

- DECODERS
- ENCODERS
- MULTIPLEXERS (Selectors)
- DEMULTIPLEXERS (Distributors)
- SHIFTERS

BINARY DECODERS

HIGH-LEVEL DESCRIPTION:

INPUTS: $\underline{x} = (x_{n-1}, \dots, x_0), \quad x_j \in \{0, 1\}$

Enable $E \in \{0, 1\}$

OUTPUTS: $\underline{y} = (y_{2^n-1}, \dots, y_0), \quad y_i \in \{0, 1\}$

FUNCTION: $y_i = \begin{cases} 1 & \text{if } (x = i) \text{ and } (E = 1) \\ 0 & \text{otherwise} \end{cases}$

$$x = \sum_{j=0}^{n-1} x_j 2^j$$

and

$$i = 0, \dots, 2^n - 1$$

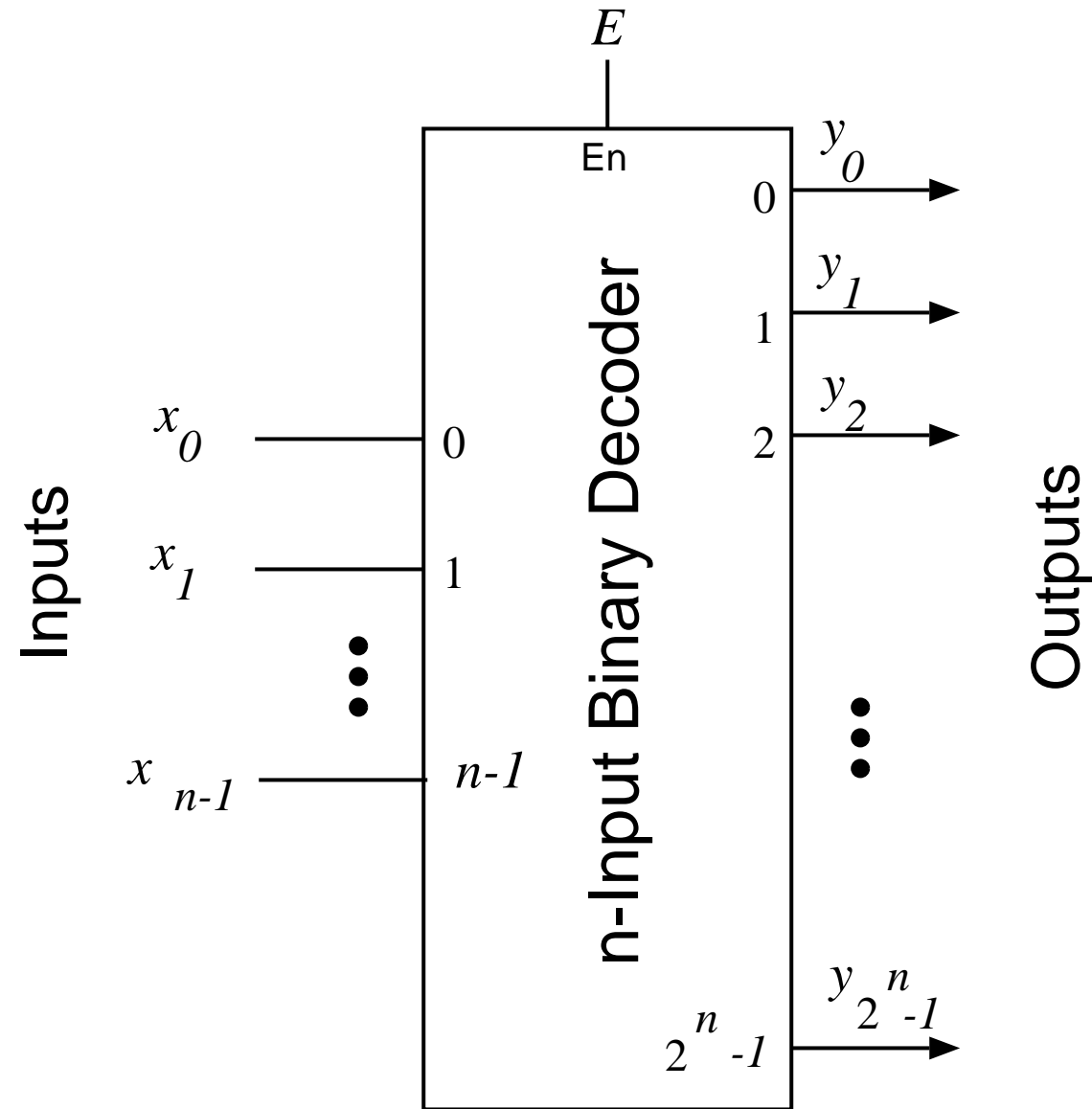


Figure 9.1: n -INPUT BINARY DECODER.

EXAMPLE 9.1: 3-INPUT BINARY DECODER

| E | x_2 | x_1 | x_0 | x | y_7 | y_6 | y_5 | y_4 | y_3 | y_2 | y_1 | y_0 |
|-----|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 3 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 4 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 5 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 6 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 7 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | - | - | - | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Example 9.1 (cont.)

BINARY SPECIFICATION:

INPUTS: $\underline{x} = (x_{n-1}, \dots, x_0), \quad x_j \in \{0, 1\}$
 $E \in \{0, 1\}$

OUTPUTS: $\underline{y} = (y_{2^n-1}, \dots, y_0), \quad y_i \in \{0, 1\}$

FUNCTION: $y_i = E \cdot m_i(\underline{x}) \quad , \quad i = 0, \dots, 2^n - 1$

EXAMPLE 9.2: IMPLEMENTATION OF 2-INPUT DECODER ⁶

$$y_0 = x_1'x_0'E \quad y_1 = x_1'x_0E \quad y_2 = x_1x_0'E \quad y_3 = x_1x_0E$$

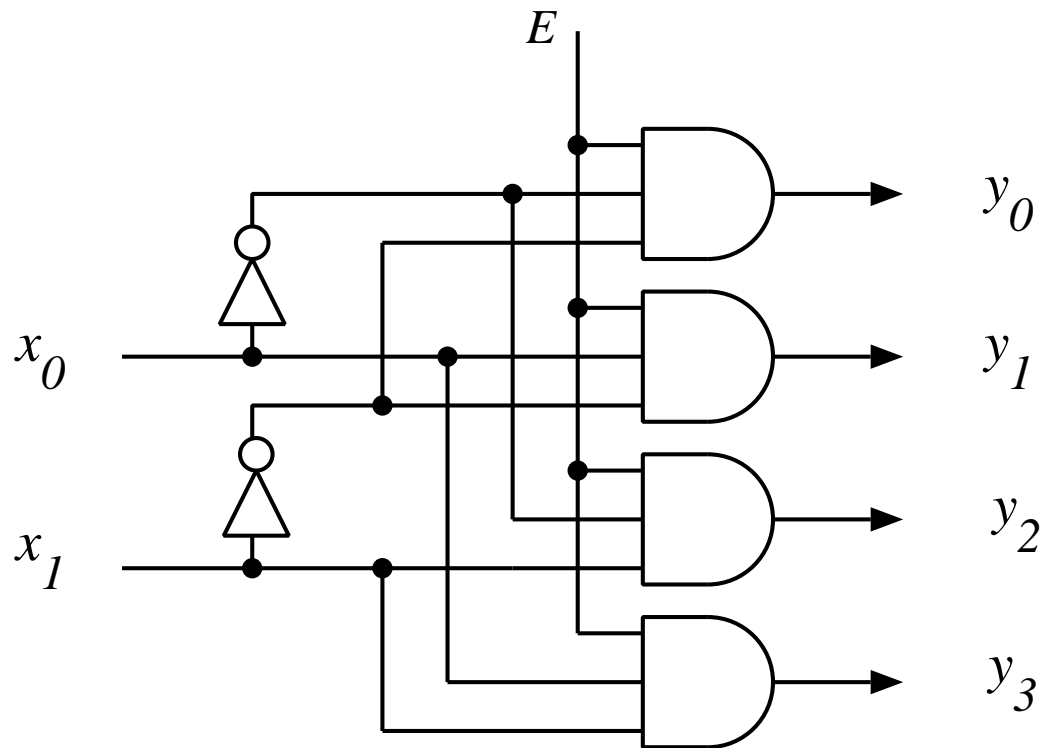


Figure 9.2: GATE NETWORK IMPLEMENTATION OF 2-INPUT BINARY DECODER.

DECODER USES

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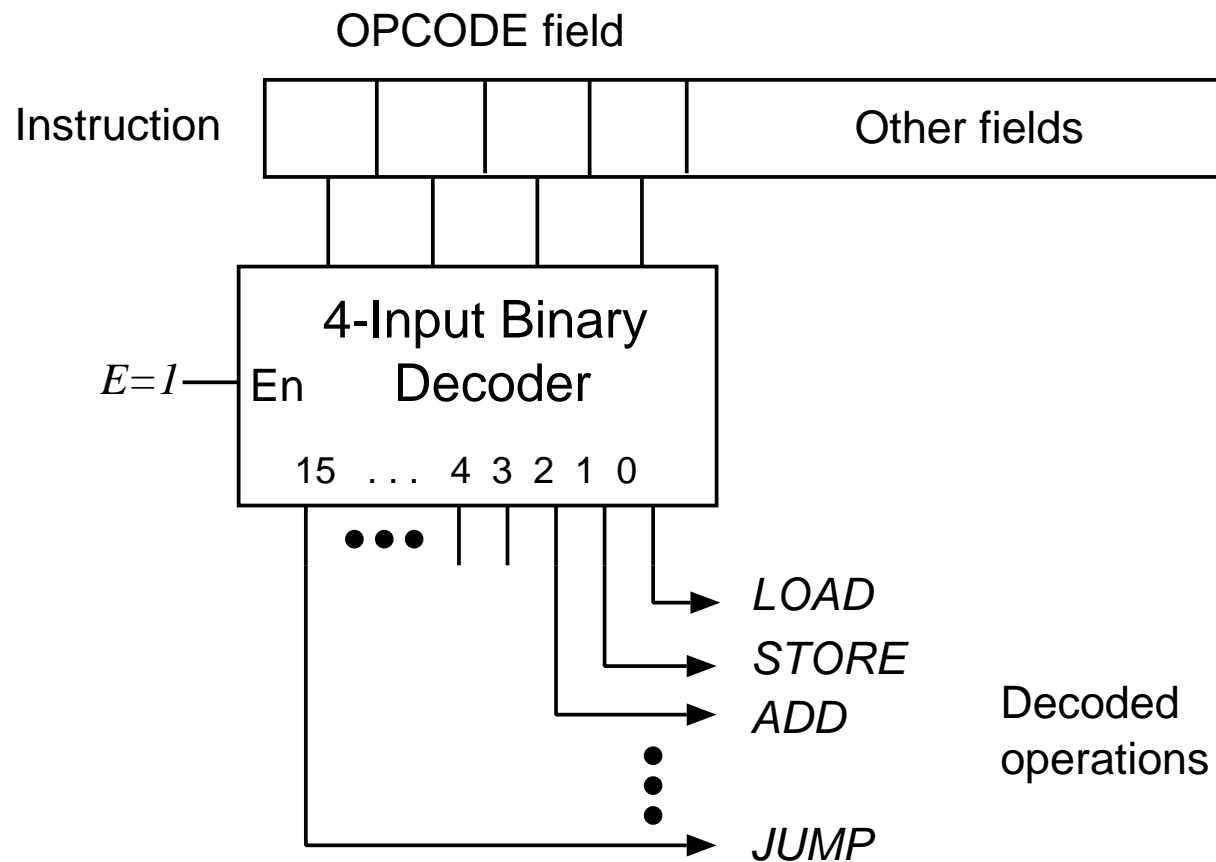


Figure 9.3: OPERATION DECODING.

