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FUNDAMENTALS OF OPTIMIZATION

Constraint Programming

CONTENT

- Constraint Satisfaction Optimization Problems
- Constraint Propagation
- Branching and Backtracking Search
- Examples

Constraint Satisfaction Problems

- Variables
 - $X = \{X_0, X_1, X_2, X_3, X_4\}$
- Domain
 - $X_0, X_1, X_2, X_3, X_4 \in \{1, 2, 3, 4, 5\}$
- Constraints
 - $C_1: X_2 + 3 \neq X_1$
 - $C_2: X_3 \leq X_4$
 - $C_3: X_2 + X_3 = X_0 + 1$
 - $C_4: X_4 \leq 3$
 - $C_5: X_1 + X_4 = 7$
 - $C_6: X_2 = 1 \Rightarrow X_4 \neq 2$

Constraint Satisfaction Problems

- CSP = (X, D, C) , in which:
 - $X = \{X_1, \dots, X_N\}$ – set of variables
 - $D = \{D(X_1), \dots, D(X_N)\}$ – domains of variables
 - $C = \{C_1, \dots, C_K\}$ – set of constraints over variables
 - Denote $X(c)$ – set of variables appearing in the constraint c

Constraint Satisfaction Problems

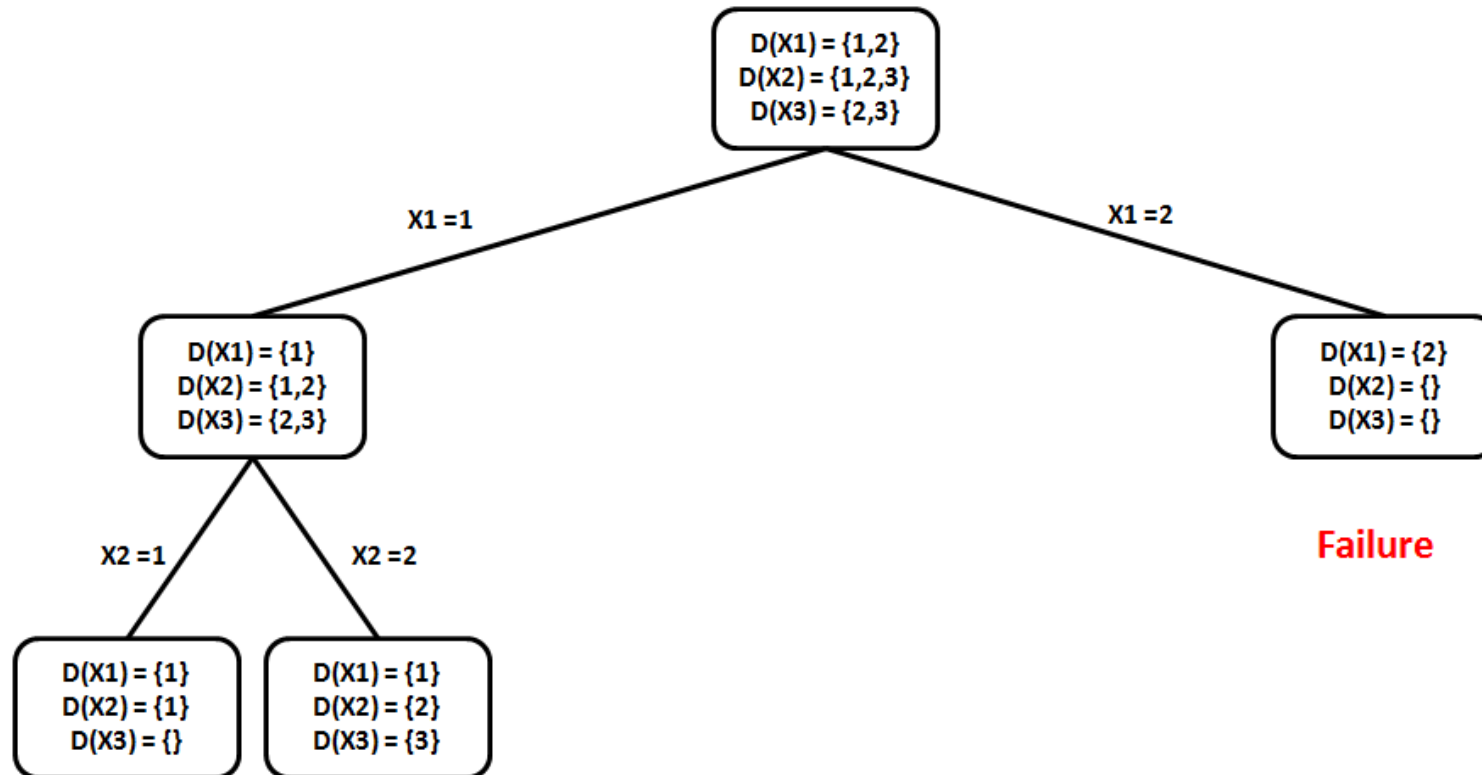
- COP = (X, D, C, f) , in which:
 - $X = \{X_1, \dots, X_N\}$ – set of variables
 - $D = \{D(X_1), \dots, D(X_N)\}$ – domains of variables
 - $C = \{C_1, \dots, C_K\}$ – set of constraints over variables
 - Denote $X(c)$ – set of variables appearing in the constraint c
 - f : objective function to be optimized

