CS4211
AY24/25 SEM 1

01. Z Specification

Background for Z Specification

- $x : T$ declares a new variable x of type T
- $:$ means "belongs to"
- $\bullet$ refers to "such that" when defining predicates

Mathematical Concepts

Predicates

A statement that is either true or false.

- Let $P(x, y)$ be $x + y = 9$
- $P(4, 5)$ is **true**.
- $P(3, 7)$ is **false**.

Logic Operators

1. Not ($\neg$)
2. And ($\wedge$)
3. Or ($\vee$)
4. Implies ($\Rightarrow$)
5. Equivalence ($\Leftrightarrow$)

Quantifiers

- $\forall x : X \bullet P(x)$ abbreviates $P(a) \wedge P(b) \wedge P(c) \wedge \ldots$
- $\exists x : X \bullet P(x)$ abbreviates $P(a) \vee P(b) \vee P(c) \vee \ldots$

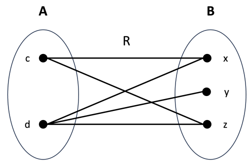
Set Theory

- A set is a collection of elements where elements are not ordered and not repeated. Examples: $\{a, b, c\} = \{b, a, c\}$ and $\{a, b, b\} = \{a, b\}$
- Well-known sets
  - $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ (The set of all natural numbers)
  - $\mathbb{N}_1 = \{1, 2, 3, \ldots\}$
  - $\mathbb{Z} = \{0, 1, -1, 2, -2, \ldots\}$ (The set of all integers)
  - $\mathbb{R}$ (The set of all real numbers)
  - $\emptyset$ (Empty Set: The set with no elements)
- Set Expressions
  - The set of natural numbers which when divided by 7 leave a remainder of 4 is $\{n : \mathbb{N} \mid \exists m : \mathbb{N} \bullet n = 7m + 4\}$
  - $\mathbb{N}$ is the set $\{z : \mathbb{Z} \mid z \geq 0\}$
  - If a, b are any natural numbers, then $a \,..\, b$ is defined as the set of all natural numbers between a and b inclusive.
- Set Relations
  - Membership: $x \in \mathbb{X}$
  - Subset ($\subseteq$): Let S and T be sets, $\forall s : S \bullet s \in T$.
  - Proper Subset ($\subset$): Let S and T be sets, $S \subseteq T \wedge S \neq T$.
  - Power Set ($\mathbb{P}$)
    - If X is a set, $\mathbb{P}\, X$ (the power set of X) is the set of all subsets of X.
    - $A \in \mathbb{P}\, B \Leftrightarrow A \subseteq B$
  - Set Operations
    - Set Union: Suppose $S, T : \mathbb{P} X$ or $S \subseteq X, T \subseteq X$, then $S \cup T = \{x : X \mid x \in S \vee x \in T\}$
    - Set Intersection: Suppose $S, T : \mathbb{P} X$, then $S \cap T = \{x : X \mid x \in S \wedge x \in T\}$
    - Set Difference: Suppose $S, T : \mathbb{P} X$, then $S - T = \{x : X \mid x \in S \wedge x \notin T\}$
    - Cardinality: $\#X$ is a natural number denoting the cardinality of (number of elements in) a finite set X.
      - $\#\{a, b, c\} = 3$

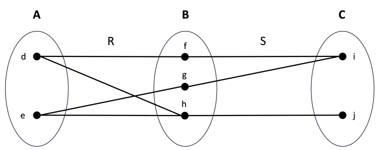
Cartesian Product and Tuples

- **Cartesian Product:** If A and B are sets, then $A \times B$ is the set of all ordered pairs (a, b) with $a \in A$ and $b \in B$.
  - $\{a, b\} \times \{a, c\} = \{(a, a), (a, c), (b, a), (b, c)\}$
- An **n-tuple** $(x_1, \ldots, x_n)$ is present in the Cartesian Product $\mathbf{A_1} \times \cdots \times \mathbf{A_n}$ if and only if each element x_i is an element of the corresponding set $\mathbf{A_i}$.
- To refer to a particular component of a tuple t, we use the projection notation $(.)$
- Suppose we have $t = (x_1, x_2, \ldots, x_n)$
  - The first component of the tuple t is written as $t.1$ which is the value $\mathbf{x_1}$.
  - The second component of the tuple t is written as $t.2$ which is the value $\mathbf{x_2}$.
  - The n-th component of the tuple t is written as $t.n$ which is the value $\mathbf{x_n}$

Relations

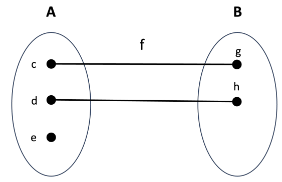


- A relation R from sets A to B, is declared as $R : A \leftrightarrow B$ is a subset of $A \times B$
- Example: $R = \{(c, x), (c, z), (d, x), (d, y), (d, z)\}$
  - The following predicates are equivalent
    1. $(c, z) \in R$
    2. $c \rightarrow z \in R$: A function that maps c to z
    3. cRz
- **Domain:** $\operatorname{dom} R$ is the set $\{a : A \mid \exists b : B \bullet aRb\}$
  - The elements in A that are related to element(s) in B
- **Range:** $\operatorname{ran} R$ is the set $\{b : B \mid \exists a : A \bullet aRb\}$
  - The elements in B that are related to element(s) in A
- **Domain and Range Restriction**
  - Let A is the domain set, B is the range set and R is the relation set.
  - Suppose $R : A \leftrightarrow B, S \subseteq A$ and $T \subseteq B$.
  - **Domain Restriction:** $S \lhd R$ is the set $\{(a, b) : R \mid a \in S\}$
  - **Range Restriction:** $R \rhd T$ is the set $\{(a, b) : R \mid b \in T\}$
  - Basically, we define a new set, either $S \subseteq A$ or $T \subseteq B$, then choose the relations $(a, b) \in R$ that **contain** elements from the new set.
- **Domain and Range Subtraction**
  - Let A is the domain set, B is the range set and R is the relation set.
  - Suppose $R : A \leftrightarrow B, S \subseteq A$ and $T \subseteq B$.
  - **Domain Subtraction:** $S \mathbin{⩤} R$ is the set $\{(a, b) : R \mid a \notin S\}$
  - **Range Subtraction:** $R \mathbin{⩥} T$ is the set $\{(a, b) : R \mid b \notin T\}$
  - Basically, we define a new set, either $S \subseteq A$ or $T \subseteq B$, then choose the relations $(a, b) \in R$ that **do not contain** elements from the new set.
  - Note: $(S \mathbin{⩤} R) \cup (S \lhd R) = R$ and $(R \mathbin{⩥} T) \cup (R \rhd T) = T$
- **Relational Image**
  - Suppose the relation $R : A \leftrightarrow B$ and $S \subseteq A$
  - $R(\!| S |\!) = \{b : B \mid \exists a : S \bullet aRb\}$ or $R(\!| S |\!) \subseteq B$
  - The relational image returns the set of all elements $b \in B$ such that there exists an $a \in S$ with $(a, b) \in R$.
  - Example: $divides(\!| \{8, 9\} |\!) = \{x : \mathbb{N} \mid \exists k : \mathbb{N} \bullet x = 8 \cdot k \vee 9 \cdot k\} = \{0, 8, 9, 16, 18, \ldots\}$
- **Inverse:** R^{-1} is the set $\{(b, a) : B \times A \mid aRb\}$ or $R^{-1} \in B \leftrightarrow A$
- **Relational Composition**
  - Suppose $R : A \leftrightarrow B$ and $S : B \leftrightarrow C$ are two relations.
  - $R \mathbin{⨾} S = \{(a, c) : A \times C \mid \exists b : B \mid aRb \wedge bSc\}$
  - $R \mathbin{⨾} S \in A \leftrightarrow C$

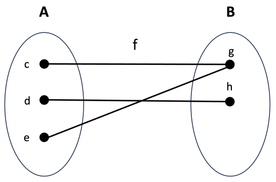


$R = \{(d,f),(d,h),(e,g),(e,h)\}$
$S = \{(f,i),(g,i),(h,j)\}$
$R;S = \{(d,i),(d,j),(e,i),(e,j)\}$

Functions



Partial Function: dom(f) ≠ A



Total Function: dom(f) = A

- A function from a set A to a set B, denoted by $f : A \rightarrow\!\!\!\!\!+ B$ is a subset f of $A \times B$ with the property that for each $a \in A$, there is **at most one** $b \in B$ with $(a, b) \in f$.
- $\operatorname{dom} f$ is the set $\{a : A \mid \exists b : B \bullet (a, b) \in f\}$
- $\operatorname{ran} f$ is the set $\{b : B \mid \exists a : A \bullet (a, b) \in f\}$
- Suppose $f : A \rightarrow\!\!\!\!\!+ B$ and $a \in \operatorname{dom} f$, then $f(a)$ denotes the unique image $b \in B$ that a is mapped to by f.
- $(a, b) \in f$ is equivalent to $f(a) = b$
- **Total Function:** $f : A \rightarrow B$ if and only if $\operatorname{dom} f = A$
- **Partial Function:** $f : A \rightarrow\!\!\!\!\!+ B$ if and only if $\operatorname{dom} f \neq A$
- **Function Overriding**
  - Suppose $f, g : A \rightarrow\!\!\!\!\!+ B$, then $f \oplus g$ is the function $(\operatorname{dom}(g) \mathbin{⩤} f) \cup g$.
  - The following predicates are true:
    1. $\operatorname{dom}(f \oplus g) = \operatorname{dom} f \cup \operatorname{dom} g$
    2. $a : \operatorname{dom} g \bullet (f \oplus g)(a) = g(a)$
    3. $\forall a : \operatorname{dom} f - \operatorname{dom} g \bullet (f \oplus g)(a) = f(a)$
    4. f ⊕ g $\in a \rightarrow\!\!\!\!\!+ b$
  - Example: $\{a \rightarrow x, b \rightarrow y, c \rightarrow x\} \oplus \{a \rightarrow y\} = \{a \rightarrow y, b \rightarrow y, c \rightarrow x\}$

Sequences

- A sequence s of elements of a set A, denoted $s : \operatorname{seq} A$, is a function $s : \mathbb{N} \rightarrow\!\!\!\!\!+ A$ where $\operatorname{dom} s = 1 \,..\, n$ for some natural number n.
- **Example**
  - $< b, c, a, b >$ denotes the sequence (function) $\{1 \rightarrow b, 2 \rightarrow c, 3 \rightarrow a, 4 \rightarrow b\} = \{(1, b), (2, c), (3, a), (4, b)\}$
  - The empty sequence is denoted by $<>$
- Since sequences are ordered mapping, $< a, b, a > \neq < a, a, b > \neq < a, b >$
- The set of all sequences of elements from A is denoted as $\operatorname{seq} A$ and is defined to be $\operatorname{seq} A = \{s : \mathbb{N} \rightarrow\!\!\!\!\!+ A \mid \exists n : \mathbb{N} \bullet \operatorname{dom} s = 1 \,..\, n\}$
- $\operatorname{seq}_1 A = \operatorname{seq} A - \{<>\}$ is defined as the set of non-empty sequences.

