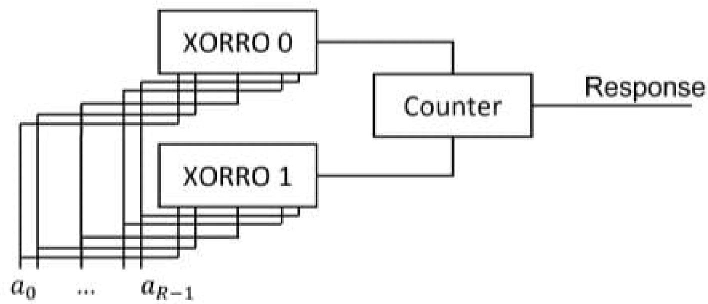
CS771 Assignment 1

Fifty Shades of ML

1 Problem 1



Delay caused by the i^{th} XOR gate :

$$t_0^i = b_i(a_i t_{11}^i + (1-a_i)t_{01}^i) + (1-b_i)(a_i t_{10}^i + (1-a_i)t_{00}^i) \quad i \in \{0, ..., R-1\}$$

where,
t_0 = total delay caused by the XOR when $b_0 = 0$ (input to the first gate)
t_1 = total delay caused by the XOR when $b_0 = 1$ (input to the first gate)
t_{xy}^i = total delay caused by the i^{th} XOR gate when $a_i = x$ and $b_i = y$
Also,

$$b_i = a_{i-1}(1-b_{i-1}) + (1-a_{i-1})b_{i-1}$$

which by rearrangement gives

$$b_i = b_{i-1}(1-2a_{i-1}) + a_{i-1}$$

Iterating over cycles:
when b_0= 0

$$b_1^0 = 0(1-2a_{i-1}) + a_0 = a_0$$

when b_0= 1

$$b_1^1 = 1(1-2a_0) + a_0 = 1 - a_0$$

when b_0= 0

$$b_2^0 = a_0(1-2a_1) + a_1 = 1 - [(1-a_0)(1-2a_1) - a_i]$$

when b_0= 1

$$b_2^1 = (1-a_0)(1-2a_1) + a_1$$

Similarly,

$$b_3^1 = 1 - b_3^0$$

Hence, if input of first XOR gate b_0 is inverted but $a_i \; \forall i \in \{0, 1, ..., R-1\}$remains same then b_i for all gates will be inverted.

$$b_i^1 = 1 - b_i^0 \quad \forall i = 0, 1, ..., R-1$$

Now,

$$t_0 = \sum_{i=0}^{R-1} b_i^0(a_i t_{11}^i + (1-a_i)t_{01}^i) + (1-b_i^0)(a_i t_{10}^i + (1-a_i)t_{00}^i) \tag{1}$$

$$t_1 = \sum_{i=0}^{R-1} b_i^1(a_i t_{11}^i + (1-a_i)t_{01}^i) + (1-b_i^1)(a_i t_{10}^i + (1-a_i)t_{00}^i) \tag{2}$$

Adding (1) and (2) gives

$$t_0 + t_1 = \sum_{i=0}^{R-1} (b_i^0 + b_i^1)((a_i t_{11}^i + (1-a_i)t_{01}^i)) + (2 - b_i^0 - b_i^1)((a_i t_{10}^i + (1-a_i)t_{00}^i))$$

But,

$$b_i^0 + b_i^1 = 1$$

Hence,

$$t_0 + t_1 = \sum_{i=0}^{R-1} (a_i t_{11}^i + (1-a_i)t_{01}^i) + (a_i t_{10}^i + (1-a_i)t_{00}^i)$$

$$t_0 + t_1 = \sum_{i=0}^{R-1} a_i(t_{11}^i + t_{10}^i - t_{01}^i - t_{00}^i) + t_{01}^i + t_{00}^i$$

Let

$$a = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ . \\ a_{R-1} \end{bmatrix}$$

$$\Delta_i = t_{11}^i + t_{10}^i - t_{01}^i - t_{00}^i$$
$$b_i = t_{01}^i + t_{00}^i$$

Next define

$$\omega = [\Delta_0 \quad \Delta_1 \quad \Delta_2 \quad . \quad . \quad . \quad \Delta_{R-1}]^\intercal$$

$$b = \sum_{i=0}^{R-1} b_i$$

Then,

$$t_0 + t_1 = \omega^\intercal a + b$$

Hence, for XORRO1,

$$t_0 + t_1 = (\omega^1)^\intercal a + b^1$$

for XORRO2,

$$t_0 + t_1 = (\omega^2)^\intercal a + b^2$$

Let f_1, f_2 be the frequency of XORRO1 and XORRO2 respectively.

