CHAPTER 8

8.1

```
>> Aug = [A eye(size(A))]
```

Here's an example session of how it can be employed.

```
>> A = rand(3)

A =
    0.9501    0.4860    0.4565
    0.2311    0.8913    0.0185
    0.6068    0.7621    0.8214

>> Aug = [A eye(size(A))]
Aug =

    0.9501    0.4860    0.4565    1.0000         0         0
    0.2311    0.8913    0.0185         0    1.0000         0
    0.6068    0.7621    0.8214         0         0    1.0000
```

8.2 (a) $[A] = 3\times2$ $\quad [B] = 3\times3$ $\quad [C] = 3\times1$ $\quad [D] = 2\times4$

$[E] = 3\times3$ $\quad [F] = 2\times3$ $\quad [G] = 1\times3$

(b) Square: $[B]$ and $[E]$
Column: $[C]$
Row: $[G]$

(c) $a_{12} = 7$ $\quad b_{23} = 7$ $\quad d_{32}$ = does not exist $\quad e_{22} = 2$ $\quad f_{12} = 0$ $\quad g_{12} = 6$

(d)

(1) $[E]+[B]=\begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}$ (2) $[A]+[F]=\text{not possible}$

(3) $[B]-[E]=\begin{bmatrix} 3 & -2 & -1 \\ -6 & 0 & 4 \\ -2 & 0 & -2 \end{bmatrix}$ (4) $7[B]=\begin{bmatrix} 28 & 21 & 49 \\ 7 & 14 & 49 \\ 14 & 0 & 28 \end{bmatrix}$

(5) $[E]\times[B]=\begin{bmatrix} 25 & 13 & 74 \\ 36 & 25 & 75 \\ 28 & 12 & 52 \end{bmatrix}$ (6) $\{C\}^T = \lfloor 3 \quad 6 \quad 1 \rfloor$

(7) $[B]\times[A]=\begin{bmatrix} 54 & 76 \\ 41 & 53 \\ 28 & 38 \end{bmatrix}$ (8) $\{D\}^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}$

(9) $[A]\times[C]=\text{not possible}$ (10) $[I]\times[B]=[B]$

(11) $[E]^T[E] = \begin{bmatrix} 66 & 19 & 53 \\ 19 & 29 & 46 \\ 53 & 46 & 109 \end{bmatrix}$ (12) $[C]^T[C] = 46$

8.3 The terms can be collected to give

$$\begin{bmatrix} 0 & -7 & 5 \\ 0 & 4 & 7 \\ -4 & 3 & -7 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 50 \\ -30 \\ 40 \end{Bmatrix}$$

Here is the MATLAB session:

```
>> A = [0 -7 5;0 4 7;-4 3 -7];
>> b = [50;-30;40];
>> x = A\b

x =
  -15.1812
   -7.2464
   -0.1449

>> AT = A'

AT =
     0     0    -4
    -7     4     3
     5     7    -7

>> AI = inv(A)

AI =
   -0.1775   -0.1232   -0.2500
   -0.1014    0.0725         0
    0.0580    0.1014         0
```

8.4 (a) Here are all the possible multiplications:

```
>> A=[6 -1;12 8;-5 4];
>> B=[4 0;0.5 2];
>> C=[2 -2;-3 1];

>> A*B
ans =
   23.5000   -2.0000
   52.0000   16.0000
  -18.0000    8.0000

>> A*C
ans =
    15   -13
     0   -16
   -22    14
```

```
>> B*C
ans =
     8    -8
    -5     1

>> C*B
ans =
    7.0000   -4.0000
  -11.5000    2.0000
```

(b) $[B][A]$ and $[C][A]$ are impossible because the inner dimensions do not match:

$(2\times2) * (3\times2)$

(c) According to **(a)**, $[B][C] \neq [C][B]$

8.5 The mass balances can be written as

$$
\begin{aligned}
&(Q_{15}+Q_{12})c_1 && && -Q_{31}c_3 && &&= Q_{01}c_{01} \\
&-Q_{12}c_1 + (Q_{23}+Q_{24}+Q_{25})c_2 && && && &&= 0 \\
& && -Q_{23}c_2 && +(Q_{31}+Q_{34})c_3 && &&= Q_{03}c_{03} \\
& && -Q_{24}c_2 && -Q_{34}c_3 + Q_{44}c_4 && -Q_{54}c_5 &&= 0 \\
&-Q_{15}c_1 && -Q_{25}c_2 && && +(Q_{54}+Q_{55})c_5 &&= 0
\end{aligned}
$$

The parameters can be substituted and the result written in matrix form as

$$
\begin{bmatrix} 6 & 0 & -1 & 0 & 0 \\ -3 & 3 & 0 & 0 & 0 \\ 0 & -1 & 9 & 0 & 0 \\ 0 & -1 & -8 & 11 & -2 \\ -3 & -1 & 0 & 0 & 4 \end{bmatrix}
\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{Bmatrix}
=
\begin{Bmatrix} 50 \\ 0 \\ 160 \\ 0 \\ 0 \end{Bmatrix}
$$

