

5.276 next_element

	DESCRIPTION	LINKS	GRAPH	AUTOMATON
Origin	N. Beldiceanu			
Constraint	next_element(THRESHOLD, INDEX, TABLE, VAL)			
Arguments	THRESHOLD : dvar INDEX : dvar TABLE : collection(index-int, value-dvar) VAL : dvar			
Restrictions	INDEX ≥ 1 INDEX ≤ TABLE THRESHOLD < INDEX required(TABLE, [index, value]) TABLE > 0 TABLE.index ≥ 1 TABLE.index ≤ TABLE distinct(TABLE, index)			
Purpose	INDEX is the smallest entry of TABLE strictly greater than THRESHOLD containing value VAL.			
Example	$\left(2, 3, \left\langle \begin{array}{ll} \text{index} - 1 & \text{value} - 1, \\ \text{index} - 2 & \text{value} - 8, \\ \text{index} - 3 & \text{value} - 9, \\ \text{index} - 4 & \text{value} - 5, \\ \text{index} - 5 & \text{value} - 9 \end{array} \right\rangle, 9 \right)$			
	The next_element constraint holds since 3 is the smallest entry located after entry 2 that contains value 9.			
Typical	TABLE > 1 range(TABLE.value) > 1			
Usage	Originally introduced for modelling the fact that a nucleotide has to be consumed as soon as possible at cycle INDEX after a given cycle represented by variable THRESHOLD.			
See also	related: minimum_greater_than (identify an element in a table), next_greater_element (allow to iterate over the values of a table).			
Keywords	characteristic of a constraint: minimum, automaton, automaton without counters, reified automaton constraint, derived collection. constraint network structure: centered cyclic(3) constraint network(1). constraint type: data constraint. modelling: table.			

Derived Collection	$\text{col} \left(\begin{array}{l} \text{ITEM} - \text{collection}(\text{index} - \text{dvar}, \text{value} - \text{dvar}), \\ [\text{item}(\text{index} - \text{THRESHOLD}, \text{value} - \text{VAL})] \end{array} \right)$
Arc input(s)	ITEM TABLE
Arc generator	$\text{PRODUCT} \mapsto \text{collection}(\text{item}, \text{table})$
Arc arity	2
Arc constraint(s)	<ul style="list-style-type: none">• $\text{item.index} < \text{table.index}$• $\text{item.value} = \text{table.value}$
Graph property(ies)	$\text{NARC} > 0$
Sets	$\text{SUCC} \mapsto \left[\begin{array}{l} \text{source}, \\ \text{variables} - \text{col} \left(\begin{array}{l} \text{VARIABLES} - \text{collection}(\text{var} - \text{dvar}), \\ [\text{item}(\text{var} - \text{TABLE.index})] \end{array} \right) \end{array} \right]$
Constraint(s) on sets	$\text{minimum}(\text{INDEX}, \text{variables})$

Graph model Parts (A) and (B) of Figure 5.594 respectively show the initial and final graph associated with the second graph constraint of the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold.