DECODER USES

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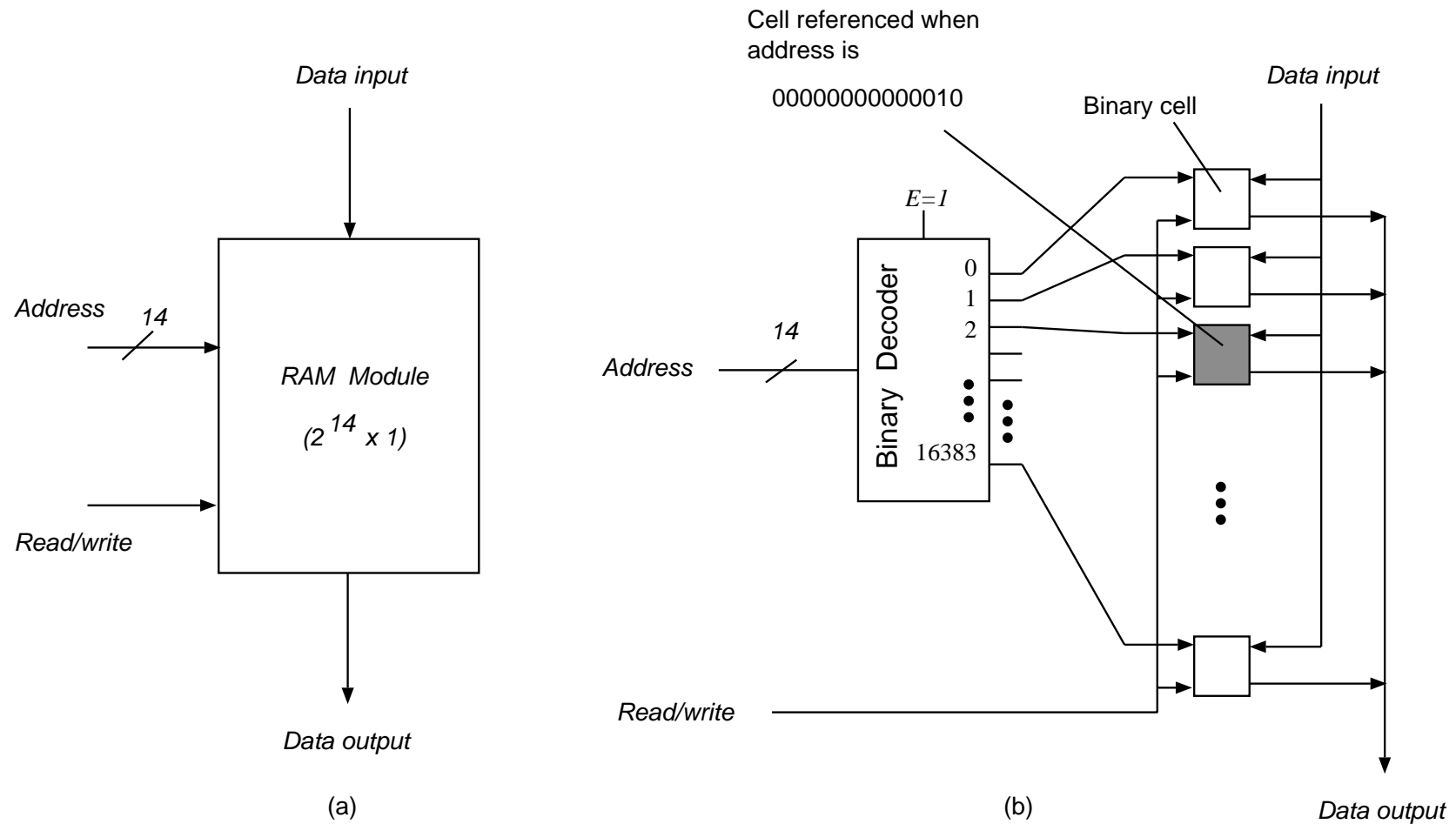


Figure 9.4: RANDOM ACCESS MEMORY (RAM): a) MODULE; b) ADDRESSING OF BINARY CELLS.

BINARY DECODER AND OR GATE

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- UNIVERSAL

EXAMPLE 9.5:

| $x_2x_1x_0$ | z_2 | z_1 | z_0 |
|-------------|-------|-------|-------|
| 000 | 0 | 1 | 0 |
| 001 | 1 | 0 | 0 |
| 010 | 0 | 0 | 1 |
| 011 | 0 | 1 | 0 |
| 100 | 0 | 0 | 1 |
| 101 | 1 | 0 | 1 |
| 110 | 0 | 0 | 0 |
| 111 | 1 | 0 | 0 |

$$(y_7, \dots, y_0) = \text{DEC}(x_2, x_1, x_0, 1)$$

$$z_2(x_2, x_1, x_0) = y_1 + y_5 + y_7$$

$$z_1(x_2, x_1, x_0) = y_0 + y_3$$

$$z_0(x_2, x_1, x_0) = y_2 + y_4 + y_5$$

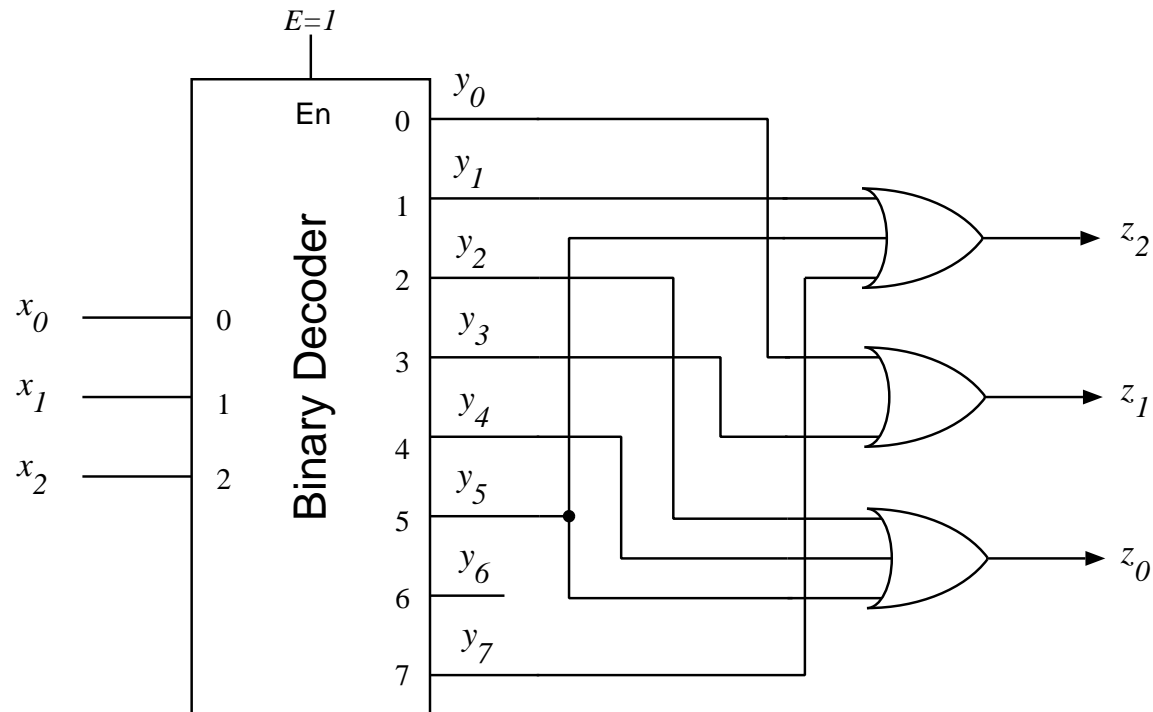


Figure 9.5: NETWORK IN EXAMPLE 9.5

$$\underline{x} = (\underline{x}_{\text{left}}, \underline{x}_{\text{right}})$$

$$\underline{x}_{\text{left}} = (x_7, x_6, x_5, x_4)$$

$$\underline{x}_{\text{right}} = (x_3, x_2, x_1, x_0)$$

$$x = 2^4 \times x_{\text{left}} + x_{\text{right}}$$

$$\underline{y} = DEC(\underline{x}_{\text{left}})$$

$$\underline{w} = DEC(\underline{x}_{\text{right}})$$

$$z_i = AND(y_s, w_t)$$

$$i = 2^4 \times s + t$$

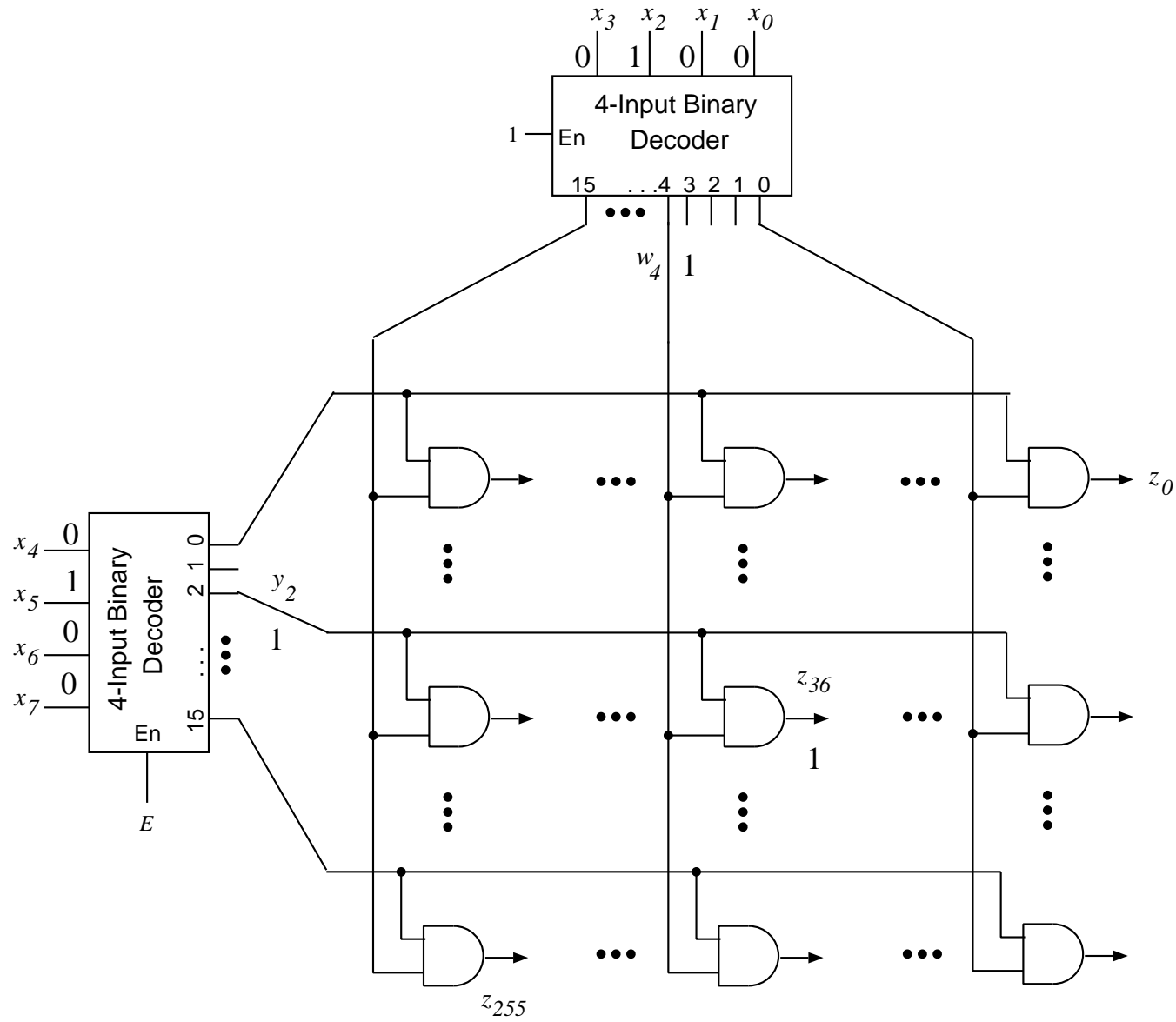


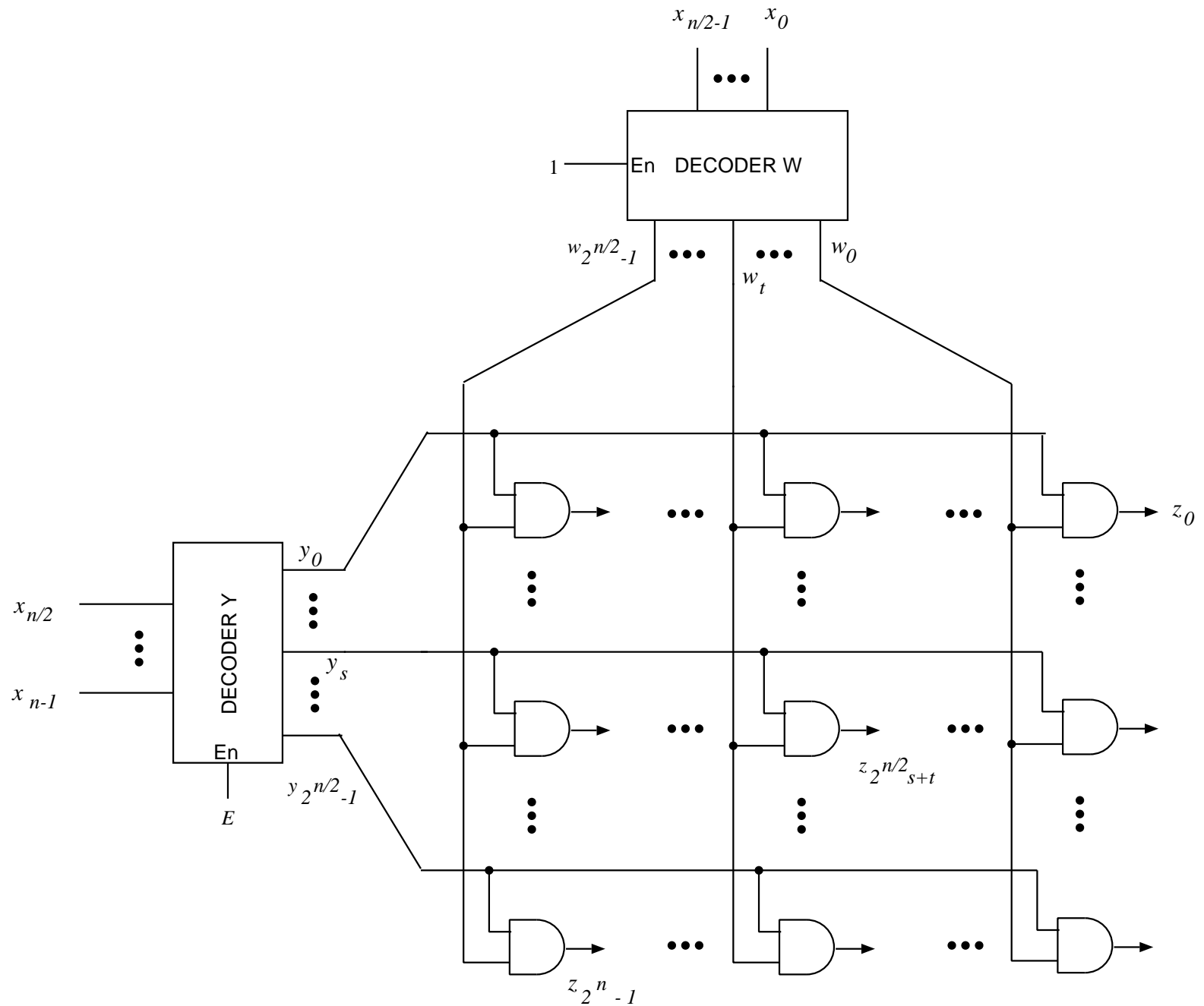
Figure 9.6: 8-INPUT COINCIDENT DECODER.

n -INPUT COINCIDENT DECODER

$$\underline{y} = \text{DEC}(\underline{x}_{\text{left}}, E)$$

$$\underline{w} = \text{DEC}(\underline{x}_{\text{right}}, 1)$$

$$\underline{z} = (\text{AND}(y_{2^{n/2}-1}, w_{2^{n/2}-1}), \dots, \text{AND}(y_s, w_t), \dots, \text{AND}(y_0, w_0))$$

Figure 9.7: n -INPUT COINCIDENT DECODER.

