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COSC 420

Exercise 03

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### Performance Analysis with Amdahl's Law

The following was produced using a Python program utilizing matplotlib.pyplot:

Speedup values for  $f_p = 0.95$ :

$$S(5) = 4.1667$$

$$S(10) = 6.8966$$

$$S(100) = 16.8067$$

$$S(1000) = 19.6271$$

$$S(10000) = 19.9621$$

Speedup values for  $f_p = 0.9$ :

$$S(5) = 3.5714$$

$$S(10) = 5.2632$$

$$S(100) = 9.1743$$

$$S(1000) = 9.9108$$

$$S(10000) = 9.9910$$

Speedup values for  $f_p = 0.75$ :

$$S(5) = 2.5000$$

$$S(10) = 3.0769$$

$$S(100) = 3.8835$$

$$S(1000) = 3.9880$$

$$S(10000) = 3.9988$$

Speedup values for  $f_p = 0.5$ :

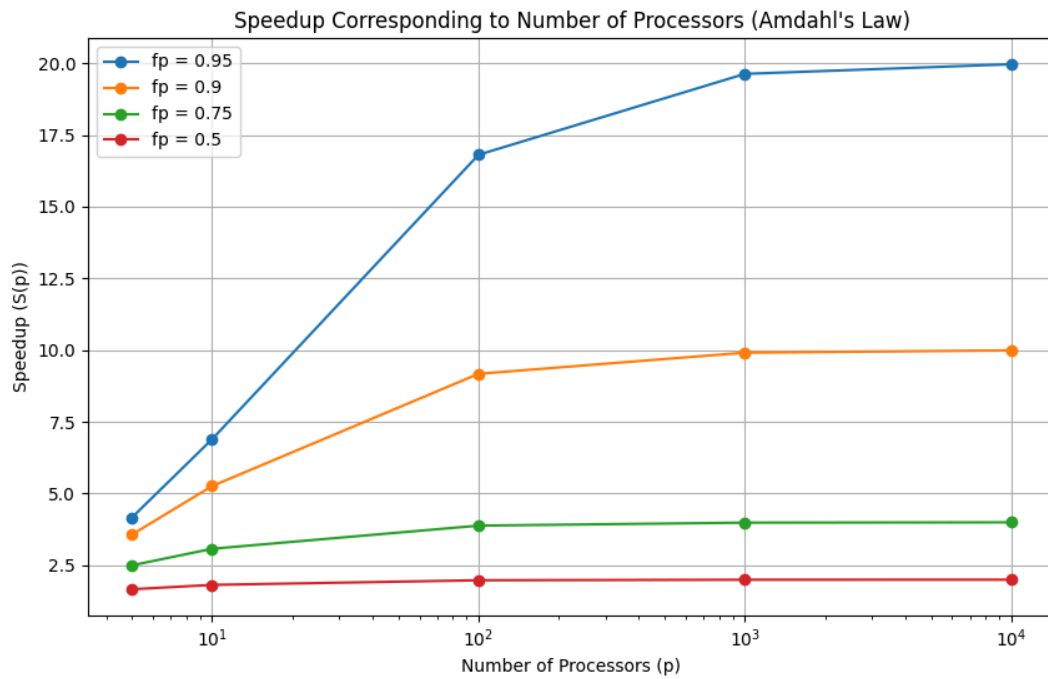
$$S(5) = 1.6667$$

$$S(10) = 1.8182$$

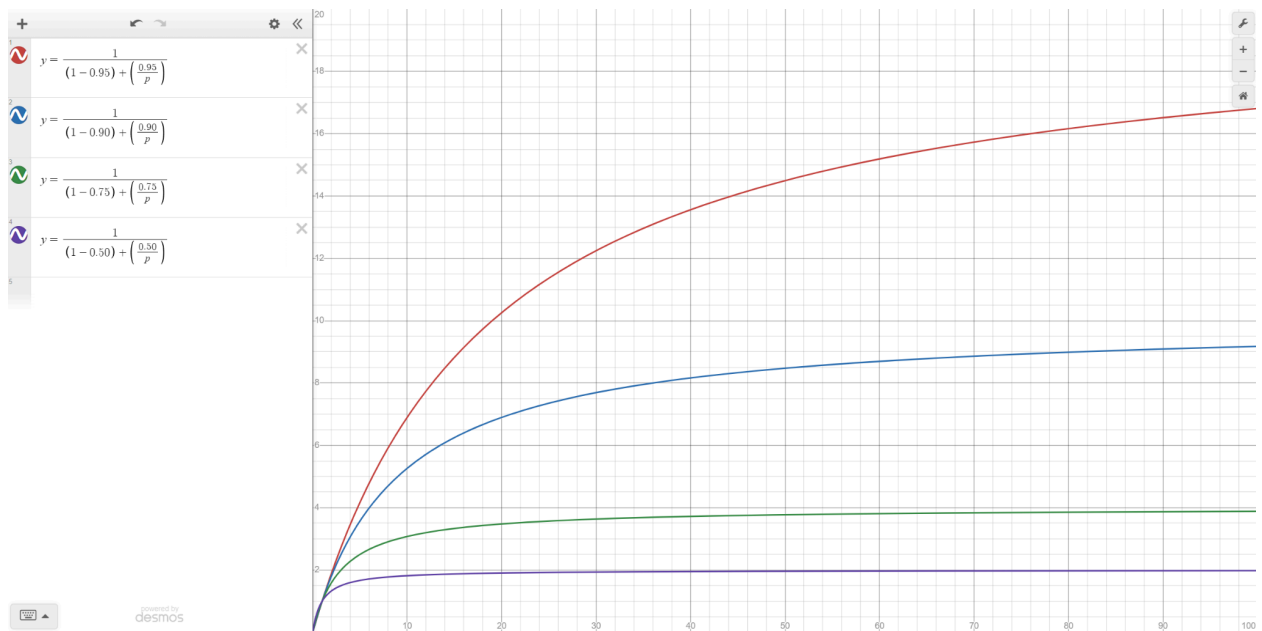
$$S(100) = 1.9802$$

$$S(1000) = 1.9980$$

$$S(10000) = 1.9998$$



The following secondary line graph was constructed using Demos Online Graphing Calculator:



Performance analysis given the following output:

Serial Program:

summation from 1 to 10000000000: 50000000005000000000

runtime: 0.98

2 core program:

MPI runtime Rank 1: 0.5

Summation from 1 to 10000000000: 50000000005000000000

MPI runtime Rank 0: 0.5

8 Core program:

MPI runtime Rank 1: 0.23

MPI runtime Rank 7: 0.23

MPI runtime Rank 4: 0.24

MPI runtime Rank 6: 0.24

MPI runtime Rank 5: 0.24

MPI runtime Rank 3: 0.24

MPI runtime Rank 2: 0.25

Summation from 1 to 10000000000: 50000000005000000000

MPI runtime Rank 0: 0.25

We can extract the following from the data:

Serial runtime:  $T_{\text{serial}} = 0.98$

2-core parallel runtime: Average  $T_{\text{parallel}} = (0.5 + 0.5) / 2 = 0.5$