Special Functions for Sequences

1. Concatenation
   - $< a, b > \frown < b, a, c >=< a, b, b, a, c >$
2. Head
   - | head: $\operatorname{seq}_1 A \rightarrow A$
     |---------------------------------
     |$\forall s : \operatorname{seq}_1 A \bullet head(s) = s(1)$
   - $head < c, b, b >= c$
3. Tail
   - | tail: $\operatorname{seq}_1 A \rightarrow \operatorname{seq} A$
     |---------------------------------
     |$\forall s : \operatorname{seq}_1 A \bullet < head(s) > \frown tail(s) = s$
   - $tail < c, b, b >=< b, b >$
4. Filter
   - $< a, b, c, d, e, d, c, b, a > \upharpoonright \{a, d\} =< a, d, d, a >$
   - Filter only keeps the element in the specified set, preserves order in the original sequence and outputs a new sequence.

Background for Z Specification

- Z specification language is **strongly typed**.
- Every expression is given a type.
- Any set can be used as a type.
- The following are equivalent declarations of variables x and y of types A and B respectively.
  - $(x, y) : A \times B$
  - $x : A, y : B$
  - $x, y : A$ (only when $B = A$)

Schemas

Schema Definition Conventions

Let `MSG` be the set of all possible messages that can be transmitted.

- **Variables** are declared and typed in the **top part of the schema**.
- A **predicate (axiom)** restraining the possible values of the declared variables are given in the **bottom part of the schema.**
- The value of the state variables before the operation are denoted by **unprimed identifiers**.
  - Example: `items : seq MSG`
- Values after the operation are denoted by **primed identifiers**.
  - Example: `items' : seq MSG`
- Hidden state values are denoted by **double primed identifiers.**
  - Example: `items'' : seq MSG`
- The decoration ? denotes an **input**
  - Example: `msg? : MSG`
- The decoration ! denotes an **output**
  - Example: `msg! : MSG`
- There is an implicit $\wedge$ between each line (predicate) in the predicate section

State Schema

- A **state schema** specifies a relationship between variable values
- It specifies a snapshot or a static view of the system.
- An instance of a schema is an assignment of values to variables consistent with their type declaration and satisfying the predicate.

$Buffer$
- $items : \operatorname{seq} MSG$

- $\#items \leq max$

Operation Schema

- Used to specify how the system can change
- An operation can be thought as taking an instance of the state schema and producing a new instance.
- To specify such an operation, we express as a predicate the relationship between the **instance of the state before the operation** and the **instance after the operation.**

$Join$
- $items, items' : \operatorname{seq} MSG$
- $msg? : MSG$

- $\#items \leq max$
- $\#items' \leq max$
- $\#items < max$
- $items' = items ^\frown < msg? >$

$Leave$
- $items, items' : \operatorname{seq} MSG$
- $msg! : MSG$

- $\#items \leq max$
- $\#items' \leq max$
- $\#items \neq \emptyset$
- $items = < msg! > ^\frown items'$

Delta (δ) and Initial State($_{INIT}$)

1. Delta (Δ): To specify a **before** and **after** instance of the state schema for any operation.
2. Initial State ($_{INIT}$): To specify a state when an instance of a state is first initialized.

$\Delta Buffer$
- $items, items' : \operatorname{seq} MSG$

- $\#items \leq max$
- $\#items' \leq max$

$Buffer_{INIT}$
- $Buffer$

- $items = <>$

- Initially the buffer would be empty.
- Then, the operations of **Join** and **Leave** can occur whenever they are enabled.
- Operations are assumed to be atomic.
- At all times, an observer will notice that the state schema is satisfied.

Schema Inclusion

- Schema Inclusion is the act of including a schema in the declaration of another schema.
- It means the included schema has its declaration added to the new schema, and its predicate cojoined to the predicate of the new schema.
- The first "S" Schema is the **short form**, while the second "S" Schema is the **long form**.

A
- $x : T_1$
- $y : T_2$

- $P(x, y)$

S
- A
- $z : T_3$

- $Q(x, y, z)$

S
- $x : T_1$
- $y : T_2$
- $z : T_3$

- $P(x, y) \wedge Q(x, y, z)$

Merging Schemas

- **Type Compatability** is needed to merge schemas.
- In this case, the variable y is common between states A and B.
- We can simply merge the two types into a new state C without further specifying any new predicates.
- The full form of state C is also provided.

A
- $x : T_1$
- $y : T_2$

- $P(x, y)$

B
- $y : T_2$
- $z : T_3$

- $Q(y, z)$

C
- A
- B

C
- $x : T_1$
- $y : T_2$
- $z : T_3$

- $P(x, y) \wedge Q(y, z)$

Conjunction and Disjunction of Schemas

- Suppose A and B are schemas
- When using the conjunction ($\wedge$) operator on two schemas, it is equivalent to merging the two schemas.
  - The declaration of $A \wedge B$ is the **union** of the declarations of A and B
  - The predicate of $A \wedge B$ is the **conjunction** of the predicates of A and B
- The disjunction ($\vee$) operator on two schemas yields a different result.
  - The declaration of $A \vee B$ is the **union** of the declarations of A and B
  - The predicate of $A \vee B$ is the **disjunction** of the predicates of A and B

A
- $x : T_1$
- $y : T_2$

- $P(x, y)$

B
- $y : T_2$
- $z : T_3$

- $Q(y, z)$

$A \wedge B$
- $x : T_1$
- $y : T_2$
- $z : T_3$

- $P(x, y) \wedge Q(y, z)$

$A \vee B$
- $x : T_1$
- $y : T_2$
- $z : T_3$

- $P(x, y) \vee Q(y, z)$

Composition of Schemas

- Using the composition operator ($\fatsemi$) on two schemas is typically used to combine the effects of two operations.

A
- $x : T_1$
- $y : T_2$

- $P(x, y)$

AOP_1
- δA
- $t_3? : T_3;\ t_4! : T_4$

- $Q_1(x, x', y, y', t_3?, t_4!)$

AOP_2
- δA
- $t_5? : T_5;\ t_6! : T_6$

- $Q_2(x, x', y, y', t_5?, t_6!)$

$AOP_1 \fatsemi AOP_2$
- δA
- $t_3? : T_3;\ t_4! : T_4;\ t_5? : T_5;\ t_6! : T_6$

- $\exists x'' : T_1;\ y'' : T_2 \bullet Q_1(x, x', y, y', t_3?, t_4!) \wedge Q_2(x, x', y, y', t_5?, t_6!)$

- Example: $JoinLeave = Join \fatsemi Leave$
  - The pre-state of $Join$ is the pre-state of $Join \fatsemi Leave$.
  - The post-state of $Join$ is identified with the pre-state of $Leave$ hidden within $Join \fatsemi Leave$.
  - The consequent post-state of $Leave$ is the post-state of $Join \fatsemi Leave$.

$JoinLeave$
- $\Delta Buffer$
- $msg?, msg! : MSG$

- $\#items < max$
- $\exists\, items'' : \operatorname{seq} MSG \bullet items'' =$
- $items ^\frown < msg? > \wedge items'' = < msg! > ^\frown items'$

02. CSP

Background

- CSP is a formal language for describing patterns of interactions in concurrent systems.
- We are mainly concerned with specifying the interaction between a system and its environment which is also the **external or visible behaviour**.
- The perceived behaviour of a process will depend on the observer.

Concepts

- **Process** is defined by what it can do (visible behaviour or observable events)
- **Events** are communication or interactions between processes.
  - A process engages in **events**.
  - Each event is an atomic action.
- **Alphabet of a process**: The **set of events** a process can possibly engage in.
  - Example: A Chocolate Vending Machine has the following events:
    1. coin - insert a coin
    2. choc - extract a chocolate
  - The alphabet of a chocolate vending machine is $\{coin, choc\}$

Notation and Convention

- **Events** are denoted in **lower case**
  - Example: x, y, z are variables that denote events.
- **Processes** are denoted in **upper case**
  - Example: X, Y, Z are variables that denote processes.
- The **alphabet** of a process P is denoted by αP
- The set of traces of P is denoted by `traces(P)`
- **Trace Notation**
  - If A is a set of events, then $\text{seq}\, A$ denotes the set of all finite sequences of events from A.
  - In this scenario, α `P = A` and `trace(P) =` $\text{seq}\, A$
  - Let $s, t : \text{seq}\, A$, $s \frown t$ be the concatenation of s with t.
  - We define the relation $\leq$ to be the **sequence prefix** of two sequences.

$$\leq : \text{seq}\, A \leftrightarrow \text{seq}\, A$$

$$s \leq t \Leftrightarrow \exists\, u : \text{seq}\, A \bullet s \frown u = t$$

  - $s^n = s \frown s \frown s \frown \ldots \frown s$ denotes the event s concatenated with itself n times.

CSP Primitives

Primitive Processes

- $STOP$: A process that communicates nothing, often the result of a deadlock.
- $SKIP$: A process that represents successful termination.

Algebraic Operators

1. Prefix: $a \rightarrow P$
2. Sequential Composition: $P;\ Q$
3. Parallel Composition (Synchronous): $P \,|[\, X \,]|\, Q$
4. Interleaving (Asynchronous): $P \,|||\, Q$
5. Choice: $a \rightarrow P \,\square\, b \rightarrow Q$
6. Interrupt Process: $P \,\nabla\, e \rightarrow Q$

Prefix

- A process which may participate in **event a** then act according to **process description P** is written as: `a -> P`.
- **a** is the event prefix to **P**.
- Examples
  1. $VMU = coin \rightarrow STOP$
  2. $SHORTLIFE = (beat \rightarrow (beat \rightarrow STOP)) = beat \rightarrow beat \rightarrow STOP$
  3. $VMS = coin \rightarrow choc \rightarrow STOP$

Sequential Composition ($P;\ Q$)

- Let $\checkmark$ be the Termination event.
- The process which may only terminate is written as $SKIP$.
- Let $SKIP = \checkmark \rightarrow STOP$.
- The sequential composition of processes P and Q, written as $P;\ Q$, acts as P until P terminates by communicating $\checkmark$ and then proceeds to act as Q.

Parallel Composition

- The parallel composition of **processes P and Q** synchronised on the **event set X** is written as $P \,|[\, X \,]|\, Q$.
- No event from X may occur in $P \,|[\, X \,]|\, Q$ unless **jointly enabled** by both P and Q.
- When events from X occur, they occur in both P and Q simultaneously, and are referred to as **synchronisations**.
- Events **not from X** may occur in either P or Q seperately **but not jointly**.
- Example: $(a \rightarrow P) \,|[\, a \,]|\, (c \rightarrow a \rightarrow Q)$
  - All **a** events must be Synchronous between the two processes.
- Often, it is simply written as $P \parallel Q$ where the common event set $X = \alpha P \cap \alpha Q$ is omitted.
  - When $P \parallel Q$ is given, we still know that all common events in $X = \alpha P \cap \alpha Q$ must be synchronous between P and Q.