TREE DECODING

$$\underline{x} = (\underline{x}_{\text{left}}, \underline{x}_{\text{right}})$$

$$\underline{x}_{\text{left}} = (x_3, x_2)$$

$$\underline{x}_{\text{right}} = (x_1, x_0)$$

4-INPUT TREE DECODER

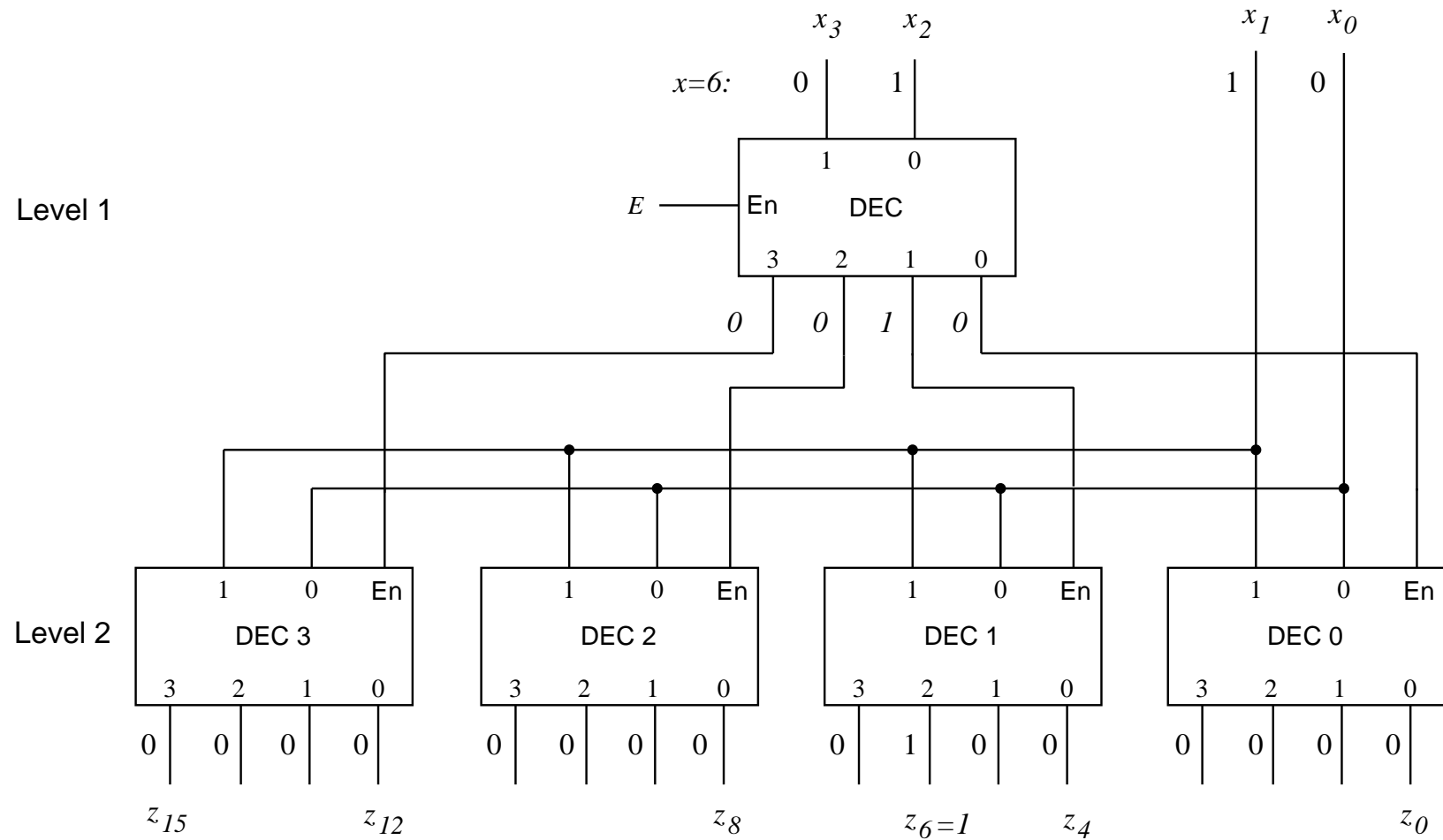
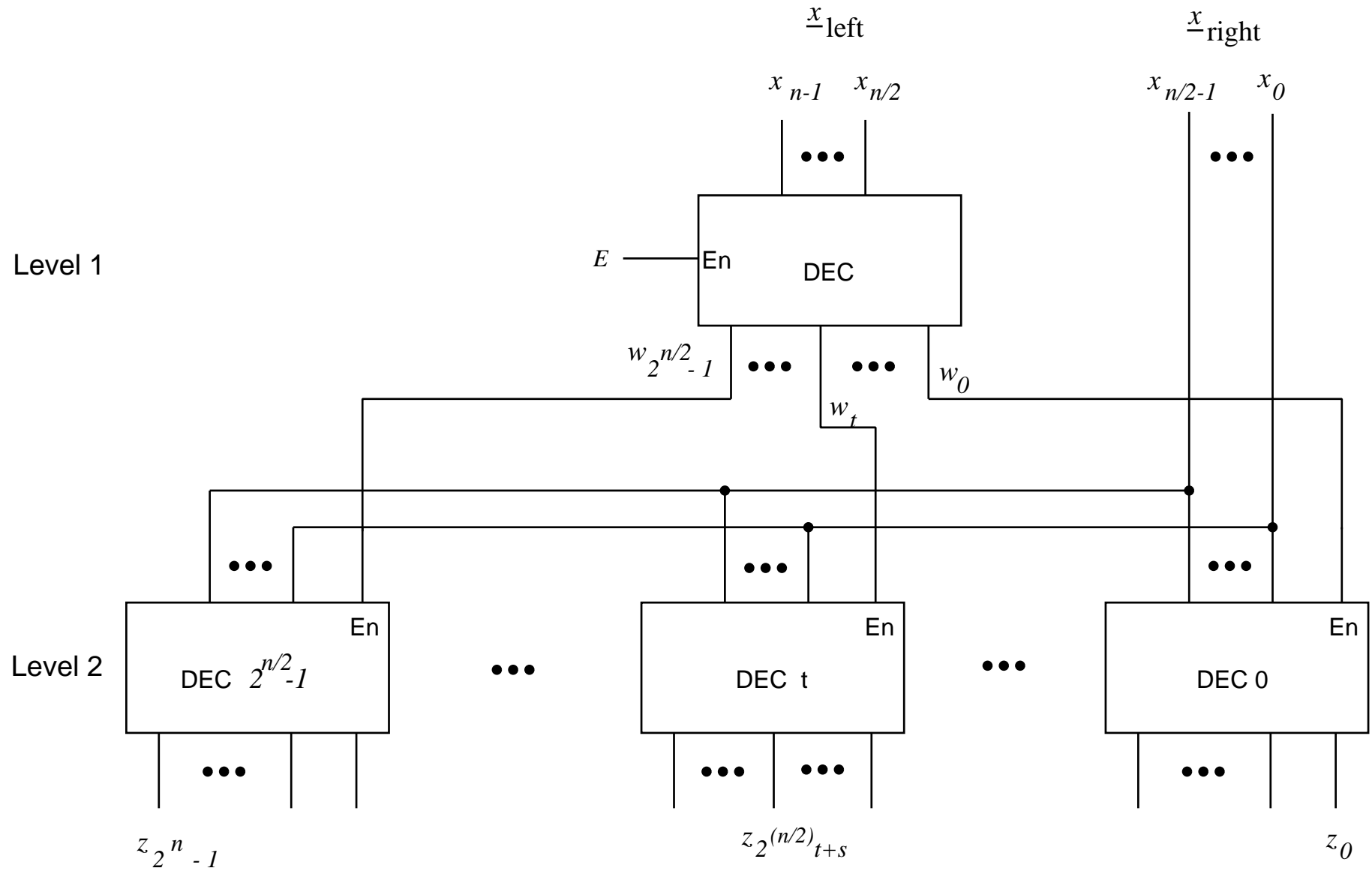


Figure 9.8: 4-INPUT TREE DECODER.

$$\underline{w} = \text{DEC}(\underline{x}_{\text{left}}, E)$$

$$\underline{z} = (\text{DEC}(\underline{x}_{\text{right}}, w_{2^{n/2}-1}), \dots, \text{DEC}(\underline{x}_{\text{right}}, w_t), \dots, \text{DEC}(\underline{x}_{\text{right}}, w_0))$$

Figure 9.9: n -INPUT TWO-LEVEL TREE DECODER.

COMPARISON OF DECODER NETWORKS

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| | Coincident | Tree |
|--|---------------------------------------|----------------------------|
| Decoder modules | 2 | $2^k + 1$ |
| AND gates | 2^{2k} | – |
| Load per network input | 1 decoder input | 2^k decoder inputs (max) |
| Fanout per decoder output | 2^k AND inputs | 1 enable input |
| Number of module inputs (related to number of connections) | $2k + 2 + 2^{2k+1}$ | $1 + k + 2^k + k2^k$ |
| Delay | $t_{\text{decoder}} + t_{\text{AND}}$ | $2t_{\text{decoder}}$ |

EXAMPLE 9.6: 6-INPUT DECODER

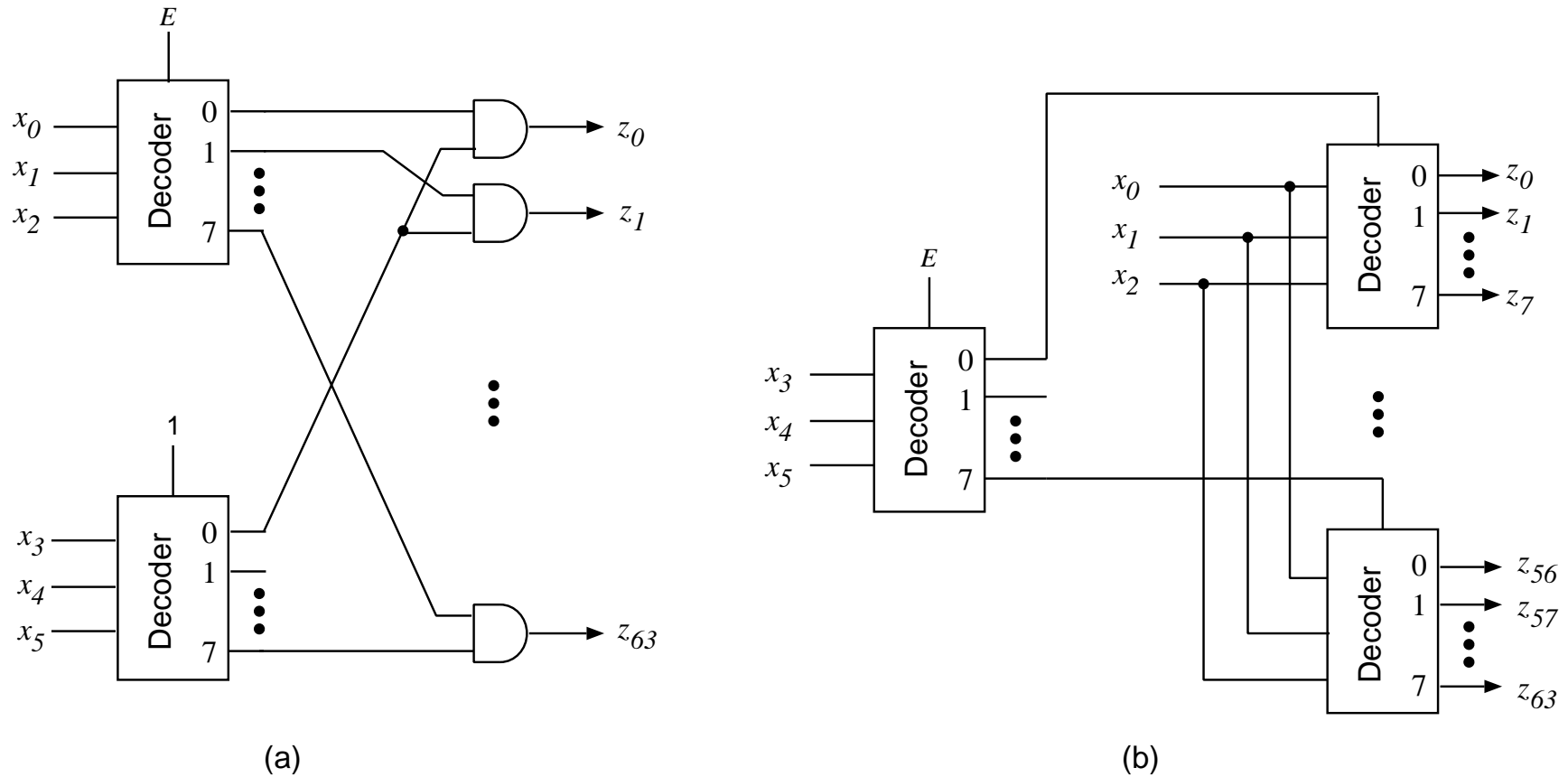


Figure 9.10: IMPLEMENTATION OF 6-INPUT DECODER. a) COINCIDENT DECODER. b) TREE DECODER.

