CHAPTER 1

1.1 You are given the following differential equation with the initial condition, $v(t = 0) = 0$,

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

Multiply both sides by m/c_d

$$\frac{m}{c_d}\frac{dv}{dt} = \frac{m}{c_d} g - v^2$$

Define $a = \sqrt{mg / c_d}$

$$\frac{m}{c_d}\frac{dv}{dt} = a^2 - v^2$$

Integrate by separation of variables,

$$\int \frac{dv}{a^2 - v^2} = \int \frac{c_d}{m} dt$$

A table of integrals can be consulted to find that

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a}\tanh^{-1}\frac{x}{a}$$

Therefore, the integration yields

$$\frac{1}{a}\tanh^{-1}\frac{v}{a} = \frac{c_d}{m}t + C$$

If $v = 0$ at $t = 0$, then because $\tanh^{-1}(0) = 0$, the constant of integration $C = 0$ and the solution is

$$\frac{1}{a}\tanh^{-1}\frac{v}{a} = \frac{c_d}{m}t$$

This result can then be rearranged to yield

$$v = \sqrt{\frac{gm}{c_d}}\tanh\left(\sqrt{\frac{gc_d}{m}}t\right)$$

1.2 This is a transient computation. For the period from ending June 1:

Balance = Previous Balance + Deposits – Withdrawals

Balance = 1512.33 + 220.13 – 327.26 = 1405.20

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	Deposit	Withdrawal	Balance
1-May			$ 1512.33
	$ 220.13	$ 327.26	
1-Jun			$ 1405.20
	$ 216.80	$ 378.61	
1-Jul			$ 1243.39
	$ 450.25	$ 106.80	
1-Aug			$ 1586.84
	$ 127.31	$ 350.61	
1-Sep			$ 1363.54

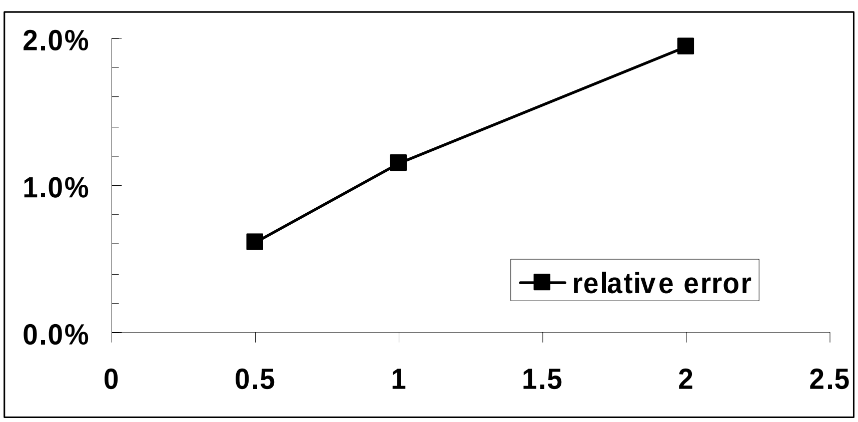
1.3 At t = 12 s, the analytical solution is 50.6175 (Example 1.1). The numerical results are:

step	v(12)	absolute relative error
1	51.2008	1.15%
0.5	50.9259	0.61%

where the relative error is calculated with

$$\text{absolute relative error} = \left|\frac{\text{analytical} - \text{numerical}}{\text{analytical}}\right| \times 100\%$$

The error versus step size can be plotted as



Thus, halving the step size approximately halves the error.

1.4 (a) The force balance is

$$\frac{dv}{dt} = g - \frac{c'}{m} v$$

Applying Laplace transforms,

$$sV - v(0) = \frac{g}{s} - \frac{c'}{m} V$$

Solve for

$$V = \frac{g}{s(s + c'/m)} + \frac{v(0)}{s + c'/m} \qquad (1)$$

The first term to the right of the equal sign can be evaluated by a partial fraction expansion,

$$\frac{g}{s(s + c'/m)} = \frac{A}{s} + \frac{B}{s + c'/m} \qquad (2)$$

$$\frac{g}{s(s + c'/m)} = \frac{A(s + c'/m) + Bs}{s(s + c'/m)}$$

Equating like terms in the numerators yields

$$A + B = 0$$

$$g = \frac{c'}{m} A$$

Therefore,

$$A = \frac{mg}{c'} \qquad B = -\frac{mg}{c'}$$

These results can be substituted into Eq. (2), and the result can be substituted back into Eq. (1) to give