Constraint Programming

- A computation paradigm for solving CSP, COP combining
 - Constraint Propagation: narrow the search space by pruning redundant values from the domains of variables
 - Branching (backtracking search): split the problem into equivalent sub-problems by
 - Instantiating some variables with values of its domain
 - Split the domain of a selected variable into sub-domains

Constraint Programming

$X = \{X1, X2, X3\}$
 $D(X1) = \{1, 2, 3, 4\}$, $D(X2) = \{1, 2, 3\}$, $D(X3) = \{1, 2, 3\}$
 $C1: X1 \leq X2$
 $C2: X3 = X1 + X2$
 $C3: X2 + X3 = 5$



Failure

Solution

Constraint Propagation

- Domain consistency (DC)
 - Given a CSP = (X, D, C) , a constraint $c \in C$ is called domain consistent if for each variable $X_i \in X(c)$ and each value $v \in D(X_i)$, there exists values for variables of $X(c) \setminus \{X_i\}$ such that c is satisfied
 - A CSP is called domain consistent if c is domain consistent for all $c \in C$

Constraint Propagation

- DC algorithms aim at pruning redundant values from the domains of variables so that the obtained equivalent CSP is domain consistent

Constraint Propagation

- Example: CSP = (X, D, C) in which:
 - $X = \{X_1, X_2, X_3, X_4\}$
 - $D(X_1) = \{1, 2, 3, 4\}$, $D(X_2) = \{1, 2, 3, 4, 5, 6, 7\}$, $D(X_3) = \{2, 3, 4, 5\}$, $D(X_4) = \{1, 2, 3, 4, 5, 6\}$
 - $C = \{c_1, c_2, c_3\}$ với
 - $c_1 \equiv X_1 + X_2 \geq 5$
 - $c_2 \equiv X_1 + X_3 \geq X_4$
 - $c_3 \equiv X_1 + 3 \geq X_3$
- CSP is domain consistent
 - When branching, consider $X_1 = 1$, a DC algorithm will transform the given CSP to an equivalent domain consistent CSP¹ having : $D^1(X_1) = \{1\}$, $D^1(X_2) = \{4, 5, 6, 7\}$, $D^1(X_3) = \{2, 3, 4\}$, $D^1(X_4) = \{1, 2, 3, 4, 5\}$

Constraint Propagation

- A domain consistent CSP does not ensure to have feasible solutions
 - Example:
 - $X = \{X_1, X_2, X_3\}$
 - $D(X_1) = D(X_2) = D(X_3) = \{0, 1\}$
 - $c_1 \equiv X_1 \neq X_2$, $c_2 \equiv X_1 \neq X_3$, $c_3 \equiv X_2 \neq X_3$
- The CSP is domain consistent but does not have any feasible solution