EXAMPLE 9.6 (cont.)

| | Coincident | Tree |
|---------------------------|-----------------|------------------------|
| Decoder modules | 2 | 9 |
| AND gates | 64 | — |
| Load per network input | 1 decoder input | 8 decoder inputs (max) |
| Fanout per decoder output | 8 AND inputs | 1 enable input |
| Number of module inputs | 136 | 36 |
| Delay | $3d$ | $4d$ |

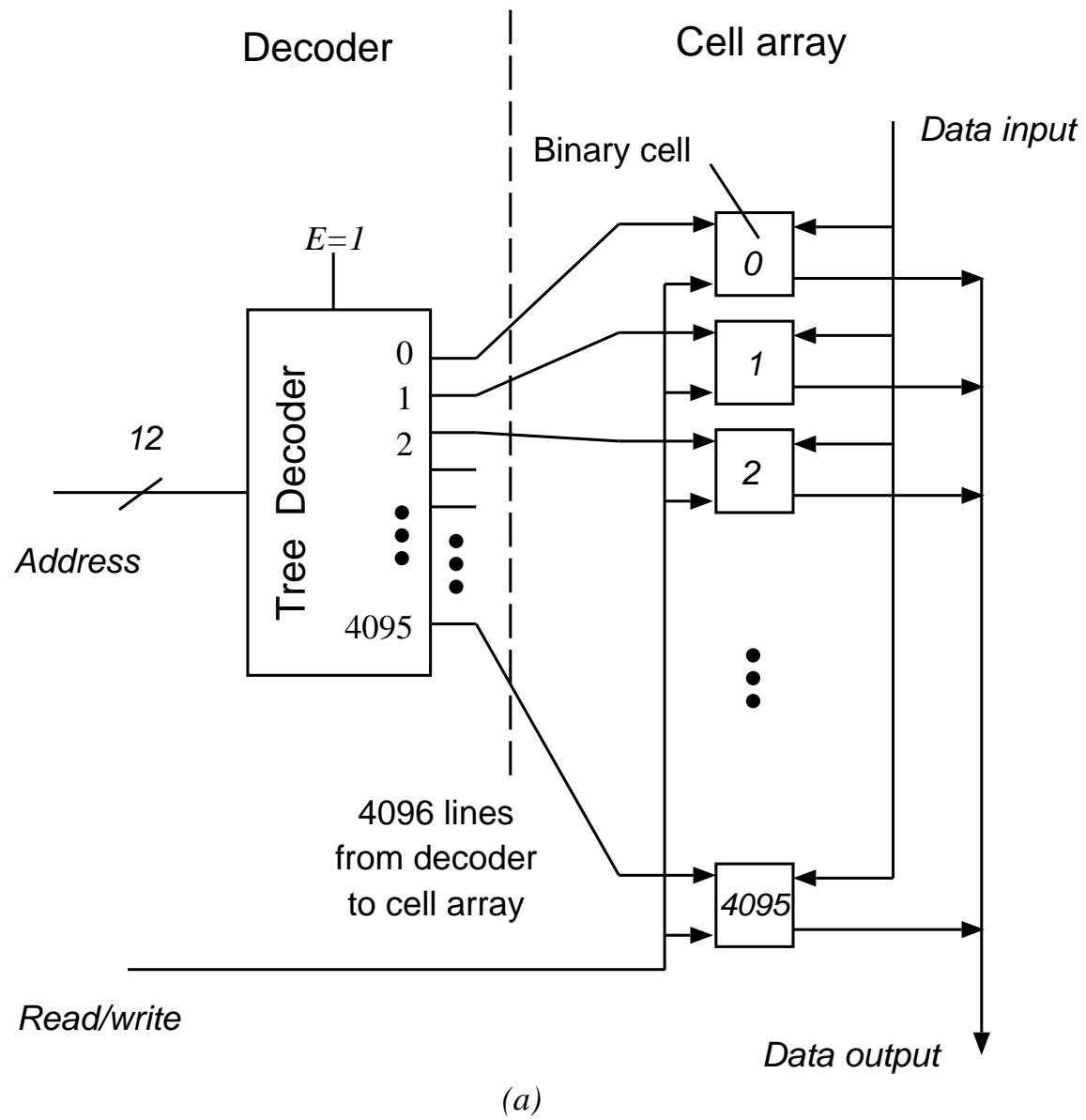
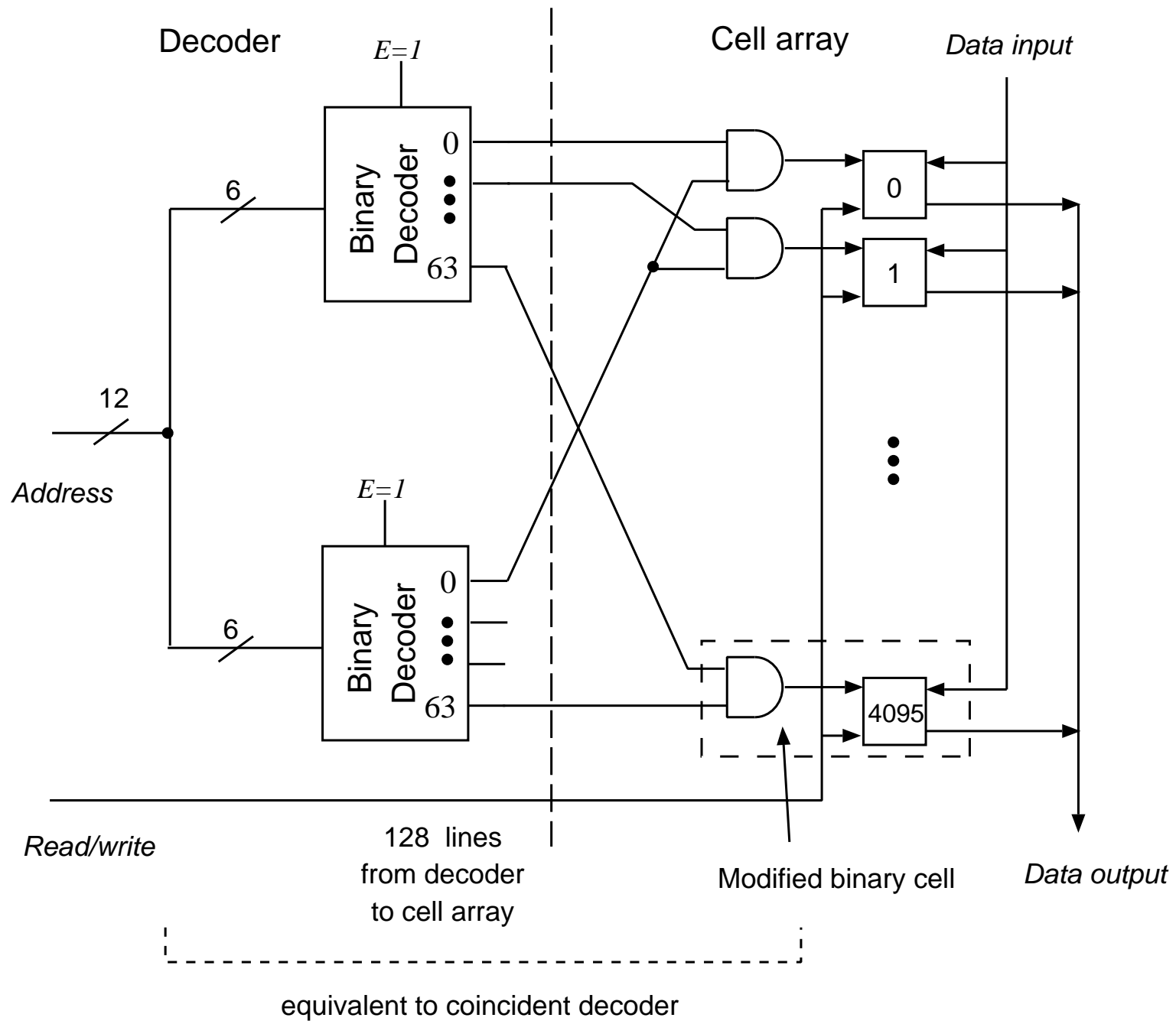


Figure 9.11: a) SYSTEM WITH TREE DECODER.



(b)

Figure 9.11: b) SYSTEM WITH COINCIDENT DECODER.

BINARY ENCODERS

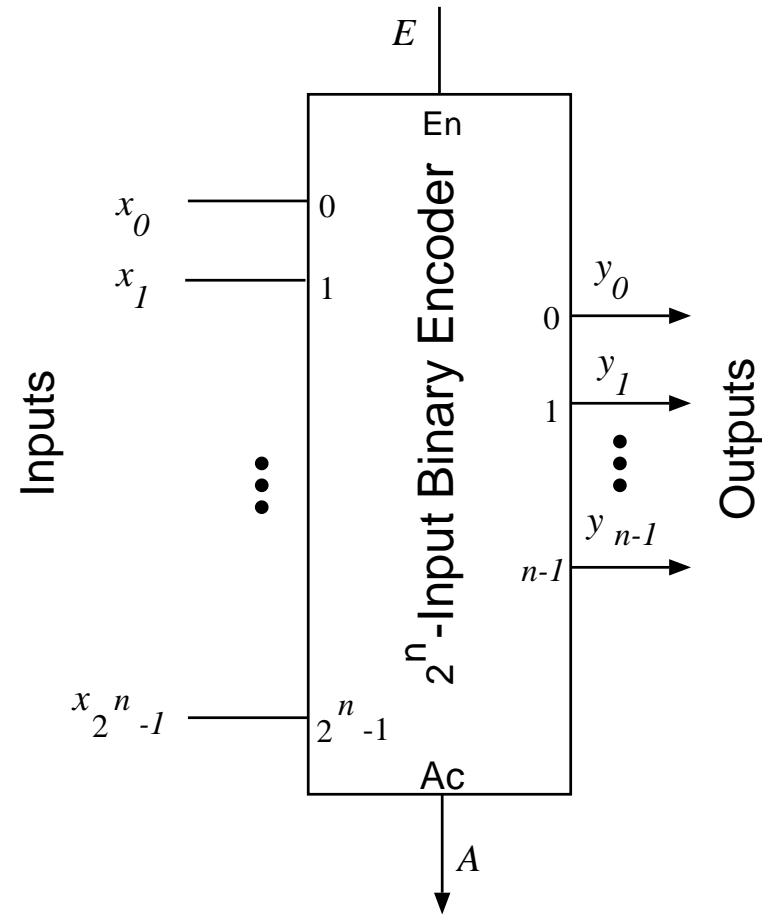


Figure 9.12: 2^n -INPUT BINARY ENCODER.