$$V = \frac{mg/c'}{s} - \frac{mg/c'}{s + c'/m} + \frac{v(0)}{s + c'/m}$$

Applying inverse Laplace transforms yields

$$v = \frac{mg}{c'} - \frac{mg}{c'} e^{-(c'/m)t} + v(0)e^{-(c'/m)t}$$

or

$$v = v(0)e^{-(c'/m)t} + \frac{mg}{c'}\left(1 - e^{-(c'/m)t}\right)$$

where the first term to the right of the equal sign is the general solution and the second is the particular solution. For our case, $v(0) = 0$, so the final solution is

$$\boxed{v = \frac{mg}{c'}\left(1 - e^{-(c'/m)t}\right)}$$

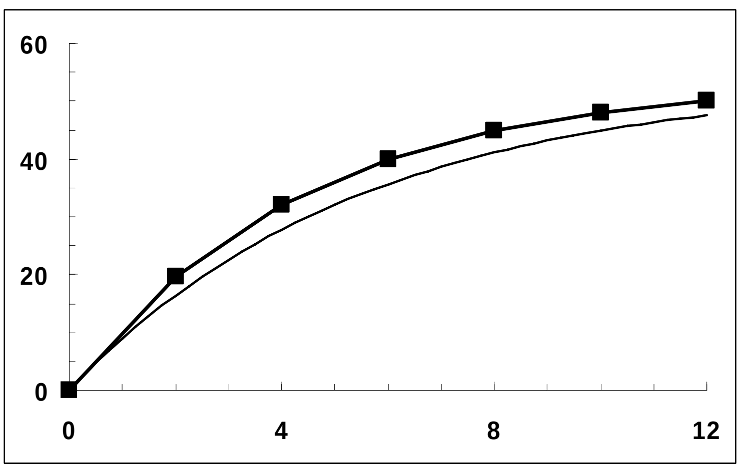
(b) The numerical solution can be implemented as

$$v(2) = 0 + \left[9.81 - \frac{12.5}{68.1}(0)\right]2 = 19.62$$

$$v(4) = 19.62 + \left[9.81 - \frac{12.5}{68.1}(19.62)\right]2 = 32.0374$$

The computation can be continued and the results summarized and plotted as:

t	*v*	*dv/dt*
0	0	9.81
2	19.6200	6.2087
4	32.0374	3.9294
6	39.8962	2.4869
8	44.8700	1.5739
10	48.0179	0.9961
12	50.0102	0.6304



Note that the analytical solution is included on the plot for comparison.

1.5 $v(t) = \frac{gm}{c}(1 - e^{-(c/m)\,t})$

jumper #1: $v(t) = \frac{9.8(70)}{12}(1 - e^{-(12/70)\,10}) = 46.8714$

jumper #2: $46.8714 = \frac{9.8(75)}{15}(1 - e^{-(15/75)\,t})$

$46.8714 = 49 - 49e^{-0.2\,t}$

$0.04344 = e^{-0.2\,t}$

$\ln 0.04344 = -0.2t$

$t = \frac{\ln 0.04344}{-0.2} = 15.6818 \text{ s}$

1.6 Before the chute opens ($t < 10$), Euler's method can be implemented as

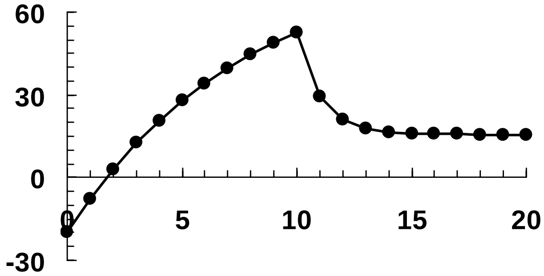
$$v(t + \Delta t) = v(t) + \left[9.8 - \frac{10}{80}v(t)\right]\Delta t$$

After the chute opens ($t \geq 10$), the drag coefficient is changed and the implementation becomes

$$v(t + \Delta t) = v(t) + \left[9.8 - \frac{50}{80}v(t)\right]\Delta t$$

Here is a summary of the results along with a plot:

