Homework 1
COSC 362

Instructions: Work individually to answer the following questions.

1. Let $\Sigma = \{a, b\}$, find a regular expression for $L = \{ab^2wb^2 : w \in \{a, b\}^*\}$.
2. Let $\Sigma = \{a, b\}$, find a regular expression for $L = \{w : |w| \equiv 2 \pmod 3\}$.
3. Let $\Sigma = \{a, b\}$, find a regular expression for $L = \{w : w \in \{a, b\}^*, w$ has exactly one pair of consecutive a's$\}$.
4. Let $\Sigma = \{a, b\}$, find a DFA for $L = \{ab^2wb^2 : w \in \{a, b\}^*\}$.
5. Let $\Sigma = \{a, b\}$, find a DFA for $L = \{w : |w| \equiv 2 \pmod 3\}$.
6. Let $\Sigma = \{a, b\}$, find a DFA for $L = \{w : n_a(w) \mod 3 < n_b(w) \mod 3\}$.
7. Let $\Sigma = \{a, b\}$, find a DFA to accept all strings of length $3n$, $n = 0, 1, 2, \cdots$
8. Consider the DFA $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \delta, q_2, \{q_2\})$ and

$$\begin{array}{ll} \delta(q_0, 0) = q_0 & \delta(q_0, 1) = q_1 \\ \delta(q_1, 0) = q_0 & \delta(q_1, 1) = q_2 \\ \delta(q_2, 0) = q_1 & \delta(q_2, 1) = q_3 \\ \delta(q_3, 0) = q_2 & \delta(q_3, 1) = q_4 \\ \delta(q_4, 0) = q_3 & \delta(q_4, 1) = q_4 \end{array}$$

   (a) Draw the transition diagram for M.

   (b) Find a string of length at least five and show that it is in the language of M by showing each step of the machine when given that string as input.

   (c) Find a string of length at least five and show that it is *not* in the language of M by showing each step of the machine when given that string as input.

Answers:

1. $ab^a(a|b)^*b^2$

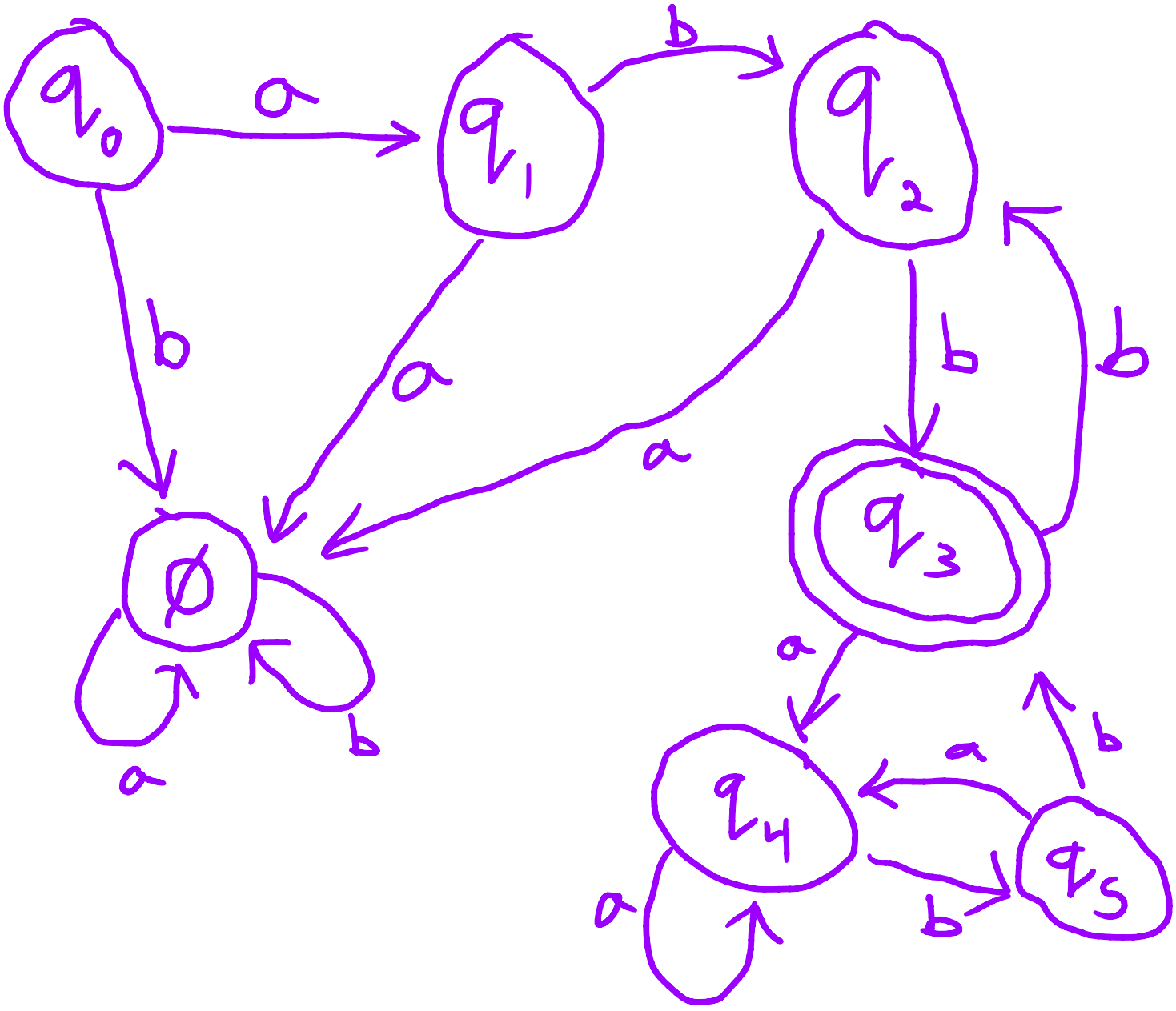
2. $((a|b)^3)^*(a|b)^2$

or

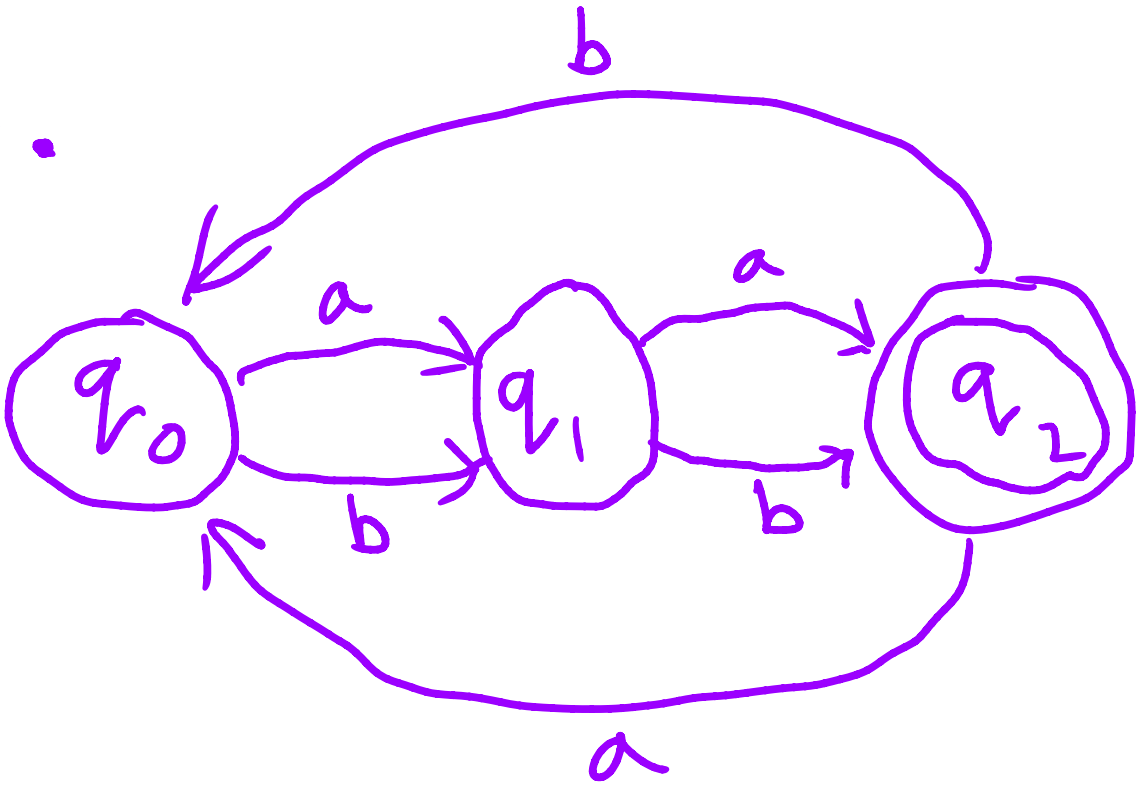