Constraint Propagation

```
Algorithm AC3(X,D,C){
  Q = {(x,c) | c ∈ C ∧ x ∈ X(c)};
  while(Q not empty){
    select and remove (x,c) from Q;
    if ReviseAC3(x,c) then{
      if D(x) = {} then
        return false;
      else
        Q = Q ∪ {(x',c') | c' ∈ C \ {c} ∧ x,x' ∈ X(c') ∧ x ≠ x'}
    }
  }
  return true;
}
```

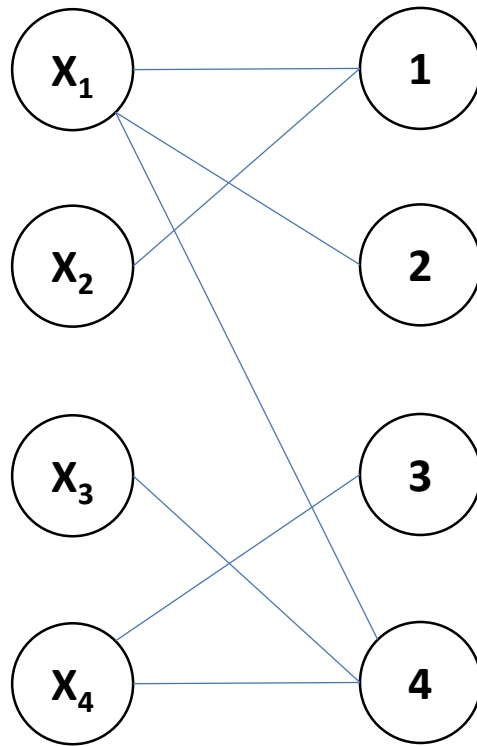
```
Algorithm ReviseAC3(x,c){
  CHANGE = false;
  for v ∈ D(x) do{
    if there does not exists other values
      of X(c) \ {x} such that c
        is satisfied then{
          remove v from D(x);
          CHANGE = true;
        }
    }
  }
  return CHANGE;
}
```

Constraint Propagation

- Some constraints, e.g., binary constraints (related 2 variables) → have efficient DC algorithm
- Constraint AllDifferent(X_1, X_2, \dots, X_N), the DC algorithm is efficient based on the matching (Max-Matching) algorithm on bipartite graphs
 - Nodes on the right-hand side are variables and nodes on the left-hand side are values
 - For each edge (X_i, v) , (với $v \in D(X_i)$), if there does not exist a matching of size N containing (X_i, v) , then v is removed from $D(X_i)$