Chute closed			Chute opened		
t	***v***	***dv/dt***	***t***	***v***	***dv/dt***
0	-20.0000	12.3000	10	52.5134	-23.0209
1	-7.7000	10.7625	11	29.4925	-8.6328
2	3.0625	9.4172	12	20.8597	-3.2373
3	12.4797	8.2400	13	17.6224	-1.2140
4	20.7197	7.2100	14	16.4084	-0.4552
5	27.9298	6.3088	15	15.9531	-0.1707
6	34.2385	5.5202	16	15.7824	-0.0640
7	39.7587	4.8302	17	15.7184	-0.0240
8	44.5889	4.2264	18	15.6944	-0.0090
9	48.8153	3.6981	19	15.6854	-0.0034
			20	15.6820	-0.0013



1.7 (a) The first two steps are

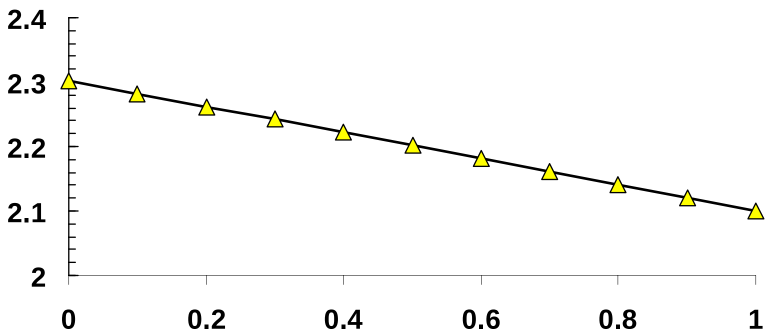
$$c(0.1) = 10 - 0.2(10)0.1 = 9.8 \text{ Bq/L}$$

$$c(0.2) = 9.8 - 0.2(9.8)0.1 = 9.604 \text{ Bq/L}$$

The process can be continued to yield

t	*c*	*dc/dt*
0	10.0000	-2.0000
0.1	9.8000	-1.9600
0.2	9.6040	-1.9208
0.3	9.4119	-1.8824
0.4	9.2237	-1.8447
0.5	9.0392	-1.8078
0.6	8.8584	-1.7717
0.7	8.6813	-1.7363
0.8	8.5076	-1.7015
0.9	8.3375	-1.6675
1	8.1707	-1.6341

(b) The results when plotted on a semi-log plot yields a straight line



The slope of this line can be estimated as

$$\frac{\ln(8.1707) - \ln(10)}{1} = -0.20203$$

Thus, the slope is approximately equal to the negative of the decay rate.

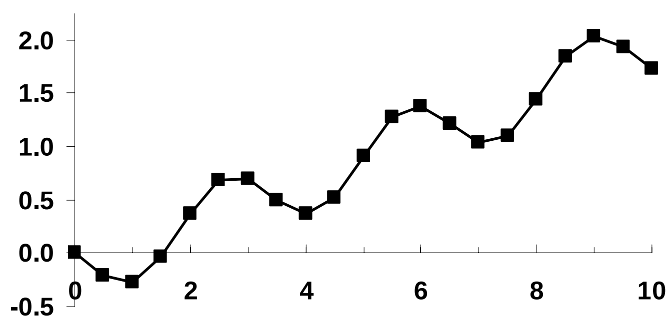
1.8 The first two steps yield

$$y(0.5) = 0 + \left[3\frac{500}{1200}\sin^2(0) - \frac{500}{1200}\right]0.5 = 0 + \left[0 - 0.41667\right]0.5 = -0.20833$$

$$y(1) = -0.20833 + \left[\sin^2(0.5) - 0.41667\right]0.5 = -0.27301$$

The process can be continued to give

t	*y*	*dy/dt*	*t*	*y*	*dy/dt*
0	0.00000	-0.41667	5.5	1.27629	0.20557
0.5	-0.20833	-0.12936	6	1.37907	-0.31908
1	-0.27301	0.46843	6.5	1.21953	-0.35882
1.5	-0.03880	0.82708	7	1.04012	0.12287
2	0.37474	0.61686	7.5	1.10156	0.68314
2.5	0.68317	0.03104	8	1.44313	0.80687
3	0.69869	-0.39177	8.5	1.84656	0.38031
3.5	0.50281	-0.26286	9	2.03672	-0.20436
4	0.37138	0.29927	9.5	1.93453	-0.40961
4.5	0.52101	0.77779	10	1.72973	-0.04672
5	0.90991	0.73275			



