AMT F2015 Final Exam

Instructions: Do not panic. Complete as much of each question as possible. For each request to "interpret", write no more than a few sentences and in plain language. Good Luck!

1. **Choice Theory (10)** Let the agent's consumption space be $X = [0, 1] \times [0, 1]$, and consider the lexicographic preference ordering. Does the lexicographic preference ordering satisfy the following properties? For each property, write TRUE or FALSE; if FALSE, then explain.

   (a) complete true
   (b) transitive true
   (c) strictly monotone true
   (d) convex true
   (e) continuous false because upper contour set is open
   (f) quasi-linear
   (g) locally non-satiated true
   (h) representable by a discontinuous utility function False

2. **Choice Theory (10)** Let $X = \{apple, orange, banana, 250g\ lingonberries\}$ be a set of fruit alternatives, and C be a choice correspondence. That is, for each subset of alternatives $S \subseteq X$, $C(S) \subseteq S$ is the agent's selection.

   (a) State the Weak Axiom of Revealed Preference (a Venn diagram may help).
   (b) Consider the following procedure that can be used to define a choice correspondence:
   For each subset of fruits,
   if there is an *odd* number of fruits in the subset, then I choose the heaviest ones,
   if there is an *even* number of fruits in the subset, then I choose the lightest ones.
   Does this satisfy WARP? Why?
   (c) State the Independence of Irrelevant Alternatives property. Give a real-life example of when it is violated.

3. **Risk Aversion (10)** Let $u(x) = \ln(x)$ be an agent's Bernoulli utility function and wealth w. Consider the lottery that pays 0 euros with probability p, and 100 euros with probability $1 - p$.

   (a) If the individual owns the lottery, what is the minimum price x he would sell it for?
   (b) If he does not own it, what is the maximum price y he would be willing to pay for it?
   (c) Are buying and selling prices equal? Interpret.

4. **Portfolio Optimization (10)** An **asset** is a divisible claim to a financial return in the future. Suppose that there are two assets: **safe** and **risky**. A safe asset returns 1 euro per euro invested, and a risky asset returns 0 euros with p probability or 3 euros with $1-p$ probability. An individual has w wealth to invest, and w can be divided between only the two assets. The agent chooses α and β—amount of his wealth invested in the safe and risky asset, respectively. Hence, if the risky asset returns $z \in \{0,3\}$, the individual's **portfolio** (α, β) pays $\alpha + \beta z$.

   (a) The agent is an expected utility maximizer with Bernoulli utility function $u(x) = \ln(x)$. Write his utility maximization problem including budget constraint.

   (b) If you have not already, using the budget constraint, rewrite the agent's problem by writing β in terms of α and w (given by the budget constraint).

   (c) Write the first order conditions for optimality. Explain in an intuitive fashion what "ratios" are being equalized in the FOC's.

5. **Classical Exchange Problem (25)** Consider the 2-agent 2-commodity case.

   (a) Construct the Pareto set when one agent has linear preferences, and the other has Leontief preferences (of the form $u(x_1, x_2) = \min[ax_1, x_2]$); you may choose the locus of kinks as you wish.

   (b) A **rule** recommends for each possible profile of preferences and endowments an allocation. Consider a rule that for each economy, selects from the Pareto-efficient and endowment lower bound set. For your economy in (a), select a point z in the Pareto-efficient and endowment lower bound set; suppose the rule selects z for your economy. Show that the agent with linear preferences can report a "lie" and be better off.

   (c) Let $X = \mathbb{R}^K_+$ be each agent's consumption space, R a profile of preferences, e a profile of endowments, and p a list of prices. Formally state the definition of a Walrasian Equilibrium. Identify the entire set of WE for your economy.

   (d) Let $z(p, e)$ be the aggregate excess demand function. Show that Walras' Law implies

$$p \cdot z(p,e) = p_1 \left(\sum_{i \in N} z_1^i(p,e) \right) + p_2 \left(\sum_{i \in N} z_2^i(p,e) \right) = 0$$

   (e) State the First and Second Welfare Theorems.

   (f) An allocation is **envy-free** if no agent prefers another agent's assignment to his own. Show that for each economy with an equal-division endowment profile, each Walrasian Equilibrium for this economy is envy-free.

   (g) Consider an economy with 3 agents and 2 commodities; the endowment profile is $\omega_1 = (0,5)$, $\omega_2 = (5,0)$, and $\omega_3 = (1,1)$. Agents have monotonic preferences. Is the envy-free allocation $x = ((2,2),(2,2),(2,2))$ in the Core for this economy?

   (h) What is the relationship between Walrasian Equilibria and the Core?

   (i) Consider the 3-agent, 2-commodity case where each agent has continuous, monotonic strictly, and strictly convex preferences. Select an endowment profile and depict a Walrasian Equilibrium in $\mathbb{R}^2_+$. Argue that this is the unique WE (a graphical argument suffices).

6. **Bonus (5)** Describe an experiment that demonstrates behavior contradicting expected utility maximization. Explain why it violates expected utility.