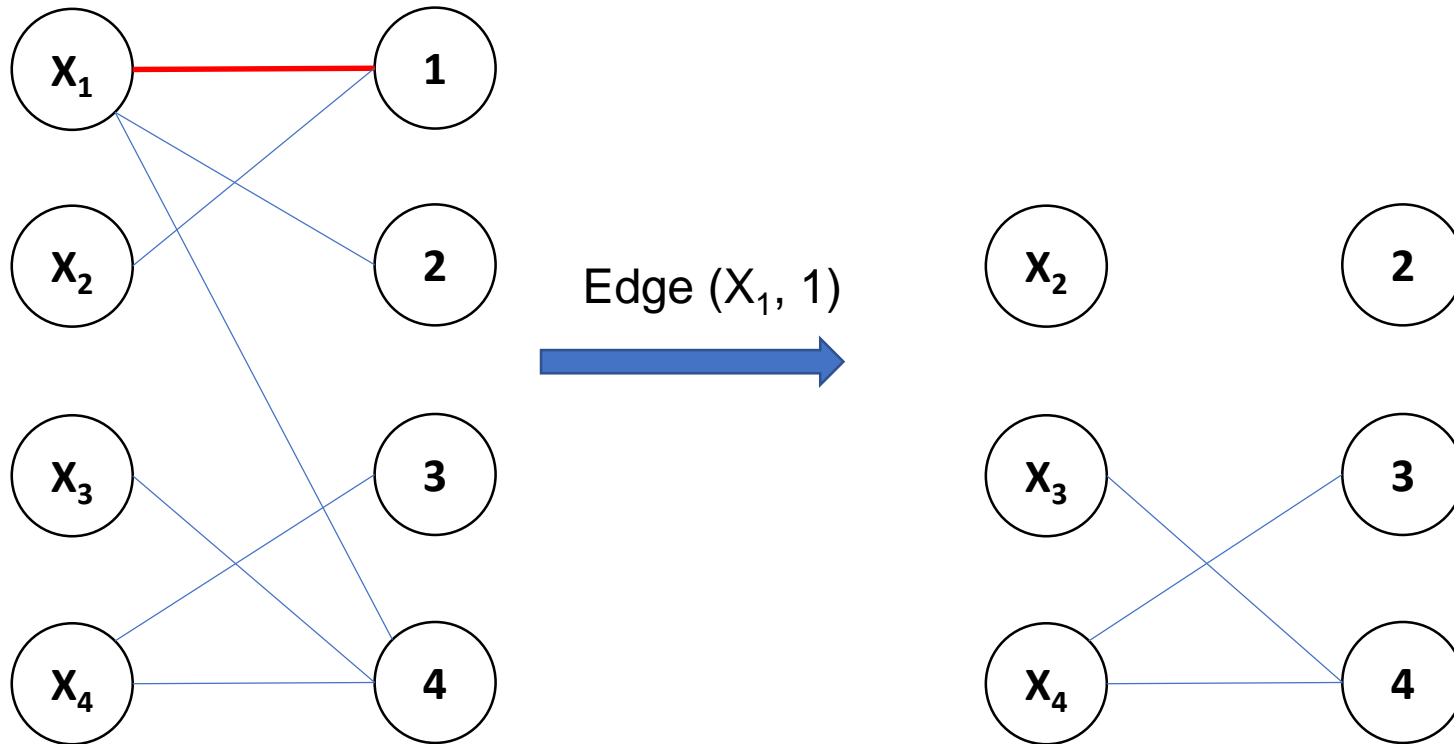
Constraint Propagation

- $X = \{X_1, X_2, X_3, X_4\}$
- $D(X_1) = \{1, 2, 4\}$, $D(X_2) = \{1\}$, $D(X_3) = \{4\}$, $D(X_4) = \{3, 4\}$



Constraint Propagation

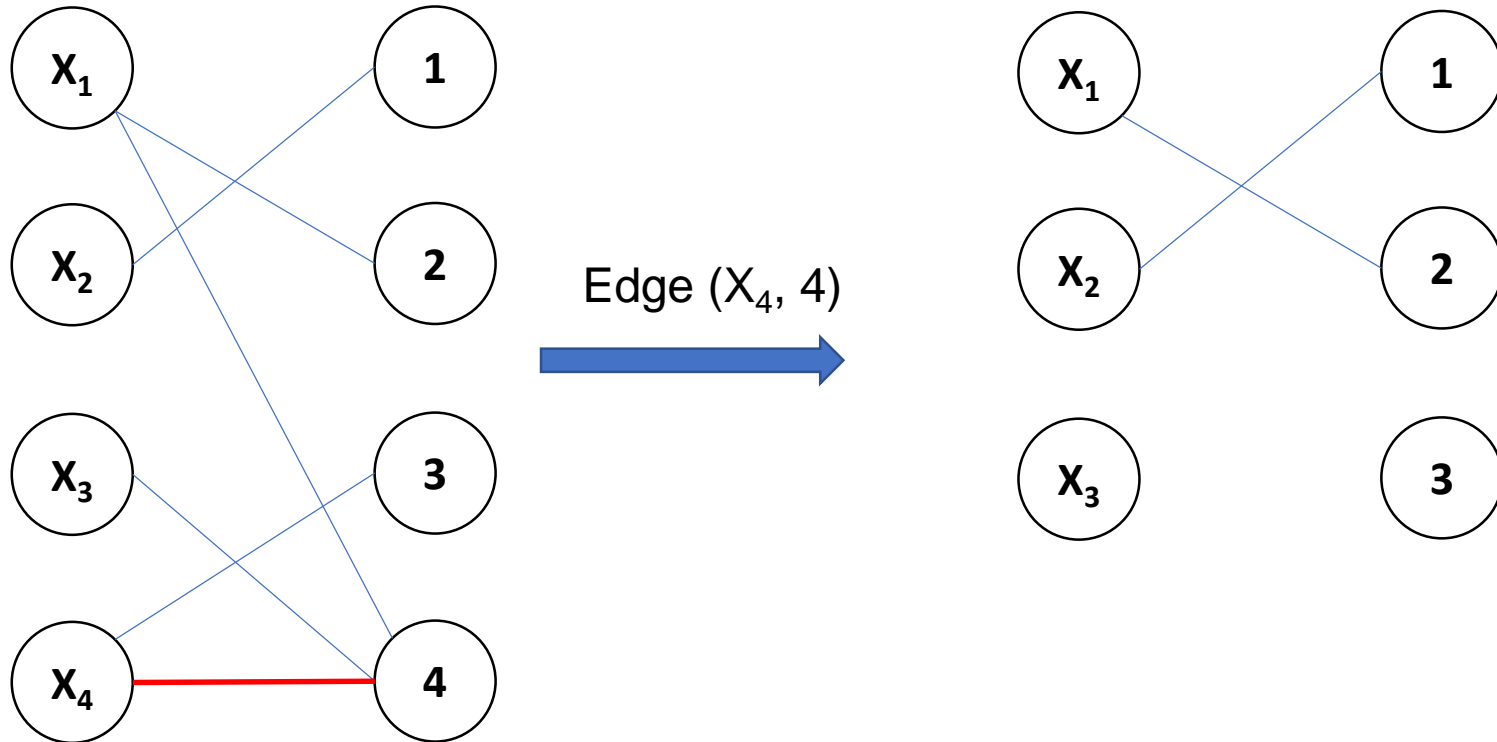
- $X = \{X_1, X_2, X_3, X_4\}$
- $D(X_1) = \{1, 2, 4\}$, $D(X_2) = \{1\}$, $D(X_3) = \{4\}$, $D(X_4) = \{3, 4\}$



