CHAPTER 9

9.1 The flop counts for the tridiagonal algorithm in Fig. 9.6 can be summarized as

	Mult/Div	Add/Subtr	Total
Forward elimination	3(n – 1)	2(n – 1)	5(n – 1)
Back substitution	2n – 1	n – 1	3n – 2
Total	5n – 4	3n – 3	**8n – 7**

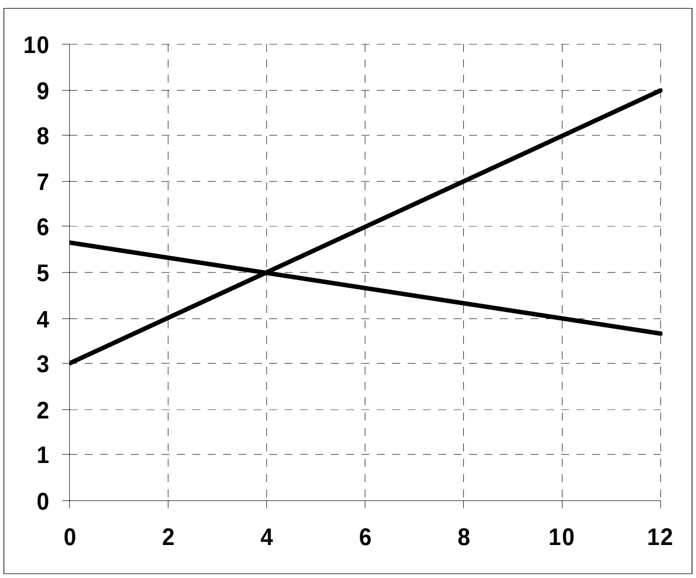
Thus, as n increases, the effort is much, much less than for a full matrix solved with Gauss elimination which is proportional to n^3.

9.2 The equations can be expressed in a format that is compatible with graphing x_2 versus x_1:

$$x_2 = 0.5x_1 + 3$$

$$x_2 = -\frac{1}{6}x_1 + \frac{34}{6}$$

which can be plotted as



Thus, the solution is $x_1 = 4$, $x_2 = 5$. The solution can be checked by substituting it back into the equations to give

$$4(4) - 8(5) = 16 - 40 = -24$$

$$4 + 6(5) = 4 + 30 = 34$$

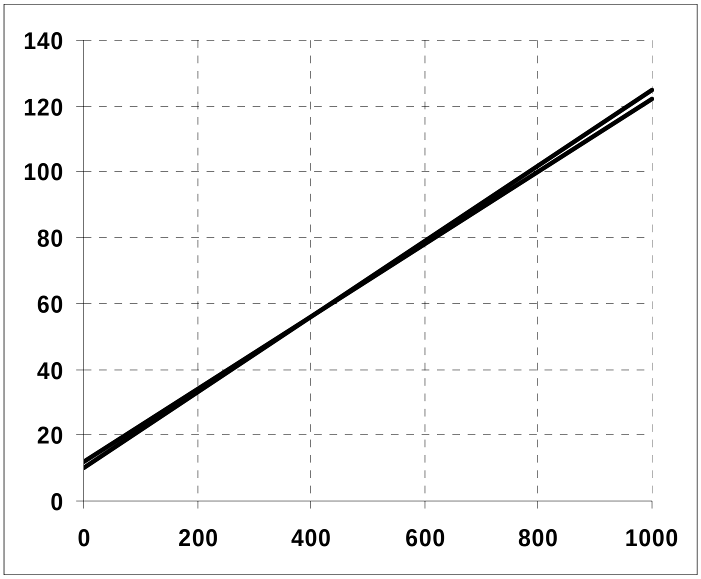
9.3 (a) The equations can be expressed in a format that is compatible with graphing x_2 versus x_1:

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$x_2 = 0.11x_1 + 12$$

$$x_2 = 0.114943x_1 + 10$$

which can be plotted as



Thus, the solution is approximately $x_1 = 400$, $x_2 = 60$. The solution can be checked by substituting it back into the equations to give

$$-1.1(400) + 10(60) = 160 \approx 120$$

$$-2(400) + 17.4(60) = 244 \approx 174$$

Therefore, the graphical solution is not very good.

