Y1. Game Theory, Autumn 2008

30.10.2008

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Answer only to **three** questions with good handwriting. All questions have equal weight in grading.

1. Consider the following game of two players: Both players choose a positive integer. If player i's integer is greater than player j's integer but less than three times his integer, player j pays one euro to player i. If player i's integer is at least three times player j's integer, player i pays one euro to player j. If the integers are equal no payment is made. Each player's preferences are represented by his monetary payoff.

   a. Show that there is no pure strategy Nash equilibrium in this game.

   b. Show that there is a mixed strategy equilibrium where both players choose integers 1, 2 and 5 with probability ⅓.

2. Consider the following extensive game with perfect information:

   There are two players, 1 and 2.

   Terminal histories are: (A, C), (A, D, F), (A, D, G) and (E).

   Player function is $P(\emptyset) = 1$, $P(A) = 2$, $P(A, D) = 1$.

   Preferences over the terminal histories are represented by the following payoffs:

$$u_1(E) = 3, u_2(E) = 1$$

$$u_1(A,C) = 1, u_2(A,C) = 2$$

$$u_1(A,D,F) = 2, u_2(A,D,F) = 0$$

$$u_1(A,D,G) = 4,\ u_2(A,D,G) = 3$$

a. Give all possible strategies for the two players.

b. Find the pure strategy Nash equilibrium/a of the game.

c. Find the pure strategy subgame perfect equilibrium/a of the game.

3. Consider the following second-price common value auction: Two bidders receive signals t_i for $i = 1,2$ about the value of the object for sale. Each bidder's signal is uniformly distributed between zero and one. Both bidders believe that the signal each bidder receives is independent of the other bidder's signal. The valuation of bidder i of the object is $v_i = t_1 + t_2$ for $i = 1,2$. Each bidder chooses a bid (a non-negative number) and the bidder who places the highest bid wins the object and pays the second highest bid for it. The bidder who does not win the object pays nothing. If abidder wins the object his payoff is $u_i = v_i - b_j$ where $j \neq i$. Show that the game has a Nash equilibrium in which both bidders' bid is given by $b_i = \alpha t_i$. Determine α.

4. There are two players: a worker (W) and an employer (E). There are two possible types for W; he can be either good (g) or bad (b). W knows his type and E's prior belief is that W is good with probability γ and bad with probability $1 - \gamma$. Prior to the employment opportunity W chooses to go to college (C) or to beach (B). After having observed W's choice, E chooses whether to hire W (H) or not (N). The payoffs of the players are as follows if W is of type g:

$$u_W(B,N;g) = 0,\ u_E(B,N;g) = 0$$

$$u_W(B,H;g) = 2,\ u_E(B,H;g) = 1$$

$$u_W(C,N;g) = -1,\ u_E(C,N;g) = 0$$

$$u_W(C,H;g) = 1,\ u_E(C,H;g) = 1$$

The payoffs of the players are as follows if W is of type b:

$$u_W(B,N;b) = 0,\ u_E(B,N;b) = 0$$

$$u_W(B,H;b) = 2,\ u_E(B,H;b) = -1$$

$$u_W(C,N;b) = -3,\ u_E(C,N;b) = 0$$

$$u_W(C,H;b) = -4,\ u_E(C,H;b) = -1$$

Show that following assesment is a weak sequential equilibrium:

1. type g chooses C and type b chooses B.
2. the employer chooses H after signal C and N after signal B
3. the employer believes that conditional on signal B the probability that W is g is zero and conditional on signal C the probability that W is g is one.