Interleaving

- $P \,|||\, Q$ denotes an asynchronous parallel composition between two processes **P** and **Q**.
- Both components **P** and **Q execute concurrently** without any synchronisation.
- Example: $((a \rightarrow P) \,|||\, (c \rightarrow a \rightarrow Q))$
  - One possible trace is $\langle c, a, a \rangle$, after which the process acts as $P \,|||\, Q$
    1. c from $c \rightarrow a \rightarrow Q$ is engaged, leaving us with $((a \rightarrow P) \,|||\, (a \rightarrow Q))$
    2. a from $a \rightarrow P$ is engaged, leaving us with $(P \,|||\, (a \rightarrow Q))$
    3. a from $a \rightarrow Q$ is engaged, leaving us with $P \,|||\, Q$
  - Another possible trace is $\langle a, c, a \rangle$, after which the process acts as $P \,|||\, Q$
    1. a from $a \rightarrow P$ is engaged, leaving us with $(P \,|||\, (c \rightarrow a \rightarrow Q))$
    2. c from $c \rightarrow a \rightarrow Q$ is engaged, leaving us with $(P \,|||\, (a \rightarrow Q))$
    3. a from $a \rightarrow Q$ is engaged, leaving us with $P \,|||\, Q$

Choice

- In a general choice, $(a \rightarrow P) \,\square\, (b \rightarrow Q)$, the process begins with both events **a** and **b** enabled.
- The subsequent behaviour depends on the event which occured.
  - If the event which occured is **a**, the process will act as **P** afterwards.
  - If the event which occured is **b**, the process will act as **Q** afterwards.
- Example: $(a \rightarrow P) \,\square\, (c \rightarrow a \rightarrow Q)$
  - If the first event is **a**, after which the process acts as P.
  - If the first event is **c**, after which the process acts as $a \rightarrow Q$.

Interrupt

- The interrupt process $P \,\nabla\, e \rightarrow Q$ behaves as **process P** until the first occurrence of **event e** which then the control passes to **process Q**.
- When coding the specification, the keyword **interrupt** is used instead of the symbol ∇.
- For the System process, the first event can be a routine or an exception.
- After that, it still behaves as a System process.

```
Err() = exception -> Err();
Routine() = routine -> Routine();
ExceptionHandling() = Routine() interrupt exception ->
     ExceptionHandling();
System = Err() || ExceptionHandling();
```

Laws for Concurrency

- Law 1: $P \parallel Q = Q \parallel P$
- Law 2: $P \parallel (Q \parallel R) = (P \parallel Q) \parallel R$
- Law 3: $P \parallel STOP_{\alpha P} = STOP_{\alpha P}$

Let...

1. $a \in (\alpha P - \alpha Q)$
2. $b \in (\alpha Q - \alpha P)$
3. $\{c, d\} \subseteq (\alpha P \cap \alpha Q)$

- Law 4A: $(c \rightarrow P) \parallel (c \rightarrow Q) = c \rightarrow (P \parallel Q)$
- Law 4B: $(c \rightarrow P) \parallel (d \rightarrow Q) = STOP$ if $c \neq d$
- Law 5A: $(a \rightarrow P) \parallel (c \rightarrow Q) = a \rightarrow (P \parallel (c \rightarrow Q))$
- Law 5B: $(c \rightarrow P) \parallel (b \rightarrow Q) = b \rightarrow ((c \rightarrow P) \parallel Q)$
- Law 6: $(a \rightarrow P) \parallel (b \rightarrow Q) = a \rightarrow (P \parallel (b \rightarrow Q)) \,\square\, b \rightarrow ((a \rightarrow P) \parallel Q)$

Channel

- Processes may communicate through channels.
- A channel is like a message buffer for one process to send a value to another process.
- A channel event is written as one of the following forms:

Form	Description
$c!n$	Channel Output. This event occurs when a process writes n (a value) to the tail of channel c's buffer
$c?n$	Channel Input. This event occurs when a process reads a value from the head of channel c's buffer to a local variable n
$c.n$	Channel output and its matching channel input are engaged together by two processes.

Channel Example

Suppose we are given the following specification in CSP.

```
channel c 1; // Channel with buffer size = 1
Sender(i) = c!i -> Sender(i);
Receiver() = c?x -> a.x -> Receiver();
System() = Sender(5) ||| Receiver();
```

- Note: A process can have optional parameters, eg: Sender(i)
- The first event must be c!5 since c's buffer is empty.
- The second event must be c?5 since c's buffer size is 1.
- The third event can either be c!5 or a.5

Channel Example: Synchronous Buffer

Suppose we are given the following specification in CSP.

```
channel c 0; // Synchronous Buffer
Sender(i) = c!i -> Sender(i);
Receiver() = c?x -> a.x -> Receiver();
System() = Sender(5) ||| Receiver();
```

- Note: A synchronous buffer is defined by setting the buffer size to 0.
- The first event must be c.5, since the sender must write to the c's buffer and the reciever must read from c's buffer simultaneously.
- The second event must be a.5

03. PAT CSP#

Operational Semantics: Primitives

- STOP (A process that does nothing)
- SKIP

$$SKIP \xrightarrow{\checkmark} STOP$$

SKIP can only engage the **termination event** ($\checkmark$), afterwards it becomes STOP.

- Prefixing

$$\overline{(a \rightarrow P) \xrightarrow{a} P}$$

$a \rightarrow P$ can only engage event **a**, afterwards it becomes process **P**.

Operational Semantics

General Choice

- If P is choosen

$$\frac{P \xrightarrow{a} P'}{(P \,\Box\, Q) \xrightarrow{a} P'}$$

- If Q is chosen

$$\frac{Q \xrightarrow{a} Q'}{(P \,\Box\, Q) \xrightarrow{a} Q'}$$

Sequential Composition

- In process P;Q, P takes control first and Q starts only when P has finished.
- Let $\checkmark$ be the termination event.

$$\frac{P \xrightarrow{a} P'}{(P;\ Q) \xrightarrow{a} (P';\ Q)}$$

$$\frac{P \xrightarrow{\checkmark} P'}{(P;\ Q) \xrightarrow{\checkmark} Q}$$

Interrupt

- In process P ∇ Q, whenever an event is engaged by Q, P is interrupted and the control is transferred to Q.

$$\frac{P \xrightarrow{a} P'}{(P \nabla Q) \xrightarrow{a} (P' \nabla Q)}$$

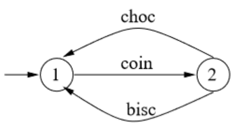
$$\frac{Q \xrightarrow{a} Q'}{(P \nabla Q) \xrightarrow{a} Q}$$

Operational Semantics: Example #1

- Let $VMS = coin \rightarrow (choc \rightarrow VMS \,\Box\, bisc \rightarrow VMS)$
  - Step 1. $VMS \xrightarrow{coin} (choc \rightarrow VMS \,\Box\, bisc \rightarrow VMS)$ (by rule prefixing)
  - Step 2. $(choc \rightarrow VMS \,\Box\, bisc \rightarrow VMS) \xrightarrow{choc} VMS$ (by rule choice 1)
  - Step 2. $(choc \rightarrow VMS \,\Box\, bisc \rightarrow VMS) \xrightarrow{bisc} VMS$ (by rule choice 2)

Labelled Transition System (LTS)

- A **Labelled Transition System** contains a set of states, an initial state (where the system starts from) and a labelled transition relation.
- The Labelled Transition System is a directed graph



- Let $VMS = coin \rightarrow (choc \rightarrow VMS \,\Box\, bisc \rightarrow VMS)$
- State 1 represents the process VMS.
- State 2 represents the process $(choc \rightarrow VMS \,\Box\, bisc \rightarrow VMS)$

Operational Semantics Continued

Interleaving

- In process $P \,|||\, Q$, P and Q behaves independently.
- The exception is the termination, hence assume a is not $\checkmark$.

$$\frac{P \xrightarrow{a} P'}{(P \,|||\, Q) \xrightarrow{a} (P' \,|||\, Q)}$$

$$\frac{Q \xrightarrow{a} Q'}{(P \,|||\, Q) \xrightarrow{a} (P \,|||\, Q')}$$

Synchronization

- In process $P \,|[\, X \,]|\, Q$, no event from X may occur unless jointly by both P and Q.
- When events from X do occur, they occur in P and Q simultaneously.

$$\frac{P \xrightarrow{a} P', a \notin X}{(P \,|[\, X \,]|\, Q) \xrightarrow{a} (P' \,|[\, X \,]|\, Q)}$$

$$\frac{Q \xrightarrow{a} Q', a \notin X}{(P \,|[\, X \,]|\, Q) \xrightarrow{a} (P \,|[\, X \,]|\, Q')}$$

$$\frac{P \xrightarrow{a} P', Q \xrightarrow{a} Q', a \in X}{(P \,|[\, X \,]|\, Q) \xrightarrow{a} (P' \,|[\, X \,]|\, Q')}$$

Operational Semantics: Example #2

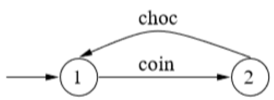
Given the process $a \rightarrow P \,|[\, a \,]|\, (c \rightarrow a \rightarrow Q)$

1. $(a \rightarrow P \,|[\, a \,]|\, (c \rightarrow a \rightarrow Q)) \xrightarrow{c} (a \rightarrow P \,|[\, a \,]|\, (a \rightarrow Q))$
   Only event **c** can be engaged at first as **a** is a common event in both $a \rightarrow P$ and $c \rightarrow a \rightarrow Q$.
2. $(a \rightarrow P \,|[\, a \,]|\, (a \rightarrow Q)) \xrightarrow{a} (P \,|[\, a \,]|\, Q)$
   Engage the common event **a** on both $a \rightarrow P$ and $a \rightarrow Q$.