(b) Because the lines have very similar slopes, you would expect that the system would be ill-conditioned

(c) The determinant can be computed as

$$\begin{vmatrix} -1.1 & 10 \\ -2 & 17.4 \end{vmatrix} = -1.1(17.2) - 10(-2) = -19.14 + 20 = 0.86$$

This result is relatively low suggesting that the solution is ill-conditioned.

9.4 (a) The determinant can be evaluated as

$$D = 0\begin{bmatrix} 2 & -1 \\ -2 & 0 \end{bmatrix} - (-3)\begin{bmatrix} 1 & -1 \\ 5 & 0 \end{bmatrix} + 7\begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$$

$$D = 0(-2) + 3(5) + 7(-12) = -69$$

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

(b) Cramer's rule

$$x_1 = \frac{\begin{vmatrix} 2 & -3 & 7 \\ 3 & 2 & -1 \\ 2 & -2 & 0 \end{vmatrix}}{-69} = \frac{-68}{-69} = 0.9855$$

$$x_2 = \frac{\begin{vmatrix} 0 & 2 & 7 \\ 1 & 3 & -1 \\ 5 & 2 & 0 \end{vmatrix}}{-69} = \frac{-101}{-69} = 1.4638$$

$$x_3 = \frac{\begin{vmatrix} 0 & -3 & 2 \\ 1 & 2 & 3 \\ 5 & -2 & 2 \end{vmatrix}}{-69} = \frac{-63}{-69} = 0.9130$$

(c) Pivoting is necessary, so switch the first and third rows,

$$\begin{aligned} 5x_1 - 2x_2 \phantom{{}-x_3} &= 2 \\ x_1 + 2x_2 - x_3 &= 3 \\ -3x_2 + 7x_3 &= 2 \end{aligned}$$

Multiply pivot row 1 by 1/5 and subtract the result from the second row to eliminate the a_{21} term.

$$\begin{aligned} 5x_1 - 2x_2 \phantom{{}-x_3} &= 2 \\ 2.4x_2 - x_3 &= 2.6 \\ -3x_2 + 7x_3 &= 2 \end{aligned}$$

Pivoting is necessary so switch the second and third row,

$$\begin{aligned} 5x_1 - 2x_2 \phantom{{}+7x_3} &= 2 \\ -3x_2 + 7x_3 &= 2 \\ 2.4x_2 - x_3 &= 2.6 \end{aligned}$$

Multiply pivot row 2 by 2.4/(–3) and subtract the result from the third row to eliminate the a_{32} term.

$$\begin{aligned} 5x_1 - 2x_2 \phantom{{}+7x_3} &= 2 \\ -3x_2 + 7x_3 &= 2 \\ 4.6x_3 &= 4.2 \end{aligned}$$

The solution can then be obtained by back substitution

$$x_3 = \frac{4.2}{4.6} = 0.913043$$

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$x_2 = \frac{2 - 7(0.913043)}{-3} = 1.463768$$

$$x_1 = \frac{2 + 2(1.463768)}{5} = 0.985507$$

(d)

$$-3(1.463768) + 7(0.913043) = 2$$

$$0.985507 + 2(1.463768) - (0.913043) = 3$$

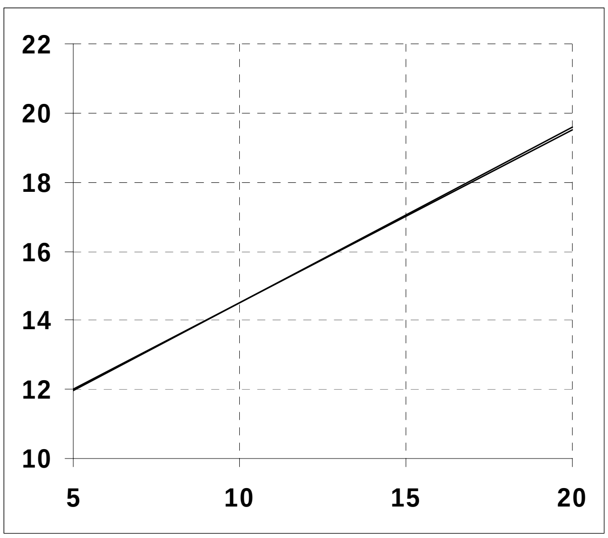
$$5(0.985507) - 2(1.463768) = 2$$

9.5 (a) The equations can be expressed in a format that is compatible with graphing x_2 versus x_1:

$$x_2 = 0.5x_1 + 9.5$$

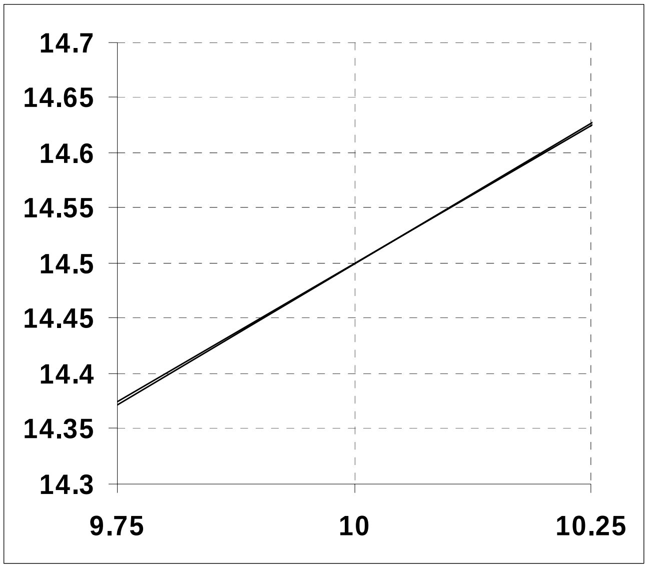
$$x_2 = 0.51x_1 + 9.4$$

The resulting plot indicates that the intersection of the lines is difficult to detect:



Only when the plot is zoomed is it at all possible to discern that solution seems to lie at about x_1 = 14.5 and x_2 = 10.

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.



(b) The determinant can be computed as

$$\begin{vmatrix} 0.5 & -1 \\ 1.02 & -2 \end{vmatrix} = 0.5(-2) - (-1)(1.02) = 0.02$$

which is close to zero.

(c) Because the lines have very similar slopes and the determinant is so small, you would expect that the system would be ill-conditioned

(d) Multiply the first equation by 1.02/0.5 and subtract the result from the second equation to eliminate the x_1 term from the second equation,

$$0.5x_1 - x_2 = -9.5$$

$$0.04x_2 = 0.58$$

The second equation can be solved for

$$x_2 = \frac{0.58}{0.04} = 14.5$$

This result can be substituted into the first equation which can be solved for

$$x_1 = \frac{-9.5 + 14.5}{0.5} = 10$$

(e) Multiply the first equation by 1.02/0.52 and subtract the result from the second equation to eliminate the x_1 term from the second equation,

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$0.52x_1 \quad - x_2 = -9.5$$
$$- 0.03846x_2 = -0.16538$$

The second equation can be solved for

$$x_2 = \frac{-0.16538}{-0.03846} = 4.3$$

This result can be substituted into the first equation which can be solved for

$$x_1 = \frac{-9.5 + 4.3}{0.52} = -10$$

Interpretation: The fact that a slight change in one of the coefficients results in a radically different solution illustrates that this system is very ill-conditioned.

