Continuous Time Dynamic Modeling in `dynr` and `OpenMx`

Michael D. Hunter

School of Psychology
Georgia Institute of Technologyr

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Plan

- Discrete vs Continuous Time
- Software
- General Modeling
- Examplles

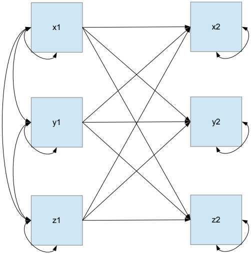
Discrete-Time Parameters Depend on the Lag

Simulation Design

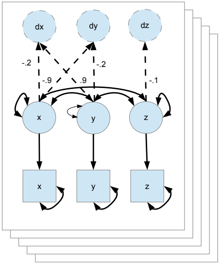
- 2-occasion data
- 3 variables
- 100 individuals
- Continuous generating process
- Sampled in discrete time
- Measurement Interval: 7 hours to 1 year
- Fit discrete- & continuous-time structural equation models to each sampling interval
- Compare parameters across conditions

Discrete-Time Parameters Depend on the Lag

Condidate Models

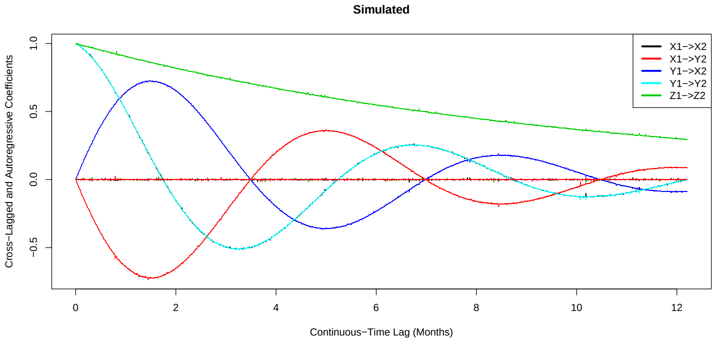


Cross-lagged Panel Model

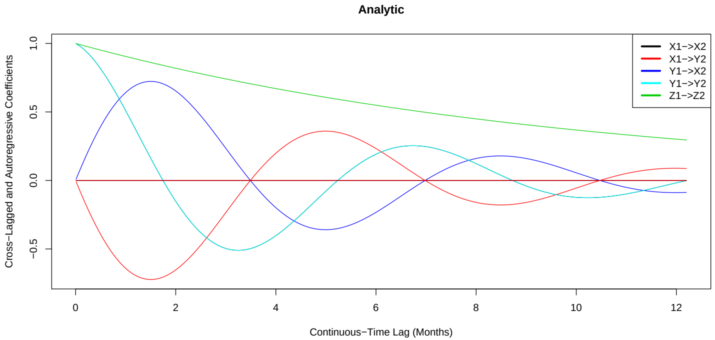


Continuous-Time Structural Equation Model

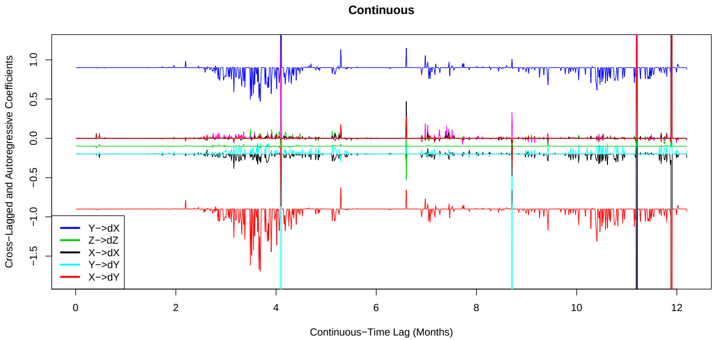
Discrete-Time Parameters Depend on the Lag



The Dependence has an Analytic Form



Continuous-Time Parameters Do Not Depend on Lag



Summary

- One-occasion mediation has no causal direction.
- Cross-lagged panel designs obtain different estimates for each measurement interval.
- Continuous-time models obtain constant estimates for every measurement interval.
- Estimated discrete-time parameters match those analytically predicted.
- Continuous-time models even work on two-occasion data!