Operational Semantics: Example #3

- $VMC = coin \rightarrow (choc \rightarrow VMC \,\Box\, bisc \rightarrow VMC)$
- $CHOCLOV = choc \rightarrow CHOCLOV \,\Box\, coin \rightarrow choc \rightarrow CHOCLOV$

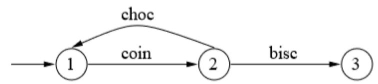
1. How would the process $VMC \,|[\, A \,]|\, CHOCLOV$ behave when $A = \{coin, choc, bisc\}$
   - Step 1: $VMC \,|[\, A \,]|\, CHOCLOV \xrightarrow{coin} (choc \rightarrow VMC \,\Box\, bisc \rightarrow VMC) \,|[\, A \,]|\, (choc \rightarrow CHOCLOV)$
   - Step 2: $(choc \rightarrow VMC \,\Box\, bisc \rightarrow VMC) \,|[\, A \,]|\, (choc \rightarrow CHOCLOV) \xrightarrow{choc} VMC \,|[\, A \,]|\, CHOCLOV$



Operational Semantics Example: #4

- $VMC = coin \rightarrow (choc \rightarrow VMC \,\Box\, bisc \rightarrow VMC)$
- $CHOCLOV = choc \rightarrow CHOCLOV \,\Box\, coin \rightarrow choc \rightarrow CHOCLOV$

2. How would the process $VMC \,|[\, \{coin, choc\} \,]|\, CHOCLOV$ or equivalently $VMC \,\|\, CHOCLOV$ behave?
   - Step 1: $VMC \,\|\, CHOCLOV \xrightarrow{coin} (choc \rightarrow VMC \,\Box\, bisc \rightarrow VMC) \,\|\, (choc \rightarrow CHOCLOV)$
   - Step 2a: $(choc \rightarrow VMC \,\Box\, bisc \rightarrow VMC) \,|[\, A \,]|\, (choc \rightarrow CHOCLOV) \xrightarrow{choc} VMC \,|[\, A \,]|\, CHOCLOV$
   - Step 2b: $(choc \rightarrow VMC \,\Box\, bisc \rightarrow VMC) \,|[\, A \,]|\, (choc \rightarrow CHOCLOV) \xrightarrow{bisc} VMC \,|[\, A \,]|\, (choc \rightarrow CHOCLOV)$



Case Study: Dining Philosophers

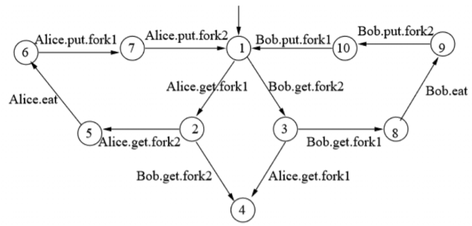
1. Specify the dining philosophers

```
Alice = Alice.get.fork1 -> Alice.get.fork2 -> Alice.eat ->
      Alice.put.fork1 -> Alice.put.fork2 -> Alice
Bob = Bob.get.fork1 -> Bob.get.fork2 -> Bob.eat -> Bob.put.
      fork1 -> Bob.put.fork2 -> Bob
Fork1 = (Alice.get.fork1 -> Alice.put.fork1 -> Fork1) [] (
      Bob.get.fork1 -> Bob.put.fork1 -> Fork1)
Fork2 = (Alice.get.fork2 -> Alice.put.fork2 -> Fork2) [] (
      Bob.get.fork2 -> Bob.put.fork2 -> Fork2)
College = Alice || Bob || Fork1 || Fork2
```

2. Get the alphabets of each process
   - $\alpha Alice = \{Alice.get.fork1, Alice.get.fork2, Alice.eat, Alice.put.fork1, Alice.put.fork2\}$
   - $\alpha Bob = \{Bob.get.fork1, Bob.get.fork2, Bob.eat, Bob.put.fork1, Bob.put.fork2\}$
   - $\alpha Fork1 = \{Alice.get.fork1, Alice.put.fork1, Bob.get.fork1, Bob.put.fork1\}$
   - $\alpha Fork2 = \{Alice.get.fork2, Alice.put.fork2, Bob.get.fork2, Bob.put.fork2\}$
3. Apply the operational semantics rule (one at a time) to build the Labelled Transition System.
   - Alice can perform Alice.get.fork1
   - Bob can perform Bob.get.Fork2
   - Fork1 can perform Alice.get.fork1 or Bob.get.fork1
   - Fork2 can perform Alice.get.fork2 or Bob.get.Fork2
   - By rule syn3, College can perform either Alice.get.fork1 or Bob.get.fork2, and then a state of the form.

$$\cdots \,\|\, \cdots \,\|\, \cdots \,\|\, \cdots$$

4. Analyze the Labelled Transition system
   - Is the system deadlock-free?
   - Will Alice or Bob starve to death?



Safety

- Safety means **something bad never happens**.
- Examples
  1. deadlock-freeness

     ```
     #assert College() deadlockfree;
     ```

     The system never deadlocks
  2. invariant

     ```
     #assert Bank() |= [] Value >= Debit;
     ```

     The savings of a bank account must always be non-negative.
- **Note:**
  - '[]' signifies 'always' in Linear Temporal Logic.
  - '|=' represents 'satisfisfaction' in Linear Temporal Logic.

Verifying Safety

- To verify safety, perform **reachability analysis** on the **Labelled Transition System**.
- A counterexample to the safety property is a finite execution which leads to a bad state.
- Perform either **Depth First Search (DFS)** or **Breadth First Search (BFS)** to search all reachable states for a 'bad' one.
- Example (using the LTS of Dining Philosophers):
  1. Depth First Search: $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 1 \rightarrow$ backtrack $\rightarrow 4 \rightarrow$ FOUND!
  2. Breadth First Search: $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 4 \rightarrow$ FOUND!

Applications of Safety Verification

Many properties can be formulated as a safety property and solved using **reachability analysis**.

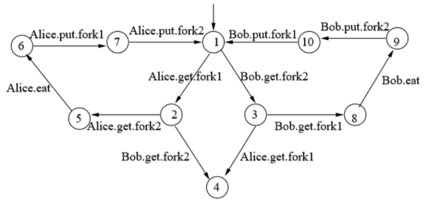
1. Mutual Exclusion: []!(more than one process accessing the critical section)
   - There will never be more than one process accessing the critical section.
2. Security: [](only the authorized user can access the information)
   - It is always the case that only the authorized user can access the information.
3. Program Analysis
   - Arrays are always bounded.
   - Pointers are always non-null
   - etc...

Liveness

- Liveness means **something good eventually happens**.
- Examples
  1. A program is eventually terminating.
  2. A file writer is eventually closed.
  3. Both Alice and Bob eventually get to eat.

Verifying Liveness

- To verify liveness, perform **loop searching** on the **Labelled Transition System**.
- A counterexample to a liveness property is an infinite system execution during which the 'good' thing never happens.
  - Example: An infinite loop fails the property that the program is eventually terminating.
- We can search through the Labelled Transition System for a bad loop using **Nested Depth First Search** or **Strongly Connected Component based Search**
- Example



Assertion: Alice will always eventually eat. (**False**)

```
#assert College() |= Alice.eat
```

Counterexamples

- $\langle Alice.get.fork1, Bob.get.fork2 \rangle$
- $< Bob.get.fork2 \rightarrow Bob.get.fork1 \rightarrow Bob.eat \rightarrow Bob.put.fork2, \rightarrow Bob.put.fork1 >^*$

CSP# Features

Global Definition

- **Constants**

```
#define max 5;
```

- **Enumerations**

```
enum {red, blue, green};
// Syntactic Sugar for the following
#define red 0;
#define blue 1;
#define green 2;
```

- **Variables**

```
var knight = 0;
```

- **Arrays**

```
// A fixed sized array may be defined as follows:
var board = [3, 5, 6, 0, 2, 7, 8, 4, 1];
// If we do not specify the elements in an array,
// all elements in array are initialized to 0
var leader[3]; // Array of size 3.
var matrix[3][2] // Multi-dimensional array (internally an
    array of 6)
```

- Often, it is desirable to provide the range of the variables / arrays explicity by giving the lower bound or upper bound or both.

```
var knight:{0..} = 0;
var board:{0..10} = [3, 5, 6, 0, 2, 7, 8, 4, 1];
```

- **Array Initialization**: To ease modelling, PAT supports fast array initialization using the following syntax

```
#define N 2;
// Initialized array with syntax shortcuts
var array = [1(2), 3..6, 7(N * 2), 12..10];
// The above is the same as the following
var array = [1, 1, 3, 4, 5, 6, 7, 7, 7, 7, 12, 11, 10];
```

- **Macro**
  - Macros are used to define system properties and processes.
  - The keyword **#define** may be used to define macros.

  ```
  #define goal x == 0;
  #assert System() reaches goal;
  // If the value of x is 0 then do P else do Q.
  if (goal) { P } else { Q };
  ```

  - **Explanation**
    - goal is the name of the macro
    - x == 0 is what the goal means.
- **Note:**
  - The constant value can only be of **integer** or **boolean** value.
  - **#define** is a keyword used for multiple purposes. Here it defines a global constant.
  - (;) **semi-colon** marks the end of the 'sentence'.
  - Multi-dimensional arrays are internally converted to one-dimension.
  - The **var** keyword is used to defined variables.
  - The scope of these variables are global if they are not within an event or process.
  - PAT only supports integer, boolean and integer arrays for the purpose of **efficient verification**.
  - However, advanced data structures (eg: Stack, Queue, Hashtable, etc...) are necessary for some models which PAT provides an interface to create user defined data types by inheriting an **abstract class** `ExpressionValue` using the **C# library**.

CSP#: Process Definition

- Event Prefixing
  1. Basic Form

     ```
     e -> p;
     VM() = coin -> coffee -> VM();
     ```

  2. Compound Form
     - For example in `x.exp1.exp2`, `x` is the event name and `exp1` and `exp2` are expressions.
     - Each expression corresponds to a variable (eg: process parameters, channel input variables or global variables).

     ```
     #define N 2;
     // Dining Philosophers Example
     Phil(i) = get.i.(i + 1) % N -> Rest();
     ```

- Statement Block inside Events (aka Data Operations)
  - An event can be attached with assignments which update global or local variables.
  - Process arguments and channel inputs can only used without being updated.
  - Semi-colons (;) mark the end of a statement in C# or end of a sentence in CSP#.

```
var array = [0, 2, 4, 7, 1, 3];
var maxi = -1;
P() = findmax {
    var index = 0;
    while (index < 6) {
        if (maxi < array[index]) { maxi = array[index]; }
        index = index + 1;
    };
} -> Skip;
```

P() = ... is equivalent to defining process P without any process parameters.

- **Conditional Choice:** A choice may depend on a Boolean expression which in turn depends on the valuation of the variables.

```
var x = 1;
Init = []i{1,2}@set.i{x = i} -> Skip;
P = if (x == 1) { a -> Stop } else { b -> Stop };
System = Init;P; // Sequential composition of two processes
```

- **Guarded Process:** A guarded process only executes when its guard condition is satisfied.

```
var x = 1;
Init = []i{1,2}@set.i{x = i} -> Skip;
P = [x == 1] a -> Stop [] [x != 1] b -> Stop;
System = Init;P; // Sequential composition of two processes
```

- `[]i:{1, 2}` means choice for variable i which can be either 1 or 2.
- In both examples, Process P behaves differently depending of the value of variable x.
- Both conditional choice and guarded process can produce the same effect.

```
var x = 0;
P = [x < 4] b{x = x + 1;} -> P;
aSys = [x == 2] a -> Stop ||| P;
```

In the example above, the trace $\langle b, b, a\rangle$ is possible but $\langle b, b, b, a\rangle$ is not possible.