$(a|b)^{3n-1}$ where $n \geq 1$

3. $(b^*ab^*)^*aa(b^*ab^*)^*$

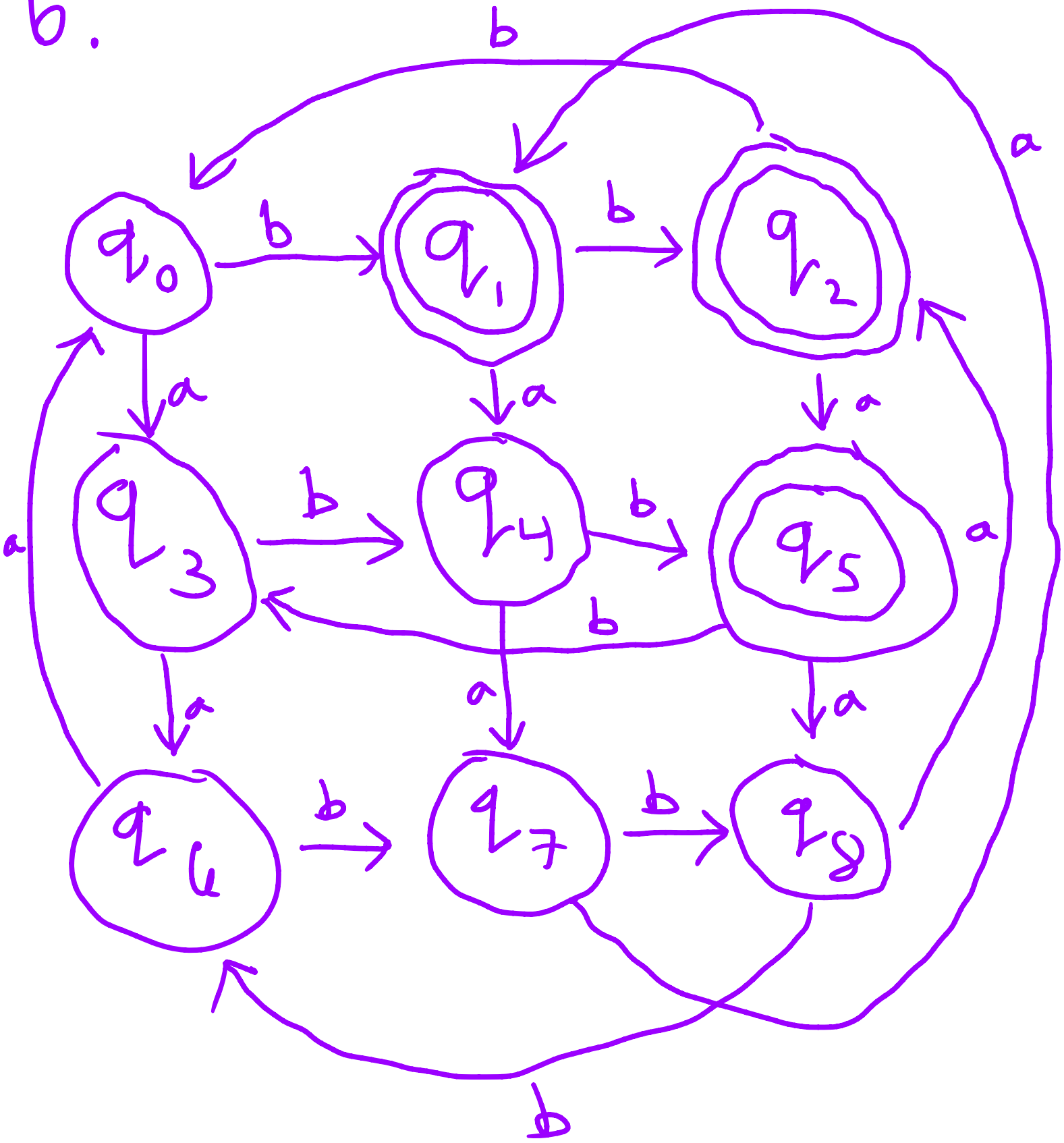
4.



