TT-ENTAILS: Inference by Enumeration in Propositional Logic

CSE 4308/5360 – Artificial Intelligence I
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Entailment and Inference

- To say that a knowledge base **KB** entails a statement **alpha** means simply that:

  KB => alpha.

- Alternative (but equivalent) definition: a knowledge base **KB** entails a statement **alpha** if and only if:
  - In every world where **KB** is true, **alpha** is also true.
- This definition is compatible with our intuition.
  - KB => alpha means that if **KB** is true (thus, if we live in a world where **KB** is true) then **alpha** is true.
  - If there exists a world where **KB** is true and **alpha** is false, then clearly we cannot say that KB => alpha.

Worlds in Propositional Logic

- A knowledge base **KB** entails a statement **alpha** if and only if:
  - In every world where **KB** is true, **alpha** is also true.
- In the above definition, we use the term “world”.
- What is a “world” in propositional logic?
- Equivalent definitions of the term “world”:
  - A world is a row in the truth table.
  - A world is an assignment of boolean values to all symbols.

Inference by Enumeration

- In propositional logic, to determine if **KB** entails **alpha**, we apply an algorithm called “inference by enumeration”.
- Inference by enumeration is a “smoking-gun” algorithm.
- What is a “smoking gun” algorithm?
  - You ask a question to the algorithm.
  - The algorithm does a loop, searching for a smoking gun.
  - If it finds a smoking gun, it returns one answer.
  - If it finds no smoking gun, it returns another answer.

Inference by Enumeration

- Inference by enumeration as a "smoking gun" algorithm:
  - You ask a question to the algorithm. What question?
  - The algorithm does a loop, searching for a smoking gun. What would be a smoking gun?
  - If it finds a smoking gun, it returns one answer. What answer?
  - If it finds no smoking gun, it returns another answer. What answer?

Inference by Enumeration

- Inference by enumeration as a “smoking gun” algorithm:
  - You ask a question to the algorithm. What question?
    - Does KB entail alpha?
  - The algorithm does a loop, searching for a smoking gun. What would be a smoking gun?
  - If it finds a smoking gun, it returns one answer. What answer?
  - If it finds no smoking gun, it returns another answer. What answer?

Inference by Enumeration

- Inference by enumeration as a “smoking gun” algorithm:
  - You ask a question to the algorithm. What question?
    - Does KB entail alpha?
  - The algorithm does a loop, searching for a smoking gun. What would be a smoking gun?
    - A row in the truth table where KB is true and alpha is false.
  - If it finds a smoking gun, it returns one answer. What answer?
  - If it finds no smoking gun, it returns another answer. What answer?

Inference by Enumeration

- Inference by enumeration as a “smoking gun” algorithm:
  - You ask a question to the algorithm. What question?
    - Does KB entail alpha?
  - The algorithm does a loop, searching for a smoking gun. What would be a smoking gun?
    - A row in the truth table where KB is true and alpha is false.
  - If it finds a smoking gun, it returns one answer. What answer?
    - False (KB does NOT entail alpha).
  - If it finds no smoking gun, it returns another answer. What answer?

Inference by Enumeration

- Inference by enumeration as a "smoking gun" algorithm:
  - You ask a question to the algorithm. What question?
    - Does KB entail alpha?
  - The algorithm does a loop, searching for a smoking gun. What would be a smoking gun?
    - A row in the truth table where KB is true and alpha is false.
  - If it finds a smoking gun, it returns one answer. What answer?
    - False (KB does NOT entail alpha).
  - If it finds no smoking gun, it returns another answer. What answer?
    - True (KB entails alpha).

Inference by Enumeration

- Inference by enumeration algorithm:
  - For each row R in the truth table:
    - If **KB** is true in R and **alpha** is false in R, return false.
  - Return true.

- This is what you will have to implement in a subsequent assignment (assignment 4 or 5).
- Seems pretty simple, a three-line piece of pseudocode.
- Problem:

Inference by Enumeration

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  - For each row R in the truth table:
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Inference by Enumeration

- Inference by enumeration algorithm:
  - For each row R in the truth table:
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  - Return true.

- This is what you will have to implement in a subsequent assignment (assignment 4 or 5).
- Seems pretty simple, a three-line piece of pseudocode.
- Problem: how do you loop through rows of the truth table?
  - Answer: the TT-Entails pseudocode.

The TT-Entails Pseudocode

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
               TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

Making Sense of TT-Entails

- Contrast the TT-Entails pseudocode to the previous pseudocode we saw for inference by enumeration:
  - For each row R in the truth table:
    - If **KB** is true in R and **alpha** is false in R, return false.
  - Return true.
- The two pseudocodes look very different.
- However, they do EXACTLY THE SAME THING.
  - It typically takes an entire lecture to convince people of that.
- TT-Entails provides a specific (but complicated) way to loop through rows in the truth table.

What We Need to Specify

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
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Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
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               TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- To understand TT-Entails, we must understand the items in red.
- Hard ones: LogicalExpression, ExtractSymbols, PL-True?
- Easy ones: concatenate, First, Rest, Extend

Implementing a Logical Expression

Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)

- We will treat all these terms as synonyms and use interchangeably:
  - (Logical) expression.
  - (Logical) statement.
  - Sentence.

- Is it reasonable that KB is the same data type as alpha?
  - After all, KB is a **set** of statements, whereas alpha is a single statement.

Implementing a Logical Expression

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- We will treat all these terms as synonyms and use interchangeably:
  - (Logical) expression.
  - (Logical) statement.
  - Sentence.
- Is it reasonable that KB is the same data type as alpha?
  - After all, KB is a **set** of statements, whereas alpha is a single statement.
- Yes! KB is simply the conjunction of all the statements it contains.

Implementing a Logical Expression

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
```

- What is a good data structure for a sentence in propositional logic?

Implementing a Logical Expression

Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)

- What is a good data structure for a sentence in propositional logic?
- Answer: a tree.
  - As an aside, trees are the typical choice for representing content specified in some language, such as:
    - Logical statements, expressed in some logical language.
    - Programs, expressed in a programming language (compilers convert them to tree representations in the process of compiling them).
    - Text written in a natural language, such as English (ever heard of a parse tree?).

Trees

- How do we implement a tree as a class?

Trees

- How do we implement a tree as a class?
- A tree is a recursive data structure.
- There is no difference between the data structure for a tree and the data structure for a node in the tree.
- A tree is represented by its root.

Trees

```
class Tree
{
   Whatever content; // Content of the node (root of the tree)
   Tree[] children;      // Children of the node.
}
```

- Again, remember that there is no distinction (in programming) between the data type for a tree and the data type for a node.
- A tree is simply its root.
- The root knows about its children, which know about their children, …
- Children can be implemented as an array or a list.

Trees

```
class Tree
{
   Whatever content; // Content of the node (root of the tree)
   Tree[] children;      // Children of the node.
}
```

- The tree is a recursive data structure.
  - Overall, programming with logic will be a big exercise in recursion.
- What is recursive about it?
- Answer: it is a data type that uses itself in its definition.
  - The children of the tree are an array (or list) of trees.

Trees

```
class Tree
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   Whatever content; // Content of the node (root of the tree)
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```

- In any recursive definition, we must identify the base case(s).
- What is the base case here?

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- In any recursive definition, we must identify the base case(s).
- What is the base case here?
  - A leaf.
- How is a leaf represented?