MATLAB can then be used to solve for the concentrations

```
>> Q = [6 0 -1 0 0;
-3 3 0 0 0;
0 -1 9 0 0;
0 -1 -8 11 -2;
-3 -1 0 0 4];
>> Qc = [50;0;160;0;0];
>> c = Q\Qc

c =
   11.5094
   11.5094
   19.0566
   16.9983
```

```
   11.5094
```

8.6 The problem can be written in matrix form as

$$\begin{bmatrix} 0.866 & 0 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.866 & 0 & 0 & 0 \\ -0.866 & -1 & 0 & -1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & -0.866 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

MATLAB can then be used to solve for the forces and reactions,

```
>> A = [0.866 0 -0.5 0 0 0;
0.5 0 0.866 0 0 0;
-0.866 -1 0 -1 0 0;
-0.5 0 0 0 -1 0;
0 1 0.5 0 0 0;
0 0 -0.866 0 0 -1];
>> b = [0 -1000 0 0 0 0]';
>> F = A\b

F =
 -500.0220
  433.0191
 -866.0381
   -0.0000
  250.0110
  749.9890
```

Therefore,

$F_1 = –500$	$F_2 = 433$	$F_3 = –866$
$H_2 = 0$	$V_2 = 250$	$V_3 = 750$

8.7

```
>> k1 = 10;k2 = 30;k3 = 30;k4 = 10;
>> m1 = 1;m2 = 1;m3 = 1;
>> km = [(1/m1)*(k2+k1), -(k2/m1),0;-(k2/m2), (1/m2)*(k2+k3), -
(k3/m2);0, -(k3/m3),(1/m3)*(k3+k4)];
>> x = [0.05;0.04;0.03];
>> kmx = km*x

kmx =

    0.8000
         0
         0
```

Therefore, $\ddot{x}_1 = –0.8$, $\ddot{x}_2 = 0$, and $\ddot{x}_3 = 0$ m/s^2.

8.8
```
>> A=[3+2*i 4;-i 1]
>> b=[2+i;3]
>> z=A\b
```

z =
  -0.5333 + 1.4000i
   1.6000 - 0.5333i

8.9 Vertical force balances can be written to give the following system of equations,

$$
\begin{aligned}
&m_1 g + k_2(x_2 - x_1) - k_1 x_1 = 0\\
&m_2 g + k_3(x_3 - x_2) - k_2(x_2 - x_1) = 0\\
&m_3 g + k_4(x_4 - x_3) - k_3(x_3 - x_2) = 0\\
&m_4 g + k_5(x_5 - x_4) - k_4(x_4 - x_3) = 0\\
&m_5 g - k_5(x_5 - x_4) = 0
\end{aligned}
$$

Collecting terms,

$$
\begin{bmatrix}
k_1 + k_2 & -k_2 & & & \\
-k_2 & k_2 + k_3 & -k_3 & & \\
& -k_3 & k_3 + k_4 & -k_4 & \\
& & -k_4 & k_4 + k_5 & -k_5 \\
& & & -k_5 & k_5
\end{bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix}
=
\begin{Bmatrix} m_1 g \\ m_2 g \\ m_3 g \\ m_4 g \\ m_5 g \end{Bmatrix}
$$

After substituting the parameters, the equations can be expressed as (g = 9.81),

$$
\begin{bmatrix}
130 & -50 & & & \\
-50 & 120 & -70 & & \\
& -70 & 170 & -100 & \\
& & -100 & 120 & -20 \\
& & & -20 & 20
\end{bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix}
=
\begin{Bmatrix} 539.55 \\ 735.75 \\ 588.60 \\ 735.75 \\ 882.90 \end{Bmatrix}
$$

The solution can then be obtained with MATLAB:

```
>> A=[130 -50 0 0 0
-50 120 -70 0 0
0 -70 170 -100 0
0 0 -100 120 -20
0 0 0 -20 20];
>> b=[539.55;735.75;588.6;735.75;882.9];
>> x=A\b
x =
   43.5319
  102.3919
  133.9240
```

```
  150.1105
  194.2555
```

8.10 The position of the three masses can be modeled by the following steady-state force balances

$$0 = k(x_2 - x_1) + m_1 g - kx_1$$

$$0 = k(x_3 - x_2) + m_2 g - k(x_2 - x_1)$$

$$0 = m_3 g - k(x_3 - x_2)$$

Terms can be combined to yield

$$2kx_1 - kx_2 \qquad\qquad = m_1 g$$

$$-kx_1 + 2kx_2 - kx_3 = m_2 g$$

$$\qquad -kx_2 + kx_3 = m_3 g$$

Substituting the parameter values

$$\begin{bmatrix} 20 & -10 & 0 \\ -10 & 20 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 19.62 \\ 29.43 \\ 24.525 \end{Bmatrix}$$

A MATLAB session can be used to obtain the solution for the displacements

```
>> K=[20 -10 0;-10 20 -10;0 -10 10];
>> m=[2;3;2.5];
>> mg=m*9.81;
>> x=K\mg

x =
    7.3575
   12.7530
   15.2055
```

