AMT 1 F2016 Final Retake

Complete as much of each question as possible. The exam will be graded on a curve. Good Luck!

1. **Choice Theory (10)** The consumption space of an agent is $\mathbb{R}^2_+$. For each property below, on separate figures, show graphically an example of TWO preference relations: one that **satisfies** the property and one that does **NOT** satisfy the property. Clearly indicate indifference curves (if there are any), direction of increasing preferences, points of intersection, etc.

   (a) monotonicity

   (b) continuity

   (c) convexity

   (d) quasi-linear

   (e) local non-satiation

2. **Choice Theory (14)** Let $X = \{apple,\ orange,\ banana,\ 250g\ lingonberries\}$ be the set of all fruits.

   (a) How is a choice function different from a preference relation? What is the meant by the "rationalizability" of a choice function?

   (b) I have the following procedure when faced with each possible subset of available fruits:

   - if the store has $\{apple,\ orange,\ banana\}$, then I choose orange;
   - otherwise, I look at the FIRST letter of each fruit, and choose the fruit with the Earliest appearing first letter

   Does my procedure satisfy WARP? If not, then give an example of two menus and my selections that violate it.

   (c) State the Independence of Irrelevant Alternatives (Irrelevant Deletions Don't Matter) property for choice functions. Give a real-life example of when it is violated.

3. **Consumer Theory (6)** Let p and p' be two lists of prices, and w be a wealth level. A policy maker wants to know how the welfare of Jane changes if prices change from one to the other. Formally describe and discuss two different ways to measure changes in her welfare *in terms of her expenditure.* (Hint: let $u = v(p, w)$, $u' = v(p', w)$, and recall $e(p, u) = w$).

4. **Portfolio Optimization (10)** An **asset** is a divisible claim to a financial return in the future. Suppose that there are two assets: **safe** and **risky**. A safe asset returns 1 euro per euro invested, and a risky asset returns 0 euros with p probability or 3 euros with $1 - p$ probability. An individual has w wealth to invest, and w can be divided between only the two assets. The agent chooses α and β—amount of her wealth invested in the safe and risky asset, respectively. Hence, if the risky asset returns $z \in \{0, 3\}$, the individual's **portfolio** (α, β) pays $\alpha + \beta z$.

   (a) The agent is an expected utility maximizer with utility function $u(x) = \ln(x)$. Given α and β,

      i. What is her utility if the risky asset returns 3 euros? 0 euros?

      ii. What is her expected utility for the lottery?

   (b) Write her utility maximization problem including budget constraint.

   (c) If you have not already, using the budget constraint, rewrite the agent's problem by writing β in terms of α and w (given by the budget constraint).

   (d) Write the first order conditions for optimality (assuming $\alpha > 0$ and $\beta > 0$). For derivatives you may just write $u'(x)$. Explain in an intuitive fashion what "ratios" are being equalized in the FOCs.

1. **Expected Utility (10)** Consider the lotteries below.

   - L_1: 0.25 chance of $3,000e$, and 0.75 chance of $0e$
   - L_2 : 0.2 chance of $4,000e$, and 0.8 chance of $0e$
   - L_3 : $3,000e$
   - L_4 : 0.8 chance of $4,000e$, and 0.2 chance of $0e$

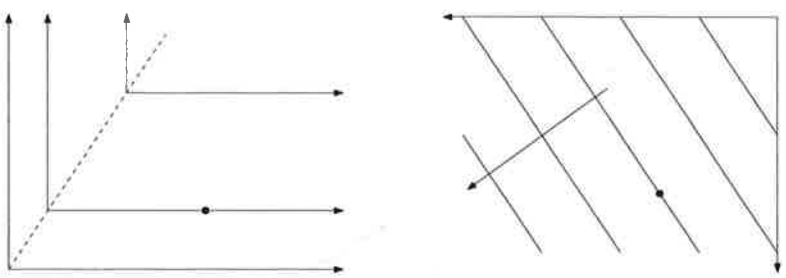
   (a) Suppose he/she has utility for money $u(m)$. Write out his/her expected utility for each lottery.

   Suppose an agent has preferences $Eu(L_2) \geq Eu(L_1)$ and $Eu(L_3) \geq Eu(L_4)$.

   (a) Show that this is not possible.

   (b) Give a verbal (or formal) definition of the Independence property for general preferences over lotteries.

2. **Classical Exchange Problem (30)** Consider the 2-agent 2-commodity case. Agent 1 has Leontief preferences, and Agent 2 has linear preferences; they are depicted below. The dot (the same point on each picture) represents the endowment profile.



   For questions require graphical answers, be clear in terms of labelling axes, points, indifference curves, etc.

   (a) (2) In the Edgeworth Box, select a point that is NOT Pareto-efficient and graphically show that another point dominates it.

   (b) (3) Identify the entire Pareto-efficient set of allocations.

   (c) (2) **Endowment Lower Bound** Identify the set of points that is preferred by each agent to his/her own endowment.

   (d) (2) You design a **rule** that recommends for each possible profile of preferences and endowments an allocation. For each economy, your rule selects from the Pareto-efficient and endowment lower bound set. For the above economy, select a point z in the Pareto-efficient and endowment lower bound set (that you have calculated); suppose your rule recommends z for this economy. However, **you don't know** each agent's preference until they tell you; so they have to report their preferences first. Suppose that the Agent 2 (with Leontief preference) always tells the truth, but Agent 1 (linear guy) does not. Construct a lie (a different linear preference) that Agent 1 may tell that would make him better off. (Hint: it will depend on z).

   (e) (5) Let $X = \mathbb{R}^2_+$ be each agent's consumption space, N be a set of agents, R their profile of preferences, e their profile of endowments, and p a list of prices. Formally state the definition of a Walrasian Equilibrium.

   (f) (5) By Walras' Law, each agent consumes on his/her budget line (and not underneath). Suppose prices p are such that the budget line (which goes through the endowment point) is "flatter" than Agent 2's indifference curve through the same point. Graphically show that...

      i. Agent 1's optimal bundle given p and e^2 lies at a "kink".

ii. Agent 2's optimal bundle given p and e^1 lies on the boundary of the Edgeworth Box.

iii. Can this price vector (and the resulting allocation) be a Walrasian Equilibrium? Why/why not?

iv. Argue that at a Walrasian Equilibrium, the budget line must have the same slope as Agent 2's indifference curves.

(g) (3) State the First and Second Welfare Theorems.

(h) (3) An allocation is **envy-free** if no agent prefers another agent's assignment to his/her own. Let e be the endowment profile where the total amount is equally divided between each agent. That is, let Ω_1 be the total amount of good 1, and Ω_2 be the total of good 2. Then, for each $i \in N$, $e^i = (\frac{\Omega_1}{N}, \frac{\Omega_2}{N})$. Write out each agent's budget set given e, and maximization problem in a Walrasian Equilibrium. Is the resulting allocation envy-free? Why/why not?

(i) (5) Consider an economy with 3 agents and 2 commodities; the endowment profile is $e_1 = (0, 5)$, $e_2 = (5, 0)$, and $e_3 = (4, 4)$. Agents have monotonic preferences. Give an example of

i. an envy-free allocation

ii. a core allocation

iii. an allocation not in the core, and the blocking coalition

You may also claim that there is not enough information to determine such an allocation.

(j) (Bonus 2) As a description of a real-world economy, what aspect do you think Walrasian Equilibrum fails to capture?