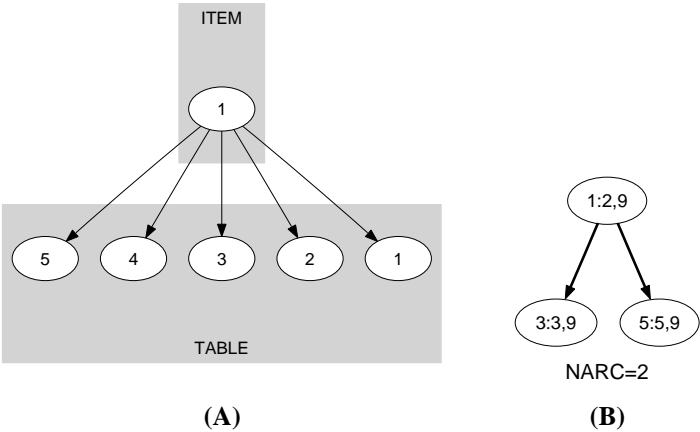


Figure 5.594: Initial and final graph of the next_element constraint

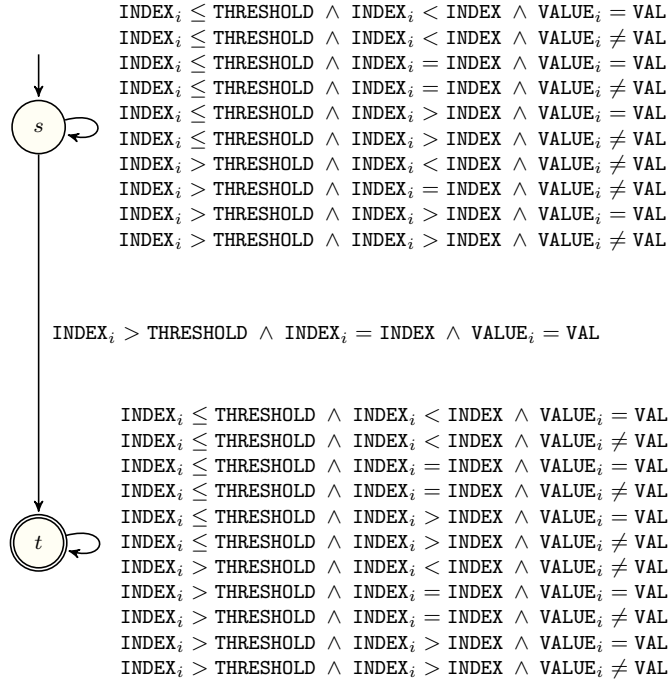
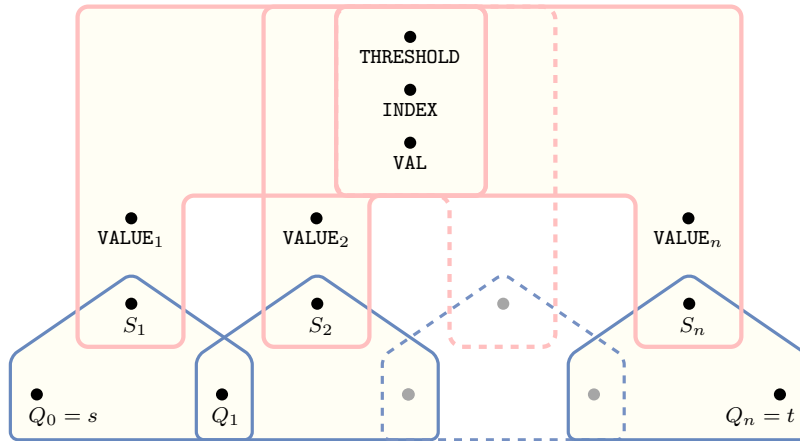
Automaton

Figure 5.595 depicts the automaton associated with the `next_element` constraint. Let I_k and V_k respectively be the `index` and the `value` attributes of the k^{th} item of the `TABLE` collections. To each quintuple $(\text{THRESHOLD}, \text{INDEX}, \text{VAL}, I_k, V_k)$ corresponds a signature variable S_k as well as the following signature constraint:

$$\begin{aligned}
((I_k \leq \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 0 \wedge \\
((I_k \leq \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 1 \wedge \\
((I_k \leq \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 2 \wedge \\
((I_k \leq \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 3 \wedge \\
((I_k \leq \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 4 \wedge \\
((I_k \leq \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 5 \wedge \\
((I_k > \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 6 \wedge \\
((I_k > \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 7 \wedge \\
((I_k > \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 8 \wedge \\
((I_k > \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 9 \wedge \\
((I_k > \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 10 \wedge \\
((I_k > \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 11.
\end{aligned}$$

The automaton is constructed in order to fulfil the following conditions:

- We look for an item of the `TABLE` collection such that $\text{INDEX}_i > \text{THRESHOLD}$ and $\text{INDEX}_i = \text{INDEX}$ and $\text{VALUE}_i = \text{VAL}$,
- There should not exist any item of the `TABLE` collection such that $\text{INDEX}_i > \text{THRESHOLD}$ and $\text{INDEX}_i < \text{INDEX}$ and $\text{VALUE}_i = \text{VAL}$.

Figure 5.595: Automaton of the `next_element` constraintFigure 5.596: Hypergraph of the reformulation corresponding to the automaton of the `next_element` constraint