

Advanced Microeconomic Theory

Exam 19.11.2014

Please answer four (4) questions. Good luck!

1. Why does economics take as the primitive the agents' preferences and not "utilities" or "happiness"? Explain what is that a utility function represents preferences. Are there preferences that cannot be represented by a utility function?  
*observed*  
*rational preferences*
2. Let there be two goods 1 and 2 with prices  $p_1$  and  $p_2$ , and a consumer with monotonic, convex preferences and income  $w$ . The government can finance a budgetary gap  $G > 0$  via either taxing the consumer's income, or via taxing his consumption of good 1. With the income tax  $T$ , the consumable income reduces to  $w - T$ . With the consumption tax, the collected tax is  $tp_1x_1$ , where  $x_1$  is the consumption of good 1 and  $t$  is the tax rate on the value of consumption of good 1. To fill the budget gap, the taxes in the two scenarios have to meet the condition  $T = G$  and  $tp_1x_1(tp_1, p_2, w) = G$ , where  $x_1(\cdot)$  is the Marshallian demand for the good 1. Argue that the consumer prefers an income tax. (Draw a picture!)
3. Consider a two person, two good exchange economy. The initial endowments are  $\omega^1 = (1, 0)$  and  $\omega^2 = (0, 1)$ . The utility functions of the agents 1 and 2 are  $u^1(x_1^1, x_2^1) = x_1^1 + \alpha x_2^1$ ,  $u^2(x_1^2, x_2^2) = x_1^2 x_2^2$ , where  $\alpha > 0$ . Find the set of Pareto efficient allocations and the Walrasian equilibrium and illustrate them in an Edgeworth box. What is the Core of this economy?
4. Explain the content of the Second Theorem of Welfare Economics. Interpret the result. You can illustrate the argument in an Edgeworth box (no need to prove it formally!).
5. Consider one consumer, one firm economy (Robinson Crusoe) where the consumer's utility function is of the form  $u(x, \ell) = x\ell$ , where  $x$  is the consumption good and  $\ell$  is the amount of leisure. Given labor input  $y$ , the firm

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produces the amount  $x = \ln y$  of the consumption good ( $\ln(\cdot)$  is the firm's production function). The resource constraint on the amount of leisure and labor is  $\ell + y = 1$ . The consumer owns the firm. The firm's objective is to maximize its profit. Denote the profit function of the firm by  $\pi(p, w) = \max_y [p \ln y - wy]$ . The consumer's optimization problem is to maximize her utility  $u(x, \ell)$  subject to the budget constraint  $px \leq w(1 - \ell) + \pi(p, w)$ .

- (a) Solve the optimization problem of the firm and that of the consumer (given the firm's optimum), and find the demand function of the consumer and the supply function of the firm (as a function of  $p$  and  $w$ ).
- (b) Solve the Walrasian equilibrium prices  $p^*$  and  $w^*$