Microeconomic Theory I
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Practice questions for Final Exam for Microeconomic theory I

The Final exam will have 4 or 5 questions of similar difficulty with the sample questions from past years provided below.

1. Consumers A and B have the same utility function $u(x, y) = xy$ over goods $x$and y. Consumer A has 3 units of x and 1 unit of y, B has 1 unit of x and 3 units of y.

   a) Illustrate the above preferences and endowments using an Edgeworth box diagram.

   b) Find the Pareto-efficient allocations and the competitive equilibrium.

   c) Suppose that there is a third consumer with linear preferences: $u^C(x_C, y_C) = x_C + 2y_C$ and initial endowments of 10 of both y and x. What is the equilibrium price ratio in this economy?

   d) Which of the consumers A and B has a higher utility in equilibrium?

2. A consumer has a utility function $u(x_1, x_2) = \ln(x_1 + 2x_2)$.

   a) Find the marginal rate of subtitution at commodity bundle $(1, 3)$.

   b) Derive the demand for x_1 as a function of the prices, p_1 and p_2 and the available income, m.

   c) Suppose that $p_1 = 2$, $p_2 = 1$ and $m = 50$. Would the consumer prefer a 1% drop in p_1 to a 1% drop in p_2?

3. A firm produces output from labour, l, and capital, k according to the following production function: $q = (lk)^{\frac{1}{3}}$, Let the wage and rental cost of capital both be equal to 1.

   a) Suppose that the firm must produce q' units of output. Determine the optimal labour and capital demands.

   b) Suppose that the firm has an alternative technology where $q = l$ (i.e. the technology uses only labour as input, and it is linear). What is the optimal way to produce Q units of output using these two technologies?

4. Find the optimal demand for a consumer with a utility function: $u(x, y, z) = x^{\alpha}y^{\beta}z^{1-\alpha-\beta}$ when the prices are p_x, p_y and p_z respectively, and the income available is I.

5. A consumer has quasilinear preferences in (l, x), where l denotes leisure and x denotes consumption and her utility function can be written as

$$u(l, x) = l + 2x^{\frac{1}{2}}.$$

   In each period, the consumer allocates her total time endowment $\bar{l}$ between work and leisure, i.e. if she consumes $l \leq \bar{l}$ units of leisure, she works for $(\bar{l} - l)$units of time. Let w_l denote the wages per unit.

(a) Formulate the consumer's maximization problem and write the first order necessary conditions for maxima.

(b) Argue that the first order conditions are also sufficient and solve the problem.

(c) Assume that a child is born to the consumer, and now she divides her time between leisure l, child care c and work $(\bar{l} - l - c)$. Suppose also that her new utility function is

$$u(l, c, x) = l + 2c^{\frac{1}{2}} + 2x^{\frac{1}{2}}.$$

How does your solution to the problem change?

6. Consider a manager of a firm whose compensation package includes a bonus for good performance. More specifically, assume that the wage w of the manager depends on the firm's profit x as follows.

$$w(x) = \begin{cases} \underline{w} & \text{if } x \leq \underline{x}, \\ \underline{w} + .05(x - \underline{x}) & \text{if } x > \underline{x}. \end{cases}$$

Suppose that the manager's Bernoulli utility function is $u(w) = w$, in other words, the manager is risk neutral in his wage.

(a) Is the manager's payoff function concave or convex in the firm's profit? (Hint: Draw $w(x)$).

(b) Suppose that the profit net of the manager's wage is distributed to the shareholders in dividends. Suppose also that the shareholders' utility function is linear in the dividends. Is the shareholders' payoff function concave or convex in x?

(c) Suppose that the manager chooses the projects that the firm undertakes. Projects can be viewed as lotteries on possible profit levels. Will the manager choose the correct projects from the shareholder's point of view?

7. Consider an exchange economy with two consumers, A and B and two goods, x and y. The utility functions of the two consumers are given by:

$$u_A(x, y) = x + 2\sqrt{y}, \quad u_B(x, y) = x + \ln y.$$

The initial allocations of the two consumers are: $x_A^E = 1,\ y_A^E = 2,\ x_B^E = 1,\ y_B^E = 1$.

(a) Draw the Edgeworth Box for this economy.

(b) Find the Pareto-efficient allocations for this economy.

(c) Determine the price ratio $\frac{p_x}{p_y}$ that is compatible with competitive equilibrium and solve for the competitive equilibrium allocations. (Hint: Think about the requirements for the price ratio when consumers choose strictly positive consumptions for both goods).

8. In this part, you are asked to provide short explanations.

(a) What are the Hicksian demand functions and how are they related to Marshallian (or Walrasian) demand functions?

(b) True or False: If a consumer has homothetic preferences, then all goods are normal goods.

(c) True or False: If the output price increases, then the optimal supply of the output by a competitive firm increases. (Why?)

(d) True or False: If the price of an input increases, then the output of a competitive firm decreases. (Why?)

(e) In choice under uncertainty, an independence (or substitution) assumption is imposed. How does this assumption affect the form of the utility representation?

9. Consider a producer with a production function $y = (2K + L)^{\frac{1}{2}}$ for the production of output, y from inputs K and L. Let the input prices be p_K and p_L.

(a) Does this production technology have decreasing, constant or increasing returns to scale? (Why?)

(b) Suppose that the producer must produce 10 units of output. What are her optimal conditional demands for the two outputs as functions of p_K and p_L?

10. Let $u(x) = x^{\frac{1}{2}}$ be the utility of a consumer from final wealth level x. Suppose that the total wealth of the consumer is given by w and there is a probability p that her house valued at l burns down.

a) Is the consumer risk averse?

b) Suppose that the consumer can buy fire insurance where amount y of coverage (i.e. payment made in case of fire) costs qy. Write down the expected utility for the consumer as a function of coverage y.

c) Solve for the optimal quantity of coverage as a function of the price q.

d) At what price will the buyer demand full insurance, i.e. $y = l$?

11. An investor has initial wealth w_0 that she can divide between a safe and risky asset; let αw_0 denote investment in the risky asset and $(1 - \alpha)w_0$ in the safe asset where $\alpha \in [0, 1]$. The risky asset pays rate of return R^H with probability p and the risky investment pays nothing with probability $(1 - p)$, (i.e. the risky investment is lost with probability $(1 - p)$). The safe asset pays R with probability 1.

(a) For an arbitrary investment portfolio $(\alpha, 1-\alpha)$, determine final wealth level of the investor (Hint: this is a random variable).

(b) Assume the investor has a Bernoulli utility function $u(w)$ where w denotes the final wealth. Assume that $u'(w) > 0$ and $u''(w) < 0$. Write down the expected utility resulting from an arbitrary portfolio $(\alpha, 1-\alpha)$.

(c) What is the first order conditions for an (interior) optimal portfolio. Solve for the optimal α when $u(w) = \ln(w)$.