AMT F2015 Final Exam Retake

Instructions: Complete as much of each question as possible. Good Luck!

1. **Choice Theory (10)** The consumption space of an agent is $\mathbb{R}^2_+$. For each property below, on separate figures, show graphically an example of TWO preference relations; one that **satisfies** the property and one that does **NOT** satisfy the property. Clearly indicate indifference curves (if there are any), direction of increasing preferences, points of intersection, etc.

   (a) monotonicity

   (b) transitivity

   (c) convexity

   (d) quasi-linearity

   (e) local non-satiation

2. **Risk Aversion (10)** Let $u(x) = \sqrt{x}$ be an agent's Bernoulli utility function for money, and w be his wealth. Consider the lottery that pays 0 euros with probability p, and 100 euros with probability $1-p$.

   (a) Write the (expected) utility of the agent if:

      i. he owns wealth w

      ii. he owns wealth w *plus* some amount of money m

      iii. he owns wealth w *minus* some amount of money m

      iv. he owns wealth w and the lottery

      v. he owns wealth w, the lottery, *minus* some amount of money m

   (b) The individual owns the lottery. What is the minimum amount of money x that the agent is willing to *sell* the lottery for?

   (c) The individual does not own the lottery. What is the maximum amount of money y that the agent is willing to *buy* the lottery for?

   (d) Are the selling and buying price different?

3. **Portfolio Optimization (10)** An **asset** is a divisible claim to a financial return in the future. Suppose that there are two assets: **safe** and **risky**. A safe asset returns 1 euro per euro invested, and a risky asset returns 0 euros with p probability or 3 euros with $1-p$ probability. An individual has w wealth to invest, and w can be divided between only the two assets. The agent chooses α and β—amount of his wealth invested in the safe and risky asset, respectively. Hence, if the risky asset returns $z \in \{0, 3\}$, the individual's **portfolio** (α, β) pays $\alpha + \beta z$.

   (a) The agent is an expected utility maximizer with Bernoulli utility function $u(x) = \ln(x)$. Given α and β,

      i. What is her utility if the risky asset returns 3 euros? 0 euros?

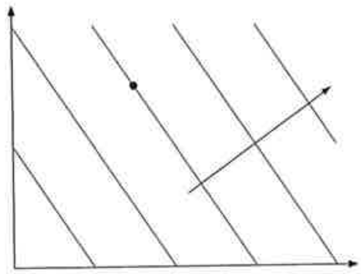
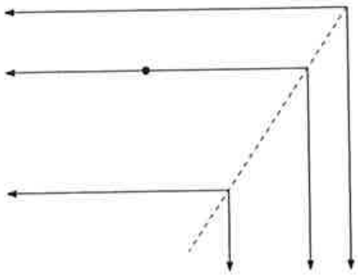
      ii. What is her expected utility for the lottery?

   (b) Write her utility maximization problem including budget constraint.

(c) If you have not already, using the budget constraint, rewrite the agent's problem by writing β in terms of α and w (given by the budget constraint).

(d) Write the first order conditions for optimality (assuming $\alpha > 0$ and $\beta > 0$). Explain in an intuitive fashion what "ratios" are being equalized in the FOCs.

4. **Classical Exchange Problem (25)** Consider the 2-agent 2-commodity case. Agent 1 has linear preferences, and Agent 2 has Leontief preferences; they are depicted below. The dot (the same point on each picture) represents the endowment profile.

(a) In the Edgeworth Box, select a point that is NOT Pareto-efficient and graphically show that another point dominates it.

(b) Identify the entire Pareto-efficient set of allocations.

(c) **Endowment Lower Bound** Identify the set of points that is preferred by each agent to his/her own endowment.

(d) You design a **rule** that recommends for each possible profile of preferences and endowments an allocation. For each economy, your rule selects from the Pareto-efficient and endowment lower bound set. For this economy, select a point z in the Pareto-efficient and endowment lower bound set (that you have calculated); suppose your rule recommends z for this economy. However, **you don't know** each agent's preference until they tell you; so they have to report their preferences first. Suppose that the Agent 2 (with Leontief preference) tells you his true preference, but Agent 1 (linear guy) does not. Construct a lie (a different linear preference) that Agent 1 may tell that would make him better off. (Hint: it will depend on z).

(e) Let $X = \mathbb{R}^2_+$ be each agent's consumption space, R a profile of preferences, e a profile of endowments, and p a list of prices. Formally state the definition of a Walrasian Equilibrium.

(f) What are some implicit assumptions of Walrasian Equilibrium?

(g) By Walras' Law, each agent consumes on his budget line (and not underneath). Suppose prices p are such that the budget line (which goes through the endowment point) is "flatter" than Agent 1's indifference curve through the same point. Graphically show that...

   i. Agent 2's optimal bundle given p and e^2 lies at a "kink".

   ii. Agent 1's optimal bundle given p and e^1 lies on the boundary of the Edgeworth Box.

   iii. Can this price vector (and the resulting allocation) be a Walrasian Equilibrium? Why or why not?

   iv. Argue that at a Walrasian Equilibrium, the budget line must have the same slope as Agent 1's indifference curves.

(h) State the First and Second Welfare Theorems.

(i) An allocation is **envy-free** if no agent prefers another agent's assignment to his own. Let e be the endowment profile where the total amount is equally divided between each agent. Using the definition of budget sets, show that each Walrasian Equilibrium for this economy is envy-free.

(j) Consider an economy with 3 agents and 2 commodities; the endowment profile is $\omega_1 = (5,0)$, $\omega_2 = (0,5)$, and $\omega_3 = (1,1)$. Agents have monotonic preferences. Is the envy-free allocation $x = ((2,2),(2,2),(2,2))$ in the Core for this economy? If it is not in the Core, then identify a blocking coalition.

(k) What is the relationship between Walrasian Equilibria and the Core?

5. **Bonus (5)** Describe an experiment that demonstrates behavior contradicting expected utility maximization. Explain why it violates expected utility.