ON 2-4-2-1 CODE (TABLE 2-3) [EXERCISE 2.9]

a) IS IT UNIQUE?

b) IF NOT UNIQUE, HOW MANY DIFFERENT 2-4-2-1 CODES ARE THERE?

	2	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
	1	0	0	0
3	0	0	1	1
	1	0	0	1
4	0	1	0	0
	1	0	1	0
5	0	1	0	1
	1	0	1	1
6	0	1	1	0
	1	1	0	0
7	0	1	1	1
	1	1	0	1
8	1	1	1	0
9	1	1	1	1

$2^6 = 64$ CODES

c) ARE ALL OF THESE CODES SELF-COMPLEMENTING?

IF NO, HOW MANY SELF-COMPLEMENTING CODES ARE THERE?

COMMENTS ON NUMBER REPRESETATION CONVERSION

CONSIDER $0 \le x < 1$

$$x_2 = 0,11101 = \frac{29}{32} = 0,90625_{10}$$

FINITE # DIGITS

$$x_{10} = 0.1 = \frac{1}{10} \Rightarrow 0,0001100110011$$

REPEATS

INFINITE # BITS

$\Rightarrow$ CONVERSION INTRODUCES AN ERROR

DECIMAL ARITHMETIC WOULD BE PREFERABLE IN FINANCIAL CALCULATIONS

EARLY COMPUTERS WERE DECIMAL. BINARY COMPUTERS, BEING MORE EFFICIENT (FASTER & SIMPLER), ARE DOMINANT.

HOWEVER, DECIMAL ARITHMETIC IS MAKING A COMEBACK BECAUSE OF IMPORTANCE OF FINANCIAL COMPUTING AND MORE EFFICIENT HARDWARE. BINARY ARITHMETIC REMAINS MAINSTREAM

EFFICIENCY OF REPRESENTATION

OF BITS NEEDED TO REPRESENT

A SET OF VALUES

EXAMPLE: $0 \leq x \leq 10^6 - 1$

IN BCD: 6 DIGITS $\Rightarrow 6 \times 4 = \underline{\underline{24}}$ BITS

IN BINARY:

$2^{20} - 1 > 10^6$, $2^{19} - 1 < 10^6 - 1$

$\Rightarrow$ NEED $\underline{\underline{20}}$ BITS

BINARY REP. IS MORE EFFICIENT THAN BCD REPRESENTATION.

WHY IS THAT?

EXAMPLE: a) SPECIFY A RADIX-3 FULL ADDER
- HIGH LEVEL

b) BINARY LEVEL
- GIVE SUM OF MINTERMS FOR EACH OUTPUT

c) ANALYZE THE COST. DISCUSS.

	3^3	3^2	3^1	3^0		
x	1	0	2	2_3	$= 35_{10}$	RADIX-3
y	0	1	2	1_3	$= 16_{10}$	ADDITION EXAMPLE
	1	2	2	0_3	51_{10}	

a) INPUTS: x_i, y_i, c_i $x_i, y_i, s_i \in \{0, 1, 2\}$

OUTPUTS: s_i, c_{i+1} $c_i, c_{i+1} \in \{0, 1\}$

FUNCTION: $s_i = (x_i + y_i + c_i) \bmod 3 = w_i \bmod 3$

$c_{i+1} = (1 \text{ IF } w_i \geq 3) \text{ OR } (0 \text{ OTHERWISE})$

s.t. $x_i + y_i + c_i = 3c_{i+1} + s_i$

j	x_i	y_i	c_i	c_{i+1}	s_i
0	0	0	0	0	0
2		1		0	1
4		2		0	2
1	0	0	1	0	1
3		1		0	2
5		2		1	0
8	1	0	0	0	1
10		1		0	2
12		2		1	0
9	1	0	1	0	2
11		1		1	0
13		2		1	1
16	2	0	0	0	2
18		1		1	0
20		2		1	1
17	2	0	1	1	0
19		1		1	1
21		2		1	2

b) BINARY LEVEL

CODE FOR DIGITS

x_i	x_{i1}	x_{i0}
0	0	0
1	0	1
2	1	0

SIM. FOR y_i & s_i

NOTE THAT: $(x_{i1}, x_{i0}) = (1, 1)$ IS A d.c.

ROW INDEX j CORRESPONDS TO $(x_{i1}, x_{i0}, y_{i1}, y_{i0}, c_i)$

- BECAUSE $(x_{i1}, x_{i0}) \neq (1,1)$ AND $(y_{i1}, y_{i0}) \neq (1,1)$

OUTPUTS ARE d.c. (NOT SPECIFIED) IN 14 OUT OF 32 ROWS

THE MINTERM MAP: m_i - i^{th} MINTERM

x_{i1} x_{i0} \ y_{i1} y_{i0} c_i	000	001	010	011	100	101	110	111
0 0	m_0	m_1	m_2	m_3	m_4	m_5	—	—
0 1	m_8	m_9	m_{10}	m_{11}	m_{12}	m_{13}	—	—
1 0	m_{16}	m_{17}	m_{18}	m_{19}	m_{20}	m_{21}	—	—
1 1	—	—	—	—	—	—	—	—

THE TRUTH TABLE:

x_{i1} x_{i0} \ y_{i1} y_{i0} c_i	000	001	010	011	100	101
0 0	000	001	001	010	010	100
0 1	001	010	010	100	100	101
1 0	010	100	100	101	101	110

c_{i+1}, s_{i1}, s_{i0}

① $c_{i+1}(x_{i1}, x_{i0}, y_{i1}, y_{i0}, c_i) = \sum m(5, 11, 12, \underline{13}, 17, 18, \underline{19}, \underline{20}, \underline{21})$

② $s_{i1}($ " $) = \sum m(3, 4, 9, 10, 16, \underline{21})$ ← SHARE etc.

③ $s_{i0}($ " $) = \sum m(1, 2, 8, \underline{13}, \underline{19}, \underline{20})$

①: 9 5-INPUT AND GATES
1 9-INPUT OR GATE

4 MINTERMS SHARED

②: 6 5-INPUT AND GATES
③: 1 6-INPUT OR GATE

+ 5 NOT GATES

TOTAL:

17 AND GATES
3 OR GATES
5 NOT GATES