- **Atomic Process:**
  - The keyword **atomic** is used to indicate that a process is of **higher priority**.
  - This means if the atomic process has an enabled event, the event will execute before any events from non-atomic processes.

```
channel ch 0;
P = atomic { a -> ch!0 -> b -> Skip };
Q = atomic { d -> ch?0 -> e -> f -> Skip };
W = g -> Skip;
System = P ||| Q ||| W;
```

  - In the example above, processes P and Q are both atomic processes, while process W is not.
  - The expected behaviour is that processes P and Q will interleave each other (only synchronised on ch.0), whereas W will execute only after event b and f have occured.

Since the channel size is 0, processes P and Q have to synchronise at the channel events.

CSP#: Assertions

- **Deadlock-freeness:** The following assertion asks if process P() is deadlock-free or not.
  - A deadlock state is a state with no further move, except for successfully terminated state.

```
P = a -> Skip;
#assert P deadlockfree; // True
```

- **Reachability:** The following assertion asks whether process P() can reach a state at which *some given condition is satisified.*
  - In this example, the assertion is True because it can reach a state where `x < 0`.

```
var x = 0;
P() = add{x = x + 1;} -> P() [] minus{x = x - 1;} -> P();
#define goal x < 0;
#assert P() reaches goal; // Note: goal condition must be a
     macro
```

- The following coin exchanging example shows how to minimize the number of coins during reachability search.

```
var x = 0;
var weight = 0;
P() = if (x <= 14) {
    coin1{x = x + 1; weight = weight + 1;} -> P();
    [] coin2{x = x + 2; weight = weight + 1;} -> P();
    [] coin5{x = x + 5; weight = weight + 1;} -> P();
};
#define goal x == 14;
#assert P() reaches goal with min(weight);
```

CSP#: Assertions and Linear Temporal Logic

- **Linear Temporal Logic (LTL)** is a formalism used for specifying and reasoning about the behavior of systems over time.
- It extends propositional logic by introducing temporal operators that describe how properties of a system evolve over time, making it suitable for reasoning about sequences of states in a system.
- In LTL, time is viewed as a linear sequence of discrete points, and temporal operators allow the expression of future and current behaviors in a system.
- Let ϕ and ψ be LTL formulaes.

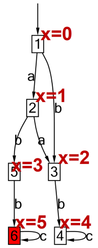
Operator	Usage	Name	Explanation
X	**X** ϕ	Next	ϕ has to hold at the next state
G	**G** ϕ	Globally	ϕ has to hold on the entire subsequent path
F	**F** ϕ	Finally	ϕ eventually has to hold somewhere on the subsequent path
U	ψ **U** ϕ	Until	ϕ holds at the current or a future position, and ψ has to hold until that position. At that position ψ does not have to hold anymore.
R	ψ **R** ϕ	Release	ϕ is true until the first position in which ψ is true, or forever if such a position does not exist.

- The LTL assertion is true if and only if every execution (**trace**) of the system satisfies the formula F, where F is an LTL formula whose syntax is defined as the following rules.
  - F = e | prop | [] F | <> F | X F | F1 U F2 | F1 R F2
  - Notations
    1. e is an event
    2. prop is a predefined propositional
    3. [] reads as "always" (or 'G' in CSP#)
    4. <> reads as "eventually" (or 'F' in CSP#)
    5. X reads as "next"
    6. U reads as "until"
    7. R reads as "release"

```
#assert P() |= F;
```

CSP#: Assertions and Linear Temporal Logic (Examples)

```
var x = 0;
P = [x < 2] a{x = x + 1} -> P
    [][x < 4] b {x = x + 2} -> P
    [][x >= 4] c -> P;
#define ge2 x >= 2;
#define lt2 x < 2;
```




```
#assert P deadlockfree; // Valid

#assert P |= lt2; // First state, x < 2?
                  // Yes

#assert P |= !c; // Init event is not c?
                 // Yes

#assert P |= X (a || b);
// Next event a or b? Yes!

#assert P |= [] ge2; // Always x >= 2? No
// Counter Example: Initial State (x = 0)

#assert P |= <> ge2; // Eventually x >= 2?
// Yes. Traces: <a, b>, <a, a>, <b>

#assert P |= [] (lt2 -> X ge2);
// It is always true that if the current state is x < 2, it
     implies the next state is x >= 2?
// No. For example, <a>, init state x = 0 (lt2), next state x =
     1(!ge2)

#assert P |= [] (lt2 -> X(Xge2));
// It is always true that if the current state is x < 2, then
     the next next state is x >= 2?
// Yes. Traces: <a, a>, <a, b>, <b>

#assert P |= (lt2 U ge2);
// Is it always x < 2 until x >= 2? Yes

#assert P |= (ge2 R le3);
// x <= 3 until the first position
// where x >= 2? Yes

#assert P |= (ge2 R lt2);
// x < 2 until the first position
// where x >= 2? No
// Counter Examples: <a, b>, <b>
```

04. Timed and Probability CSP

Real-Time System Module

Let P and Q be processes, while d represents a duration of d time units.

```
P = Wait[t] // delay
P = P timeout[t] Q // timeout
P = P interrupt[t] Q // timed interrupt
P = P deadline [t] // deadline
P = P within[t] // within
```

Timed Process Definition: Wait

- A wait process `Wait[t]` delays the system execution for a period of t time units then terminates.
- Each (V, P) is an ordered pair of values and processes.

Definition 1

$$\frac{t \leq d}{(V, Wait[d]) \xrightarrow{t} (V, Wait[d-t])}$$

Definition 2

$$\frac{}{(V, Wait[0]) \xrightarrow{\tau} (V, Skip)}$$

```
P = Wait[t]; P
```

- The starting time of process P is delayed by exactly t time units.
- If the amount of time elapsed is less than the specified duration, then the Wait process will be still active.

Timed Process Definition: Timeout

- The process `P timeout[t] Q` passes control to process `Q` if no event has occured in process `P` before t time units have elapsed.
- For instance if process `a -> P timeout [t] Q` engages in event `a` before t time units have elapsed since the process is enabled, then the process is transformed to P.
- If event `a` has not occured by time t, the process transforms to Q (by silent tau-transition).
- **Invisible Events**
  1. Let τ (tau) be an invisible event. Example: `tau{pv = x}`
  2. { pv = x } (Event with no name is also an invisible event)
  3. **Note:** Invisible events are not observable.
- The timeout constraint is removed if an event in P is engaged before d time units has elapsed.

$$\frac{(V, P) \xrightarrow{x} (V', P')}{(V, P \text{ timeout[d] } Q) \xrightarrow{x} (V', P')}$$

- If P engages in an invisible event τ, the timeout constraint is not removed, but the process becomes `P' timeout[d] Q`

$$\frac{(V, P) \xrightarrow{\tau} (V, P')}{(V, P \text{ timeout[d] } Q) \xrightarrow{\tau} (V', \text{P' timeout[d] } Q)}$$

- If time has not passed d units, the timeout constraint remains.

$$\frac{(V, P) \xrightarrow{t} (V, P'), t \leq d}{(V, P \text{ timeout[d] } Q) \xrightarrow{t} (V, \text{P' timeout[d - t] } Q)}$$

- If no event has occured by timeout, the process transforms to Q by silent-tau transition.

$$\frac{}{(V, P \text{ timeout[0] } Q) \xrightarrow{\tau} (V, Q)}$$

Timed Process Definition: Timed Interrupt

- Process `P interrupt[t] Q` behaves as P until t time unit elapse and then switches to Q
- For example process `(a -> b -> c -> . . . ) interrupt[t] Q` may engage in events a, b, c, ...as long as t time units has not elapsed.
- Once t time units have elapse, then the process transforms to Q by silent-tau transition.

Definition 1

$$\frac{(V, P) \xrightarrow{x} (V', P')}{(V, P \text{ interrupt[d] } Q) \xrightarrow{x} (V', P' \text{ interrupt[d] } Q)}$$

Definition 2

$$\frac{(V, P) \xrightarrow{t} (V', P'), t \leq d}{(V, P \text{ interrupt[d] } Q) \xrightarrow{t} (V', P' \text{ interrupt[d - t] } Q)}$$

Definition 3

$$\frac{}{(V, P \text{ interrupt[0] } Q) \xrightarrow{\tau} (V', Q)}$$

Timed Process Definition: Deadline

Process `P deadline[t]` is constrained to terminate within t time units.

Definition 1

$$\frac{(V, P) \xrightarrow{x} (V', P')}{(V, P \text{ deadline[d]}) \xrightarrow{x} (V', P' \text{ deadline[d]})}$$

Definition 2

$$\frac{(V, P) \xrightarrow{t} (V', P'), t \leq d}{(V, P \text{ deadline[d]}) \xrightarrow{t} (V', P' \text{ deadline[d - t]})}$$

Definition 3

$$\frac{}{(V, P \text{ deadline[0]}) \rightarrow Skip}$$

Timed Process Definition: Within

- The within operator forces the process to make an observable move within the given time frame.
- In `P within[t]` says the first event of P must engage within t time units.

Probability CSP Module: Probability Processes

- The PCSP module adds **probability processes** to existing process definitions in the CSP# module.
- **Probability processes** are a special kind of process with probabilistic characteristic defined using the keyword `pcase`.
- It is a compositional process made up of probabilistic branches.

```
P = pcase{
  [prob1] : Q1
  [prob2] : Q2
  ...
  default : Qn
}
```

- `prob1` and `prob2` are floating point probability values.
- `default = 1 - prob1 - prob2 - ...`

```
P = pcase{
  weight1 : Q1
  weight2 : Q2
  ...
  weightn : Qn
}
```

- PAT will add up and normalize the weights.
- For example, the probability of P to Q1 is $\frac{weight1}{weight1+weight2+\cdots+weightn}$

Probabilty Processes Assertion

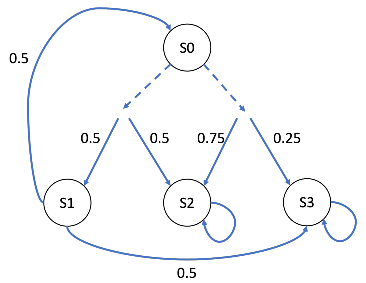
```
#assert P reaches cond with prob/pmin/pmax
```

- This assertion asks the (min/max/both) probability that the process `P()` can reach a state at which some given condition is satisifed.
- The keyword `prob` provides both the minimum and maximum probability a process `P()` can reach a certain state. It provides a range of probabilities.

