[CS M51A F14] Solution to Quiz 1

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Quiz Problems (50 points total)

Problem 1 (10 points)

Find x such that the following equation holds. Show your work.

$$(10835)_9 = (x)_{13}$$

Solution

$$\begin{aligned}
(10835)_9 &= 5 \cdot 9^0 + 3 \cdot 9^1 + 8 \cdot 9^2 + 0 \cdot 9^3 + 9^4 \\
&= (7241)_{10} \\
&= (33B0)_{13}
\end{aligned}$$

Problem 2 (10 points)

Simplify the following expression by using postulates of Boolean Algebra.

$$a + a'b + a'b'c + a'b'c'd$$

Solution

$$\begin{aligned}
a + a'b + a'b'c + a'b'c'd &= a + b + a'b'c + a'b'c'd \\
&= b + (a + a'b'c) + a'b'c'd \\
&= b + b'c + a + a'b'c'd \\
&= b + c + (a + b'c'd) \\
&= a + (b + c + b'c'd) \\
&= a + (b + c + b'd) \\
&= a + c + (b + b'd) \\
&= a + b + c + d
\end{aligned}$$

Problem 3 (30 points)

$f(x, y)$ is a function which accepts inputs $x, y \in \{0, 1, 2\}$, and outputs the parity of $x + y$. That is,

$$f(x, y) = \begin{cases} 0, & \text{if } x + y \text{ is even} \\ 1, & \text{if } x + y \text{ is odd} \end{cases}$$

1. **(5 points)** Suppose that you use the binary code to encode x and y. Write the function f in tabular form.
Solution

x_1	x_0	y_1	y_0	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	-
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	-
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	-
1	1	0	0	-
1	1	0	1	-
1	1	1	0	-
1	1	1	1	-

2. **(5 points)** Describe f by a pair of sets chosen from: one-set, zero-set, and dc-set.
Solution
Any pair of the following:

$$[\textit{one-set}(1,4,6,9), \textit{zero-set}(0,2,5,8,10)]$$

$$[\textit{one-set}(1,4,6,9), \textit{dc-set}(3,7,11,12,13,14,15)]$$

$$[\textit{zero-set}(0,2,5,8,10), \textit{dc-set}(3,7,11,12,13,14,15)]$$

3. **(20 points)** Let $F(x,y,z)$ be a function that accepts inputs $x, y \in \{0,1,2\}$, $z \in \{0,1\}$, and outputs the parity of $x+y+z$. (a) Describe F using f as a subroutine. (b) Without using the table, describe F in m-notation. Hint: you can use the result from 2. Show your work.
Solution
(a)

$$F = f(x,y) \oplus z$$

(b) if $f(x,y) = 1$, then $F = 1$ if and only if z=0. We know from 2 that $f(x,y) = 1$ when the assignment of $x_1x_0y_1y_0$ is 0001, 0100, 0110, 1001. So $F = 1$ when the assignment of $x_1x_0y_1y_0z$ is 00010,01000,01100,10010.

if $f(x,y) = 0$, then $F = 1$ if and only if z=1. We know from 2 that $f(x,y) = 0$ when the assignment of $x_1x_0y_1y_0$ is 0000, 0010, 0101, 1000, 1010. So $F = 1$ when the assignment of $x_1x_0y_1y_0z$ is 00001,00101,01011,10001,10101. Therefore,

$$F = \sum m(2,8,12,18,1,5,11,17,21)$$