No matching of size 3 \rightarrow remove 1 from $D(X_1)$

Constraint Propagation

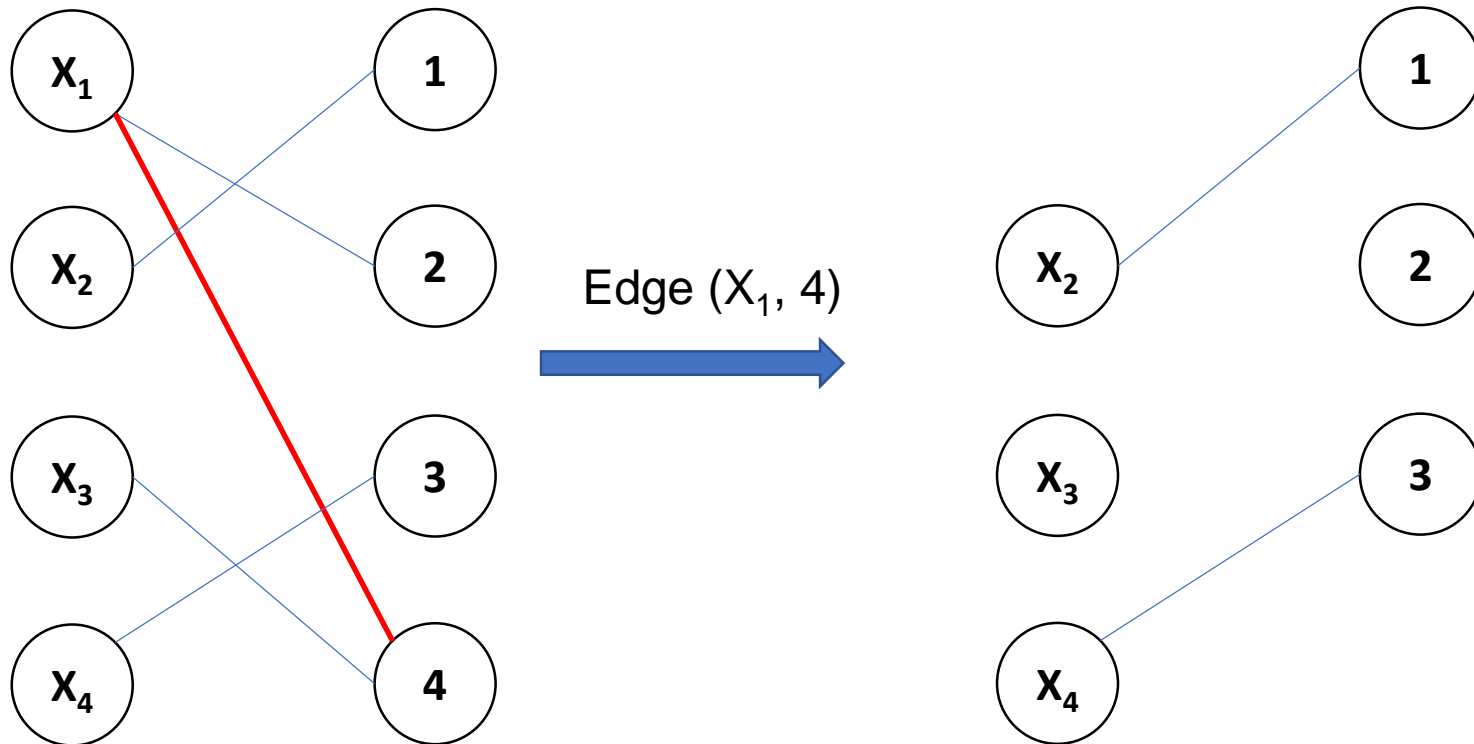
- $X = \{X_1, X_2, X_3, X_4\}$
- $D(X_1) = \{2,4\}$, $D(X_2) = \{1\}$, $D(X_3) = \{4\}$, $D(X_4) = \{3,4\}$