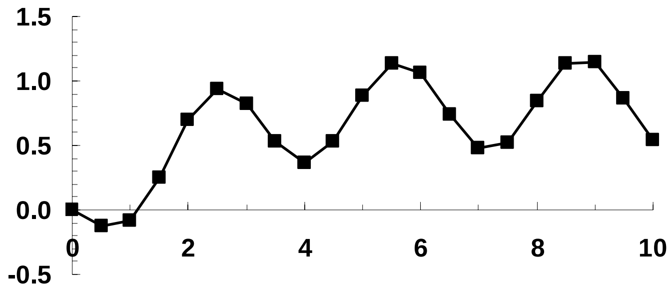
1.9 The first two steps yield

$$y(0.5) = 0 + \left[3\frac{500}{1200}\sin^2(0) - \frac{300(1+0)^{1.5}}{1200}\right]0.5 = 0 + \left[0 - 0.25\right]0.5 = -0.125$$

$$y(1) = -0.125 + \left[3\frac{500}{1200}\sin^2(0.5) - \frac{300(1-0.125)^{1.5}}{1200}\right]0.5 = -0.08366$$

The process can be continued to give

t	*y*	*dy/dt*	*t*	*y*	*dy/dt*
0	0.00000	-0.25000	5.5	1.13864	-0.15966
0.5	-0.12500	0.08269	6	1.05881	-0.64093
1	-0.08366	0.66580	6.5	0.73834	-0.51514
1.5	0.24924	0.89468	7	0.48077	0.08906
2	0.69658	0.48107	7.5	0.52530	0.62885
2.5	0.93711	-0.22631	8	0.83973	0.59970
3	0.82396	-0.59094	8.5	1.13958	0.01457
3.5	0.52849	-0.31862	9	1.14687	-0.57411
4	0.36918	0.31541	9.5	0.85981	-0.62702
4.5	0.52689	0.72277	10	0.54630	-0.11076
5	0.88827	0.50073			



1.10 $Q_{1,\text{in}} = Q_{2,\text{out}} + v_{3,\text{out}} A_3$

$$A_3 = \frac{Q_{1,\text{in}} - Q_{2,\text{out}}}{v_{3,\text{out}}} = \frac{40\text{ m}^3/\text{s} - 20\text{ m}^3/\text{s}}{6\text{ m/s}} = 3.333\,\text{m}^2$$

1.11

$$Q_{\text{students}} = 30\text{ ind} \times 80\frac{\text{J}}{\text{ind s}} \times 15\text{ min} \times 60\frac{\text{s}}{\text{min}} \times \frac{\text{kJ}}{1000\text{ J}} = 2160\text{ kJ}$$

$$m = \frac{PV\text{Mwt}}{RT} = \frac{(101.325\text{ kPa})(10\text{m} \times 8\text{m} \times 3\text{m} - 30 \times 0.075\text{ m}^3)(28.97\text{ kg/kmol})}{(8.314\text{ kPa m}^3/(\text{kmol K})((20+273.15)\text{K})} = 286.3424\text{ kg}$$

$$\Delta T = \frac{Q_{\text{students}}}{mC_v} = \frac{2160\text{ kJ}}{(286.3424\text{ kg})(0.718\text{ kJ/(kg K)})} = 10.50615\text{K}$$

Therefore, the final temperature is 20 + 10.50615 = 30.50615$^{\text{o}}$C**.**

1.12 $\sum \text{M}_{\text{in}} - \sum \text{M}_{\text{out}} = 0$

Food + Drink + Air In + Metabolism = Urine + Skin + Feces + Air Out + Sweat

$$\text{Drink} = \text{Urine} + \text{Skin} + \text{Feces} + \text{Air Out} + \text{Sweat} - \text{Food} - \text{Air In} - \text{Metabolism}$$

$$\text{Drink} = 1.4 + 0.35 + 0.2 + 0.4 + 0.2 - 1 - 0.05 - 0.3 = 1.2 \text{ L}$$

1.13 (a) The force balance can be written as:

$$m\frac{dv}{dt} = -mg(0)\frac{R^2}{(R+x)^2} + c'v$$

Dividing by mass gives

$$\frac{dv}{dt} = -g(0)\frac{R^2}{(R+x)^2} + \frac{c'}{m}v$$

(b) Recognizing that $dx/dt = v$, the chain rule is

$$\frac{dv}{dt} = v\frac{dv}{dx}$$

Setting drag to zero and substituting this relationship into the force balance gives

