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COSC 420

Exercise 03

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Performance Analysis with Amdahl's Law

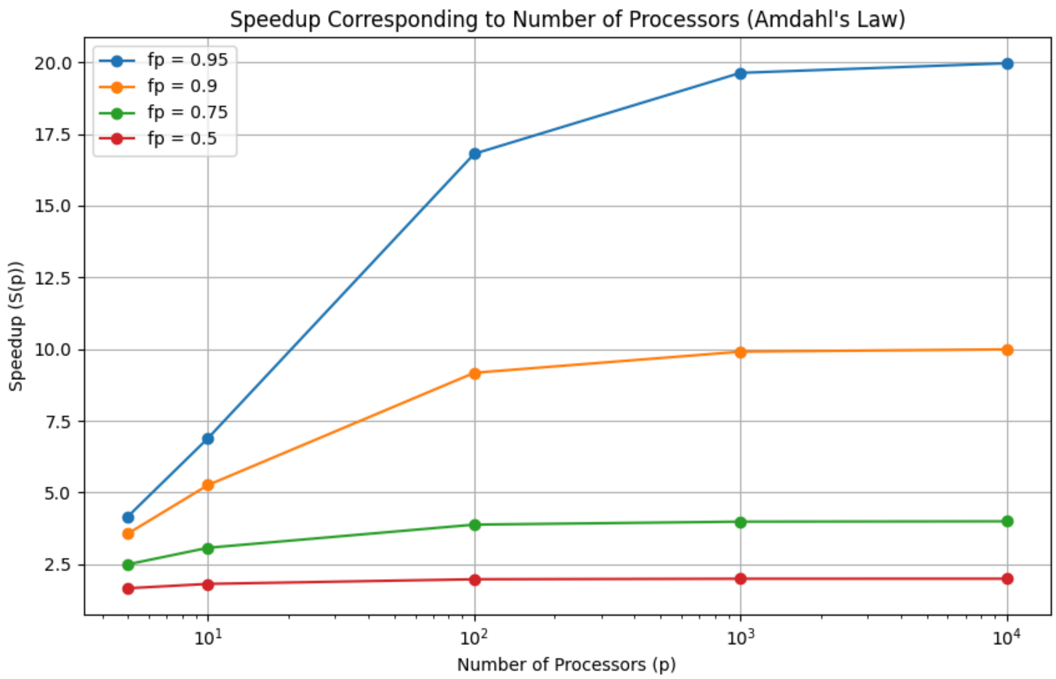
The following was produced using a Python program utilizing matplotlib.pyplot:

Speedup values for fp = 0.95:
S(5) = 4.1667
S(10) = 6.8966
S(100) = 16.8067
S(1000) = 19.6271
S(10000) = 19.9621

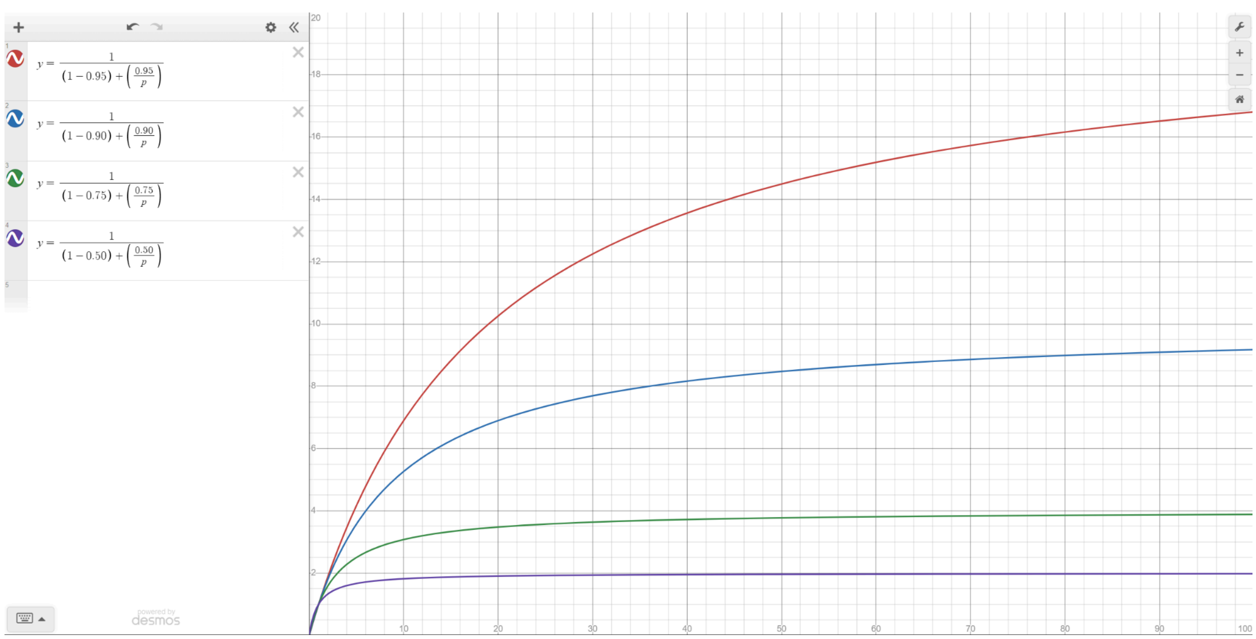
Speedup values for fp = 0.9:
S(5) = 3.5714
S(10) = 5.2632
S(100) = 9.1743
S(1000) = 9.9108
S(10000) = 9.9910