$$\begin{aligned} f_1 - f_2 > 0 &\implies \frac{1}{(t_0+t_1)_{XORRO1}} - \frac{1}{(t_0+t_1)_{XORRO2}} > 0 \\ &\implies (t_0+t_1)_{XORRO2} - (t_0+t_1)_{XORRO1} > 0 \\ &\implies (\omega^2)^\intercal a + b^2 - (\omega^1)^\intercal a - b^1 > 0 \\ &\implies (\omega^2 - \omega^1)^\intercal a + (b^2 - b^1) > 0 \end{aligned}$$

Define

$$W = \omega^2 - \omega^1$$
$$B = b^2 - b^1$$

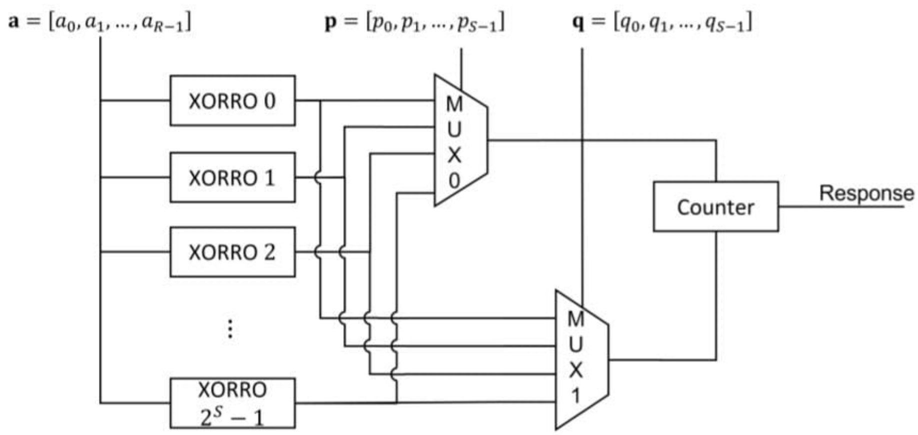
and

$$W^\intercal a + B = Y \quad W \in \mathcal{R}^R \quad a \in \{0, 1\}$$

Now if $f_1 > f_2$ then $Y > 0$ and response is 1. Similarly if $f_1 < f_2$ then $Y < 0$ and response is 0. Hence the response is

$$\frac{sign(Y) + 1}{2}$$

2 Problem 2



$$p = [p_0, p_1, ..., p_{S-1}]$$

$$q = [q_0, q_1, ..., q_{S-1}]$$

Converting p,q to decimal system

$$\implies (p, q) \in \{0, 1, 2, ..., 2^S - 1\}^2, p \neq q$$

So pth and qth XORRO are chosen and their frequency will be compared. Hence, if $f_p > f_q$ then response will be 1, otherwise 0.
Let (ω_{ij}, b_{ij}) be the linear model if ith and jth XORRO are chosen and $i = p, j = q$. So we can write,

$$(\omega, b)_{pq} = \sum_{i,j \in \{0,1,2,...,2^S-1\}} (i == p) \cdot (j == q) \cdot ((\omega_{ij}, b_{ij}))$$

Let $(i == p) \cdot (j == q) = f_{ij}$. Then $f_{ij} = 1 : i = p, j = q$ and 0 otherwise.
Then

$$(\omega, b)_{pq} = \sum_{i,j \in \{0,1,2,...,N\}} f_{ij} \cdot ((\omega_{ij}, b_{ij}))$$

where $N = 2^S - 1$. Let

$$f = \begin{bmatrix} f_{00} \\ f_{01} \\ f_{02} \\ ... \\ f_{0N} \\ f_{10} \\ f_{11} \\ ... \\ f_{1N} \\ ... \\ f_{NN} \end{bmatrix}$$

which is a mtrix of order $(N+1)^2 * 1$.
We can write

$$(\omega, b)_{pq} = \begin{bmatrix} \omega_{00} & \omega_{01} & ... & \omega_{NN} \\ b_{00} & b_{01} & ... & b_{NN} \end{bmatrix} \cdot \begin{bmatrix} f_{00} \\ f_{01} \\ f_{02} \\ ... \\ f_{0N} \\ f_{10} \\ f_{11} \\ ... \\ f_{1N} \\ ... \\ f_{NN} \end{bmatrix}$$

$$(\omega, b)_{pq} = \begin{bmatrix} W \\ B \end{bmatrix} \cdot F = \begin{bmatrix} \omega_{pq} \\ b_{pq} \end{bmatrix}$$