Trees

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class Tree
{
   Whatever content; // Content of the node (root of the tree)
   Tree[] children;      // Children of the node.
}
```

- In any recursive definition, we must identify the base case(s).
- What is the base case here?
  - A leaf.
- How is a leaf represented?
  - No children (empty array or list of children, or you can make children a NULL pointer).

Logical Expressions as Trees

- What are the building blocks of sentences in propositional logic?

Logical Expressions as Trees

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  - Symbols.
  - Connectives.
- To figure out how to represent logical expressions as trees, we should start with the base case.
- Which logical expressions are leafs?

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- Which logical expressions are leafs?
  - Symbols.
- Then, how do we represent a connective?

Logical Expressions as Trees

- What are the building blocks of sentences in propositional logic?
  - Symbols.
  - Connectives.
- To figure out how to represent logical expressions as trees, we should start with the base case.
- Which logical expressions are leafs?
  - Symbols.
- Then, how do we represent a connective?
- A connective is a tree node.
  - The sentences it connects are the children of the node.

Example

- How do we translate this statement in English?

B12 <=> (P11 OR P22 OR P13)

Example

- How do we translate this statement in English?

B12 <=> (P11 OR P22 OR P13)

- There is a breeze at square 1,2 if and only if there is: a pit at square 1,1, or a pit at square 2,2, or a pit at square 1,3.

Example

B12 <=> (P11 OR P22 OR P13)

- How do we represent this statement as a tree?
- What goes to the root? What is the top-level connective?

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- The root contains connective <=> as content.
- What are the children of the root?

Example

B12 <=> (P11 OR P22 OR P13)

- How do we represent this statement as a tree?
- What goes to the root? What is the top-level connective?
- The root contains connective <=> as content.
- What are the children of the root?
  - B12 on the left.
  - (P11 OR P22 OR P13) on the right.
- How do we represent (P11 OR P22 OR P13)?

Example

B12 <=> (P11 OR P22 OR P13)

- How do we represent this statement as a tree?
- What goes to the root? What is the top-level connective?
- The root contains connective <=> as content.
- What are the children of the root?
  - B12 on the left.
  - (P11 OR P22 OR P13) on the right.
- How do we represent (P11 OR P22 OR P13)?
- It is a node with:
  - OR as the connective
  - Three children, P11, P22, and P13.
- How do we represent B12, P11, P22, P13?

Example

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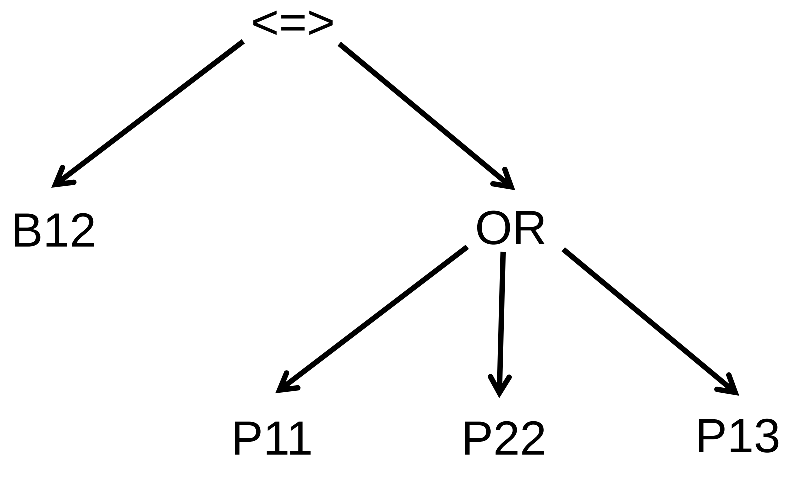
- It is a node with:
  - OR as the connective
  - Three children, P11, P22, and P13.
- How do we represent B12, P11, P22, P13?
  - They are leaf nodes.

Example

B12 <=> (P11 OR P22 OR P13)

- The above statement becomes:



The LogicalExpression Class

- This is the Tree class from a previous slide:

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class Tree
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- How can we convert this to represent logical expressions?

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class Tree
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   Whatever content; // Content of the node (root of the tree)
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}
```

- How can we convert this to represent logical expressions?

```
class LogicalExpression
{
   String symbol;
   String connective;
   LogicalExpression[] children;
}
```

The LogicalExpression Class

```
class LogicalExpression
{
   String symbol;
   String connective;
   LogicalExpression[] children;
}
```

- The **symbol** and **connective** member variables are the content of the tree node.
  - Of course, in practice, one of the two has to be NULL. The node cannot contain both a symbol and a connective.
- If the sentence is a symbol, then:
  - symbol = ???
  - connective = ???
  - children = ???

The LogicalExpression Class

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class LogicalExpression
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- The **symbol** and **connective** member variables are the content of the tree node.
  - Of course, in practice, one of the two has to be NULL. The node cannot contain both a symbol and a connective.
- If the sentence is a symbol, then:
  - symbol = the symbol (very straightforward)
  - connective = NULL
  - children = NULL (or empty array, whatever you prefer).

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- If the sentence is NOT a symbol, then:
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- If the sentence is NOT a symbol, then:
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  - connective = the top-level connective
  - children = the sentences that the connective connects.

Back to TT-Entails

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Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
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    if Empty?(symbols):
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        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- At this point, we have established what the LogicalExpression data type is.

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- Arguments: ???
- Return type: ???
- Task: ???

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- Arguments: one argument, **sentence**, of type LogicalExpression.
  - In other words, it is a sentence (or statement) in propositional logic.
- Return type: ???
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ExtractSymbols

- Next task: implementing ExtractSymbols.
- Arguments: one argument, **sentence**, of type LogicalExpression.
  - In other words, it is a sentence (or statement) in propositional logic.
- Return type: a list of strings.
- Task: ???

ExtractSymbols

- Next task: implementing ExtractSymbols.
- Arguments: one argument, **sentence**, of type LogicalExpression.
  - In other words, it is a sentence (or statement) in propositional logic.
- Return type: a list of strings.
- Task: Return the list of all symbols appearing in **sentence**.

ExtractSymbols

```
List<string> ExtractSymbols(LogicalExpression sentence)
    List result = empty list;
    if (sentence.symbol != NULL):
        ???
    else:
        ???
```

ExtractSymbols

```
List<string> ExtractSymbols(LogicalExpression sentence)
   List result = empty list;
   if (sentence.symbol != NULL):
      result.add(sentence.symbol);
   else:
      ???
```

ExtractSymbols

```
List<string> ExtractSymbols(LogicalExpression sentence)
    List result = empty list;
    if (sentence.symbol != NULL):
        result.add(sentence.symbol);
    else:
        for each child in sentence.children:
            result = concatenate(result, ExtractSymbols(child));
    return result;
```

- This is a classic example of how recursion makes life simple.
- A few lines of pseudocode.
- They translate into a few lines in real code.
- They can process an arbitrarily long and complicated sentence in propositional logic.

ExtractSymbols

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        result.add(sentence.symbol);
    else:
        for each child in sentence.children:
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    return result;
```

- What is the base case?

ExtractSymbols

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List<string> ExtractSymbols(LogicalExpression sentence)
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    if (sentence.symbol != NULL):
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        for each child in sentence.children:
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```

- What is the base case?
  - A sentence that is a symbol.
- How is it handled?

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List<string> ExtractSymbols(LogicalExpression sentence)
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- What is the base case?
  - A sentence that is a symbol.
- How is it handled?
  - We return a list of that symbol.
- Why do we need to return a list, and not just the symbol?

ExtractSymbols

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List<string> ExtractSymbols(LogicalExpression sentence)
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- What is the base case?
  - A sentence that is a symbol.
- How is it handled?
  - We return a list of that symbol.
- Why do we need to return a list, and not just the symbol?
  - Because ExtractSymbols returns a list of strings.

ExtractSymbols

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List<string> ExtractSymbols(LogicalExpression sentence)
   List result = empty list;
   if (sentence.symbol != NULL):
      result.add(sentence.symbol);
   else:
      for each child in sentence.children:
         result = concatenate(result, ExtractSymbols(child));
   return result;
```

- What is the recursive case?

ExtractSymbols

```
List<string> ExtractSymbols(LogicalExpression sentence)
   List result = empty list;
   if (sentence.symbol != NULL):
      result.add(sentence.symbol);
   else:
      for each child in sentence.children:
         result = concatenate(result, ExtractSymbols(child));
   return result;
```

- What is the recursive case?
  - A sentence that is NOT a symbol.
- How is it handled?

ExtractSymbols

```
List<string> ExtractSymbols(LogicalExpression sentence)
    List result = empty list;
    if (sentence.symbol != NULL):
        result.add(sentence.symbol);
    else:
        for each child in sentence.children:
            result = concatenate(result, ExtractSymbols(child));
    return result;
```

- What is the recursive case?
  - A sentence that is NOT a symbol.
- How is it handled?
  - We get the results of calling ExtractSymbols on all the children.
  - We concatenate those results.

Back to TT-Entails

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    else:
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```

- Next task: implementing PL-True?

PL-True?