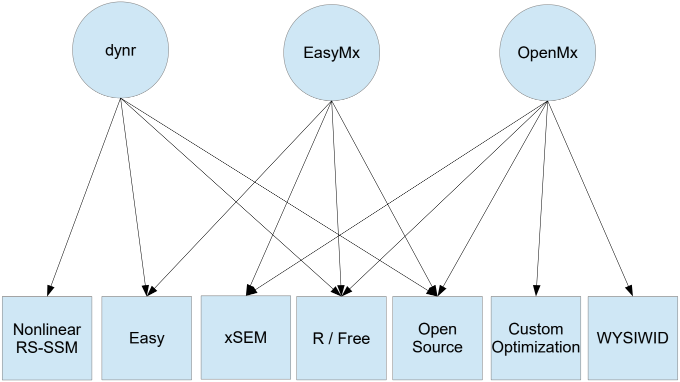
Implications

- Researchers using different lags will find different results, even when the process is the same!
- Researchers using the same lag will find different results, depending on which lag they pick!
- Also applies to pre-test post-test designs!

Limitations

- Continuous convergence problems?
- Operation with real data?
- Mis-specified/imperfect models?

Software


dynr
EasyMx
OpenMx
Nonlinear RS-SSM
Easy
xSEM
R / Free
Open Source
Custom Optimization
WYSIWID

OpenMx $\geq$ 2.0

Recent Features

- objective function $= \underbrace{\text{expectation function}}_{\text{data generation}} + \underbrace{\text{fit function}}_{\text{data comparison}}$
- Multiple optimizers: SLSQP, NPSOL, CSOLNP
- New Models: Item factor analysis, GREML, state space, LISREL
- New Helpers: `mxFactorScores`, `mxGenerateData`, `mxMI`, `mxRefModels`, `mxTryHard`

OpenMx $\geq$ 2.7

More Recent Features

- Handy S3 methods: `logLik`, `confint`, `anova`, `coef`, `simulate`
- New Fit Functions for multigroup, mixtures, hidden Markov models, weighted least squares
- Utilities: `mxSE`, `mxAutoStart`, nonparametric and parametric `mxBootstrap`, `mxCheckIdentification`, `mxGetExpected`

Why dynr?

Why make a new package for this?

- Linear and nonlinear models
- Multiple subjects
- Intuitive interface
- Fast
- Great reporting
- Combine dynamics for continuous and categorical latent variables

dynr preparation

- Gather data with `dynr.data()`
- Prepare *recipes* with
  - `prep.measurement()`
  - `prep.*Dynamics()`
  - `prep.initial()`
  - `prep.noise()`
  - `prep.regimes()` (optional)
- *Mix* recipes and data into a model with `dynr.model()`
- *Cook* model with `dynr.cook()`
- *Serve* results with
  - `summary()`
  - `plot()`
  - `dynr.ggplot()`
  - `plotFormula()`
  - `printex()`

Example 0

OpenMx

See `StateSpaceExpectationCodeListings.R` for OpenMx examples of

1. Process factor analysis in discrete and continuous time
2. Use of `mxSE` to compute standard error on transformed parameters
3. Hypothesis testing of nested models
4. Kalman scores
5. Data generation
6. Multisubject models

State Space Model

Measurement

- Structural Equation Measurement Model

$$\boldsymbol{y}_i = \Lambda \boldsymbol{\eta}_i + K\boldsymbol{x}_i + \boldsymbol{\varepsilon}_i \quad \text{with} \quad \boldsymbol{\varepsilon}_i \sim \mathcal{N}\left(\mathbf{0}, \Theta\right) \tag{1}$$

- State Space Measurement Model

$$\boldsymbol{y}_i = \Lambda \boldsymbol{\eta}_i + K\boldsymbol{x}_i + \boldsymbol{\varepsilon}_i \quad \text{with} \quad \boldsymbol{\varepsilon}_i \sim \mathcal{N}\left(\mathbf{0}, \Theta\right) \tag{2}$$

State Space Model

Transition/Structural

- Structural Equation Structural Model

$$\boldsymbol{\eta}_i = B\boldsymbol{\eta}_i + \Gamma\boldsymbol{x}_i + \boldsymbol{\zeta}_i \quad \text{with} \quad \boldsymbol{\zeta}_i \sim \mathcal{N}(\mathbf{0}, \Psi) \tag{3}$$

