11.1 Some Basics

- Know how to sketch xy-plane, xz-plane, yz-plane
- 3 axes divide 3D space into 8 octants, the first octant is where x,y,z > 0
- Know how to plot a point in 3D space (First locate x,y on xy-plane, then go up/down)
- Distance formula: $P = (x_0, y_0, z_0), Q = (x_1, y_1, z_1)$

dist =
$$|PQ| = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}$$

- Circle/Disk vs. Sphere/Closed. Distinguish equations for them and sketch
- Sphere: Supposed $P=(x_0,y_0,z_0)$ is the center of a sphere and R is the radius $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$

11.2 Vectors

- Notation: $\bar{u} = 8\hat{i} + 2\hat{j} 3\hat{k}$ where $\hat{i} = (1,0,0)$, $\hat{j} = (0,1,0)$, $\hat{k} = (0,0,1)$
- Note: Write $\bar{v} = 4\hat{\imath} + 0\hat{\jmath} 1\hat{k}$ instead of $\bar{v} = 4\hat{\imath} 1\hat{k}$
- Must know to compute 2D vectors with trigonometry (Memorize values of sin/cos from $0-\pi$)
- Associated Definitions:
 - (A) Zero-vector is all 0's, denoted by $\bar{0}$
 - (B) Supposed P = (x_0, y_0, z_0) and Q = (x_1, y_1, z_1) $\overrightarrow{PQ} = "Q - P" = <math>(x_1 - x_0)\hat{\imath} + (y_1 - y_0)\hat{\jmath} + (z_1 - z_0)\hat{k}$
 - (C) Given $\bar{v}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$, we define the length (aka magnitude/norm): $||\bar{v}||=\sqrt{a^2+b^2+c^2}$
 - (D) A unit vector is a vector with length 1
 - (E) Supposed we have a vector $\bar{\mathbf{v}}$ and we want the unit vector which points in the same direction as $\bar{\mathbf{v}}$:

$$\frac{\overline{v}}{||\overline{v}||}$$

(F) Two nonzero vectors are parallel if one is a scalar multiple of the other

11.3 Dot Product (Scalar Product)

- Definition: Let $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ $\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

$$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

- Product Properties:
 - $(A) \ \bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$
 - (B) $\bar{a} \cdot (\bar{b} \pm \bar{c}) = \bar{a} \cdot \bar{b} \pm \bar{a} \cdot \bar{c}$

(C)
$$\bar{a} \cdot \bar{a} = ||a||^2 = a_1^2 + a_2^2 + a_3^2$$

- Additional Properties:
 - (A) $\bar{a}\cdot\bar{b}=\left||\bar{a}||.\left||\bar{b}|\right|.\cos\theta$, where θ = angle between \bar{a},\bar{b}
 - (B) $\bar{a} \cdot \bar{b} = 0$ iff $\bar{a} \perp \bar{b}$
 - (C) $\cos \theta = \frac{\bar{a} \cdot \bar{b}}{||\bar{a}||.||\bar{b}||}$
- Vector Projection:

$$Proj_{\bar{b}}\bar{a} = \frac{\bar{a}\cdot\bar{b}}{\bar{b}\cdot\bar{b}}.\bar{b}$$

11.4 Cross Product

- Pre-definition: Define $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad bc$
- Definition: Given $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\bar{b} = b_1\hat{i} + b_2\hat{i} + b_3\hat{k}$

$$\begin{split} & \vec{b} = b_1 \hat{\imath} + b_2 \hat{\jmath} + b_3 \hat{k} \\ & \vec{a} \times \vec{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \hat{\imath} + \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \hat{k} \text{ (Trick } \begin{vmatrix} a_2 & a_3 & a_1 & a_2 \\ b_2 & b_3 & b_1 & b_2 \end{vmatrix}) \end{split}$$

- Product Properties:
 - (A) $\bar{a} \times (\bar{b} \pm \bar{c}) = \bar{a} \times \bar{b} \pm \bar{a} \times \bar{c}$
 - (B) $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$ /anticommutativity/
- Additional Properties:
 - (A) $|a \times b| = |\bar{a}| |\bar{b}| \sin \theta$
 - (B) $\bar{a} \times \bar{b} = \bar{0}$ iff \bar{a} and \bar{b} are parallel
 - (C) $\bar{a} \times \bar{b}$ is \bot to both \bar{a} and \bar{b} via right-hand rule!

11.5 Lines in Space

- Idea: Start with a single point $P=(x_0,y_0,z_0)$ and a direction vector $\bar{L}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$ If we attached \bar{L} to P, we see a line that goes forever!
- Parametric form: Suppose we have $P=(x_0,y_0,z_0)$ and $\bar{L}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$, the parametric equations of the corresponding line are:

$$x = x_0 + at$$

$$y = y_0 + bt$$
 where $t = any number$
$$z = z_0 + ct$$

- Vector equation of a line: All we do is put x,y,z from above into a vector:

$$\bar{r}(t) = (x_0 + at)\hat{i} + (y_0 + bt)\hat{j} + (z_0 + ct)\hat{k}$$

- Symmetric Equation:
 - + Normal case $(a, b, c \neq 0)$ $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$
 - + Special case A: (either one of a,b,c=0)

Ex:
$$P = (1,2,3)$$
 and $\bar{L} = 0\hat{i} + 8\hat{j} + 7\hat{k}$

Here's the parametric form: x = 1 + 0t

$$y = 2 + 8t$$

$$z = 3 + 7t$$

=> Symmetric form: $\frac{y-2}{8} = \frac{z-3}{7}, x = 1$

+ Special case B: (2 of a,b,c=0)

Ex: P = (1,2,3) and $\bar{L} = 42\hat{i} + 0\hat{j} + 0\hat{k}$

Here's the parametric form: x = 1 + 42t

$$y = 2$$
$$z = 3$$

=> Symmetric form: y = 2, z = 3 (No need to mention x)

- Distance between a point and line: Suppose we have a line with point P and direction \bar{L} and suppose Q is some other point, then the perpendicular distance from Q to the line:

 $dist = \frac{||\overline{PQ} imes \overline{L}||}{||\overline{L}||}$ (Note: We can extract P and \overline{L} from a line given its form)