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- Arguments:
  - LogicalExpression **sentence**.
  - Map<String, Boolean> **model**.
- Return type: Boolean.
- Task: determine if **sentence** is true or false in the row of the truth table represented by **model**.
- From the above, it should be clear that **model** represents a row in the truth table.
- What kind of data structure is good for that?

PL-True?

- Next task: implementing PL-True?.
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  - LogicalExpression **sentence**.
  - Map<String, Boolean> **model**.
- Return type: Boolean.
- Task: determine if **sentence** is true or false in the row of the truth table represented by **model**.
- From the above, it should be clear that **model** represents a row in the truth table.
- What kind of data structure is good for **model**?
- Any kind of association map (like dictionaries in Python, HashMaps in Java, stl::map in C++) that can map symbols to boolean values.

PL-True?

```
Boolean PL-True?(LogicalExpression sentence, Map model)
  if sentence.symbol != NULL:
    ???
```

PL-True?

```
Boolean PL-True?(LogicalExpression sentence, Map model)
   if sentence.symbol != NULL:
      return model[sentence.symbol];

   else ???
```

PL-True?

```
Boolean PL-True?(LogicalExpression sentence, Map model)
   if sentence.symbol != NULL:
      return model[sentence.symbol];

   else if sentence.connective == "and":
      ???
```

PL-True?

```
Boolean PL-True?(LogicalExpression sentence, Map model)
   if sentence.symbol != NULL:
      return model[sentence.symbol];

   else if sentence.connective == "and":
      for each child in sentence.children:
         if (PL-True(child, model) == false):
            return false;
      return true;

   else if sentence.connective == "or":
      ???
```

PL-True?

```
Boolean PL-True?(LogicalExpression sentence, Map model)
   if sentence.symbol != NULL:
      return model[sentence.symbol];

   else if sentence.connective == "and":
      for each child in sentence.children:
         if (PL-True(child, model) == false):
            return false;
      return true;

   else if sentence.connective == "or":
      for each child in sentence.children:
         if (PL-True(child, model) == true):
            return true;
      return false;
```

PL-True?

```
Boolean PL-True?(LogicalExpression sentence,
                 Map model)
  if sentence.symbol != NULL:
    return model[sentence.symbol];

  else if sentence.connective == "and":
    for each child in sentence.children:
      if (PL-True(child, model) == false):
        return false;
    return true;

  else if sentence.connective == "or":
    for each child in sentence.children:
      if (PL-True(child, model) == true):
        return true;
    return false;


and so on.
```

- How do we handle "if"?

PL-True?

```
Boolean PL-True?(LogicalExpression sentence,
                 Map model)
  if sentence.symbol != NULL:
    return model[sentence.symbol];

  else if sentence.connective == "and":
    for each child in sentence.children:
      if (PL-True(child, model) == false):
        return false;
    return true;

  else if sentence.connective == "or":
    for each child in sentence.children:
      if (PL-True(child, model) == true):
        return true;
    return false;


and so on.
```

- How do we handle "if"?
- left = sentence.children[0];
- right = sentence.children[1];
- Return false if:
  - PL-True(left, model) == true
  - PL-True(right, model) == false
- Return true otherwise.

PL-True?

```
Boolean PL-True?(LogicalExpression sentence,
                 Map model)
  if sentence.symbol != NULL:
    return model[sentence.symbol];

  else if sentence.connective == "and":
    for each child in sentence.children:
      if (PL-True(child, model) == false):
        return false;
    return true;

  else if sentence.connective == "or":
    for each child in sentence.children:
      if (PL-True(child, model) == true):
        return true;
    return false;


and so on.
```

- How do we handle "iff"?

PL-True?

```
Boolean PL-True?(LogicalExpression sentence,
                        Map model)
  if sentence.symbol != NULL:
    return model[sentence.symbol];

  else if sentence.connective == "and":
    for each child in sentence.children:
      if (PL-True(child, model) == false):
        return false;
    return true;

  else if sentence.connective == "or":
    for each child in sentence.children:
      if (PL-True(child, model) == true):
        return true;
    return false;


and so on.
```

- How do we handle "iff"?
- left = sentence.children[0];
- right = sentence.children[1]
- Return true if:
  - PL-True(left, model) and

  PL-True(right, model) return the same thing.
- Return false otherwise.

PL-True?

```
Boolean PL-True?(LogicalExpression sentence,
                 Map model)
  if sentence.symbol != NULL:
    return model[sentence.symbol];

  else if sentence.connective == "and":
    for each child in sentence.children:
      if (PL-True(child, model) == false):
        return false;
    return true;

  else if sentence.connective == "or":
    for each child in sentence.children:
      if (PL-True(child, model) == true):
        return true;
    return false;


and so on.
```

- How do we handle "not"?
- child = sentence.children[0];
- Return the opposite of what PL-True?(child, model) returns.

Back to TT-Entails

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                          List symbols, Map model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Next task: understanding what TT-Entails actually does.

TT-Entails

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])
```

- TT-Entails goes through the truth table.
- If it finds a smoking gun (any row where KB is true and alpha is false), it returns false.
- If it goes through **<u>all the rows</u>** in the truth table and does NOT find a smoking gun, it returns true.
- Clearly, the actual work is done by TT-Check-All.
- So, to understand TT-Entails, we must understand TT-Check-All.

TT-Check-All

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- TT-Check-All arguments:
  - KB and alpha, the same arguments as in TT-Entails.
  - model: this is an association map, mapping symbols to true/false values.
  - symbols: this is a list of symbols.

TT-Check-All: Model and Symbols

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                            List symbols, Map model)
```

- What is the value of **model** when TT-Check-All gets called from TT-Entails?
  - It is empty. In other words, it does not map any symbol to a value.
- What is the value of **symbols** when TT-Check-All gets called from TT-Entails?
  - It is the list of all symbols appearing in KB or appearing in alpha.

TT-Check-All

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
  if Empty?(symbols):
    if PL-True?(KB,model) then return PL-True?(alpha, model)
    else return true
  else:
    P = First(symbols);
    rest = Rest(symbols);
    return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
           TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- As you see, TT-Check-All calls itself.
  - Each time it calls itself, a symbol is removed from **symbols**, and it is assigned a value in **model**.
- Overall, argument **symbols** is the list of symbols that still do **not** have a value in **model.**
- The model is empty initially, and gets extended at each recursive call.

