Advanced Microeconomic Theory II, retake 19.1 2016/Kultti

1. Construct a three-player extensive form game where there are multiple subgame perfect equilibria, and at least one Nash-equilibrium that is not subgame perfect.

2. There is a risk neutral principal and an agent whose utility is given by $u(w,e) = \sqrt{w} - e^2$ where w is wage and $e \in \{e_L, e_h\}$ is the effort level. The agent's outside option is worth $\underline{u} = 2$. There are two outcomes x_1 and x_2 where the latter one is better. The probability that x_2 is realised is given by $p_L = \frac{1}{4}$ if the agent chooses $e_L = 1$, and by $p_H = \frac{1}{2}$ if the agent chooses $e_H = 2$. Assuming that the principal wants the agent to choose high effort determine the optimal contract that the principal offers the agent.

3. A king has two sons. His objective is to get the sons married. If none of the sons marries the king's utility is zero. If one of them marries his utility is unity and if both marry his utility is $x > 1$. To get married the sons must go and find a princess and there are two princesses waiting to be married. The sons are quite independent fellows and if both are allowed to go and look for princesses they are equally likely to find either of the two princesses. If the sons happen to contact the same princess they will fight. Regardless of who wins this option gives utility $y < 0$ to the king. Determine whether the king should send zero, one or two of his sons to look for a princess.