- State Space Structural Model in Discrete Time

$$\boldsymbol{\eta}_{i+1} = B\boldsymbol{\eta}_i + \Gamma\boldsymbol{x}_i + \boldsymbol{\zeta}_i \quad \text{with} \quad \boldsymbol{\zeta}_i \sim \mathcal{N}(\mathbf{0}, \Psi) \tag{4}$$

- State Space Structural Model in Continuous Time

$$\frac{d\boldsymbol{\eta}}{dt} = B\boldsymbol{\eta}_i + \Gamma\boldsymbol{x}_i + \boldsymbol{\zeta}_i \quad \text{with} \quad \boldsymbol{\zeta}_i \sim \mathcal{N}(\mathbf{0}, \Psi) \tag{5}$$

Notation

Structure

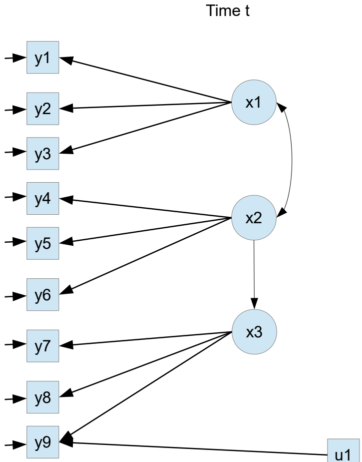
Durbin & Koopman	$\boldsymbol{\alpha}_t$	$=$	$\boldsymbol{T}_t \boldsymbol{\alpha}_{t-1} + \boldsymbol{R}_t \boldsymbol{\eta}_t$
LISCOMP	$\boldsymbol{\eta}_t$	$=$	$\boldsymbol{B}_t \boldsymbol{\eta}_{t-1} + \boldsymbol{\Gamma}_t \boldsymbol{x}_t + \boldsymbol{\zeta}_t$
West & Harrison	$\boldsymbol{\theta}_t$	$=$	$\boldsymbol{G}_t \boldsymbol{\theta}_{t-1} + \boldsymbol{w}_t$
Kalman	$\boldsymbol{x}_t$	$=$	$\boldsymbol{F}_t \boldsymbol{x}_{t-1} + \boldsymbol{B}_t \boldsymbol{u}_t + \boldsymbol{w}_t$
Åstrom & Murray	$\boldsymbol{x}_t$	$=$	$\boldsymbol{A}_t \boldsymbol{x}_{t-1} + \boldsymbol{B}_t \boldsymbol{u}_t + \boldsymbol{v}_t$
OpenMx	$\boldsymbol{x}_t$	$=$	$\boldsymbol{A}_t \boldsymbol{x}_{t-1} + \boldsymbol{B}_t \boldsymbol{u}_t + \boldsymbol{q}_t$