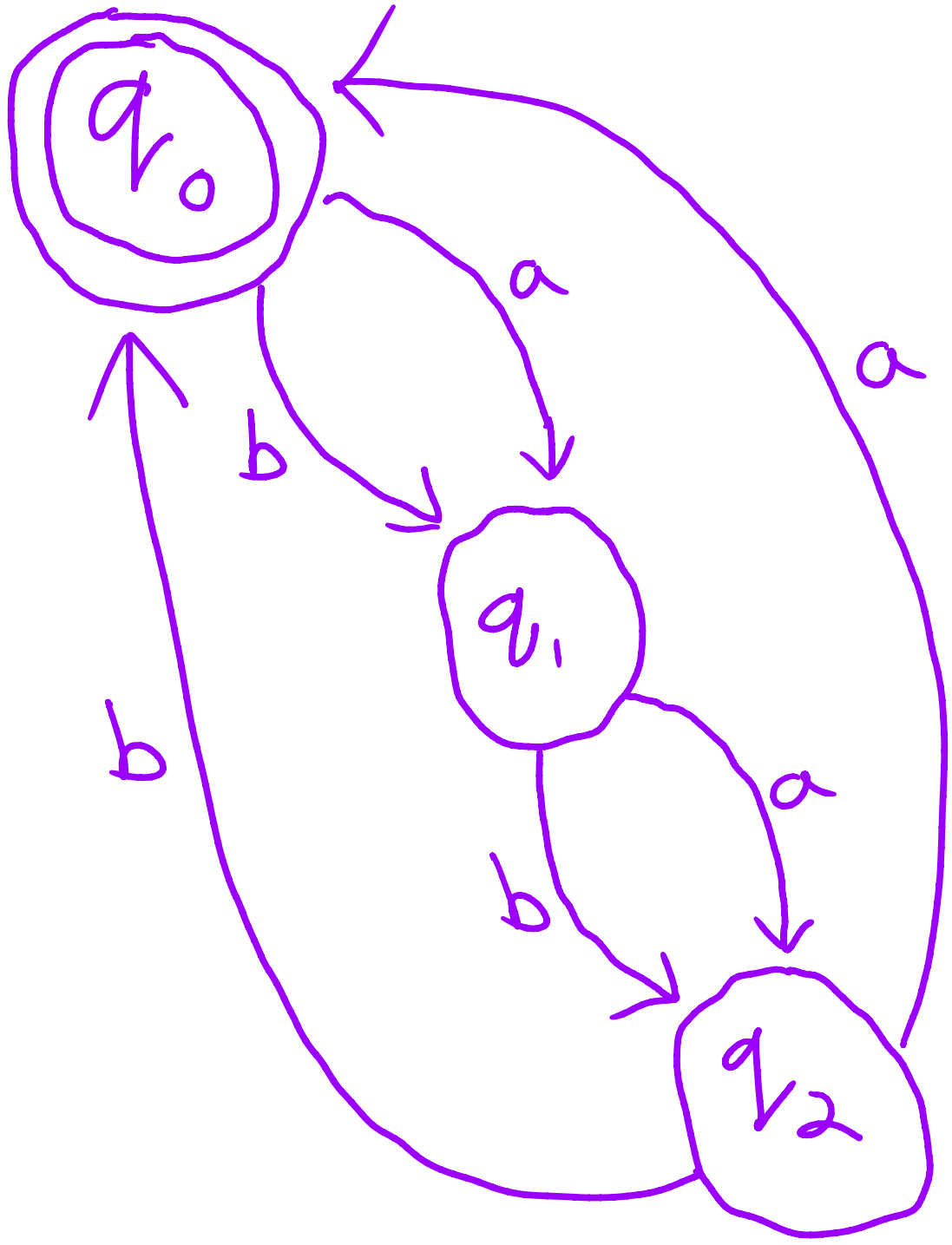
5.



6.