Speedup values for fp = 0.75:
S(5) = 2.5000
S(10) = 3.0769
S(100) = 3.8835
S(1000) = 3.9880
S(10000) = 3.9988

Speedup values for fp = 0.5:
S(5) = 1.6667
S(10) = 1.8182
S(100) = 1.9802
S(1000) = 1.9980
S(10000) = 1.9998



The following secondary line graph was constructed using Demos Online Graphing Calculator:



Performance analysis given the following output:

Serial Program:

summation from 1 to 1000000000: 500000000500000000

runtime: 0.98

2 core program:

MPI runtime Rank 1: 0.5

Summation from 1 to 1000000000: 500000000500000000

MPI runtime Rank 0: 0.5

8 Core program:

MPI runtime Rank 1: 0.23

MPI runtime Rank 7: 0.23

MPI runtime Rank 4: 0.24

MPI runtime Rank 6: 0.24

MPI runtime Rank 5: 0.24

MPI runtime Rank 3: 0.24

MPI runtime Rank 2: 0.25

Summation from 1 to 1000000000: 500000000500000000

MPI runtime Rank 0: 0.25

We can extract the following from the data:

Serial runtime: Tserial = 0.98

2-core parallel runtime: Average Tparallel = (0.5 + 0.5) / 2 = 0.5

8-core parallel runtime: Average Tparallel = (0.23 + 0.23 + 0.24 + 0.24 + 0.24 + 0.24 + 0.25 + 0.25) / 8 = 0.24

(a)

Since speedup is the ratio of the serial runtime to the parallel runtime, we can use

$S(p) = (Ts / Tp)$

Ts is the runtime of the serial program, and Tp is the parallel runtime.

So,

$S(2) = Ts / Tp = 0.98 / 0.5 = 1.96$

$S(8) = Ts / Tp = 0.98 / 0.24 \approx 4.08$

(b)

Since efficiency is the ratio of speedup to the number of processors, which measures how effectively the processors are used, we can use

$E(p) = S(p) / p$

So,

$E(2) = S(2) / 2 = 1.96 / 2 = 0.98 >> 98\%$

$E(8) = S(8) / 8 = 4.08 / 8 \approx 0.51 >> 51\%$

(c)

Since the parallel fraction is the portion of the program that can be parallelized and calculated using Amdahl's Law, we can manipulate Amdahl’s original equation to determine the process speedup for p processors to solve for Fp instead of S(p),

$S(p) = 1/((1-Fp) + (Fp / p))$

$S(p) \bullet ((1-Fp) + (Fp / p)) = 1$

$S(p) - (S(p) \bullet Fp) + (S(p) \bullet Fp / p) = 1$

$S(p) - 1 = (S(p) \bullet Fp) - (S(p) \bullet Fp / p)$

$S(p) - 1 = Fp \bullet (S(p) - (S(p) / p))$

$S(p) - 1 = (Fp \bullet S(p)) \bullet (1 - (1 / p))$

Thus, after isolating Fp, we finally derive

$Fp = (S(p) - 1) / (S(p) \bullet (1 - (1 / p))$

So,

$(S(2) - 1) / (S(2) \bullet (1 - (1 / 2)) = (1.96 - 1) / (1.96 \bullet 0.5) = 0.96 / 0.98 \approx 0.98 >> 98\%$

$(S(8) - 1) / (S(8) \bullet (1 - (1 / 8)) = (4.08 - 1) / (4.08 \bullet 0.875) = 3.08 / 3.57 \approx 0.86 >> 86\%$

(d)

Finally, since we know that $Fp(p) + Fs(p) = 1$ and that Fs is the fraction of the program that is serial and cannot be parallelized, then

$Fs(p) = 1 - Fp(p)$

So,

$Fs(2) = 1 - Fp(2) = 1 - 0.98 = 0.2 >> 2\%$

$Fs(8) = 1 - Fp(8) = 1 - 0.86 = 0.14 >> 14\%$

In summary, we computed the following results:

Speedup (2 cores): 1.96

Speedup (8 cores): 4.08

Efficiency (2 cores): 98%

Efficiency (8 cores): 51%

Parallel Fraction (2 cores): 98%

Parallel Fraction (8 cores): 86%

Serial Fraction (2 cores): 2%

Serial Fraction (8 cores): 14%