PCSP Example: Simple pcase

```
var current = 0;
aSystem = State0;

State0 = pcase{
  [0.5] : e05{current = 1} -> State1
  default : e05{current = 2} -> State2
} [] pcase{
  [0.25] : e025{current = 3} -> State3
  default : e075{current = 2} -> State2
};

State1 = pcase{
  [0.5] : e05{current = 0} -> State0
  default : e05{current = 3} -> State3
}

State2 = e -> State2;
State3 = e -> State3;

#define predicate current == 2;
#assert aSystem reaches predicate with pmax; // 0.75
#assert aSystem reaches predicate with pmin; // 0.67
```

PCSP Example: Calculating Max Reachability for Simple pcase

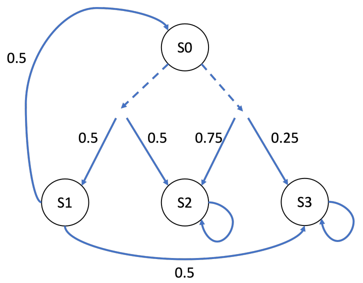


Let x_i be the max reachability from S_i to S_2:

- $x_0 = max(0.5x_1 + 0.5x_2, 0.75x_2 + 0.25x_3)$
- $x_1 = 0.5x_0 + 0.5x_3$
- $x_2 = 1$
- $x_2 = 0$
- x_0, x_1 dependent on other reachability values.
- Initially, assume $x_0, x_1 = 0$.
- When calculating each current iteration, use the previous iteration's x_0 value when calculating x_1. That is why in the table below, we start from iteration 0.
- Stop iterating only when the values are stabilized.

Iteration	x_0	x_1
0	0	0
1	0.75	0
2	0.75	0.375
3	0.75	0.375

PCSP Example: Calculating Min Reachability for Simple pcase



Let x_i be the min reachability from S_i to S_2:

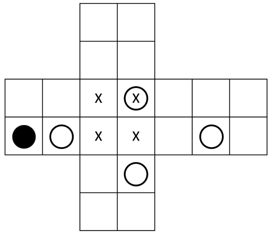
- $x_0 = min(0.5x_1 + 0.5x_2, 0.75x_2 + 0.25x_3)$
- $x_1 = 0.5x_0 + 0.5x_3$
- $x_2 = 1$
- $x_2 = 0$
- x_0, x_1 dependent on other reachability values.
- Initially, assume $x_0, x_1 = 1$.
- Use the previous iteration's x_0 value when calculating x_1.
- Stop iterating only when the values are stabilized.

Iteration	x_0	x_1
0	1	1
1	0.75	0.5
2	0.75	0.375
3	0.6875	0.375
4	0.6875	0.3438
5	0.6719	0.3438
...		
	0.6667	0.3333

Z Specification Example(s)

Z Specification Case Study: Shunting Game

- The diagram illustrates the board and the starting state for a shunting game.
- The black piece is the shunter.
- A move consists of the black piece (the shunter) moving one position either vertically or horizontally provided either:
  - The position moved to is empty, or
  - The position moved to is occupied by a white piece but the position beyond the white piece is empty, in which case the white piece is pushed into the empty position.
- The shunter cannot push two white pieces at the same time.
- At each stage, a score is kept of the number of moves made so far.
- The game ends when the white pieces occupy the four positions marked with a cross.



- $Board == (1..7 \times 3..4) \cup (3..4 \times 1..6)$
- $over == \{(3,3),(4,3),(3,4),(4,4)\}$

$$next : Board \leftrightarrow Board$$
$$\forall (i,j),(k,l) : Board \bullet (i,j)\ \underline{next}\ (k,l) \Leftrightarrow (i = k \land (j = l+1 \lor j = l-1)) \lor (j = 1 \land (i = k+1 \lor i = k-1))$$

$$beyond : Board \times Board \mathrel{+\!\!\!\rightarrow} \mathbb{N} \times \mathbb{N}$$
$$\operatorname{dom} beyond = b, w : Board \mid b\ \underline{next}\ w$$
$$\forall\, b, w : \operatorname{dom} beyond \bullet beyond(b,w) = 2w - b$$

Shunting

$$bposn : Board$$
$$wposn : \mathbb{P}\,Board$$
$$score : \mathbb{N}$$
$$bposn \notin wposn \land \#wposn = 4$$

ShuntingInit

$$bposn = (1,4)$$
$$wposn = \{(2,4),(4,3),(4,5),(6,4)\}$$
$$score = 0$$

Over

$$\Xi Shunting$$
$$score! : \mathbb{N}$$
$$wposn = over$$
$$score! = score$$

Move

$$\Delta Shunting$$
$$wposn \neq over$$
$$bposn'\ \underline{next} bposn$$
$$bposn' \notin wposn \Rightarrow wposn' = wposn$$
$$bposn' \in wposn \Rightarrow wposn' = (wposn - bposn') \cup beyond(bposn, bposn')$$
$$score' = score + 1$$

PAT CSP# Examples

Concurrency Example #1

We are given the following system specification in CSP

```
VMC = coin -> ((choc -> VMC)[](bisc -> VMC));
CHOCLOV = choc -> CHOCLOV [] coin -> choc -> CHOCLOV;
#alphabet VMC{coin, choc, bisc};
#alphabet CHOCLOV{coin, choc, bisc};
System = VMC || CHOCLOV;
```

- When coding process specifications in CSP, we use the keyword **#alphabet**, instead of the symbol α.
- The only possible trace for this example is $\langle coin, choc\rangle^n$ for $n : \mathbb{N}_1$ after which the system acts as $VMC \parallel CHOCLOV$.
  - As defined in lines 3 and 4 of the specification, the common events are the alphabets of VMC and CHOCLOV which are **coin**, **choc** and **bisc**.
  - However, the process **CHOCLOV** does not have the event **bisc**.
  - In VMC, the **coin** event has to be engaged first before we can engage $choc \rightarrow VMC$
  - Hence, both VMC and CHOCLOV engages in the **coin** event first then the **choc** event before acting as $VMC \parallel CHOCLOV$ again.

Concurrency Example #2

We are given the same specification as the previous slide in CSP except that we now do not define the alphabet for the **VMC** and **CHOCLOV** processes.

```
VMC = coin -> ((choc -> VMC)[](bisc -> VMC));
CHOCLOV = choc -> CHOCLOV [] coin -> choc -> CHOCLOV;
System = VMC || CHOCLOV;
```

- If we did not explicity define the alphabet for each process, it can be **auto inferred**.
  - $\alpha VMC = \{coin, choc, bisc\}$
  - $\alpha CHOCLOV = \{coin, choc\}$
- In this specification the system may deadlock with the following trace $\langle coin, bisc\rangle$.
  - After $\langle coin, bisc\rangle$, no event is possible.
  - This is because now the **bisc** event is not common to both **VMC** and **CHOCLOV** processes.
  - Hence, the **bisc** event can occur seperately.
  - After $\langle coin, bisc\rangle$ has occurred, the system would be stuck at $VMC \parallel choc \rightarrow CHOCLOV$.
  - Since **coin** and **choc** are common events, neither events can be engaged synchronously as **coin** is a prefix for **choc**.

Concurrency Example #3

We are given the following system specification in CSP

```
VMH = on -> coin -> choc -> off -> VMH;
CUST = on -> ((coin -> bisc -> CUST) [] (curse -> coin -> choc -> CUST));
System = VMH || CUST;
```

- The common events between **VMH** and **CUST** are **on**, **coin** and **choc** and they must occur synchronously in the two processes.
- $\langle on, curse, coin, choc, off\rangle$ is a possible trace.
  - After the trace, the process will still behave as a System process.
- Deadlocks can occur with the following trace $\langle on, coin, bisc\rangle$
  - The system will be stuck at $choc \rightarrow off \rightarrow VMH \parallel CUST$ and no event can be engaged.

Concurrency Example #4

We are given the following system specification in CSP

```
SLOWALK = left -> rest -> SLOWALK [] right -> rest -> SLOWALK;
SLOCLIMB = up -> rest -> SLOCLIMB [] down -> SLOCLIMB;
System = SLOWALK || SLOCLIMB;
```

Are the following traces possible?

1. $\langle up, rest\rangle$
   - No. The common event between **SLOWALK** and **SLOCLIMB** is **rest**.
   - $\langle up, left, rest\rangle$ and $\langle up, right, rest\rangle$ are possible traces.
2. $\langle \ldots, up, rest\rangle$ where up may not the first event.
   - Yes. For example $\langle left, up, rest\rangle$ and $\langle right, up, rest\rangle$.

CSP# Example: Patterson's Algorithm

Patterson's Algorithm is a concurrent programming algorithm for mutual exclusion that allows two or more processes to share a single-use resource without conflict, using only shared memory for communication.

```
bool flag[2] = {false, false};
int turn;
```

```
P0:      flag[0] = true;
P0_gate: turn = 1;
        while (flag[1] && turn == 1) {
          // busy wait
        }
        // critical section
        ...
        // end of critical section
        flag[0] = false;
```

```
P1:      flag[1] = true;
P1_gate: turn = 0;
        while (flag[0] && turn == 0) {
            // busy wait
        }
        // critical section
        ...
        // end of critical section
        flag[1] = false;
```

```
#define N 2;
var pos[N]; // pos[N] is the flag[N]
var step[N]; // step[1] is the !turn
var counter = 0; // how many are in CS

Process0() = Repeat0(1); cs.0{counter = counter + 1;} -> reset{pos[0] = 0; counter = counter -
     1;} -> Process0();
Repeat0(j) = [j < N] update.0.1{pos[0] = j;} -> update.0.2{step[j] = 0;} -> ([step[j] != 0 || (
     pos[1] < j)]idle.j -> Repeat0(j + 1))
  [] [j == N] Skip;

Process1() = Repeat1(1); cs.1{counter = counter + 1;} -> reset{pos[1] = 0; counter = counter -
     1;} -> Process1();
Repeat1(j) = [j < N] update.1.1{pos[1] = j;} -> update.1.2{step[j] = 1;} -> ([step[j] != 1 || (
     pos[0] < j)]idle.j -> Repeat1(j + 1))
  [] [j == N] Skip;

Peterson() = Process0() ||| Process1();
#define goal counter > 1;
#assert Peterson() reaches goal; // Should be not valid
#assert Peterson() |= <> cs.0; // Should be not valid
#assert Peterson() |= [] (update.0.1 -> <> cs.0) // Should be valid
```

CSP# Example: Alternating Bit Protocol

- We are trying to model a simple network protocol that retransmits lost messages between a Sender (A) and Receiver (B).
- A sends a bit to B and message may get lost. Using an internal timer, A should retransmit if there is no ACK when **the time is out**. A should **continue to listen** for the correct ACK if it receives a wrong one.
- B will send ACK if and only if it receives a correct bit. We assume the ACK never gets lost. B should **ignore the wrong bit** received.
- After finishing one bit, A and B should move on to send and ACK the alternating bit.

```
channel c 1; // unreliable channel.
channel d 1; // perfect channel.
channel tmr 0; // sender's internal timer.

Sender(alterbit) = (c!alterbit -> Skip [] lost -> Skip); tmr!1 -> Wait4Response(alterbit);

Wait4Response(alterbit) = (d?x -> ifa (x == alterbit) {
  tmr!0 -> Sender(1 - alterbit)
} else { Wait4Response(alterbit) }) // Time not out, wait
[] tmr?2 -> Sender(alterbit); // Time is out, retransmit

Timer = tmr?1 -> (tmr?0 -> Timer [] tmr!2 -> Timer);

Receiver(alterbit) = c?x -> ifa (x == alterbit) {
  d!alterbit -> Receiver(1 - alterbit)
} else { Receiver(alterbit) }; // Wait to receive

ABP = (Sender(0) ||| Receiver(0) ||| Timer);

#assert ABP |= [](c!0 -> <>d?0);
```

Is it always true, that when a Sender process writes 0 into the unreliable channel c, a Receiver process will read the value 0 from the perfect channel d?