BINARY ENCODER: HIGH-LEVEL SPECIFICATION

INPUTS: $\underline{x} = (x_{2^n-1}, \dots, x_0)$, $x_i \in \{0, 1\}$, with at most one $x_i = 1$
 $E \in \{0, 1\}$

OUTPUTS: $\underline{y} = (y_{n-1}, \dots, y_0)$, $y_j \in \{0, 1\}$
 $A \in \{0, 1\}$

FUNCTION: $y = \begin{cases} i & \text{if } (x_i = 1) \text{ and } (E = 1) \\ 0 & \text{otherwise} \end{cases}$
 $A = \begin{cases} 1 & \text{if (some } x_i = 1) \text{ and } (E = 1) \\ 0 & \text{otherwise} \end{cases}$

$$y = \sum_{j=0}^{n-1} y_j 2^j, \quad i = 0, \dots, 2^n - 1$$

EXAMPLE 9.7: FUNCTION OF AN 8-INPUT BINARY ENCODER

| E | x_7 | x_6 | x_5 | x_4 | x_3 | x_2 | x_1 | x_0 | y | y_2 | y_1 | y_0 | A |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-----|-------|-------|-------|-----|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 3 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 4 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 5 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | - | - | - | - | - | - | - | - | 0 | 0 | 0 | 0 | 0 |

BINARY SPECIFICATION OF ENCODER

INPUTS: $\underline{x} = (x_{2^n-1}, \dots, x_0), \quad x_i \in \{0, 1\}$
 , with at most one $x_i = 1$

$$E \in \{0, 1\}$$

OUTPUTS: $\underline{y} = (y_{n-1}, \dots, y_0), \quad y_j \in \{0, 1\}$

$$A \in \{0, 1\}$$

FUNCTION: $y_j = E \cdot \Sigma(x_k), \quad j = 0, \dots, n - 1$

$$A = E \cdot \Sigma(x_i), \quad i = 0, \dots, 2^n - 1$$

EXAMPLE 9.8

$$y_0 = E \cdot (x_1 + x_3 + x_5 + x_7)$$

$$y_1 = E \cdot (x_2 + x_3 + x_6 + x_7)$$

$$y_2 = E \cdot (x_4 + x_5 + x_6 + x_7)$$

$$A = E \cdot (x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7)$$

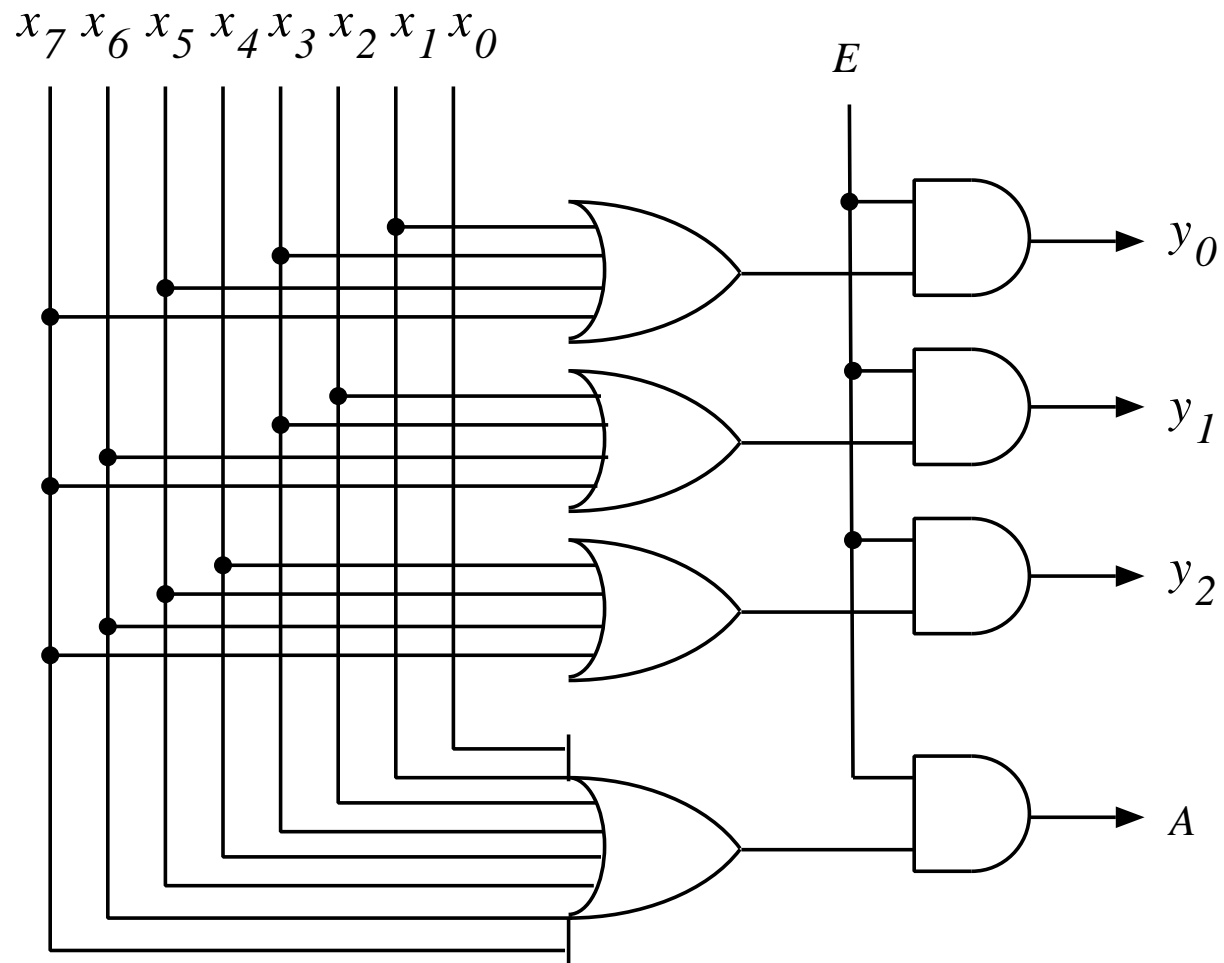


Figure 9.13: IMPLEMENTATION OF AN 8-INPUT BINARY ENCODER.

USES OF BINARY ENCODERS

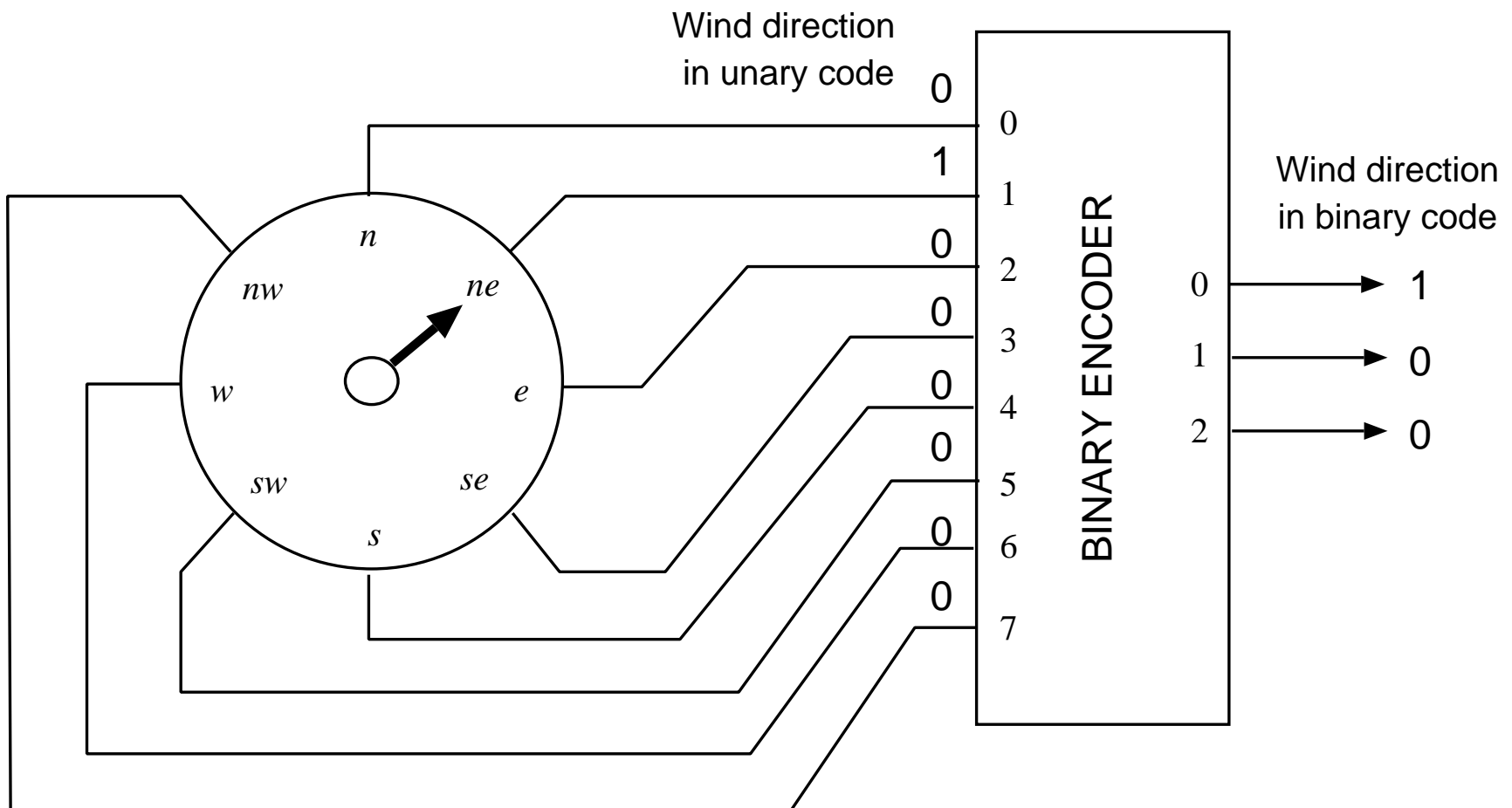


Figure 9.14: WIND DIRECTION ENCODER.

PRIORITY ENCODERS: HIGH-LEVEL DESCRIPTION

INPUTS: $\underline{x} = (x_{2^n-1}, \dots, x_0), \quad x_i \in \{0, 1\}$

OUTPUTS: $\underline{y} = (y_{n-1}, \dots, y_0), \quad y_j \in \{0, 1\}$

FUNCTION:

$$y_i = \begin{cases} i & \text{if } (x_i = 1) \text{ and } (x_k = 0, k > i) \text{ and } (E = 1) \\ 0 & \text{otherwise} \end{cases}$$

$$A = \begin{cases} 1 & \text{if (some } x_i = 1) \text{ and } (E = 1) \\ 0 & \text{otherwise} \end{cases}$$

$$y = \sum_{j=0}^{n-1} y_j 2^j, \quad i, k \in \{0, 1, \dots, 2^n - 1\}$$

8-INPUT PRIORITY ENCODER

| E | x_7 | x_6 | x_5 | x_4 | x_3 | x_2 | x_1 | x_0 | y_2 | y_1 | y_0 | A |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | - | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | - | - | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | - | - | - | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | - | - | - | - | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | - | - | - | - | - | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | - | - | - | - | - | - | 1 | 1 | 0 | 1 |
| 1 | 1 | - | - | - | - | - | - | - | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | - | - | - | - | - | - | - | - | 0 | 0 | 0 | 0 |

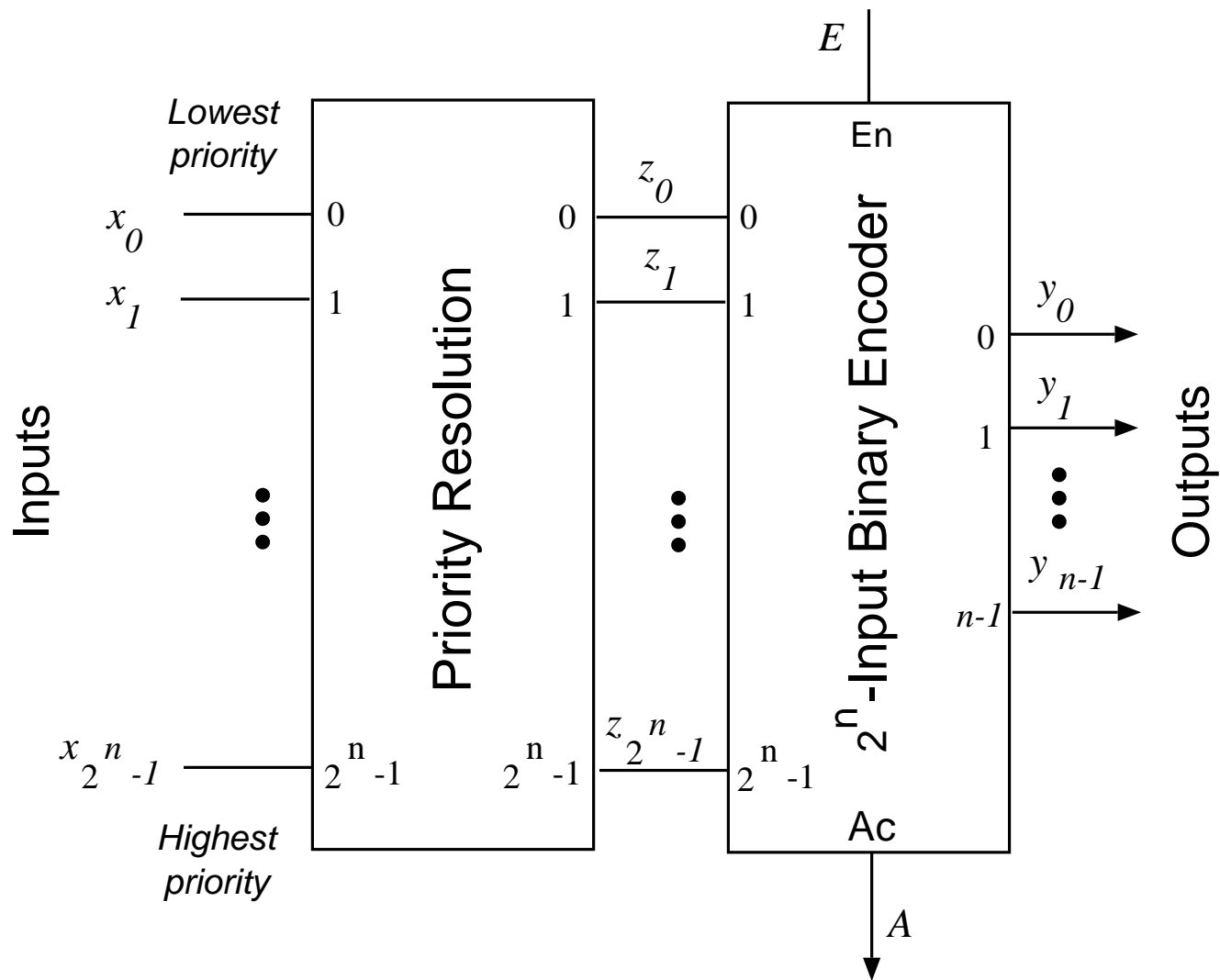


Figure 9.15: PRIORITY ENCODER.