No matching of size 3 \rightarrow removed 4 from $D(X_4)$

Constraint Propagation

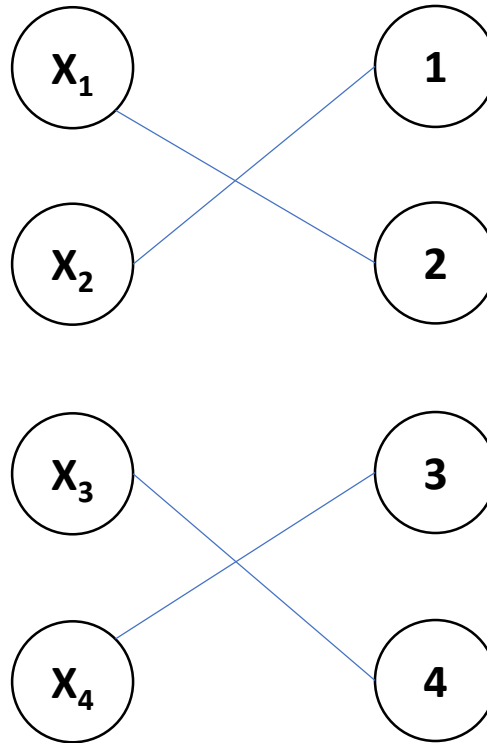
- $X = \{X_1, X_2, X_3, X_4\}$
- $D(X_1) = \{2, 4\}$, $D(X_2) = \{1\}$, $D(X_3) = \{4\}$, $D(X_4) = \{3\}$