- Not valid
- Counter Example: $\langle c!0, c?0, d!0, tmr.1, tmr.2, c!0, c?0, tmr.1, tmr.2, c!0, (c!0, c?0, tmr.1, tmr.2, c!0)^* \rangle$

CSP# Example: Shunting Game

Modelling the game board using a 1-D array

```
#define M 7;
#define N 6;
#define o -1;
#define a 1;
#define w 0;
  // col number: 0 1 2 3 4 5 6
var board[N][M] = [o,o,a,a,o,o,o, // 0 row number starting from 0
                   o,o,a,a,o,o,o, // 1
                   a,a,a,w,a,a,a, // 2
                   a,w,a,a,a,w,a, // 3
                   a,w,a,a,a,w,a, // 4
                   o,o,a,w,o,o,o, // 5
                   o,o,a,a,o,o,o];// 6

// Black Position
var r:{0..N-1} = 3; // row
var c:{0..M-1} = 0; // column
```

- Modelling the moves

```
Game = [r - 1 >= 0] MoveUp [] [r - 2 >= 0] PushUp
  [] [r + 1 < N] MoveDown [] [r + 2 < N] PushDown
  [] [c - 1 >= 0] MoveLeft [] [c - 2 >= 0] PushLeft
  [] [c + 1 < M] MoveRight [] [c + 2 < M] PushRight;

MoveUp = [board[r - 1][c] == a] go_up{r = r - 1} -> Game;
PushUp = [board[r - 2][c] == a && board[r - 1][c] == w] push_up{
  board[r - 2][c] = w;
  board[r - 1][c] = a;
  r = r - 1;
} -> Game
...
```

- Modelling goal and trouble state.

```
#define goal board[2][2] == w && board[2][3] == w && board[3][2] == w && board[3][3] == w;
#assert Game reaches goal;
#define trouble board[0][3] == w;
#assert Game reaches trouble;
#assert Game |= [](trouble -> ! <> goal); // trouble prevents reaching goal
```

CSP# Example: Keyless System

- One of the latest automotive technologies, a push-button keyless system, allows you to start your car's engine without the hassle of key insertion and offers great convenience.
- These systems are designed so it is possible to start the engine without the owner's key-fob and it cannot lock your key fob inside the car because the system will sense it and prevent the user from locking them in.
- However, the keyless system can also suprise you as it may allow you to drive the car without a key-fob, meaning you can drive without physically having the key.
- In this example, we will model such a Keyless System and use assertions to check if one can drive the car without having a key with them.

Constants and variables

```
#define N 2; // number of owners
#define far 0; // owner is out and far away from the car
#define near 1; // owner is close enough to open / lock the car if he has the keyfob
#define in 2; // owner is in the car
#define off 0; // engine is off
#define on 1; // engine is on
#define unlock 0; // door is unlocked but closed
#define lock 1; // door is locked (and must be closed)
#define open 2; // door is open
```

```
#define incar -1; // keyfob is put inside car
#define faralone -2; // keyfob is put outside and far

var owner[N]; // owners' position, initially all users are far away from the car
var engine = off; // engine status, intially off
var door = lock; // door status, initally locked
var key = 0; // key fob position, initially with its first owner
var moving = 0; // car moving status, 0 for stop and 1 for moving
var fuel = 10; // energy costs, say 1 for a short drive and 5 for long driving
```

- Owner positions

```
car = (||i:{0..N-1} @ (owner_pos(i) || motor(i) || door_op(i) || key_pos(i)));

owner_pos(i) = [owner[i] == far] towards.i{owner[i] = near} -> owner_pos(i)
          [] [owner[i] == near] goaway.i{owner[i] = far} -> owner_pos(i)
          [] [owner[i] == near && door == open && moving == 0]
              getin.i{owner[i] = in} -> owner_pos(i)
          [] [owner[i] == in && door == open && moving == 0]
          goout.i{owner[i] = near} -> owner_pos(i);
```

- Key-fob positions

```
key_pos(i) =
  [key == i && owner[i] == in] putincar.i{key = incar} -> key_pos(i)
[] [key == i && owner[i] == far] putaway.i{key = faralone} -> key_pos(i)
[] [(key == faralone && owner[i] == far) || (key == incar && owner[i] == in)] getkey.i{key = i}
     -> key_pos(i);
```

Door Operation

```
door_op(i) =
  [key == i && owner[i] == near && door == lock && moving == 0]
  unlockopen.i{door = open} -> door_op(i)
[] [owner[i] == near && door == unlock && moving == 0] justopen.i{door = open} -> door_op(i)
[] [door != open && owner[i] == in] insideopen.i{door = open} -> door_op(i)
[] [door == open] close.i{door = unlock} -> door_op(i)
[] [door == unlock && owner[i] == in] insidelock.i{door = lock} -> door_op(i)
[] [door == unlock && owner[i] == near && key == i] outsidelock.i{door = lock} -> door_op(i);
```

Motor

```
motor(i) =
  [owner[i] == in && (key == i || key == incar) && engine == off && fuel != 0] turnon.i{engine =
       on} -> motor(i)
[] [engine == on && owner[i] == in && moving == 0] startdrive.i{
      moving = 1;
  } -> motor(i)
[] [moving == 1 && fuel != 0] shortdrive.i{
      fuel = fuel - 1;
      if (fuel == 0) {engine = off; moving = 0;}
  } -> motor(i)
[] [moving == 1 && fuel > 5] longdrive.i{
      fuel = fuel - 5;
      if (fuel == 0) { engine = off; moving = 0; }
  } -> motor(i)
[] [engine == on && moving == 1 && owner[i] == in] stop.i{moving = 0;} -> motor(i)
[] [fuel == 0 && engine == off] refill{fuel = 10} -> motor(i)
[] [engine == on && moving == 0 && owner[i] == in] turnoff.i{
      engine = off;
  } -> motor(i);
```

Reasoning

```
#define keylockinside (key == incar && door == lock && owner[0] != in && owner[1] != in);
#define drivewithoutengineon (moving == 1 && engine == off);
#define drivewithoutkeyholdbyother (moving == 1 && owner[1] == in && owner[0] == far && key == 0)
     ;

#assert car deadlockfree;
#assert car reaches keylockinside; // False
#assert car reaches drivewithoutengineon; // False
#assert car reaches drivewithoutkeyholdbyother; // True
```

CSP# Example: Multi-lift System (using C#)

In this example, we are modelling a multiple lift system in a building with multiple floors.

```
#define NoOfFloors 2; // floor-0, floor-1
#define NoOfLifts 2; // lift-0, lift-1
#define NoOfUsers 2; // user-0, user-1

// this array models the external requests; extrequestsUP[i] = 1 denotes there is an upward floor
      request at floor-i; 0 for no request;
var extrequestsUP[NoOfFloors];
var extrequestsDOWN[NoOfFloors];
// this array models the internal requests;
// intrequests[i][j] = 1 if there is a request for floor-i for lift-j; 0 for no request;
var intrequests[NoOfLifts][NoOfFloors]; // [-1, -1]

// the level of the lift door opens at. -1 means the door is closed
var door = [-1(NoOfLifts)];
```

Data Operations to clear internal and external requests when the door of the lift-i is open at each level.

```
door[i] = level;
intrequests[i][level] = 0;
if (direction > 0) {
  extrequestsUP[level] = 0;
} else {
  extrequestsDOWN[level] = 0;
}
```

Data Operations (using a function defined in an external C# library) to check whether to continue travelling on the same direction or to change direction.

```
public static bool CheckIfToMove(int level, int direction, int i, int NoOfFloors, int[]
     intrequests, int[] extrequestsUP, int[] extrequestsDOWN) {
  int Counter = level + direction;
  while (Counter >= 0 && Counter < NoOfFloors) {
      if (extrequestsUP[Counter] != 0 || extrequestsDOWN[Counter] != 0 || intrequests[i *
          NoOfFloors + Counter] != 0) {
          return true;
      } else {
          Counter = Counter + direction;
      }
  }
  return false;
}
```

Modelling the Lift

```
Lift(i, level, direction) = ifa(intrequests[i][level] != 0 || (direction == 1 && extrequestsUP[
     level] == 1) || (direction == -1 && extrequestsDOWN[level] == 1)) {
  opendoor.i.level{ *data operations to clear request* } -> close.i.level{door[i] = -1;} -> Lift(
       i, level direction)
} else {
  checkIfToMove.i.level -> ifa(call(CheckIfToMove, level, direction, i,
  NoOfFloors, intrequests, extrequestsUP, extrequestsDOWN)) {
      moving.i.level.direction ->
      ifa (level + direction == 0 || level + direction == NoOfFloors - 1) {
          Lift(i, level + direction, -1 * direction)
      } else {
          Lift(i, level + direction, direction)
      }
  } else {
      ifa ((level == 0 && direction == 1) ||
           (level == NoOfFloors - 1 && direction == -1)) {
          Lift(i, level, direction)
      } else {
          changedir.i.level -> Lift(i, level, -1 * direction)
      }
  }
}
```

Modelling the Users

```
User() = []pos:{0..NoOfFloors - 1}@ (ExternalPush(pos); UserWaiting(pos));

// The following models the behaviours of the user pushing external buttons
ExternalPush(pos) = ifa(pos != 0) {pushdown.pos{extrequestsDOWN[pos] = 1;} -> Skip}
                  else {pushup.pos{extrequestsUP[pos] = 1;} -> Skip}
[] ifa(pos != NoOfFloors - 1) {pushup.pos{extrequestsUP[pos] = 1;} -> Skip}
  else {pushdown.pos{extrequestsDOWN[pos] = 1;} -> Skip};

// The following models the behaviours of the user waiting and entering the lift
UserWaiting(pos) = []i:{0..NoOfLifts - 1}@([door[i] == pos]enter.i ->
  ([]y:{0..NoOfFloors - 1}@push.y{intrequests[i][y] = 1;} ->
      ([door[i] == y]exit.i -> User())));
```

Questioning the System

```
// A Lift System consisting of multiple users and lifts running in parallel
LiftSystem() = (||| {NoOfUsers} @ User()) |||
  (||| x:{0..NoOfLifts - 1} @ Lift(x, 0, 1));

// If there is an external request at the first floor, it will eventually be served.
#define on extrequestsUP[0] == 1;
#define off extrequestsUP[0] == 0;
#assert LiftSystem() |= [](on -> <>off);
```