9.6 (a) Multiply the first equation by –3/10 and subtract the result from the second equation to eliminate the x_1 term from the second equation. Then, multiply the first equation by 1/10 and subtract the result from the third equation to eliminate the x_1 term from the third equation.

$$10x_1 + 2x_2 \quad - x_3 = 27$$
$$- 5.4x_2 + 1.7x_3 = -53.4$$
$$0.8x_2 + 5.1x_3 = -24.2$$

Multiply the second equation by 0.8/(–5.4) and subtract the result from the third equation to eliminate the x_2 term from the third equation,

$$10x_1 + 2x_2 \quad - x_3 = 27$$
$$- 5.4x_2 \quad + 1.7x_3 = -53.4$$
$$5.351852x_3 = -32.11111$$

Back substitution can then be used to determine the unknowns

$$x_3 = \frac{-32.11111}{5.351852} = -6$$

$$x_2 = \frac{(-53.4 - 1.7(-6))}{-5.4} = 8$$

$$x_1 = \frac{(27 - 6 - 2(8))}{10} = 0.5$$

(b) Check:

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$10(0.5)+2(8)-(-6)=27$$

$$-3(0.5)-6(8)+2(-6)=-61.5$$

$$0.5+8+5(-6)=-21.5$$

9.7 (a) Pivoting is necessary, so switch the first and third rows,

$$-8x_1+x_2-2x_3=-20$$

$$-3x_1-x_2+7x_3=-34$$

$$2x_1-6x_2-x_3=-38$$

Multiply the first equation by –3/(–8) and subtract the result from the second equation to eliminate the a_{21} term from the second equation. Then, multiply the first equation by 2/(–8) and subtract the result from the third equation to eliminate the a_{31} term from the third equation.

$$\begin{aligned}-8x_1 \quad +x_2 \quad -2x_3 &= -20\\ -1.375x_2+7.75x_3 &= -26.5\\ -5.75x_2-1.5x_3 &= -43\end{aligned}$$

Pivoting is necessary so switch the second and third row,

$$\begin{aligned}-8x_1 \quad +x_2 \quad -2x_3 &= -20\\ -5.75x_2-1.5x_3 &= -43\\ -1.375x_2+7.75x_3 &= -26.5\end{aligned}$$

Multiply pivot row 2 by –1.375/(–5.75) and subtract the result from the third row to eliminate the a_{32} term.

$$\begin{aligned}-8x_1 \quad +x_2 \quad -2x_3 &= -20\\ -5.75x_2 \quad -1.5x_3 &= -43\\ 8.108696x_3 &= -16.21739\end{aligned}$$

The solution can then be obtained by back substitution

$$x_3=\frac{-16.21739}{8.108696}=-2$$

$$x_2=\frac{-43+1.5(-2)}{-5.75}=8$$

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$x_1 = \frac{-20 + 2(-2) - 1(8)}{-8} = 4$$

(b) Check:

$$2(4) - 6(8) - (-2) = -38$$

$$-3(4) - (8) + 7(-2) = -34$$

$$-8(4) + (8) - 2(-2) = -20$$

9.8 Multiply the first equation by –0.4/0.8 and subtract the result from the second equation to eliminate the x_1 term from the second equation.

$$\begin{bmatrix} 0.8 & -0.4 & \\ & 0.6 & -0.4 \\ & -0.4 & 0.8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 41 \\ 45.5 \\ 105 \end{Bmatrix}$$

Multiply pivot row 2 by –0.4/0.6 and subtract the result from the third row to eliminate the x_2 term.

$$\begin{bmatrix} 0.8 & -0.4 & \\ & 0.6 & -0.4 \\ & & 0.533333 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 41 \\ 45.5 \\ 135.3333 \end{Bmatrix}$$

The solution can then be obtained by back substitution

$$x_3 = \frac{135.3333}{0.533333} = 253.75$$

$$x_2 = \frac{45.5 - (-0.4)253.75}{0.6} = 245$$

$$x_1 = \frac{41 - (-0.4)245}{0.8} = 173.75$$

(b) Check:

$$0.8(173.75) - 0.4(245) = 41$$

$$-0.4(173.75) + 0.8(245) - 0.4(253.75) = 25$$

$$-0.4(245) + 0.8(253.75) = 105$$

9.9 Mass balances can be written for each of the reactors as

$$500 - Q_{13}c_1 - Q_{12}c_1 + Q_{21}c_2 = 0$$
$$Q_{12}c_1 - Q_{21}c_2 - Q_{23}c_2 = 0$$

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$200 + Q_{13}c_1 + Q_{23}c_2 - Q_{33}c_3 = 0$$

