Typical

$\texttt{SOFFSET} \geq -1$
$\texttt{SOFFSET} \leq 1$
$\texttt{POFFSET} \geq -1$
$\texttt{POFFSET} \leq 1$
$|\texttt{NODES}| > 1$

Symmetry

Items of NODES are permutable.

Arg. properties

- Functional dependency: `NODES.succ` determined by `SOFFSET`, `POFFSET`, `NODES.index` and `NODES.pred`.
- Functional dependency: `NODES.pred` determined by `SOFFSET`, `POFFSET`, `NODES.index` and `NODES.succ`.

Remark

The `inverse_offset` constraint is called `channel` in **Gecode** (`http://www.gecode.org/`). Having two offsets was motivated by the fact that it is possible to declare arrays at any position in the **MiniZinc** modelling language.

Systems

`inverseChanneling` in **Choco**, `channel` in **Gecode**.

See also

specialisation: `inverse` *(assume that* `SOFFSET` *and* `POFFSET` *are both equal to* 0*)*.

Keywords

constraint arguments: pure functional dependency.

constraint type: graph constraint.

filtering: arc-consistency.

heuristics: heuristics.

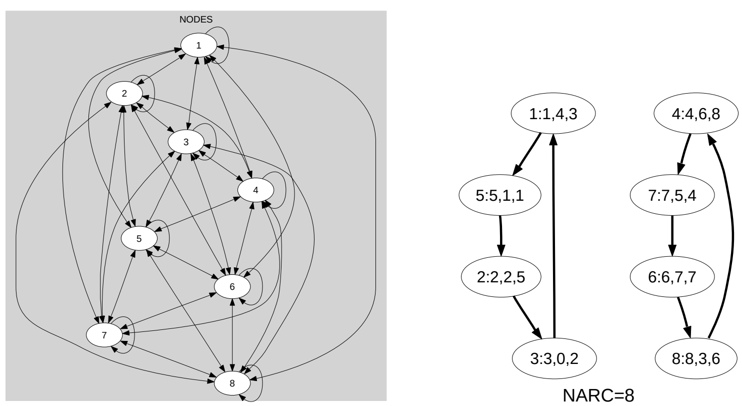
modelling: channelling constraint, dual model, functional dependency.

Arc input(s)	NODES
Arc generator	$CLIQUE \mapsto$ collection(nodes1, nodes2)
Arc arity	2
Arc constraint(s)	• nodes1.succ − SOFFSET = nodes2.index • nodes2.pred − POFFSET = nodes1.index
Graph property(ies)	**NARC**= |NODES|

Graph model In order to express the binary constraint that links two vertices one has to make explicit the identifier of the vertices. This is why the `inverse_offset` constraint considers objects that have three attributes:

- One fixed attribute `index` that is the identifier of the vertex,
- One variable attribute `succ` that is the successor of the vertex,
- One variable attribute `pred` that is the predecessor of the vertex.

Parts (A) and (B) of Figure 5.468 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold.



(A) **(B)**

Figure 5.468: Initial and final graph of the `inverse_offset` constraint