

Answer no more than three (3) out of the four (4) problems.

1. There are two firms with marginal costs  $mc = 4$ . They face demand  $q = 8 - p$ .

- i) Determine the Cournot-equilibrium.
- ii) Show that there is an equilibrium where the firms price like a monopoly if they make the following promise: We pay the difference in price if you can find the product cheaper somewhere. Be careful to establish the equilibrium strategies of all the parties.

2. Assume that a worker sells a submarine. To make a sale s/he has to exert effort  $x \in [0, \infty]$ , which results in a sale with probability  $1 - e^{-x}$ . A sale is worth 100 to the employer. Effort is costly to the employee, and his/her utility in effort and money is given by  $u(x, m) = -x + \sqrt{m}$ . Effort is not observable and cannot be contracted upon, and the worker cannot be made to pay to the employer, i.e. his/her remuneration has to be non-negative.

- i) Determine the outcome if the employer pays the worker a flat wage  $w > 0$ .
- ii) Determine the socially efficient outcome.
- iii) Determine the optimal contract designed by the employer.

3. Assume that there are 100 homeowners. The values of the homes are uniformly distributed on the set  $\{11, 12, 13, \dots, 109, 110\}$  so that there is exactly one home of each value. Each homeowner must pay tax of  $x$  euros. The tax authority, however, has limited resources and if there are  $n$  homeowners who have not paid the tax the probability of being caught is  $\frac{1}{n}$ . If caught the homeowner loses the whole value of the home to the tax authority. Assume everyone is risk neutral.

- i) What is the largest value of  $x$  that induces a homeowner with home worth 110 to pay the tax if no-one else pays the tax.
- ii) What is the largest value that induces the homeowner with the home worth 11 to pay the tax if no-one else pays the tax.
- iii) What are the incentives of the remaining homeowners if the tax is set at the level where the owner of the home of worth 110 pays the tax?

4. Consider an infinitely repeated prisoner's dilemma. Show that for sufficiently high discount factors there exists a Nash-equilibrium in which the equilibrium path (choices that are observed in equilibrium) consists for the row player of two  $D$ s and then  $C$ s, while for the column player it consists of all  $C$ s. (You must construct the strategies that generate this kind of play; determine the discount factors that support the strategies)