Values for the flows can be substituted and the system of equations can be written in matrix form as

$$\begin{bmatrix} 130 & -30 & 0 \\ -90 & 90 & 0 \\ -40 & -60 & 120 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \begin{Bmatrix} 500 \\ 0 \\ 200 \end{Bmatrix}$$

The solution can then be developed using MATLAB,

```
>> A=[130 -30 0;-90 90 0;-40 -60 120];
>> B=[500;0;200];
>> C=A\B

C =
    5.0000
    5.0000
    5.8333
```

9.10 Let x_i = the volume taken from pit i. Therefore, the following system of equations must hold

$$\begin{aligned} 0.55x_1 + 0.25x_2 + 0.25x_3 &= 4800 \\ 0.30x_1 + 0.45x_2 + 0.20x_3 &= 5800 \\ 0.15x_1 + 0.30x_2 + 0.55x_3 &= 5700 \end{aligned}$$

MATLAB can be used to solve this system of equations for

```
>> A=[0.55 0.25 0.25;0.3 0.45 0.2;0.15 0.3 0.55];
>> b=[4800;5800;5700];
>> x=A\b

x =
  1.0e+003 *
    2.4167
    9.1933
    4.6900
```

Therefore, we take x_1 = 2416.667, x_2 = 9193.333, and x_3 = 4690 m^3 from pits 1, 2 and 3 respectively.

9.11 Let c_i = component i. Therefore, the following system of equations must hold

$$\begin{aligned} 15c_1 + 17c_2 + 19c_3 &= 3890 \\ 0.30c_1 + 0.40c_2 + 0.55c_3 &= 95 \\ 1.0c_1 + 1.2c_2 + 1.5c_3 &= 282 \end{aligned}$$

These can then be solved for c_1 = 90, c_2 = 60, and c_3 = 80.

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

9.12 Centered differences (recall p. 101 and 103) can be substituted for the derivatives to give

$$0 = D\frac{c_{i-1} - 2c_i + c_{i+1}}{\Delta x^2} - U\frac{c_{i+1} - c_{i-1}}{2\Delta x} - kc_i$$

collecting terms yields

$$-(D + 0.5U\Delta x)c_{i-1} + (2D + k\Delta x^2)c_i - (D - 0.5U\Delta x)c_{i+1} = 0$$

Assuming Δx = 1 and substituting the parameters gives

$$-2.5c_{i-1} + 4.2c_i - 1.5c_{i+1} = 0$$

For the first interior node (i = 1),

$$4.2c_1 - 1.5c_2 = 200$$

For the last interior node (i = 9)

$$-2.5c_8 + 4.2c_9 = 30$$

These and the equations for the other interior nodes can be assembled in matrix form as

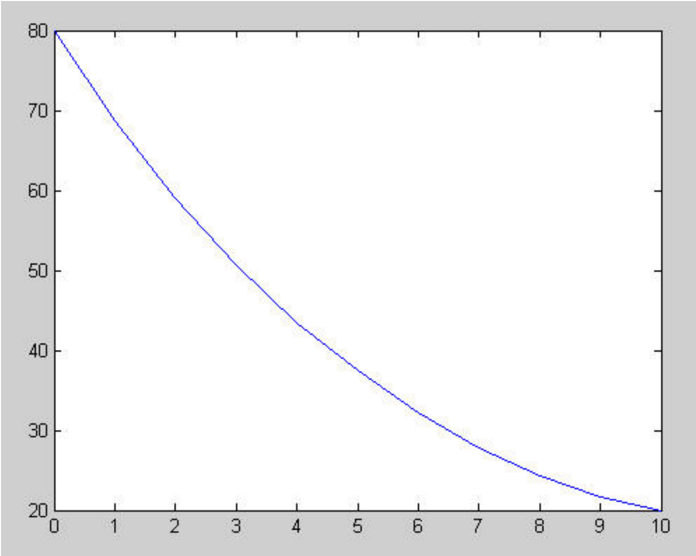
$$\begin{bmatrix} 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{Bmatrix} = \begin{Bmatrix} 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 30 \end{Bmatrix}$$