What TT-Check-All Does

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                       List symbols, Map model)
```

- The task of TT-Check-All is to look for a smoking gun in all rows of the truth table **compatible with its model**.
  - If it finds a smoking gun, it returns false.
  - Otherwise, it returns true.
- When is a row of the truth table compatible with the model?

What TT-Check-All Does

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- The task of TT-Check-All is to look for a smoking gun in all rows of the truth table <u>**compatible with its model**</u>.
  - If it finds a smoking gun, it returns false.
  - Otherwise, it returns true.
- When is a row of the truth table compatible with the model?
  - A row of the truth table can also be represented as a Map. It assigns values to symbols.
  - If the assignments in the row of the truth table do not contradict any assignments in the model, then the row is compatible with the model.

What TT-Check-All Does

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- The task of TT-Check-All is to look for a smoking gun in all rows of the truth table **compatible with its model**.
  - If it finds a smoking gun (a row where KB is true and alpha is false), it returns false.
  - Otherwise, it returns true.
- What should model be equal to, to make TT-Check-All search through the entire truth table?

What TT-Check-All Does

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- The task of TT-Check-All is to look for a smoking gun in all rows of the truth table **compatible with its model**.
  - If it finds a smoking gun (a row where KB is true and alpha is false), it returns false.
  - Otherwise, it returns true.
- What should model be equal to, to make TT-Check-All search through the entire truth table?
  - An empty model is compatible with every row in the truth table.

What TT-Check-All Does

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- The task of TT-Check-All is to look for a smoking gun in all rows of the truth table **compatible with its model**.
- When it is first called from TT-Entails, TT-Check-All needs to check the entire truth table.
- Therefore, the **model** argument in that first call is the empty model, shown as [].
- Why? Because the empty model is compatible with all rows in the truth table.

What TT-Check-All Does

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
  if Empty?(symbols):
     if PL-True?(KB,model) then return PL-True?(alpha, model)
     else return true
  else:
     P = First(symbols);
     rest = Rest(symbols);
     return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
            TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Let’s look at what TT-Check-All does when called from TT-Entails:
  - The **model argument** is empty.
  - The **symbols** argument contains all symbols.
- What will Empty?(symbols) return?

What TT-Check-All Does

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
  if Empty?(symbols):
    if PL-True?(KB,model) then return PL-True?(alpha, model)
    else return true
  else:
    P = First(symbols);
    rest = Rest(symbols);
    return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
           TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Let’s look at what TT-Check-All does when called from TT-Entails:
  - The **model argument** is empty.
  - The **symbols** argument contains all symbols.
- What will Empty?(symbols) return? False.
- So, we move on to the else part.

What TT-Check-All Does

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                                 List symbols, Map model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                    TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Let’s look at what TT-Check-All does when called from TT-Entails:
  - The **model argument** is empty.
  - The **symbols** argument contains all symbols.
- **P** is set equal to one of the symbols (the first in the list, it does not really matter which one).
- **Rest** is set equal to the rest of the symbols (all symbols except for **P**).

What TT-Check-All Does

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                              List symbols, Map model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                    TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- TT-Check-All calls itself twice.

What TT-Check-All Does

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                                   List symbols, Map model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                  TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- In the first call:
  - For the third argument (**symbols**), it uses value **rest**. So, it passes all the symbols <u>**except for the first one**</u> (which was assigned to P).
  - For the fourth argument (**model**), it passes an extended model, which includes the assignments of the previous model, plus a new assignment: ???

What TT-Check-All Does

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
   if Empty?(symbols):
      if PL-True?(KB,model) then return PL-True?(alpha, model)
      else return true
   else:
      P = First(symbols);
      rest = Rest(symbols);
      return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
             TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- In the first call:
  - For the third argument (**symbols**), it uses value **rest**. So, it passes all the symbols **<u>except for the first one</u>** (which was assigned to P).
  - For the fourth argument (**model**), it passes an extended model, which includes the assignments of the previous model, plus a new assignment: P=true.

What TT-Check-All Does

```
Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
               TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- In the second call:
  - For the third argument (**symbols**), it uses value **rest** (same as in the first call).
  - For the fourth argument (**model**), it passes an extended model, which includes the assignments of the previous model, plus a new assignment: P=false.
  - Note that in the first call P is set to true, in the second call P is set to false.

Prefix Notation

- To see how TT-Entails actually works, we should do an example.
- To make notation look like that of assignment 4, we will use prefix notation.
  - Instead of writing **A and B and C** we will write (and A B C).
  - Instead of writing **A or B or C** we will write (or A B C).
  - Instead of writing **not A** we will write (not A).
  - Instead of writing **A => B** we will write (if A B).
  - Instead of writing **A <=> B** we will write (iff A B).
- In prefix notation, to write a statement that has a connective:
  - We write a left parenthesis.
  - We write the connective.
  - We write the statements that the connective connects.
  - We write a right parenthesis.
- Example: (and M_1_2 S_1_1 (not (or M_1_3 M_1_4)))

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- What is the knowledge base saying in English?

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- What is the knowledge base saying in English?
  - There is a breeze at square (1, 1) if and only if: there is a pit at (1, 2), or there is a pit at (2, 1).
  - There is a breeze at square (1, 1).

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- What is the knowledge base saying in English?
  - There is a breeze at square (1, 1) if and only if: there is a pit at (1, 2), or there is a pit at (2, 1).
  - There is a breeze at square (1, 1).
- What does alpha say in English?

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- What is the knowledge base saying in English?
  - There is a breeze at square (1, 1) if and only if: there is a pit at (1, 2), or there is a pit at (2, 1).
  - There is a breeze at square (1, 1).
- What does alpha say in English?
  - There is no pit at (1, 2).

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- If we call TT-Entails? with these arguments, what are we asking the computer?

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- If we call TT-Entails? with these arguments, what are we asking the computer?
  – Given this knowledge base, can we infer that there is no pit at square (1,2)?
- Let's see how TT-Entails works on this question.

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols1 = ???

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols1 = [B_1_1, P_1_2, P_2_1]

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols1 = [B_1_1, P_1_2, P_2_1]
- symbols2 = ???

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols1 = [B_1_1, P_1_2, P_2_1]
- symbols2 = [P_1_2]

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols1 = [B_1_1, P_1_2, P_2_1]
- symbols2 = [P_1_2]
- symbols = ???

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols1 = [B_1_1, P_1_2, P_2_1]
- symbols2 = [P_1_2]
- symbols = [B_1_1, P_1_2, P_2_1]