No matching of size 3 \rightarrow removed 4 from $D(X_1)$

Constraint Propagation

- $X = \{X_1, X_2, X_3, X_4\}$
- $D(X_1) = \{2\}, D(X_2) = \{1\}, D(X_3) = \{4\}, D(X_4) = \{3\}$

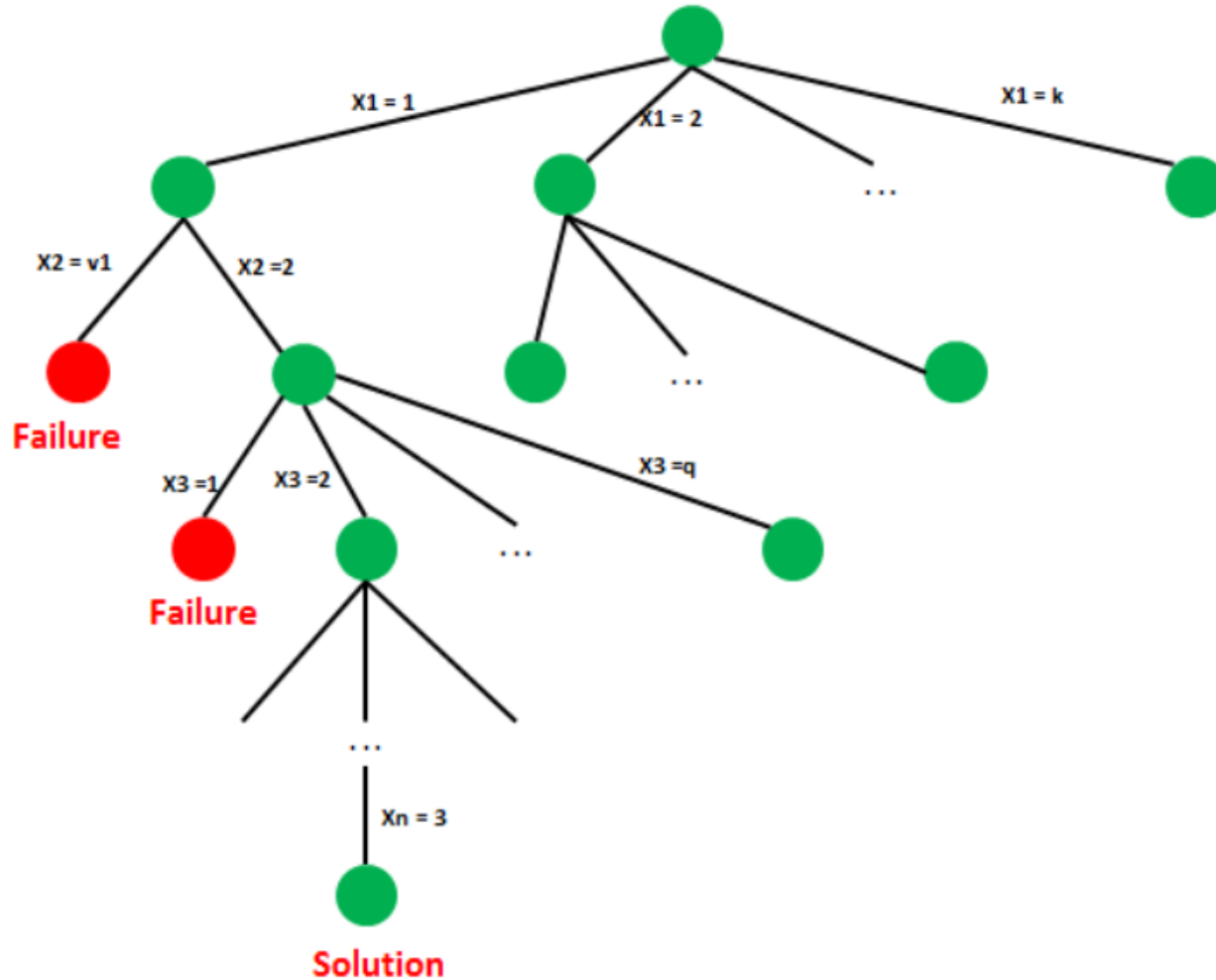


Feasible solution!

