

Game theory exam 19.11. 2012/Kultti

1. Construct a  $2 \times 2$  stage game such that i) there is a Pareto-optimal outcome that is not a Nash-equilibrium, ii) if the game is played twice the Pareto-optimal outcome is not played in either period/stage, and iii) if the game is played thrice (three times) then there exists a Nash-equilibrium where the Pareto-optimal outcome is played in the first period/stage.
2. Consider a symmetric two-player strategic form game where both players have three actions. Determine the pay-offs such that no action dominates another action but there exists a mixed strategy that dominates at least one action.
3. There is a circular plate of radius  $r > 0$  and two players. The players have an unlimited number of titanium balls of radius  $0 < R < r$ . Player-1 first places a ball on the plate, then player-2, then player-1, and so on in turns until the a player cannot place a ball on the place without moving other balls on the plate. The winner is the player who can place the last ball on the plate. Is there a strategy that guarantees a victory to one of the players?
4. In ultimatum game two players have to divide 10 euros. The rules are such that player 1 makes a proposition about the division, and player 2 either accepts or rejects. In the first case the division is implemented. In the latter case both players get zero. i) Determine a subgame perfect equilibrium of the game. ii) Determine a Nash-equilibrium different from the equilibrium in i).