8.11 The simultaneous equations are

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 5 & -15 & 0 & -5 & -2 \\ 10 & -5 & 0 & -25 & 0 & 0 \end{bmatrix} \begin{Bmatrix} i_{12} \\ i_{52} \\ i_{32} \\ i_{65} \\ i_{54} \\ i_{43} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 200 \end{Bmatrix}$$

This system can be solved with MATLAB,

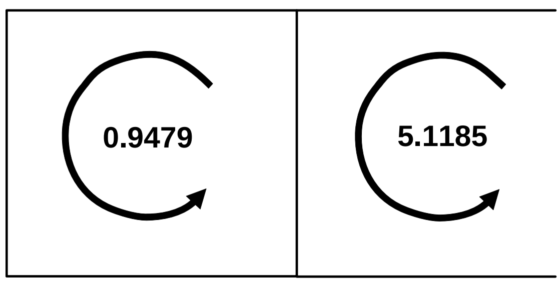
```
>> A=[1 1 1 0 0 0;
0 -1 0 1 -1 0;
```

```
0 0 -1 0 0 1;
0 0 0 0 1 -1;
0 5 -15 0 -5 -2;
10 -5 0 -25 0 0];
>> B=[0 0 0 0 0 200]';
>> I=A\B

I =
    5.1185
   -4.1706
   -0.9479
   -5.1185
   -0.9479
   -0.9479
```

$i_{21} = 5.1185$ $\quad i_{52} = -4.1706$ $\quad i_{32} = -0.9479$ $\quad i_{65} = -5.1185$ $\quad i_{54} = -0.9479$ $\quad i_{43} = -0.9479$

Here are the resulting currents superimposed on the circuit:



8.12 The current equations can be written as

$$-i_{21} - i_{23} + i_{52} = 0$$
$$i_{23} - i_{35} + i_{43} = 0$$
$$-i_{43} + i_{54} = 0$$
$$i_{35} - i_{52} + i_{65} - i_{54} = 0$$

Voltage equations:

$$i_{21} = \frac{V_2 - 10}{35} \qquad i_{54} = \frac{V_5 - V_4}{15}$$

$$i_{23} = \frac{V_2 - V_3}{30} \qquad i_{35} = \frac{V_3 - V_5}{7}$$

$$i_{43} = \frac{V_4 - V_3}{8} \qquad i_{52} = \frac{V_5 - V_2}{10}$$

$$i_{65} = \frac{150 - V_5}{5}$$

$$\begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 35 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_{21} \\ i_{23} \\ i_{52} \\ i_{35} \\ i_{43} \\ i_{54} \\ i_{65} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 150 \end{Bmatrix}$$

This system can be solved for

$i_{21} = 2.9291$ $i_{23} = -0.6457$ $i_{52} = 2.2835$ $i_{35} = -0.4950$ $i_{43} = 0.1507$
$i_{54} = 0.1507$ $i_{65} = 2.9291$ $V_2 = 112.5196$ $V_3 = 131.8893$ $V_4 = 133.0945$
$V_5 = 135.3543$

8.13

```
function X=mmult(Y,Z)
% mmult: matrix multiplication
%   X=mmult(Y,Z)
%     multiplies two matrices
% input:
%   Y = first matrix
%   Z = second matrix
% output:
%   X = product

if nargin<2,error('at least 2 input arguments required'),end
[m,n]=size(Y);[n2,p]=size(Z);
if n~=n2,error('Inner matrix dimensions must agree.'),end
for i=1:m
  for j=1:p
    s=0.;
    for k=1:n
      s=s+Y(i,k)*Z(k,j);
    end
    X(i,j)=s;
  end
end
```

Test of function for cases from Prob. 8.4:

```
>> A=[6 -1;12 8;-5 4];
>> B=[4 0;0.5 2];
>> C=[2 -2;-3 1];
>> mmult(A,B)
```

```
ans =
   23.5000   -2.0000
   52.0000   16.0000
  -18.0000    8.0000

>> mmult(A,C)
ans =
    15   -13
     0   -16
   -22    14

>> mmult(B,C)
ans =
     8    -8
    -5     1

>> mmult(C,B)
ans =
    7.0000   -4.0000
  -11.5000    2.0000

>> mmult(B,A)
??? Error using ==> mmult
Inner matrix dimensions must agree.

>> mmult(C,A)
??? Error using ==> mmult
Inner matrix dimensions must agree.
```

8.14

```
function AT=matran(A)
% matran: matrix transpose
%   AT=mtran(A)
%      generates the transpose of a matrix
% input:
%   A = original matrix
% output:
%   AT = transpose

[m,n]=size(A);
for i = 1:m
  for j = 1:n
    AT(j,i) = A(i,j);
  end
end
```

Test of function for cases from Prob. 8.4:

```
>> matran(A)
ans =
     6    12    -5
    -1     8     4

>> matran(B)
ans =
```

```
    4.0000    0.5000
         0    2.0000

>> matran(C)
ans =
     2    -3
    -2     1
```