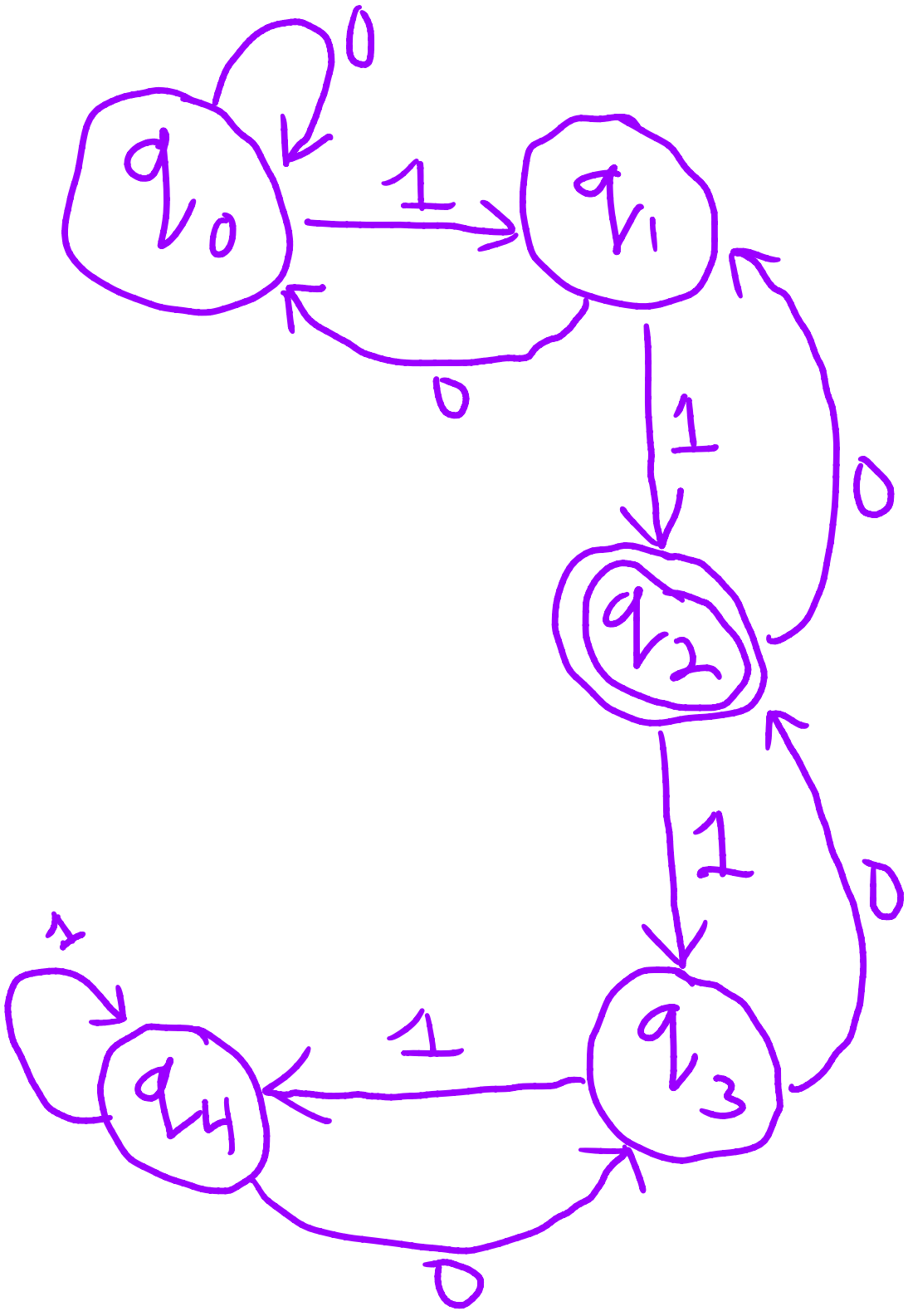


7.



8.

a)



8.

b) $\boxed{0101110} = w, |w| = 7$

Machine steps:

$q_0, q_0, q_1, q_0, q_1, q_2, q_3, q_2$ – accepted

c) $\boxed{1111101} = w, |w| = 7$

Machine steps:

$q_0, q_1, q_2, q_3, q_4, q_4, q_3, q_4$