Now our response must be
response = $\omega_{pq}^{\intercal} a + b_{pq}$ where (ω_{pq}, b_{pq}) is the model where pth and qth XORRO are chosen.

$$\implies a = \begin{bmatrix} a_0 \\ a_1 \\ ... \\ a_{R-1} \end{bmatrix}$$

Let

$$a' = \begin{bmatrix} a_0 \\ a_1 \\ ... \\ a_{R-1} \\ 1 \end{bmatrix}$$

then response will be

$$a^{'\intercal} \begin{bmatrix} W \\ B \end{bmatrix} \cdot F$$

This response can be written as

$$\mathcal{ABC}$$

where $\mathcal{A} = a^{'\intercal}, \mathcal{B} = \begin{bmatrix} W \\ B \end{bmatrix}, \mathcal{C} = F$
It can also be written as

$$\begin{bmatrix} \mathcal{B}_{00} & \mathcal{B}_{01} & ... & \mathcal{B}_{10} & \mathcal{B}_{11} & ...\mathcal{B}_{(R+1)N} & \end{bmatrix} \begin{bmatrix} \mathcal{C}_{00}\mathcal{A}_{00} & \mathcal{C}_{00}\mathcal{A}_{01} & ... & \mathcal{C}_{00}\mathcal{A}_{0R} & ... & \mathcal{C}_{((N+1)^2-1)0}\mathcal{A}_{0R} \end{bmatrix}$$

This can we compressed as

$$X^{\intercal} Y$$

where X =

$$\begin{bmatrix} \mathcal{B}_{00} & \mathcal{B}_{01} & ... & \mathcal{B}_{10} & \mathcal{B}_{11} & ...\mathcal{B}_{(R+1)N} & \end{bmatrix}$$

Y =

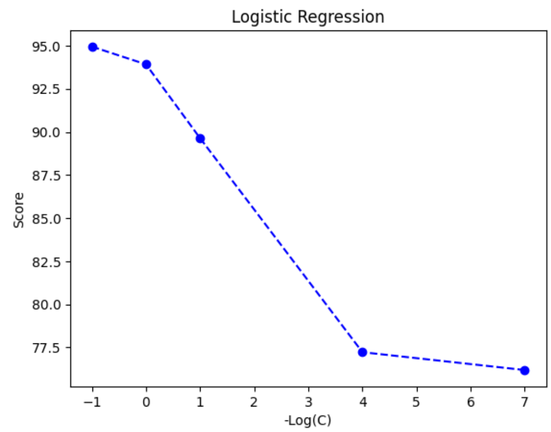
$$\begin{bmatrix} \mathcal{C}_{00}\mathcal{A}_{00} & \mathcal{C}_{00}\mathcal{A}_{01} & ... & \mathcal{C}_{00}\mathcal{A}_{0R} & ... & \mathcal{C}_{((N+1)^2-1)0}\mathcal{A}_{0R} \end{bmatrix}$$

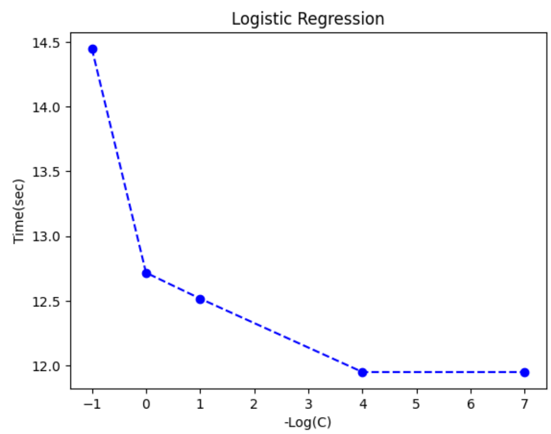
where Y is variable and X is model. Note that Y is not a but a function of a, p, q.

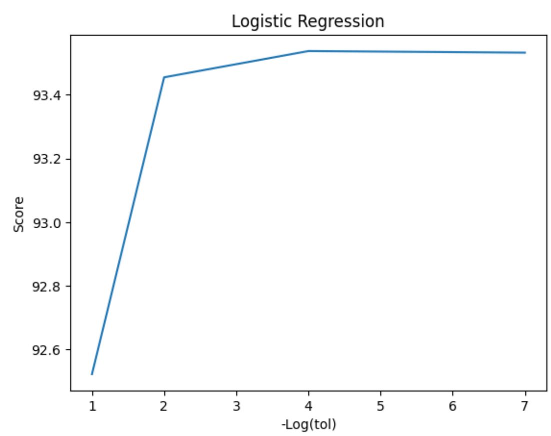
$$Y : a, p, q \rightarrow \mathcal{R}^{(N+1)^2(R+1)}$$