CSP# Example: 2-phase Commit Protocol

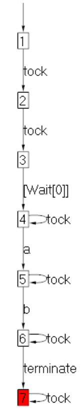
```
#define N 2; // number of participants
enum {Yes, No, Commit, Abort}; // constants
channel vote 0;
var hasNo = false;

// The following models the coordinator
Coord() = (|||{N}@ request -> Skip);
        (|||{N}@ vote?vo{if (vo == No) {hasNo = true;}} -> Skip);
        decide -> (([hasNo == false] (|||{N}@inform.Commit -> Skip);
        CoordPhaseTwo(Commit))
        [] ([hasNo == true] (|||{N}@inform.Abort -> Skip); CoordPhaseTwo(Abort)));
CoordPhaseTwo(decC) = |||{N}@acknowledge -> Skip;

// The following models a participant
Part() = request -> execute -> (vote!Yes -> PhaseTwo() [] vote!No -> PhaseTwo());
PhaseTwo() = inform.Commit -> complete -> result.Commit -> acknowledge -> Skip
         [] inform.Abort -> undo -> result.Abort -> acknowledge -> Skip;

#alphabet Coord {request, inform.Commit, inform.Abort, acknowledge};
#alphabet Part {request, inform.Commit, inform.Abort, acknowledge};
System = Coord() || (|||{N}@Part()); // Note: request is a common event between Coord and Part
     and has to be synchronised.
```

Timed CSP# Examples

Timed Process Definition: Wait (Example)

```
P = Wait[2]; (a -> b -> Skip);
```

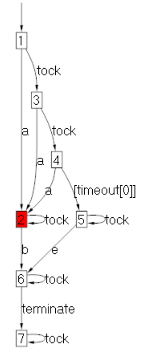
- The Wait process is active for exactly 2 time units.
- In this example, the first event **a** begins only at least after 2 time units.



Timed Process Definition: Timeout (Example)

```
P = (a -> b -> Skip) timeout[2] (e -> Skip);
```

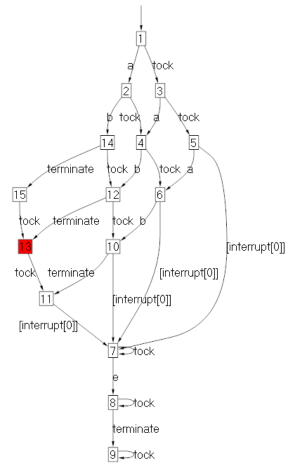
- As long as process P engages event **a** within 2 time units and before `timeout[0]` is engaged, the timeout constrained is removed and `(e -> Skip)` will never be engaged.
- `(e -> Skip)` can only be engaged after `timeout[0]` is engaged.



Timed Process Definition: Timed Interrupt (Example)

```
P = (a -> b -> Skip) interrupt[2] (e -> Skip);
```

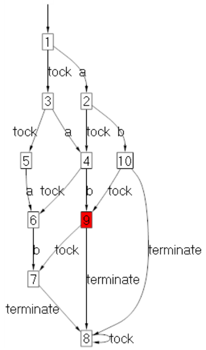
- Process P can engage in the events in `(a -> b -> Skip)` until 2 time unit has elapsed.
- After 2 time units has elapsed, Process P can only engage in events in `(e -> Skip)`.



Timed Process Definition: Deadline (Example)

```
P = (a -> b -> Skip) deadline[2];
```

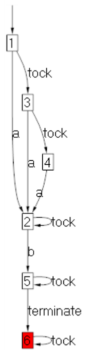
- Process P must engage in the events in `(a -> b -> Skip)` before 2 time unit has elapsed.
- After 2 time units has elapsed, Process P should terminate.



Timed Process Definition: Within (Example)

```
P = (a -> b -> Skip) within[2];
```

- In the example below, event a has to be engaged by latest $t = 2$ which is at **State 4**.



CSP# Example: Fischer's Protocol

Mutual exclusion in Fischer's Protocol is guaranteed by carefully placing bounds on the execution times of the instructions, leading to a protocol which is very simple, and relies heavily on time aspects.

```
#define N 2;
#define Delta 3;
#define Epsilon 4;
#define Idle -1;
var x = Idle;
var counter;

P(i) = ifb(x == Idle) {
  ((update.i{x = i} -> Wait[Epsilon]) within[Delta]);
  ([x == i](cs.i{counter++} -> exit.i{counter--; x=Idle} -> P(i))
  [] [x != i]P(i))
};

FischersProtocol = ||| i:{0..N-1}@P(i);
```

CSP# Example: Railway Crossing System

Control trains passing a critical point (a bridge)

```
#import "PAT.Lib.Queue";
#define N 2;
channel appr 0;
channel go 0;
channel leave 0;
channel stop 0;
var<Queue> queue;

Train(i) = appr!i -> ((stop?i -> StopS(i) within[10]) timeout[10] Cross(i));
Cross(i) = Wait[4]; (leave!i -> Train(i)) within[11];
StopS(i) = go?i -> Start(i);
Start(i) = Wait[10]; (leave!i -> Train(i)) within[10];
Gate = if (queue.Count() ==0) {
  appr?i -> atomic{tau{queue.Enqueue(i)} -> Skip}; Occ
} else {
  (go!queue.First() -> Occ) within[10]
};

Occ = (leave?[i == queue.First()]i -> atomic{tau{queue.Dequeue()} -> Skip}; Gate) [] (appr?i ->
     atomic{tau{queue.Enqueue(i)} -> stop!queue.Last() -> Skip}; Occ);
System = (||| x:{0..N-1}@Train(x)) ||| Gate;
```

CSP# Example: Light Control System

```
var dim : {0..100};
var on = false;
channel button 0;
channel dimmer 0;
channel motion 0;

TurningOn = turnOn{ on = true; dim = 100;} -> Skip;
TurningOff = turnOff{ on = false; dim = 0; } -> Skip;

ButtonPushing = button?1 -> atomic{if (dim > 0) { TurningOff } else { TurningOn }};
DimChange = dimmer?n -> atomic{setdim{dim = n} -> Skip};
ControlledLight = (ButtonPushing [] DimChange); ControlledLight;

// The motion detector
NoUser = move -> motion!1 -> User [] nomove -> Wait[1]; NoUser;
User = nomove -> motion!0 -> NoUser [] move -> Wait[1]; User;
MotionDetector = NoUser;

// The room controller
Ready = motion?1 -> button!1 -> On;
Regular = adjust -> dimmer!50 -> Regular;
On = Regular interrupt motion?0 -> OnAgain;
OnAgain = (motion?1 -> On) timeout[20] Off;
Off = button!1 ->> Ready; // Note: ->> is shortcut for atomic
Controller = Ready;

System = MotionDetector ||| ControlledLight ||| Controller;
```

Probability CSP# Examples

PCSP Example: Monty Hall

```
enum{Door1, Door2, Door3};;
var car = -1;
var guess = -1;
var goat = -1;
var final = false;

#define goal guess == car && final;

PlaceCar = []i:{Door1, Door2, Door3}@ placecar.i{car = i} -> Skip;
Goat = []i:{Door1, Door2, Door3}@
    ifb (i != car && i != guess) { hostopen.i{goat = i} -> Skip };

TakeOffer = []i:{Door1, Door2, Door3}@
    ifb (i != guess && i != goat) { changeguess{guess = i; final = true} -> Stop };
NotTakeOffer = keepguess{final = true} -> Stop;

Sys_Take_Offer = PlaceCar; Guest; Goat; TakeOffer;
#assert Sys_Take_Offer reaches goal with prob; // Max Prob = 2/3

Sys_Not_Take_Offer = PlaceCar; Guest; Goat; NotTakeOffer;
#assert Sys_Not_Take_Offer reaches goal with prob; // Max Prob = 1/3
```

PCSP Example: Monty Hall (Explanation)

- What happens if we changed **line 14** to

```
if (i != car && i != guess) { hostopen.i{goat = i} -> Skip };
```

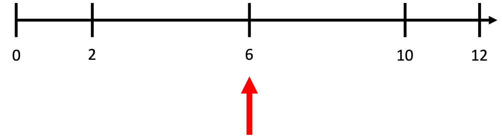
  - goat will remain as `-1`
  - In the original code, `ifb` was used. The process will wait at the `ifb` block when the condition is not true.
  - However for `if`, the process will terminate if the condition is not true.
  - Alternatively, we can change `ifb` to a guard condition.
- The code above **can deadlock**.
  - The `TakeOffer` and `NotTakeOffer` processes have a `STOP` process inside its definition.

PCSP Example: Consensus

```
#define N 2;
#define K 2;
#define range 12; // Range
#define counter_init 6; // Middle
#define left 2; // Left Target
#define right 10; // Right Target
var counter : {0..range} = counter_init; // shared coin
var Pcounter; // record the process number
var coin0counter = N; // number of coints which are 0;
var coin1counter; // number of coints which are 1;
// local variable: 0 - flip, 1 - write, 2 - check, 3 - finished
// processij means this process's coin is i and its local variable is j
process00 = pcase{ [0.5] : process01
                    default : tau {coin0counter--; coin1counter++;} -> process11 };
process01 = [counter > 0] tau {counter--;} -> process02;
process02 = [(counter <= left)] tau {Pcounter++;} -> process03
  [] [(counter >= right)] {Pcounter++; coin0counter--; coin1counter++;} -> process13
  [] [(counter > left) && (counter < right)] process00;
process03 = [Pcounter==N] done -> process03;
process13 = [Pcounter==N] done -> process13;

System = |||{N}@ process00;
```

PCSP Example: Consensus (Explanation)

- In this example, N processes come to a consensus by deciding whether the agreed value should be 0 or 1.
- In `process00`, there is a 50% chance that the process gets coin 0 in the coin flip which goes to `process01`. In the remaining 50% chance, the process gets coin 1 in the coin flip and proceeds to `process11`.
- If a process flips coin 0, the counter's value is reduced by 1.
- If a process flips coin 1, the counter's value is increased by 1.
- If the counter's value is below the left boundary of 2, then the agreed value is 0.
- If the counter's value is below the above boundary of 10, then the agreed value is 1.
- Note that in this system, all N processes are **executing concurrently** without synchronisation. Hence, it is possible that the majority decision on the coin value, may differ from the counter value.



PCSP Example: Consensus (Simplified)

```
#define N 2;
#define K 2;
#define range 12;
#define counter_init 6;
#define left 2;
#define right 6;
var counter: {0..range} = counter_init;
Var Pcounter;

process00 = pcase{
    [0.5] : tau{counter--;} -> process02
     default: tau{counter++;} -> process02
};
process02 = [(counter <= left)] tau{Pcounter++} -> process03;
            [] [(counter >= right)] tau{Pcounter++} -> process13;
            [] [(counter > left) && (counter < right)] process00;
process03 = [Pcounter == N] done -> process03;
Process13 = [Pcounter == N] done -> process13;
System = |||{N}@ process00;
```