PRIORITY RESOLUTION: HIGH-LEVEL AND BINARY-LEVEL DESCRIPTION ³⁴

Inputs: $\underline{x} = (x_{2^n-1}, \dots, x_0), \quad x_i \in \{0, 1\}$

Outputs: $\underline{z} = (z_{2^n-1}, \dots, z_0), \quad z_i \in \{0, 1\}$

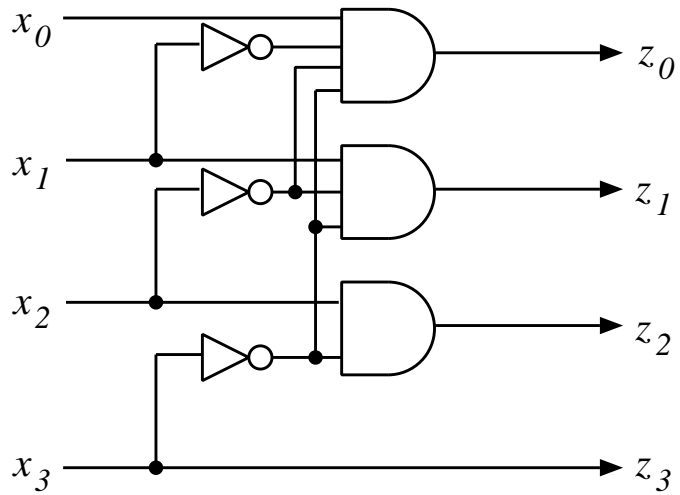
Function: $z_i = \begin{cases} 1 & \text{if } (x_i = 1) \text{ and } (x_k = 0, \quad k > i) \\ 0 & \text{otherwise} \end{cases}$
with $i, k = 0, 1, \dots, 2^n - 1$

- BINARY DESCRIPTION:

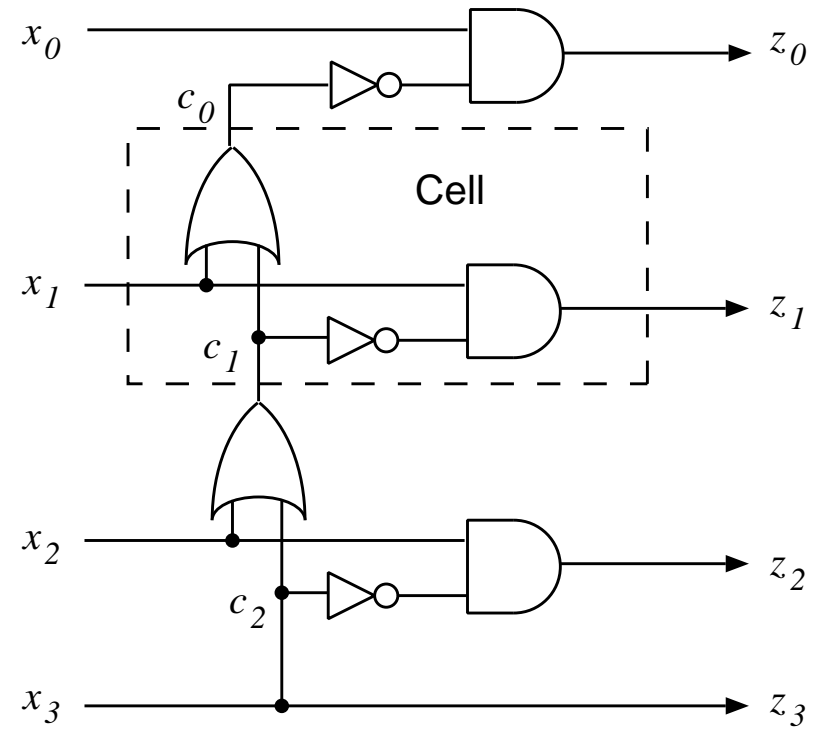
$$z_i = x'_{2^n-1} x'_{2^n-2} \dots x'_{i+1} x_i, \quad i = 0, 1, \dots, 2^n - 1$$

OR ITERATIVELY

$$\begin{aligned} c_{i-1} &= c_i \oplus x_i \\ z_i &= c'_i x_i \end{aligned}$$



(a)



(b)

Figure 9.16: 4-BIT PRIORITY RESOLUTION NETWORKS: a) PARALLEL; b) ITERATIVE.

USES OF PRIORITY ENCODERS

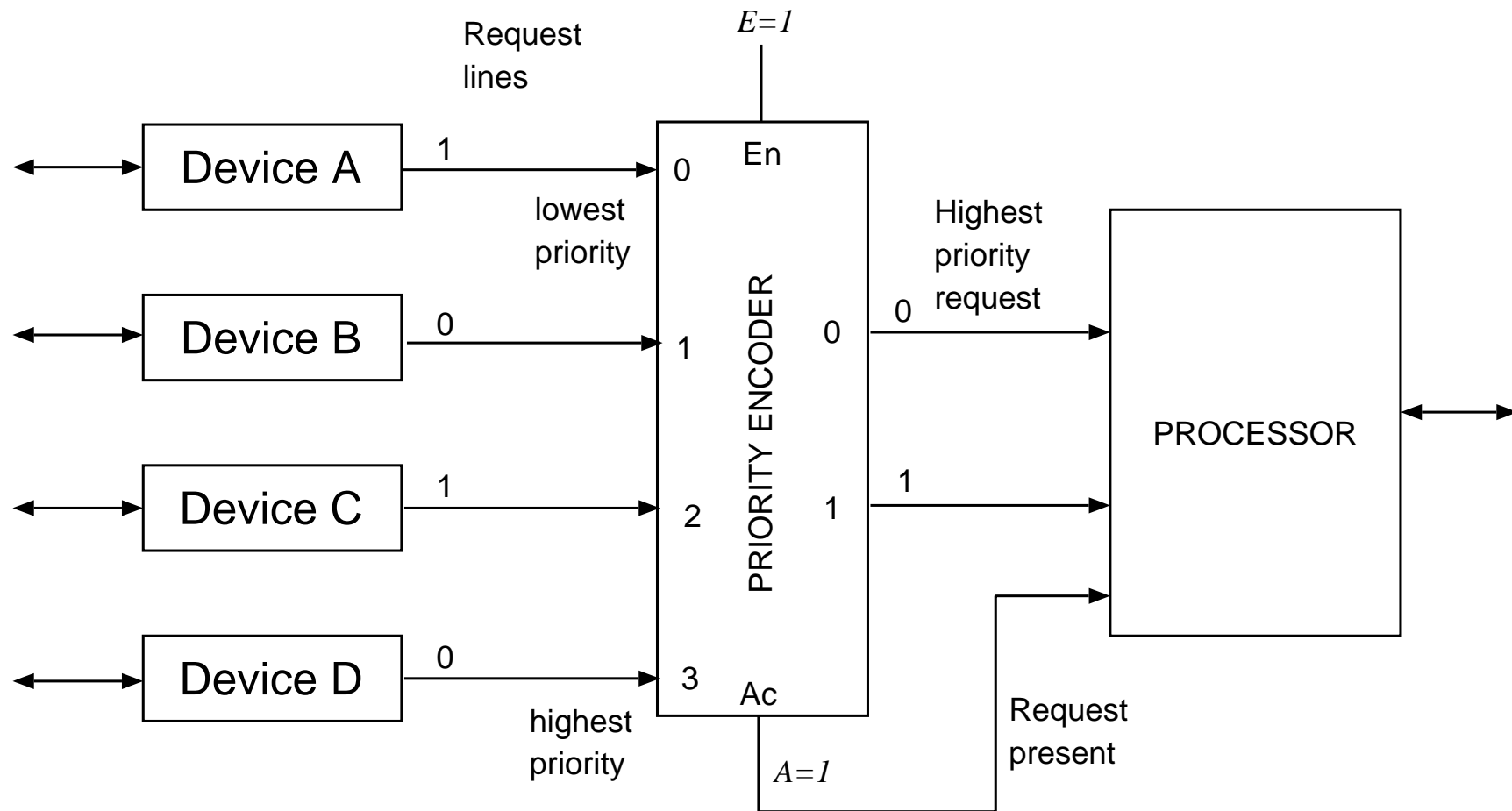


Figure 9.17: RESOLVING INTERRUPT REQUESTS USING A PRIORITY ENCODER.

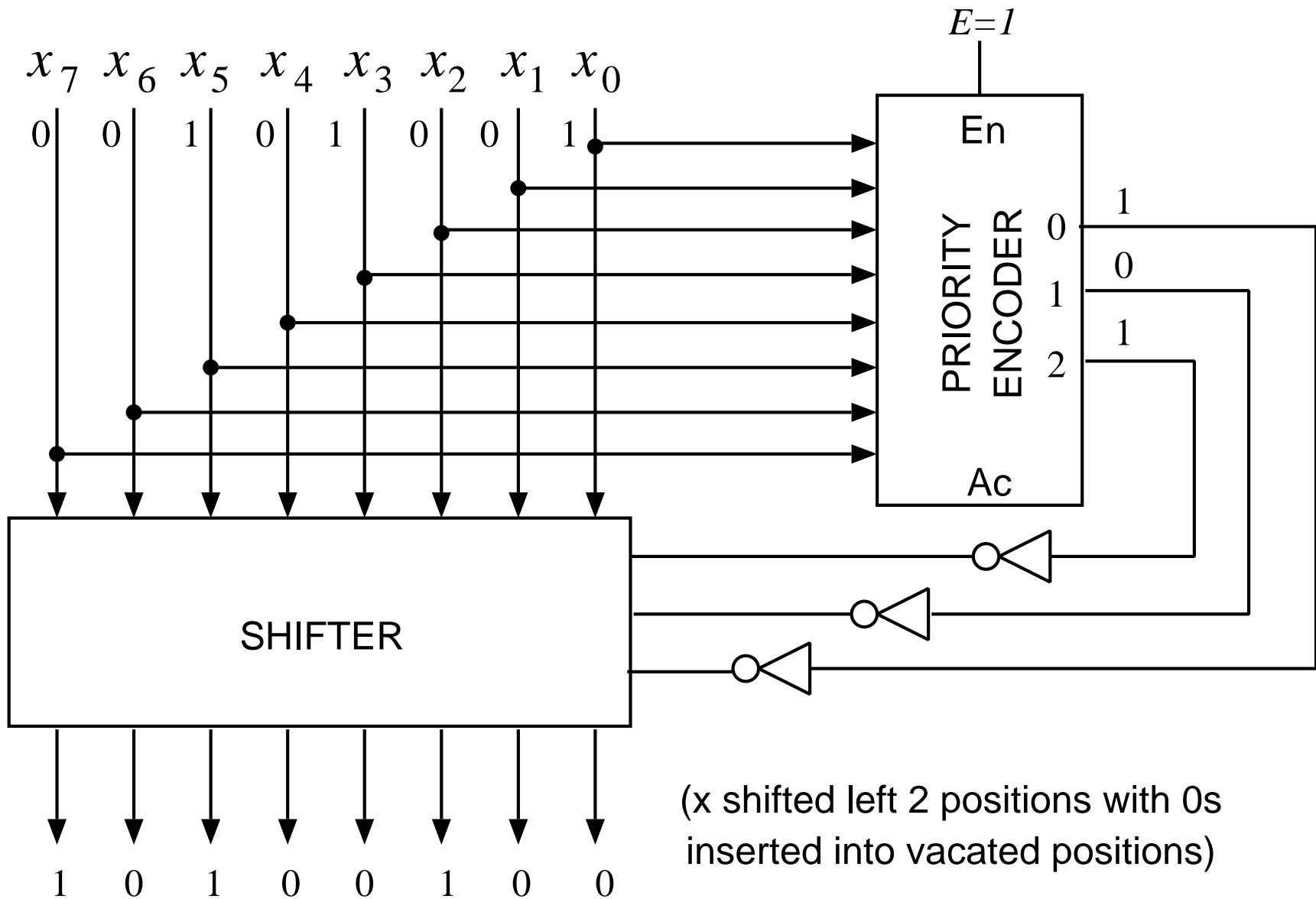


Figure 9.18: DETECTING THE LEFTMOST 1 IN A BIT-VECTOR AND REMOVING LEADING ZEROES.

