Advanced microeconomics part II 15.1. 2014/Kultti

1. There are two firms. Firm 1 has a marginal cost $mc_1 = 4$ and Firm 2 has a marginal cost $mc_2 = 3$. They face the demand $q = 8 - p$.

i) Determine the Cournot-equilibrium (firms choose quantities).

ii) Determine the Bertrand-equilibrium (firms choose prices).

iii) Determine the equilibrium if Firm1 announces that it will choose the same price as Firm2.

2. There is an employer and a worker. The worker's job is to sell a submarine. To make a sale she has to exert effort $x \in [0, \infty)$, which results in a sale with probability $1 - e^{-x}$. A sale is worth 100 to the risk neutral employer. The effort is costly to the employee, and her utility in effort and money is given by $u(x, m) = -x + \sqrt{m}$. The effort is not observable and cannot be contracted upon, and the worker cannot be made to pay to the employer, i.e. her remuneration has to be non-negative.

i) Determine the outcome if the employer pays the worker a flat wage $w > 0$.

ii) Determine the effort level that the employer would choose if s/he were to sell the submarine him/herself. Notice that her utility is given by $v(x, m) = -x + m$

iii) Determine the optimal contract that implements the effort level that is the solution to ii).

3. Notice that the following extensive form game with three players is not of perfect information: the decision nodes of player-3 belong to the same information set.

i) If this were a game of perfect information what would be the subgame perfect equilibrium.

ii) Determine the two pure strategy equilibria of the game.

iii) Show that one of these equilibria is not good in the following sense. Instead of playing the equilibrium strategy with probability 1 players make mistakes. Assume that player-1 plays the equilibrium strategy with probability $1 - \rho$ and the other strategy with probability ρ, while player-3 plays the equilibrium strategy with probability $1 - \omega$ and the other strategy with probability ω. Show that in this situation player-2's equilibrium strategy is not a best response if the mistake probabilities are small enough.

$$P1$$

l r

$P2$

a b

$P3$ $P3$

$(1,1,1)$

x y x y

$(4,4,0)$ $(0,0,1)$ $(3,2,2)$ $(0,0,0)$
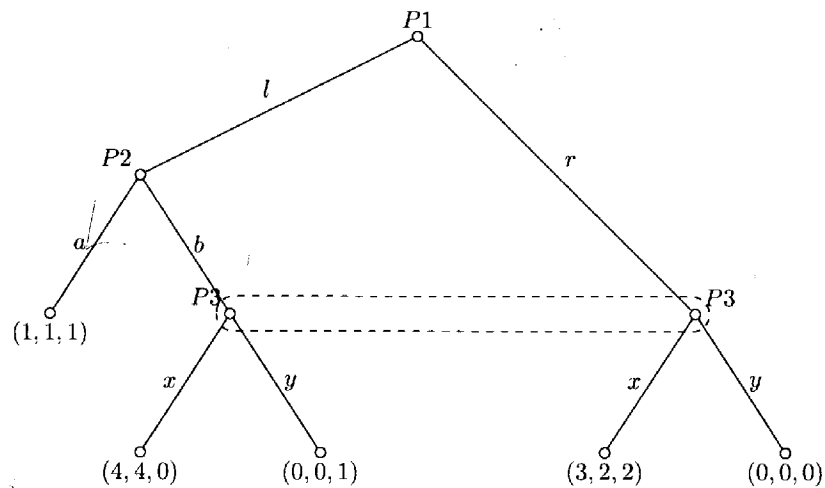
4. Assume that there are 3 buyers and a seller who has an object for sale. Each buyer's valuation of the object is 10 with probability π and 20 with probability $1 - \pi$. The seller values the object at zero. When is it more profitable to sell the object via an auction than by a posted price?