$$\frac{dv}{dx} = -\frac{g(0)}{v}\frac{R^2}{(R+x)^2}$$

(c) Using separation of variables

$$v\,dv = -g(0)\frac{R^2}{(R+x)^2}dx$$

Integrating gives

$$\frac{v^2}{2} = g(0)\frac{R^2}{R+x} + C$$

Applying the initial condition yields

$$\frac{v_0^2}{2} = g(0)\frac{R^2}{R+0} + C$$

which can be solved for $C = v_0^2/2 - g(0)R$, which can be substituted back into the solution to give

$$\frac{v^2}{2} = g(0)\frac{R^2}{R+x} + \frac{v_0^2}{2} - g(0)R$$

or

$$v = \pm\sqrt{v_0^2 + 2g(0)\frac{R^2}{R+x} - 2g(0)R}$$

Note that the plus sign holds when the object is moving upwards and the minus sign holds when it is falling.

(d) Euler's method can be developed as

$$v(x_{i+1}) = v(x_i) + \left[-\frac{g(0)}{v(x_i)}\frac{R^2}{(R+x_i)^2}\right](x_{i+1} - x_i)$$

The first step can be computed as

$$v(10{,}000) = 1{,}400 + \left[-\frac{9.8}{1{,}400}\frac{(6.37\times10^6)^2}{(6.37\times10^6+0)^2}\right](10{,}000-0) = 1{,}400 + (-0.007)10{,}000 = 1{,}330$$

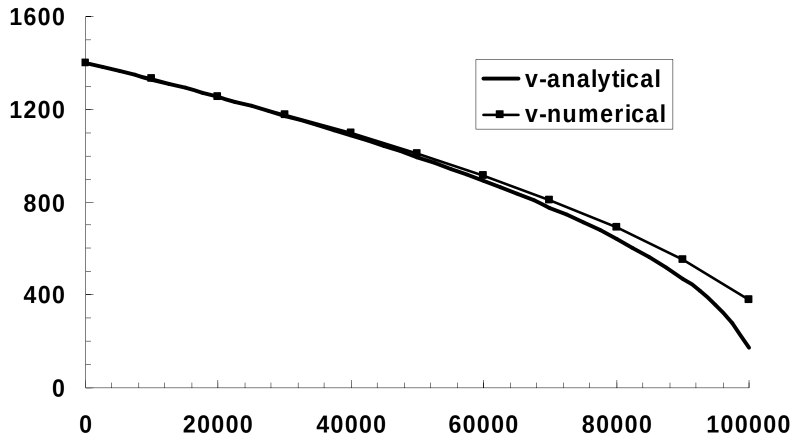
The remainder of the calculations can be implemented in a similar fashion as in the following table

x	***v***	***dv*/*dx***	***v*-analytical**
0	1400.000	-0.00700	1400.000
10000	1330.000	-0.00735	1328.272
20000	1256.547	-0.00775	1252.688
30000	1179.042	-0.00823	1172.500
40000	1096.701	-0.00882	1086.688
50000	1008.454	-0.00957	993.796
60000	912.783	-0.01054	891.612
70000	807.413	-0.01188	776.473
80000	688.661	-0.01388	641.439
90000	549.864	-0.01733	469.650
100000	376.568	-0.02523	174.033

For the analytical solution, the value at 10,000 m can be computed as

$$v = \sqrt{1{,}400^2 + 2(9.8)\frac{(6.37\times10^6)^2}{(6.37\times10^6+10{,}000)} - 2(9.8)(6.37\times10^6)} = 1{,}328.272$$

The remainder of the analytical values can be implemented in a similar fashion as in the last column of the above table. The numerical and analytical solutions can be displayed graphically.



1.14

Errata: In the first printing, the rate of evaporation should be changed to 0.1 mm/min. Subsequent printings should show the correct value.

The volume of the droplet is related to the radius as

$$V = \frac{4\pi r^3}{3} \qquad (1)$$

This equation can be solved for radius as

$$r = \sqrt[3]{\frac{3V}{4\pi}} \qquad (2)$$

The surface area is

$$A = 4\pi r^2 \qquad (3)$$

Equation (2) can be substituted into Eq. (3) to express area as a function of volume