Notation

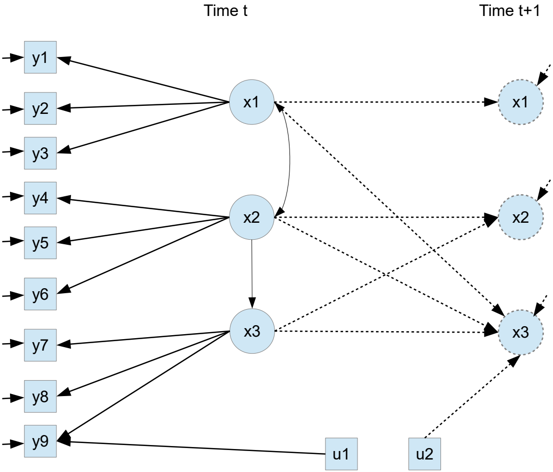
Measurement

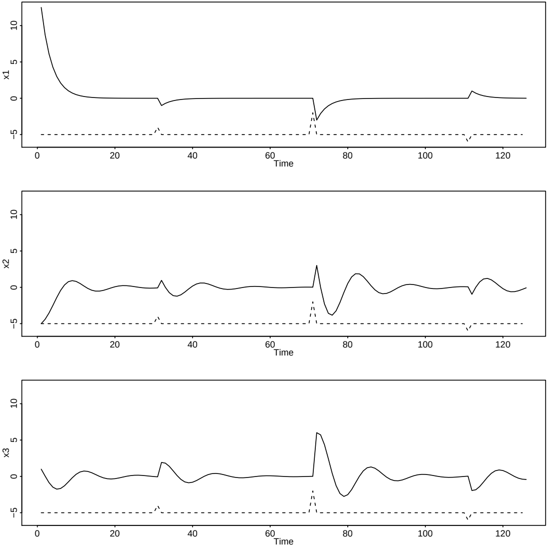
Durbin & Koopman	$\boldsymbol{y}_t$	$=$	$\boldsymbol{Z}_t\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t$
LISCOMP	$\boldsymbol{y}_t$	$=$	$\boldsymbol{\Lambda}_t\boldsymbol{\eta}_t + \boldsymbol{K}_t\boldsymbol{u}_t + \boldsymbol{\varepsilon}_t$
West & Harrison	$\boldsymbol{a}_t$	$=$	$\boldsymbol{F}_t\boldsymbol{\theta}_t + \boldsymbol{v}_t$
Kalman	$\boldsymbol{z}_t$	$=$	$\boldsymbol{H}_t\boldsymbol{x}_t + \boldsymbol{v}_t$
Åstrom & Murray	$\boldsymbol{y}_t$	$=$	$\boldsymbol{C}_t\boldsymbol{x}_t + \boldsymbol{D}_t\boldsymbol{u}_t + \boldsymbol{w}_t$
OpenMx	$\boldsymbol{y}_t$	$=$	$\boldsymbol{C}_t\boldsymbol{x}_t + \boldsymbol{D}_t\boldsymbol{u}_t + \boldsymbol{r}_t$

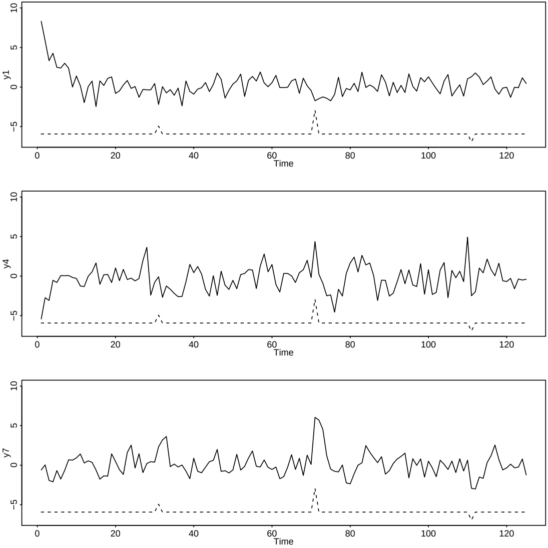
Structural Equation Models



State Space Models




x1
10
5
0
−5
0
20
40
60
80
100
120
Time
x2
10
5
0
−5
0
20
40
60
80
100
120
Time
x3
10
5
0
−5
0
20
40
60
80
100
120
Time


y1
y4
y7
Time
Time
Time
10
5
0
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60
80
100
120

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TEACHER'S CORNER

State Space Modeling in an Open Source, Modular, Structural Equation Modeling Environment

Michael D. Hunter
University of Oklahoma Health Sciences Center

Georgia Tech

Damped and Forced Harmonic Oscillator

- As a second-order system

$$\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt} + \zeta \tag{6}$$

- As a vector of first-order systems

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{d^2x}{dt^2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -k & -c \end{pmatrix} \begin{pmatrix} x \\ \frac{dx}{dt} \end{pmatrix} + \begin{pmatrix} 0 \\ \zeta \end{pmatrix} \tag{7}$$

- The Measurement Model

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ \frac{dx}{dt} \end{pmatrix} + \epsilon \tag{8}$$

Example 1a

dynr

See `LinearSDE.R` for dynr examples of

1. Model specification of a damped and forced harmonic oscillator
2. Bound setting
3. Model reporting with `printex`, `plotFormula`, `plot`, and `autoplot`
4. Kalman scores

Example 1b

OpenMx

See `StateSpaceContinuous.R` for OpenMx examples of

1. Undamped linear oscillator
2. Damped linear oscillator
3. Process factor analysis

Example 2

dynr

See `PFA.R` for dynr examples of

1. Process factor analysis
2. `prep.loadings()` function for easier factor loadings specification

Example 3

OpenMx

See `ModificationIndexCheck.R` for OpenMx examples of

1. Using modification indices with state space models
2. Model modification "on the fly"
3. Data generation from a model

Questions?

Thank You

`mhunter43@gatech.edu`