MULTIPLEXERS (selectors)

- HIGH-LEVEL AND BINARY-LEVEL DESCRIPTION

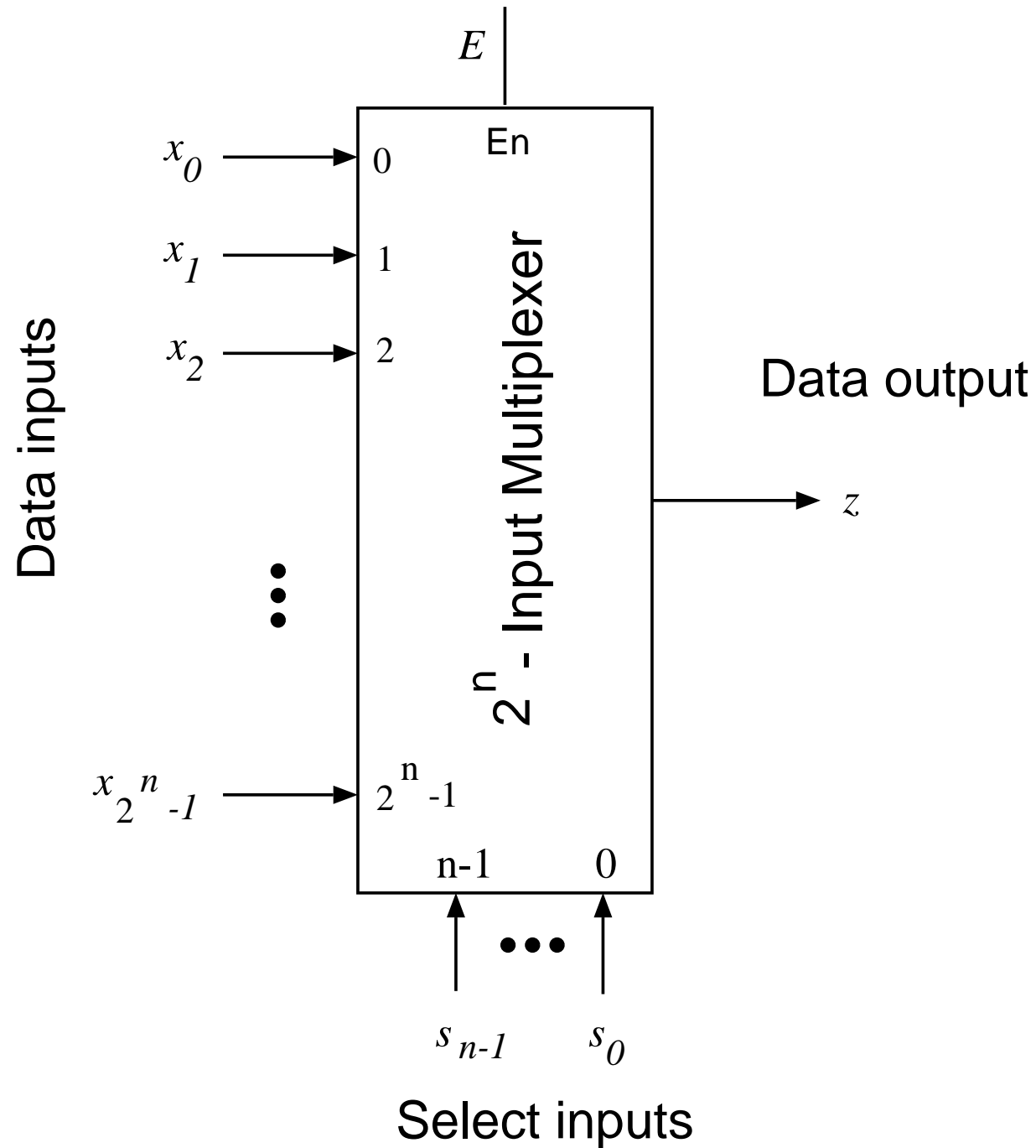
INPUTS: $\underline{x} = (x_{2^n-1}, \dots, x_0), \quad x_i \in \{0, 1\}$
 $\underline{s} = (s_{n-1}, \dots, s_0), \quad s_j \in \{0, 1\}$
 $E \in \{0, 1\}$

OUTPUTS: $z \in \{0, 1\}$

FUNCTION: $z = \begin{cases} x_s & \text{if } E = 1 \\ 0 & \text{if } E = 0 \end{cases}$

$$s = \sum_{j=0}^{n-1} s_j 2^j$$

$$z = E \cdot \left[\sum_{i=0}^{2^n-1} x_i \cdot m_i(\underline{s}) \right]$$



EXAMPLE 9.11: 4-INPUT MULTIPLEXER

| E | s_1 | s_0 | z |
|-----|-------|-------|-------|
| 1 | 0 | 0 | x_0 |
| 1 | 0 | 1 | x_1 |
| 1 | 1 | 0 | x_2 |
| 1 | 1 | 1 | x_3 |
| 0 | - | - | 0 |

$$\begin{aligned}
 z &= E \cdot (x_0 m_0(s_1, s_0) + x_1 m_1(s_1, s_0) + x_2 m_2(s_1, s_0) + x_3 m_3(s_1, s_0)) \\
 &= E \cdot (x_0 s_1' s_0' + x_1 s_1' s_0 + x_2 s_1 s_0' + x_3 s_1 s_0)
 \end{aligned}$$

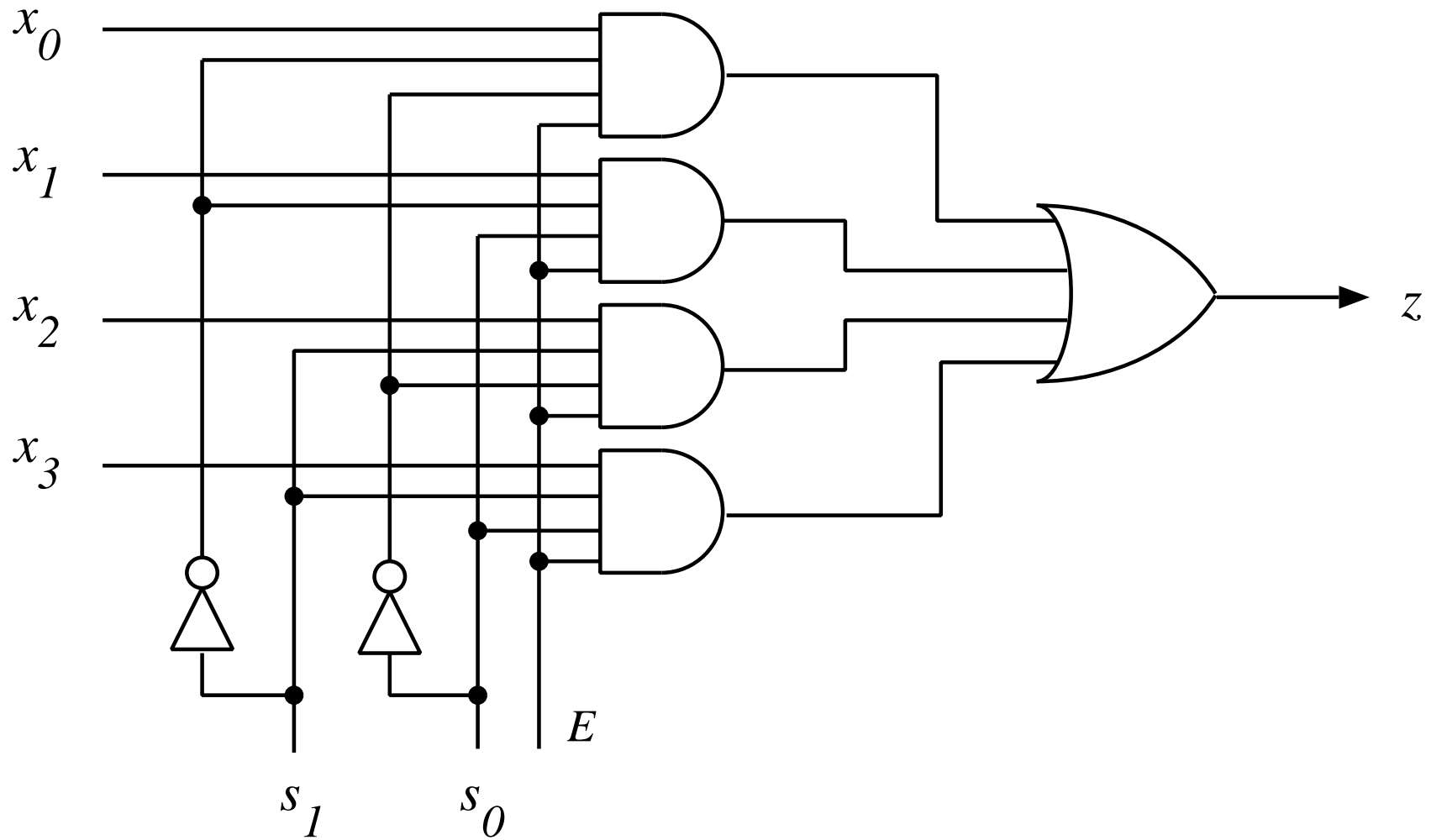


Figure 9.20: GATE IMPLEMENTATION OF 4-INPUT MULTIPLEXER

TYPICAL USES

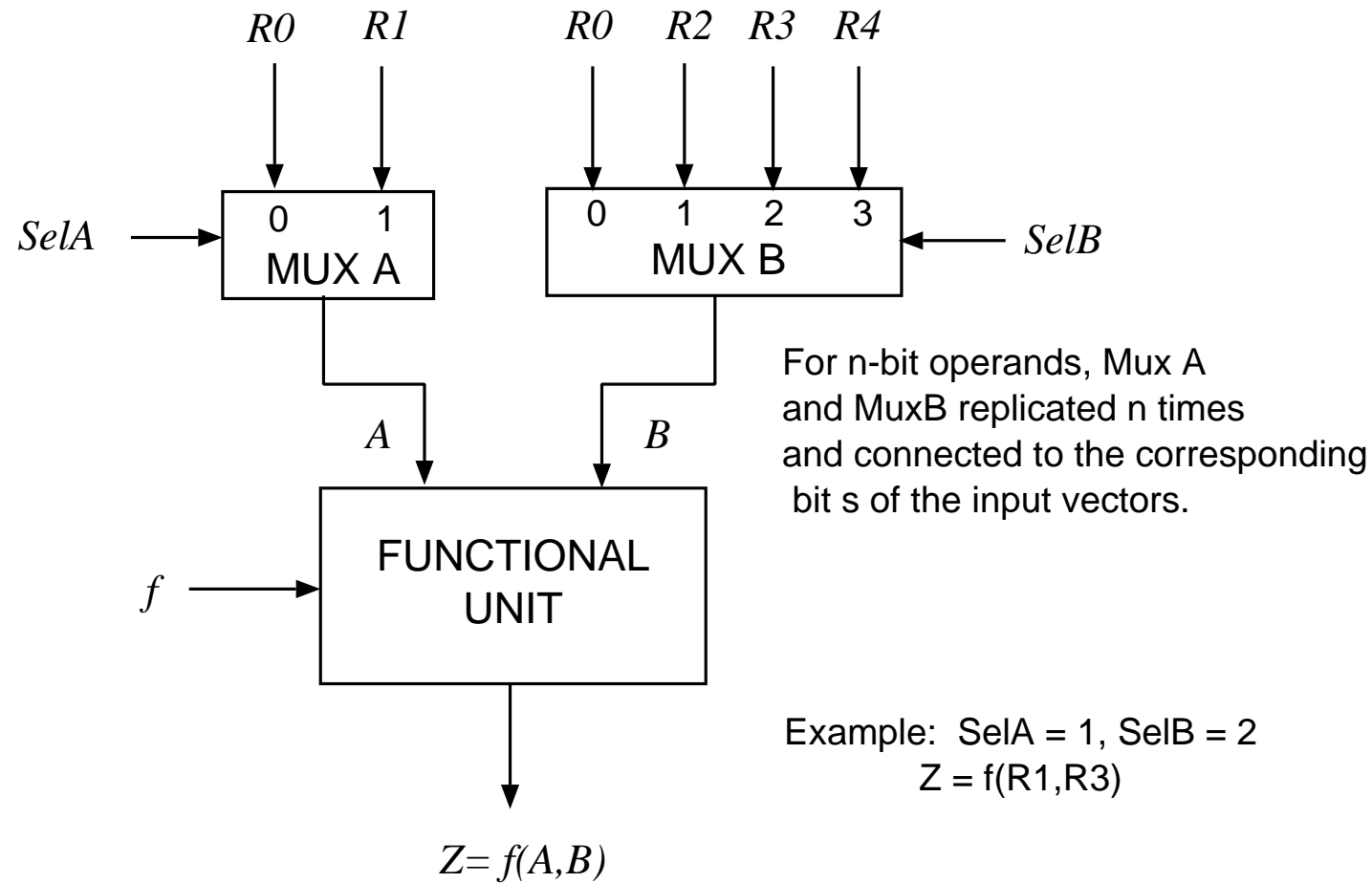


Figure 9.21: MULTIPLEXER: EXAMPLE OF USE.