$$A = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$$

This result can then be substituted into the original differential equation,

$$\frac{dV}{dt} = -k4\pi\left(\frac{3V}{4\pi}\right)^{2/3} \qquad (4)$$

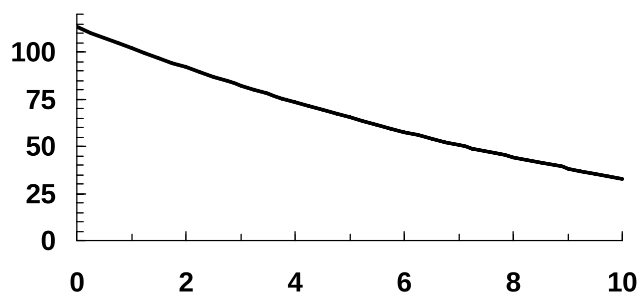
The initial volume can be computed with Eq. (1),

$$V = \frac{4\pi r^3}{3} = \frac{4\pi(3)^3}{3} = 113.0973\,\text{mm}^3$$

Euler's method can be used to integrate Eq. (4). Here are the beginning and last steps

t	*V*	*dV/dt*
0	113.0973	-11.3097
0.25	110.2699	-11.1204
0.5	107.4898	-10.9327
0.75	104.7566	-10.7466
1	102.07	-10.5621
•		
•		
•		
9	38.29357	-5.49416
9.25	36.92003	-5.36198
9.5	35.57954	-5.2314
9.75	34.27169	-5.1024
10	32.99609	-4.97499

A plot of the results is shown below:



Eq. (2) can be used to compute the final radius as

$$r = \sqrt[3]{\frac{3(32.99609)}{4\pi}} = 1.9897$$

Therefore, the average evaporation rate can be computed as

$$k = \frac{(3 - 1.9897)\,\text{mm}}{10\,\text{min}} = 0.10103\frac{\text{mm}}{\text{min}}$$

which is approximately equal to the given evaporation rate of 0.1 mm/min.

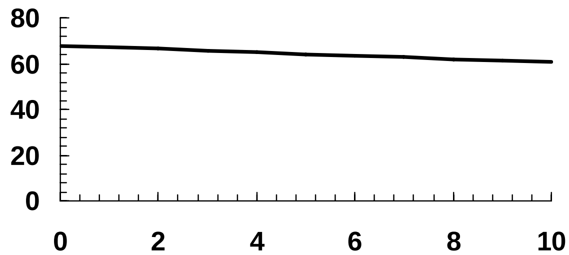
1.15 The first two steps can be computed as

$$T(1) = 68 + \left[-0.017(68-21)\right]1 = 68 + (-0.799)1 = 67.201$$

$$T(2) = 67.201 + \left[-0.017(67.201 - 21)\right]1 = 68 + (-0.78542)1 = 66.41558$$

The remaining results are displayed below along with a plot

t	*T*	*dT/dt*	*t*	*T*	*dT/dt*
0	68.00000	-0.79900	6	63.40519	-0.72089
1	67.20100	-0.78542	7	62.68430	-0.70863
2	66.41558	-0.77206	8	61.97566	-0.69659
3	65.64352	-0.75894	9	61.27908	-0.68474
4	64.88458	-0.74604	10	60.59433	-0.67310
5	64.13854	-0.73336			



1.16 Continuity at the nodes can be used to determine the flows as follows:

$$Q_1 = Q_2 + Q_3 = 0.6 + 0.4 = 1\frac{\text{m}^3}{\text{s}}$$

$$Q_{10} = Q_1 = 1\frac{\text{m}^3}{\text{s}}$$

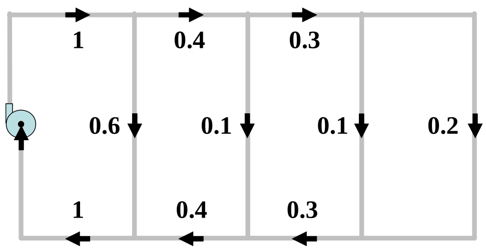
$$Q_9 = Q_{10} - Q_2 = 1 - 0.6 = 0.4\frac{\text{m}^3}{\text{s}}$$

$$Q_4 = Q_9 - Q_8 = 0.4 - 0.3 = 0.1\frac{\text{m}^3}{\text{s}}$$

$$Q_5 = Q_3 - Q_4 = 0.4 - 0.1 = 0.3\frac{\text{m}^3}{\text{s}}$$

$$Q_6 = Q_5 - Q_7 = 0.3 - 0.2 = 0.1\frac{\text{m}^3}{\text{s}}$$

Therefore, the final results are


1
0.4
0.3
0.6
0.1
0.1
0.2
1
0.4
0.3