The following script generates and plots the solution:

```
A=[4.2 -1.5 0 0 0 0 0 0 0
     -2.5 4.2 -1.5 0 0 0 0 0 0
     0 -2.5 4.2 -1.5 0 0 0 0 0
     0 0 -2.5 4.2 -1.5 0 0 0 0
     0 0 0 -2.5 4.2 -1.5 0 0 0
     0 0 0 0 -2.5 4.2 -1.5 0 0
     0 0 0 0 0 -2.5 4.2 -1.5 0
     0 0 0 0 0 0 -2.5 4.2 -1.5
     0 0 0 0 0 0 0 -2.5 4.2];
 b=[200 0 0 0 0 0 0 0 30]';
 c=A\b;
```

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

```
c=[80 c' 20];
x=0:1:10;
plot(x,c)
```



9.13 For the first stage, the mass balance can be written as

$$F_1 y_{\text{in}} + F_2 x_2 = F_2 x_1 + F_1 x_1$$

Substituting $x = Ky$ and rearranging gives

$$-\left(1 + \frac{F_2}{F_1} K\right) y_1 + \frac{F_2}{F_1} K y_2 = -y_{\text{in}}$$

Using a similar approach, the equation for the last stage is

$$y_4 - \left(1 + \frac{F_2}{F_1} K\right) y_5 = -\frac{F_2}{F_1} x_{\text{in}}$$

For interior stages,

$$y_{\text{i-1}} - \left(1 + \frac{F_2}{F_1} K\right) y_{\text{i}} + \frac{F_2}{F_1} K y_{i+1} = 0$$

These equations can be used to develop the following system,

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$\begin{bmatrix} 9 & -8 & 0 & 0 & 0 \\ -1 & 9 & -8 & 0 & 0 \\ 0 & -1 & 9 & -8 & 0 \\ 0 & 0 & -1 & 9 & -8 \\ 0 & 0 & 0 & -1 & 9 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{Bmatrix} = \begin{Bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

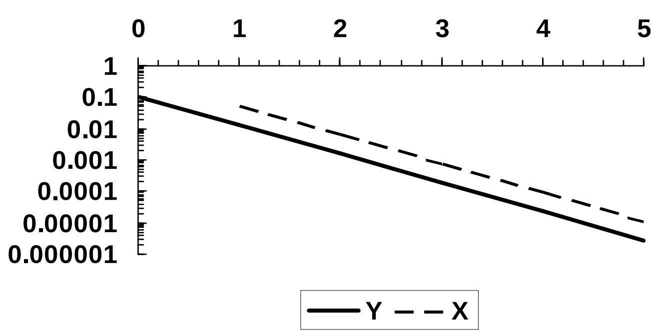
The solution can be developed with MATLAB,

```
>> format long
>> A=[9 -8 0 0 0;
-1 9 -8 0 0;
0 -1 9 -8 0;
0 0 -1 9 -8;
0 0 0 -1 9];
>> B=[0.1;0;0;0;0];
>> Y=A\B

Y =
   0.01249966621272
   0.00156212448931
   0.00019493177388
   0.00002403268445
   0.00000267029827
```

Note that the corresponding values of X can be computed as

```
>> X=4*Y

X =
   0.04999866485086
   0.00624849795722
   0.00077972709552
   0.00009613073780
   0.00001068119309
```

Therefore, y_{out} = 0.0000026703 and x_{out} = 0.05. In addition, here is a logarithmic plot of the simulation results versus stage,



9.14 Assuming a unit flow for Q_1, the simultaneous equations can be written in matrix form as

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -2 & 3 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{Bmatrix} Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

These equations can then be solved with MATLAB,

```
>> A=[-2 1 2 0 0 0;
0 0 -2 1 2 0;
0 0 0 0 -2 3;
1 1 0 0 0 0;
0 1 -1 -1 0 0;
0 0 0 1 -1 -1];
>> B=[0 0 0 1 0 0 ]';
>> Q=A\B

Q =
    0.5059
    0.4941
    0.2588
    0.2353
    0.1412
    0.0941
```