An Example

```
Boolean TT-Entails?(LogicalExpression KB, LogicalExpression alpha)
    List symbols1 = ExtractSymbols(KB);
    List symbols2 = ExtractSymbols(alpha);
    List symbols = concatenate(symbols1, symbols2);
    return TT-Check-All(KB, alpha, symbols, [ ])

Boolean TT-Check-All(LogicalExpression KB, LogicalExpression alpha,
                     List symbols, Map model)
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols1 = [B_1_1, P_1_2, P_2_1]
- symbols2 = [P_1_2]
- symbols = [B_1_1, P_1_2, P_2_1]
- We now call TT-Check-All.

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = ???
- model = ???

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- What is the task of this call to TT-Check-All?
  - Which rows of the truth table should it check?

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- What is the task of this call to TT-Check-All?
  - Which rows of the truth table should it check?
  - All rows (since the model is empty).

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- if Empty?(symbols):
  - What happens here?

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- if Empty?(symbols):
  – What happens here?
  – The condition is false, we move to the **else** part.

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- P = ???

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- P = B_1_1

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- P = B_1_1
- rest = ???

An Example

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

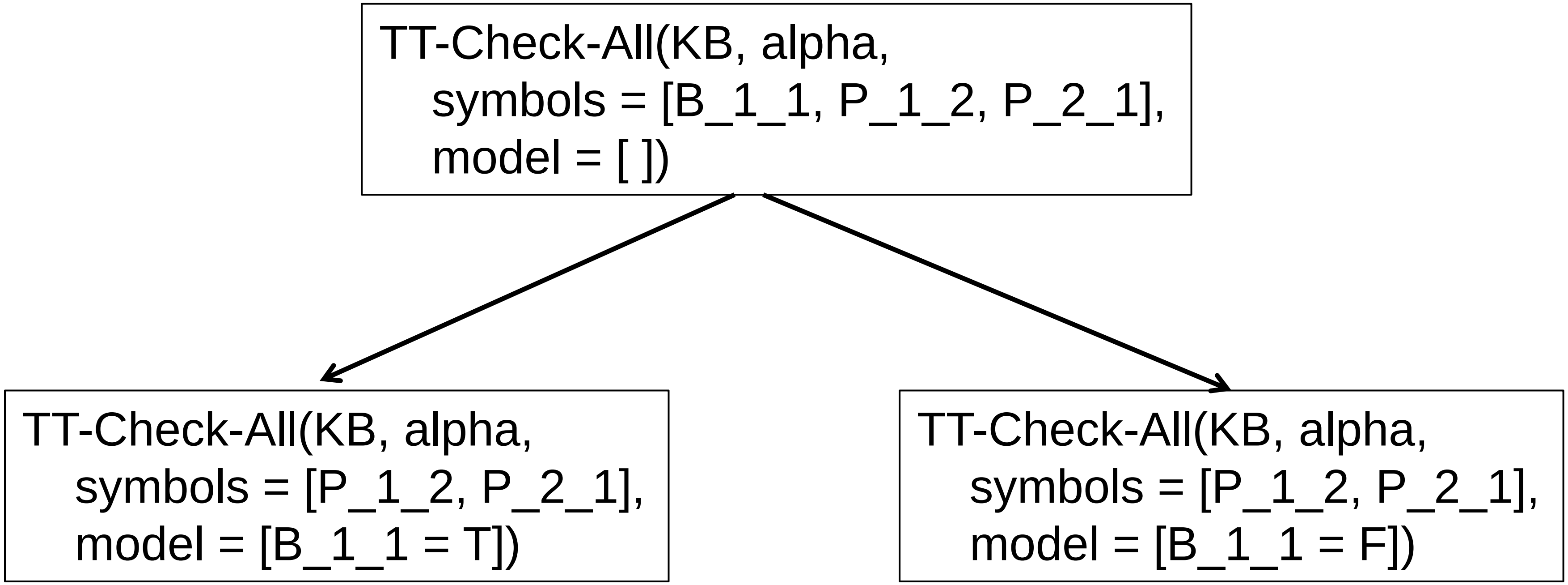
- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- P = B_1_1
- rest = [P_1_2, P_2_1]

An Example

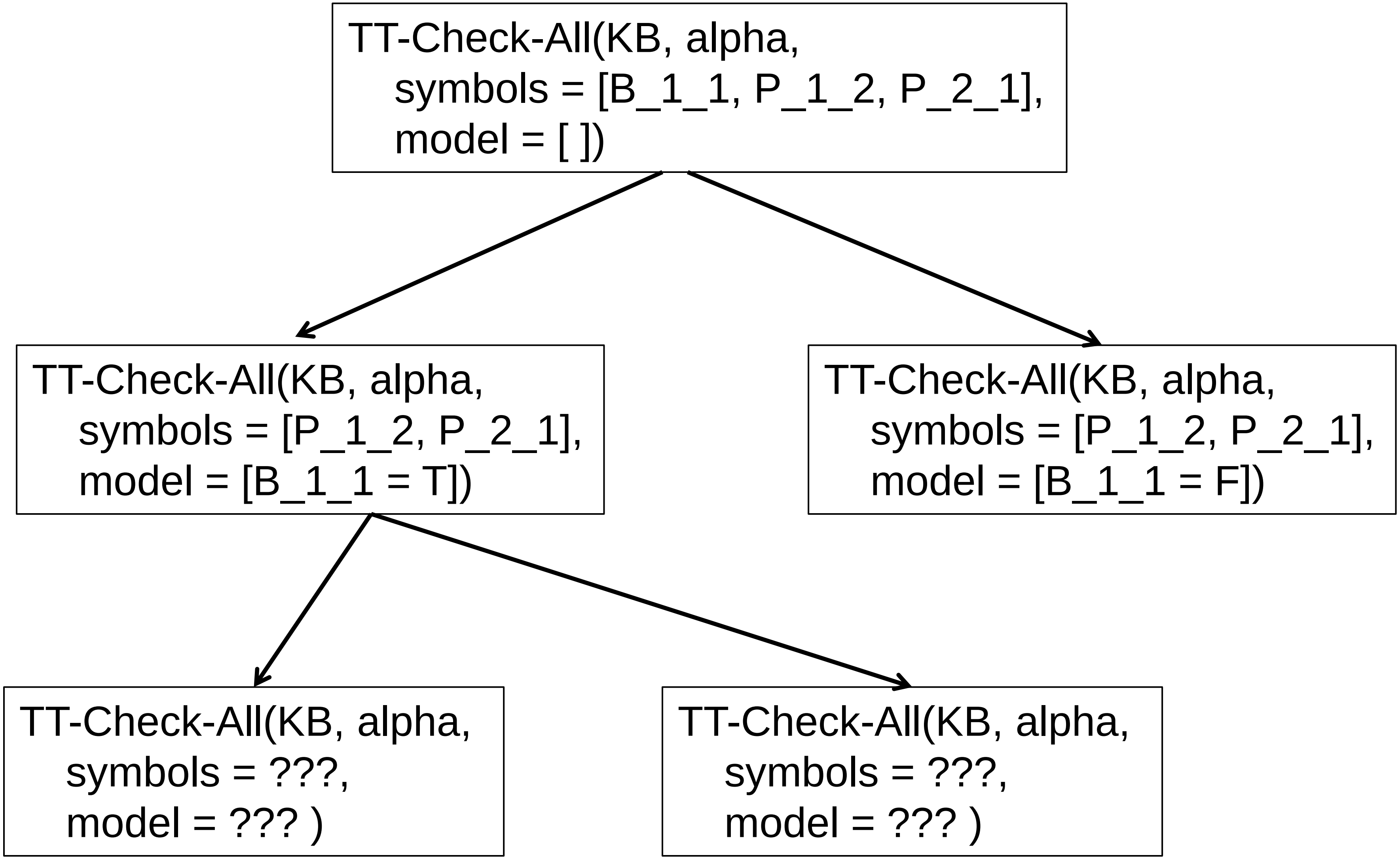
```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        P = First(symbols);
        rest = Rest(symbols);
        return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                TT-Check-All(KB, alpha, rest, Extend(P, false, model))
```

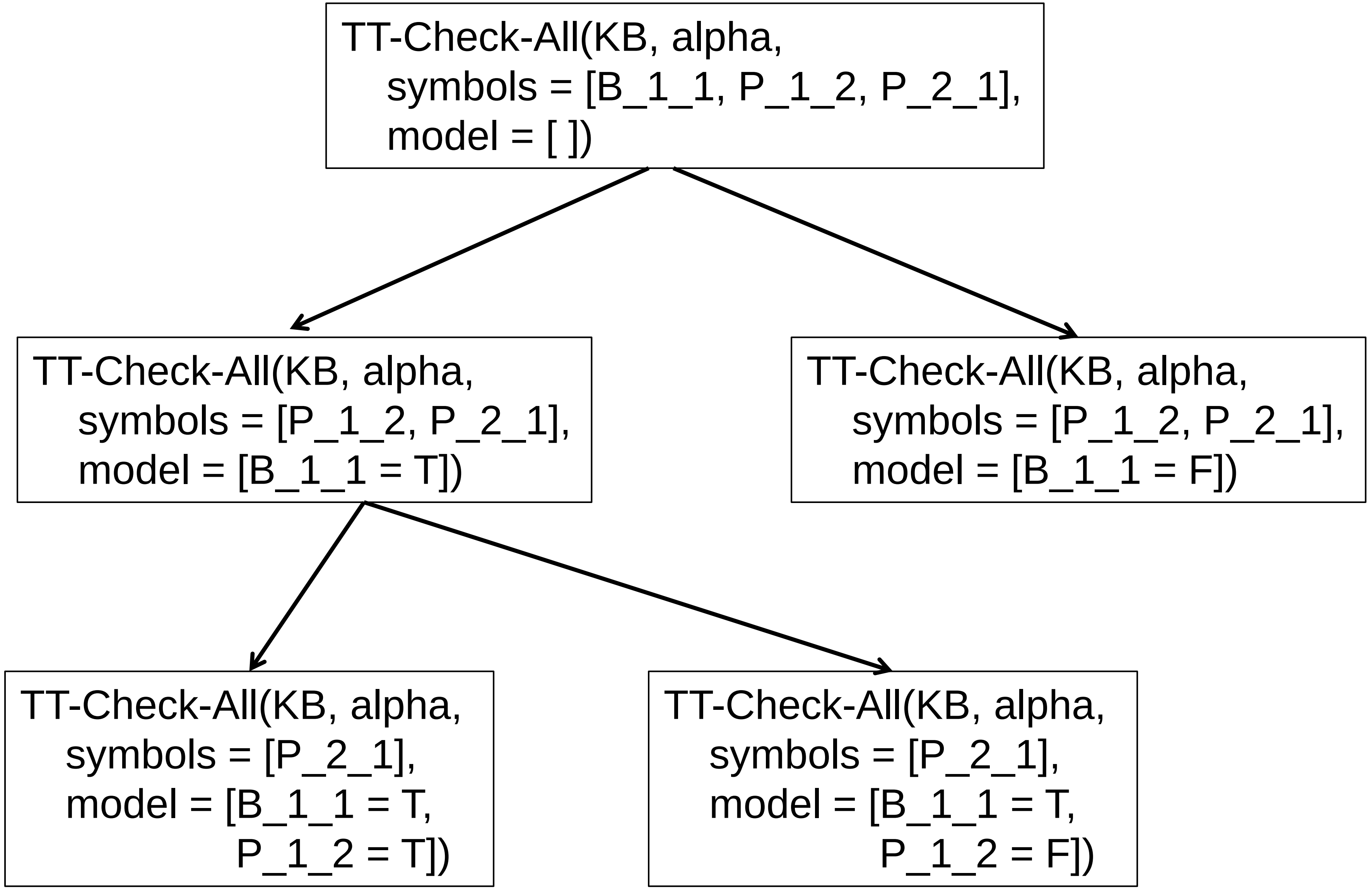
- Example KB:
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha:
  (not P_1_2)

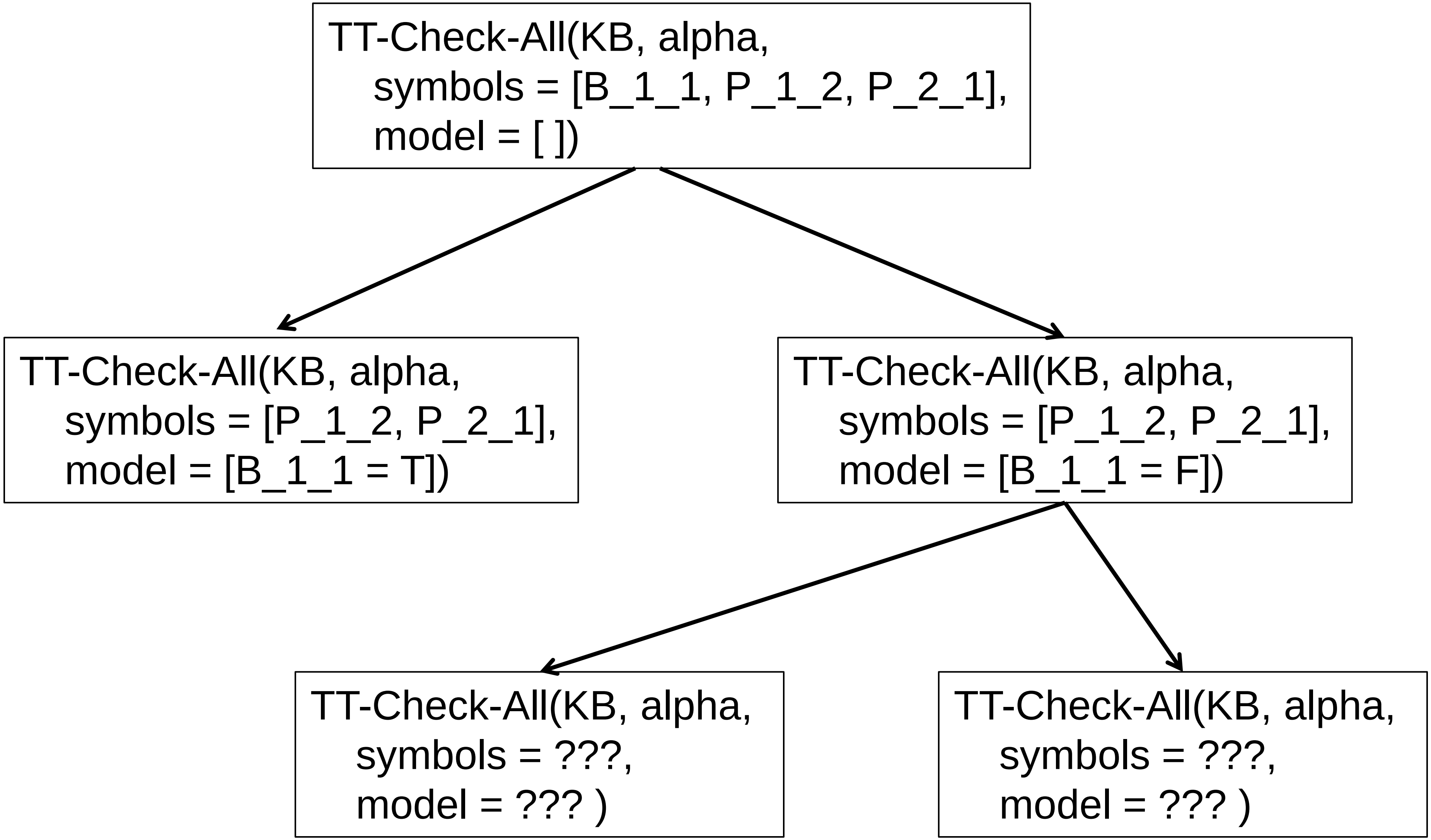
- symbols = [B_1_1, P_1_2, P_2_1]
- model = [] (the empty model)
- P = B_1_1
- rest = [P_1_2, P_2_1]
- How can we interpret this return statement?

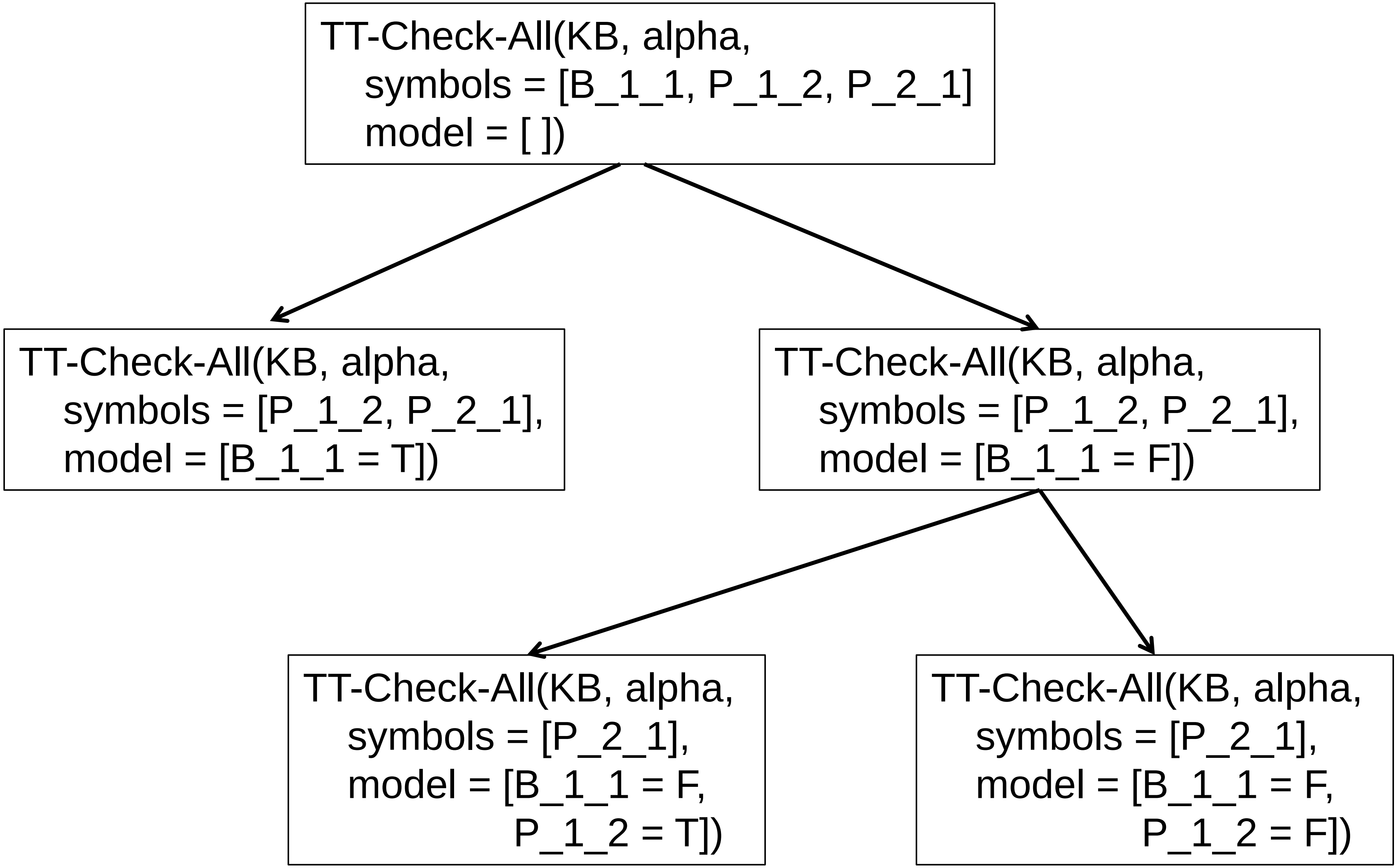


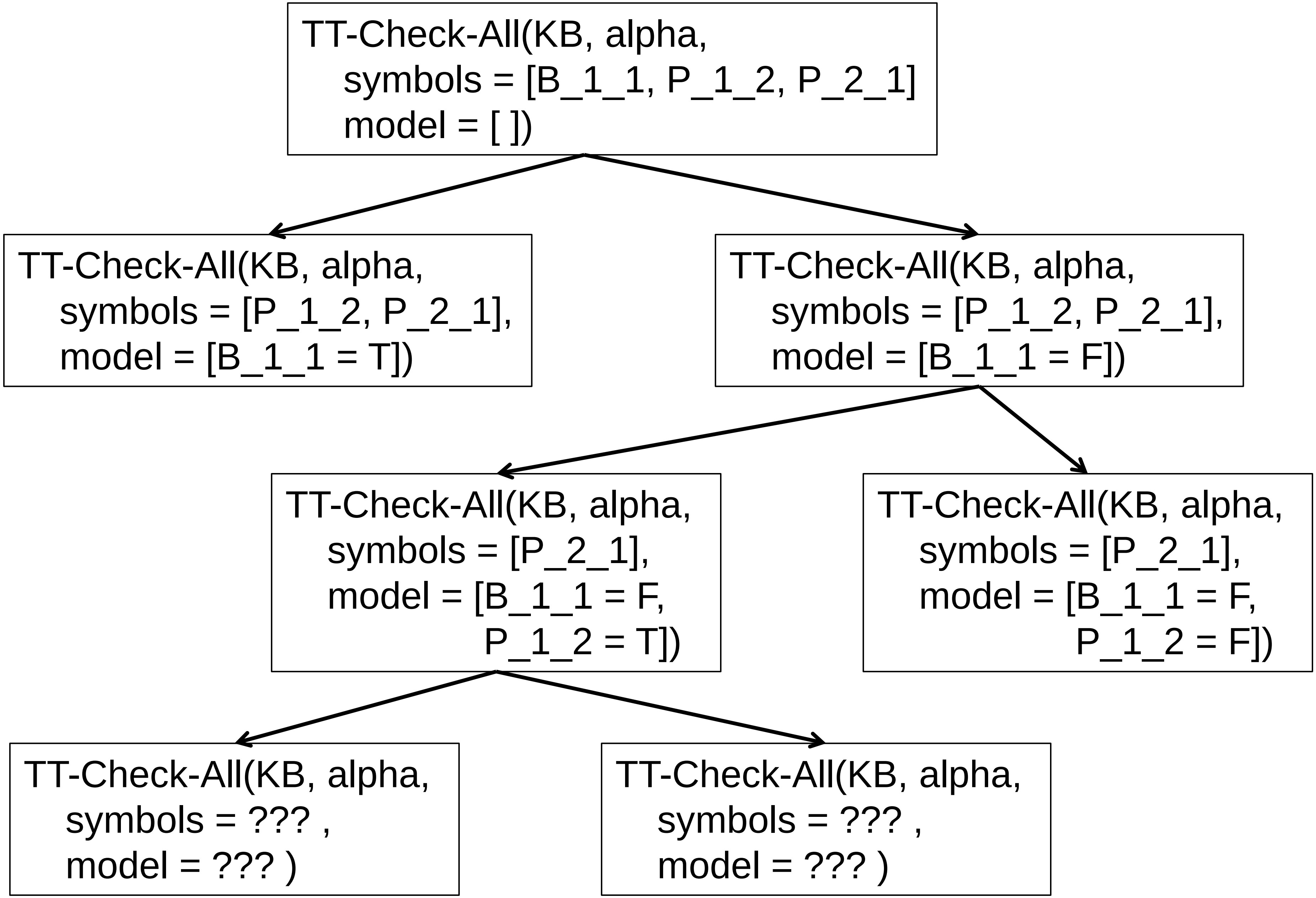
- When the model is empty, the job of TT-Check-All is to search the entire truth table for a smoking gun (a counterexample, where KB is true and alpha is false).
- TT-Check-All divides this task into two:
  - First function call: Check all rows in the truth table where B_1_1 = true.
  - Second function call: Check all rows in the truth table where B_1_1 = false.
- The final result is an AND of the results of the two function calls.
  - If none of the two calls finds a smoking gun, we return true.

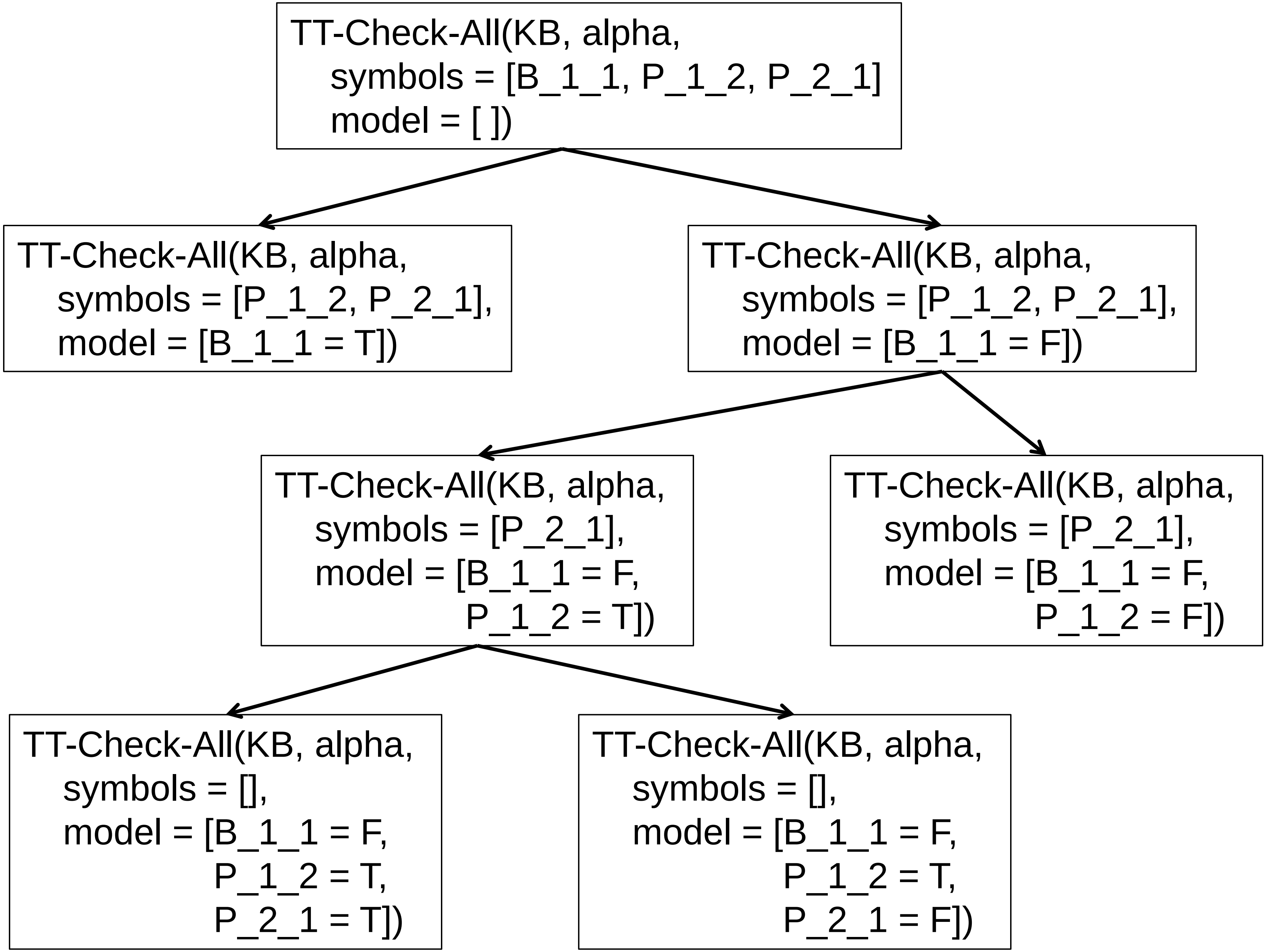

TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1],
model = [ ])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])
TT-Check-All(KB, alpha,
symbols = ???,
model = ???)
TT-Check-All(KB, alpha,
symbols = ???,
model = ???)


TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1],
model = [ ])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = T])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = F])


TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1],
model = [ ])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])
TT-Check-All(KB, alpha,
symbols = ???,
model = ???)
TT-Check-All(KB, alpha,
symbols = ???,
model = ???)


TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])


TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])
TT-Check-All(KB, alpha,
symbols = ??? ,
model = ???)
TT-Check-All(KB, alpha,
symbols = ??? ,
model = ???)


TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])
TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])
TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])
TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = T])
TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = F])

Entire tree of recursive calls to TT-Check-All

TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = F])

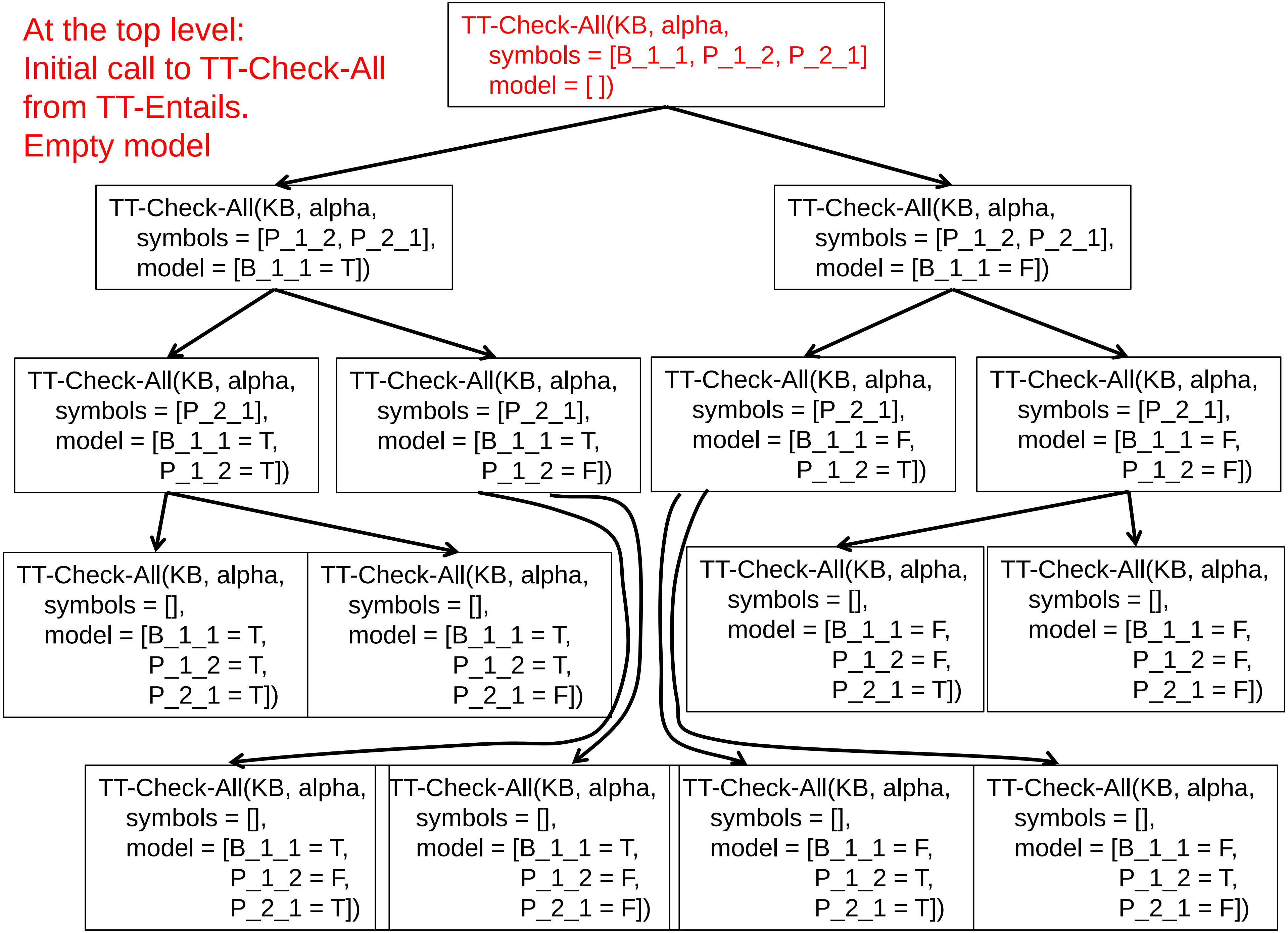
TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = F])

At the top level:
Initial call to TT-Check-All
from TT-Entails.
Empty model



At the second level:
Two function calls.
Model assigns value to
B_1_1.

TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = F])

At the third level:
Four function calls.
Model assigns values to
B_1_1, P_1_2.

TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = F])

At the fourth level:
Eight function calls.
Base case: Model assigns values to all symbols.