8-core parallel runtime: Average  $T_{\text{parallel}} = (0.23 + 0.23 + 0.24 + 0.24 + 0.24 + 0.24 + 0.25 + 0.25) / 8 = 0.24$

(a)

Since speedup is the ratio of the serial runtime to the parallel runtime, we can use

$$S(p) = (T_{\text{serial}} / T_{\text{parallel}})$$

$T_{\text{serial}}$  is the runtime of the serial program, and  $T_{\text{parallel}}$  is the parallel runtime.

So,

$$S(2) = T_{\text{serial}} / T_{\text{parallel}} = 0.5 / 0.98 = 1.96$$

$$S(8) = T_{\text{serial}} / T_{\text{parallel}} = 0.98 / 0.24 \approx 4.08$$

(b)

Since efficiency is the ratio of speedup to the number of processors, which measures how effectively the processors are used, we can use

$$E(p) = S(p) / p$$

So,

$$E(2) = S(2) / 2 = 1.96 / 2 = 0.98 \gg 98\%$$

$$E(8) = S(8) / 8 = 4.08 / 8 \approx 0.51 \gg 51\%$$

(c)

Since the parallel fraction is the portion of the program that can be parallelized and calculated using Amdahl's Law, we can use

$$F_p = (S(p) - 1) / (S(p) \cdot (1 - (1 / p)))$$

So,

$$(S(2) - 1) / (S(2) \cdot (1 - (1 / 2))) = (1.96 - 1) / (1.96 \cdot 0.5) = 0.96 / 0.98 \approx 0.98 \gg 98\%$$

$$(S(8) - 1) / (S(8) \cdot (1 - (1 / 8))) = (4.08 - 1) / (4.08 \cdot 0.875) = 3.08 / 3.57 \approx 0.86 \gg 86\%$$

(d)

Finally, since we know that  $F_p + F_s = 1$  and that  $F_s$  is the fraction of the program that is serial and cannot be parallelized, then

$$F_s = 1 - F_p$$

So,

$$F_s(2) = 1 - F_p(2) = 1 - 0.98 = 0.02 \gg 2\%$$

$$F_s(8) = 1 - F_p(8) = 1 - 0.86 = 0.14 \gg 14\%$$

In summary, we computed the following results:

Speedup (2 cores): 1.96

Speedup (8 cores): 4.08

Efficiency (2 cores): 98%

Efficiency (8 cores): 51%

Parallel Fraction (2 cores): 98%

Parallel Fraction (8 cores): 86%

Serial Fraction (2 cores): 2%

Serial Fraction (8 cores): 14%