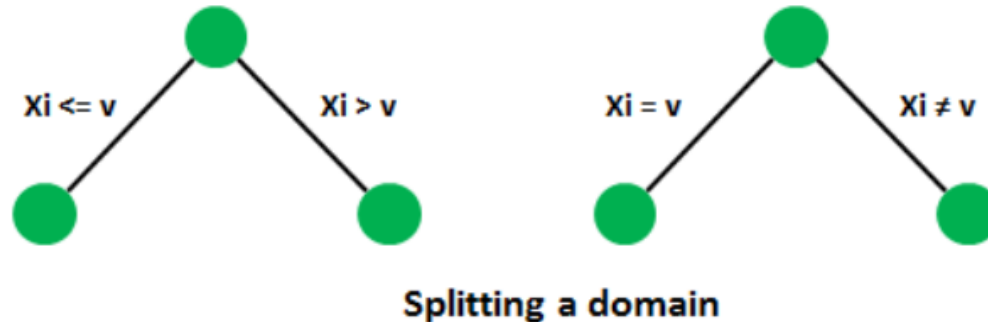
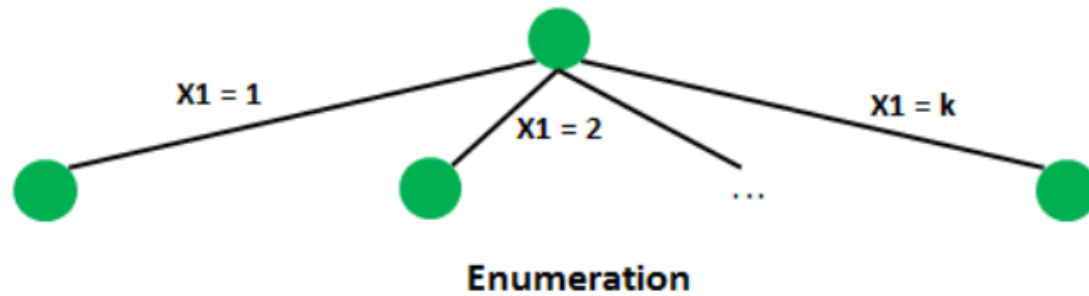
Branching & Backtracking Search

- Constraint propagation is not enough for finding feasible solutions
- Combine constraint propagation with branching and backtracking search
 - Split the original CSP P_0 into sub-problems CSP P_1, \dots, P_M
 - Set of solutions of P_0 is equivalent to the union of sets of solutions to P_1, \dots, P_M
 - Domain of each variable in P_1, \dots, P_M is not greater than the domain of that variable in P_0
 - Search Tree
 - Root is the original CSP P_0
 - Each node of the tree is a CSP
 - If P_1, \dots, P_M are children of P_0 then the set of solutions of P_0 is equivalent to the union of sets of solutions to P_1, \dots, P_M
 - Leaves
 - A feasible solution
 - Failure (a variable has an empty domain)

Branching & Backtracking Search



Branching & Backtracking Search



Branching & Backtracking Search

- Search strategies
 - Variable selection
 - **dom** heuristic: select a variable having the smallest domain
 - **deg** heuristic: select a variable participating in most of the constraints
 - **dom+deg** heuristic: first apply **dom**, then use **deg** when tie break (when there are more than one variable with the same smallest domain size)
 - **dom/deg**: select a variable having the smallest dom/deg
 - Value selection
 - Increasing order
 - Decreasing order

Example

- Variables
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Example

```
'''
If-Then-Else expression
if x[2] = 1 then x[4] != 2
'''

from ortools.sat.python import cp_model

class VarArraySolutionPrinter(cp_model.CpSolverSolutionCallback):
    #print intermediate solution
    def __init__(self, variables):
        cp_model.CpSolverSolutionCallback.__init__(self)
        self.__variables = variables
        self.__solution_count = 0

    def on_solution_callback(self):
        self.__solution_count += 1
        for v in self.__variables:
            print('%s = %i' % (v, self.Value(v)), end = ' ')

        print()

    def solution_count():
        return self.__solution_count
```

Example

```
model = cp_model.CpModel()
x = {}
for i in range(5):
    x[i] = model.NewIntVar(1,5,'x[' + str(i) + ']')

c1 = model.Add(x[2] + 3 != x[1])
c2 = model.Add(x[3] <= x[4])
c3 = model.Add(x[2] + x[3] == x[0] + 1)
c4 = model.Add(x[4] <= 3)
c5 = model.Add(x[1] + x[4] == 7)
b = model.NewBoolVar('b')

#constraints
model.Add(x[2] == 1).OnlyEnforceIf(b)
model.Add(x[2] != 1).OnlyEnforceIf(b.Not())
model.Add(x[4] != 2).OnlyEnforceIf(b)
```

Example

```
solver = cp_model.CpSolver()

#Force the solver to follow the decision strategy exactly
solver.parameters.search_branching = cp_model.FIXED_SEARCH

vars = [x[i] for i in range(5)]

solution_printer = VarArraySolutionPrinter(vars)
solver.SearchForAllSolutions(model,solution_printer)
```



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for your
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