```
TT-Check-All(KB, alpha,
    symbols = [B_1_1, P_1_2, P_2_1]
    model = [ ])
```

```
TT-Check-All(KB, alpha,
    symbols = [P_1_2, P_2_1],
    model = [B_1_1 = T])
```

```
TT-Check-All(KB, alpha,
    symbols = [P_1_2, P_2_1],
    model = [B_1_1 = F])
```

```
TT-Check-All(KB, alpha,
    symbols = [P_2_1],
    model = [B_1_1 = T,
             P_1_2 = T])
```

```
TT-Check-All(KB, alpha,
    symbols = [P_2_1],
    model = [B_1_1 = T,
             P_1_2 = F])
```

```
TT-Check-All(KB, alpha,
    symbols = [P_2_1],
    model = [B_1_1 = F,
             P_1_2 = T])
```

```
TT-Check-All(KB, alpha,
    symbols = [P_2_1],
    model = [B_1_1 = F,
             P_1_2 = F])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = T,
             P_1_2 = T,
             P_2_1 = T])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = T,
             P_1_2 = T,
             P_2_1 = F])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = F,
             P_1_2 = F,
             P_2_1 = T])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = F,
             P_1_2 = F,
             P_2_1 = F])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = T,
             P_1_2 = F,
             P_2_1 = T])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = T,
             P_1_2 = F,
             P_2_1 = F])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = F,
             P_1_2 = T,
             P_2_1 = T])
```

```
TT-Check-All(KB, alpha,
    symbols = [],
    model = [B_1_1 = F,
             P_1_2 = T,
             P_2_1 = F])
```

How does the return value at the top level depend on the eight return values at the fourth level?

TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = F])

The top level returns true if and only if all eight calls at the bottom level return true.

TT-Check-All(KB, alpha,
symbols = [B_1_1, P_1_2, P_2_1]
model = [ ])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = T])

TT-Check-All(KB, alpha,
symbols = [P_1_2, P_2_1],
model = [B_1_1 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = T,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = T])

TT-Check-All(KB, alpha,
symbols = [P_2_1],
model = [B_1_1 = F,
P_1_2 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = F,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = T,
P_2_1 = F])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = T])

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = F,
P_1_2 = F,
P_2_1 = F])

What do the bottom levels return?

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        …
```

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = T])

- Example KB: ???
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha: ???
  (not P_1_2)
- TT-Check-All returns ???.

What do the bottom levels return?

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        …
```

TT-Check-All(KB, alpha,
symbols = [],
model = [B_1_1 = T,
P_1_2 = T,
P_2_1 = T])

- Example KB: true
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha: false
  (not P_1_2)
- TT-Check-All returns false.

Since this call to TT-Check-All returns false, what can you tell about the return value of TT-Entails in this example?

What do the bottom levels return?

```
Boolean TT-Check-All(KB, alpha, symbols, model)
    if Empty?(symbols):
        if PL-True?(KB,model) then return PL-True?(alpha, model)
        else return true
    else:
        ...
```

TT-Check-All(KB, alpha,
  symbols = [],
  model = [B_1_1 = T,
    P_1_2 = T,
    P_2_1 = T])

→

- Example KB: true
  (iff B_1_1 (or P_1_2 P_2_1))
  B_1_1
- Example alpha: false
  (not P_1_2)
- TT-Check-All returns false.

Since this call to TT-Check-All returns false, what can you tell about the return value of TT-Entails in this example?

TT-Entails will return false.
If **any call** at the bottom level returns false, TT-Entails returns false.
If **all calls** at the bottom level return true, TT-Entails returns true.