Prove that the language: L = $\{a^m b^n c^m \mid m,n \geq 1\}$ is not a regular language.

Step 1: Assumption of Regularity

We will prove this by contradiction. So, assume L is regular.

Step 2: Apply Pumping Lemma

Since L is regular, there exists a pumping length m such that for any string w in L with $|w| \geq m$, we can split w into three parts: w = xyz

Such that,

1. $|xy| \leq m$ (the pumping occurs within the first m characters)
2. $|y| \geq 1$ (the y section is non-empty)
3. $xy^i z \in$ L for all $i \geq 0$ (after i repetitions of y, the string belongs to L)

Step 3: Choose a String in L

Let's choose the string w = $a^m b^n c^m$ where n ≥ 1.

The choice of w ensures that the length is at least m; thus, this satisfies the conditions of the Pumping Lemma.

Step 4: Split w = xyz

Given that the first m characters of the string are a and |xy| ≤ m, we know xy consists of only a's

Thus, we can split w = xyz such that

1. |xy| ≤ m (xy consists only of a's)
2. |y| ≥ 1 (y contains at least one a)

Thus, we set the following:

- x = a^s
- y = a^t

- z = $a^{m-s-t}b^{n}c^{m}$

Step 5: Pumping the y Term

Using the Pumping Lemma, we know $xy^{i}z \in L$.

Since y = a^{t}, if we set i = 2, we get that

w' = $xy^{2}z$ = $a^{s}a^{2t}a^{m-s-t}b^{n}c^{m}$ = $a^{m+t}b^{n}c^{m}$

Now, the number of a's is m + t, and the number of c's remains m.

Step 6: Contradiction

The new string w' has more a's than c's (m + t > m). However, every valid string in L must have exactly the same number of a's and c's as defined by L = { $a^{m}b^{n}c^{m}$ | m,n $\geq$ 1}

Since w' $\notin$ L, we have contradicted the Pumping Lemma assumption. ⇒⇐

Step 7: Conclusion

Since our Pumping Lemma assumption that L is regular led to a contradiction, L cannot be a regular language. <u>Therefore, L is not regular.</u>