9.15 The solution can be generated with MATLAB,

```
>> A=[1 0 0 0 0 0 0 0 1 0;
      0 0 1 0 0 0 0 1 0 0;
      0 1 0 3/5 0 0 0 0 0 0;
      -1 0 0 -4/5 0 0 0 0 0 0;
      0 -1 0 0 0 0 3/5 0 0 0;
      0 0 0 0 -1 0 -4/5 0 0 0;
      0 0 -1 -3/5 0 1 0 0 0 0;
      0 0 0 4/5 1 0 0 0 0 0;
      0 0 0 0 0 -1 -3/5 0 0 0;
      0 0 0 0 0 0 4/5 0 0 1];
>> B=[0 0 -74 0 0 24 0 0 0 0]';
>> x=A\B

x =
   37.3333
  -46.0000
   74.0000
  -46.6667
   37.3333
   46.0000
  -76.6667
  -74.0000
  -37.3333
   61.3333
```

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

Therefore, in kN

AB = 37.3333	BC = –46	AD = 74	BD = –46.6667	CD = 37.3333
DE = 46	CE = –76.6667	A_x = –74	A_y = –37.33333	E_y = 61.3333

9.16

```
function x=pentasol(A,b)
% pentasol: pentadiagonal system solver banded system
%   x=pentasol(A,b):
%      Solve a pentadiagonal system Ax=b
% input:
%   A = pentadiagonal matrix
%   b = right hand side vector
% output:
%   x = solution vector

% Error checks
[m,n]=size(A);
if m~=n,error('Matrix must be square');end
if length(b)~=m,error('Matrix and vector must have the same number of
rows');end
x=zeros(n,1);

% Extract bands
d=[0;0;diag(A,-2)];
e=[0;diag(A,-1)];
f=diag(A);
g=diag(A,1);
h=diag(A,2);
delta=zeros(n,1);
epsilon=zeros(n-1,1);
gamma=zeros(n-2,1);
alpha=zeros(n,1);
c=zeros(n,1);
z=zeros(n,1);

% Decomposition
delta(1)=f(1);
epsilon(1)=g(1)/delta(1);
gamma(1)=h(1)/delta(1);
alpha(2)=e(2);
delta(2)=f(2)-alpha(2)*epsilon(1);
epsilon(2)=(g(2)-alpha(2)*gamma(1))/delta(2);
gamma(2)=h(2)/delta(2);
for k=3:n-2
  alpha(k)=e(k)-d(k)*epsilon(k-2);
  delta(k)=f(k)-d(k)*gamma(k-2)-alpha(k)*epsilon(k-1);
  epsilon(k)=(g(k)-alpha(k)*gamma(k-1))/delta(k);
  gamma(k)=h(k)/delta(k);
end
alpha(n-1)=e(n-1)-d(n-1)*epsilon(n-3);
delta(n-1)=f(n-1)-d(n-1)*gamma(n-3)-alpha(n-1)*epsilon(n-2);
epsilon(n-1)=(g(n-1)-alpha(n-1)*gamma(n-2))/delta(n-1);
alpha(n)=e(n)-d(n)*epsilon(n-2);
```

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

```
delta(n)=f(n)-d(n)*gamma(n-2)-alpha(n)*epsilon(n-1);
% Forward substitution
c(1)=b(1)/delta(1);
c(2)=(b(2)-alpha(2)*c(1))/delta(2);
for k=3:n
  c(k)=(b(k)-d(k)*c(k-2)-alpha(k)*c(k-1))/delta(k);
end
% Back substitution
x(n)=c(n);
x(n-1)=c(n-1)-epsilon(n-1)*x(n);
for k=n-2:-1:1
  x(k)=c(k)-epsilon(k)*x(k+1)-gamma(k)*x(k+2);
end
```

Test of function:

```
>> A=[8 -2 -1 0 0
-2 9 -4 -1 0
-1 3 7 -1 -2
0 -4 -2 12 -5
0 0 -7 -3 15];
>> b=[5 2 1 1 5]';
>> x=pentasol(A,b)

x =
    0.7993
    0.5721
    0.2503
    0.5491
    0.5599
```

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.