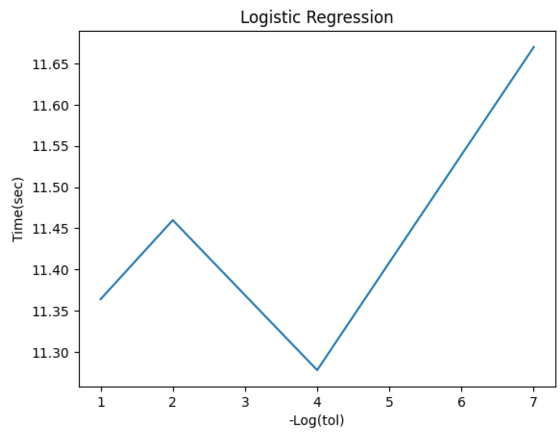
Hence our response would be $X^{\intercal} Y$, where Y is the input for the linear model and Y can be derived from training data set.

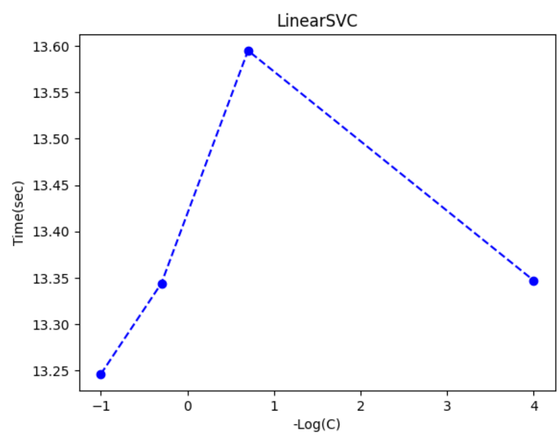
3 Problem 4

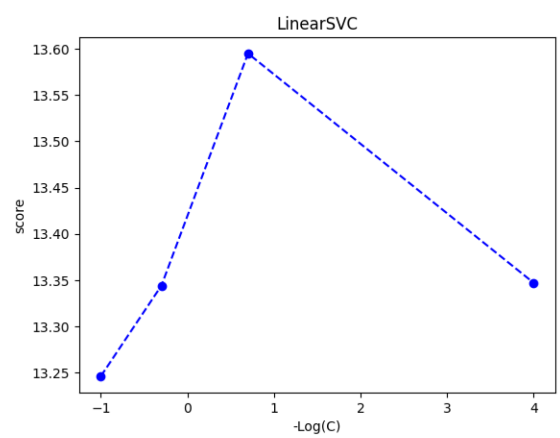

Logistic Regression
Score
95.0
92.5
90.0
87.5
85.0
82.5
80.0
77.5
−1
0
1
2
3
4
5
6
7
-Log(C)

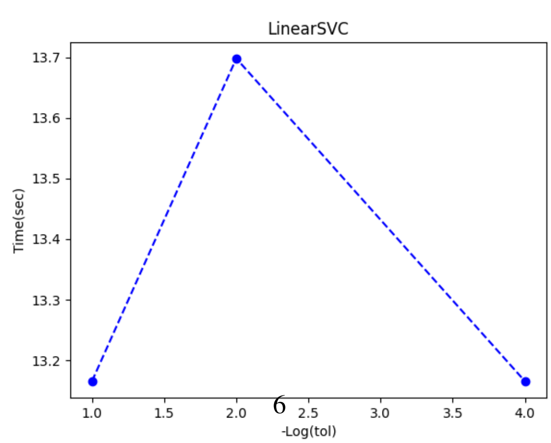

Logistic Regression
Time(sec)
14.5
14.0
13.5
13.0
12.5
12.0
−1
0
1
2
3
4
5
6
7
-Log(C)


Logistic Regression
Score
93.4
93.2
93.0
92.8
92.6
1
2
3
4
5
6
7
-Log(tol)


Logistic Regression
11.65
11.60
11.55
11.50
11.45
11.40
11.35
11.30
Time(sec)
1
2
3
4
5
6
7
-Log(tol)


LinearSVC
13.60
13.55
13.50
13.45
13.40
13.35
13.30
13.25
Time(sec)
−1
0
1
2
3
4
-Log(C)


LinearSVC
13.60
13.55
13.50
13.45
13.40
13.35
13.30
13.25
score
−1
0
1
2
3
4
-Log(C)


LinearSVC
13.7
13.6
13.5
13.4
13.3
13.2
Time(sec)
1.0
1.5
2.0
2.5
3.0
3.5
4.0
-Log(tol)