MULTIPLEXER AS UNIVERSAL COMBINATIONAL MODULE⁴³

- connect input variables \underline{x} to select inputs of multiplexer \underline{s}
- set data inputs to multiplexer equal to values of function for corresponding assignment of select variables
- using a variable at data inputs reduces size of the multiplexer

EXAMPLE

$$\begin{aligned}
 E(x_2, x_1, x_0) &= \Sigma m(1, 2, 4, 6, 7) \\
 &= x'_2(x'_1x_0) + x'_2(x_1x'_0) + x_2(x'_1x'_0) + x_2(x_1x'_0) + x_2(x_1x_0) \\
 &= x'_2m_1(x_1, x_0) + x'_2m_2(x_1, x_0) \\
 &\quad + x_2m_0(x_1, x_0) + x_2m_2(x_1, x_0) + x_2m_3(x_1, x_0) \\
 &= x_2m_0(x_1, x_0) + x'_2m_1(x_1, x_0) \\
 &\quad + 1 \cdot m_2(x_1, x_0) + x_2m_3(x_1, x_0)
 \end{aligned}$$

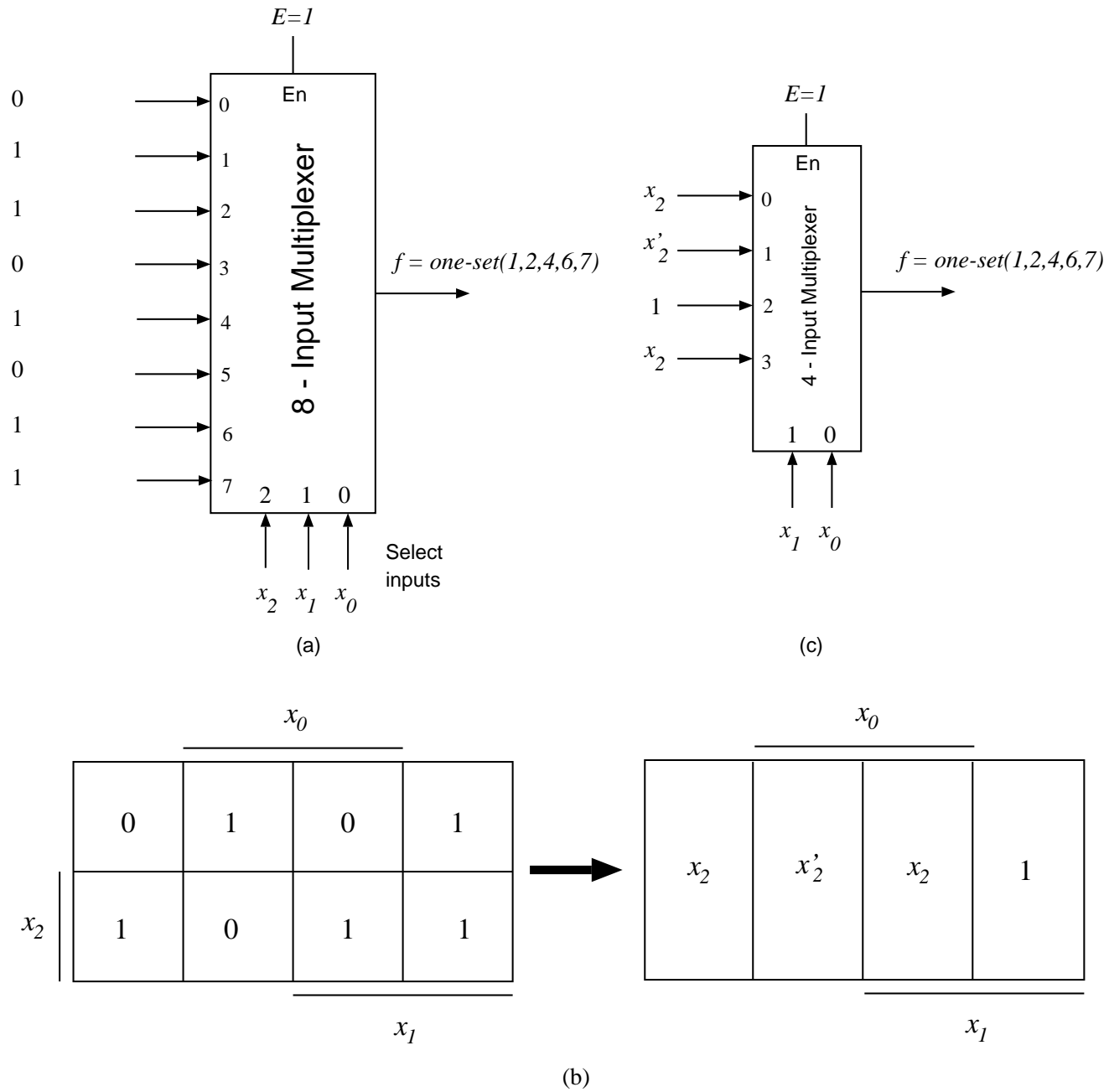


Figure 9.22: IMPLEMENTATION OF $f(x_2, x_1, x_0) = \text{one-set}(1,2,4,6,7)$: a) 8-INPUT MULTIPLEXER; b) K-map; c) 4-INPUT MULTIPLEXER.

EXAMPLE 9.12: ONE-BIT ADDER

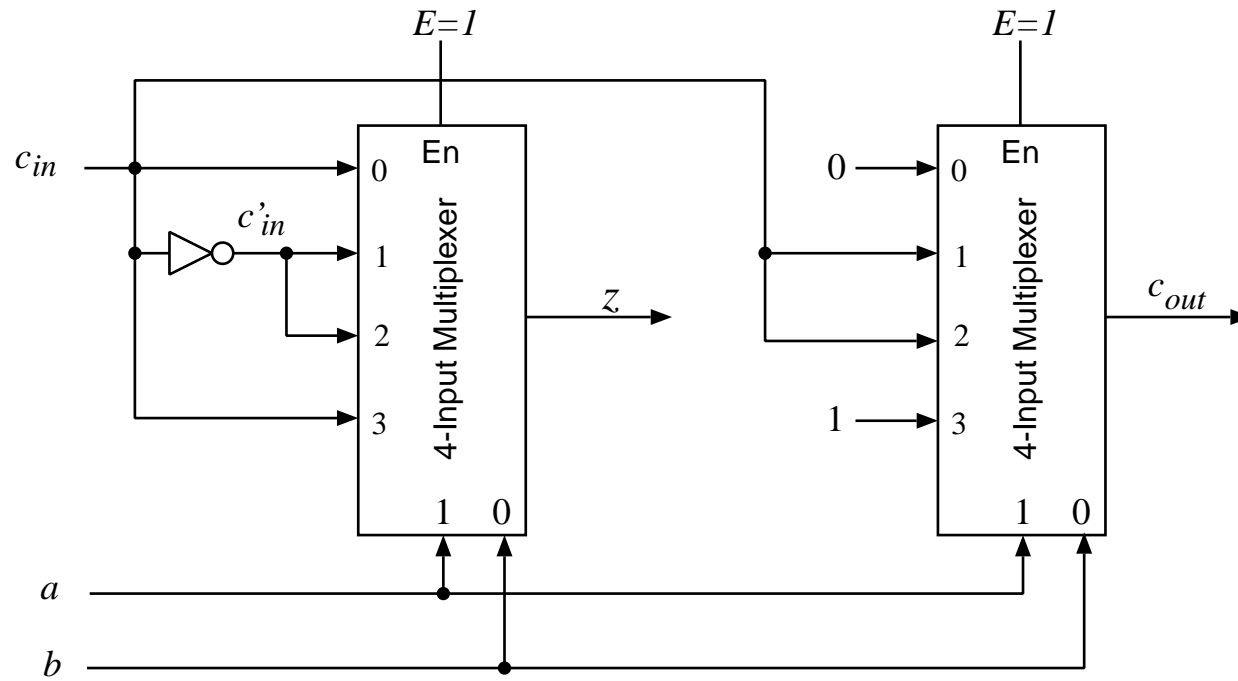
INPUTS: $a, b, c_{in} \in \{0, 1\}$

OUTPUTS: $z, c_{out} \in \{0, 1\}$

| a | b | c_{in} | z | c_{out} |
|-----|-----|----------|-----|-----------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$$\begin{aligned}
 z &= (a'b') \cdot c_{in} + (a'b) \cdot c'_{in} + (ab') \cdot c'_{in} + (ab) \cdot c_{in} \\
 &= c_{in}m_0(a, b) + c'_{in}m_1(a, b) + c'_{in}m_2(a, b) + c_{in}m_3(a, b)
 \end{aligned}$$

$$c_{out} = 0 \cdot m_0(a, b) \mathbf{+} c_{in}m_1(a, b) \mathbf{+} c_{in}m_2(a, b) \mathbf{+} 1 \cdot m_3(a, b) \quad ^{47}$$



z :

| | | | | |
|-----|----------|---|---|---|
| | c_{in} | | | |
| | 0 | 1 | 0 | 1 |
| a | 1 | 0 | 1 | 0 |
| | b | | | |

c_{out} :

| | | | | |
|-----|----------|---|---|---|
| | c_{in} | | | |
| | 0 | 0 | 1 | 0 |
| a | 0 | 1 | 1 | 1 |
| | b | | | |

| | | |
|-----|-----------|-----------|
| | c_{in} | c'_{in} |
| a | c'_{in} | c_{in} |
| | b | |

| | | |
|-----|----------|----------|
| | 0 | c_{in} |
| a | c_{in} | 1 |
| | b | |

MULTIPLEXER TREES

$$\underline{s}_{\text{left}} = (s_3, s_2)$$

$$\underline{s}_{\text{right}} = (s_1, s_0)$$

$$w_j = x_{(4j + s_{\text{right}})} \quad , \quad 0 \leq j \leq 3$$

$$z = w_{s_{\text{left}}}$$

$$s = 4s_{\text{left}} + s_{\text{right}}$$

$$z = x_{4s_{\text{left}} + s_{\text{right}}} = x_s$$

16-INPUT TREE MULTIPLEXER

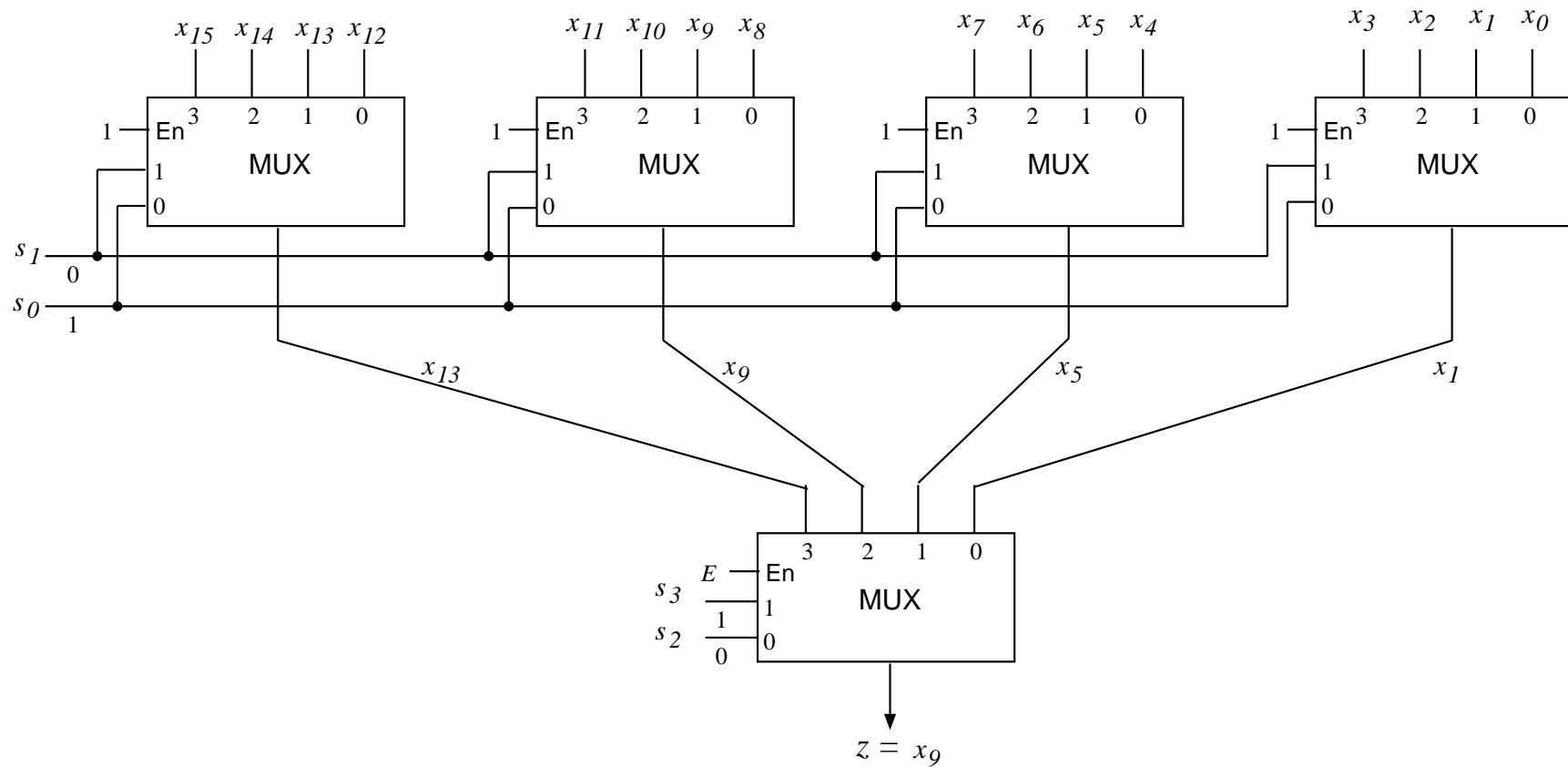


Figure 9.24: TREE IMPLEMENTATION OF A 16-INPUT MULTIPLEXER.

DEMULTIPLEXERS (distributors)

- A HIGH-LEVEL DESCRIPTION:

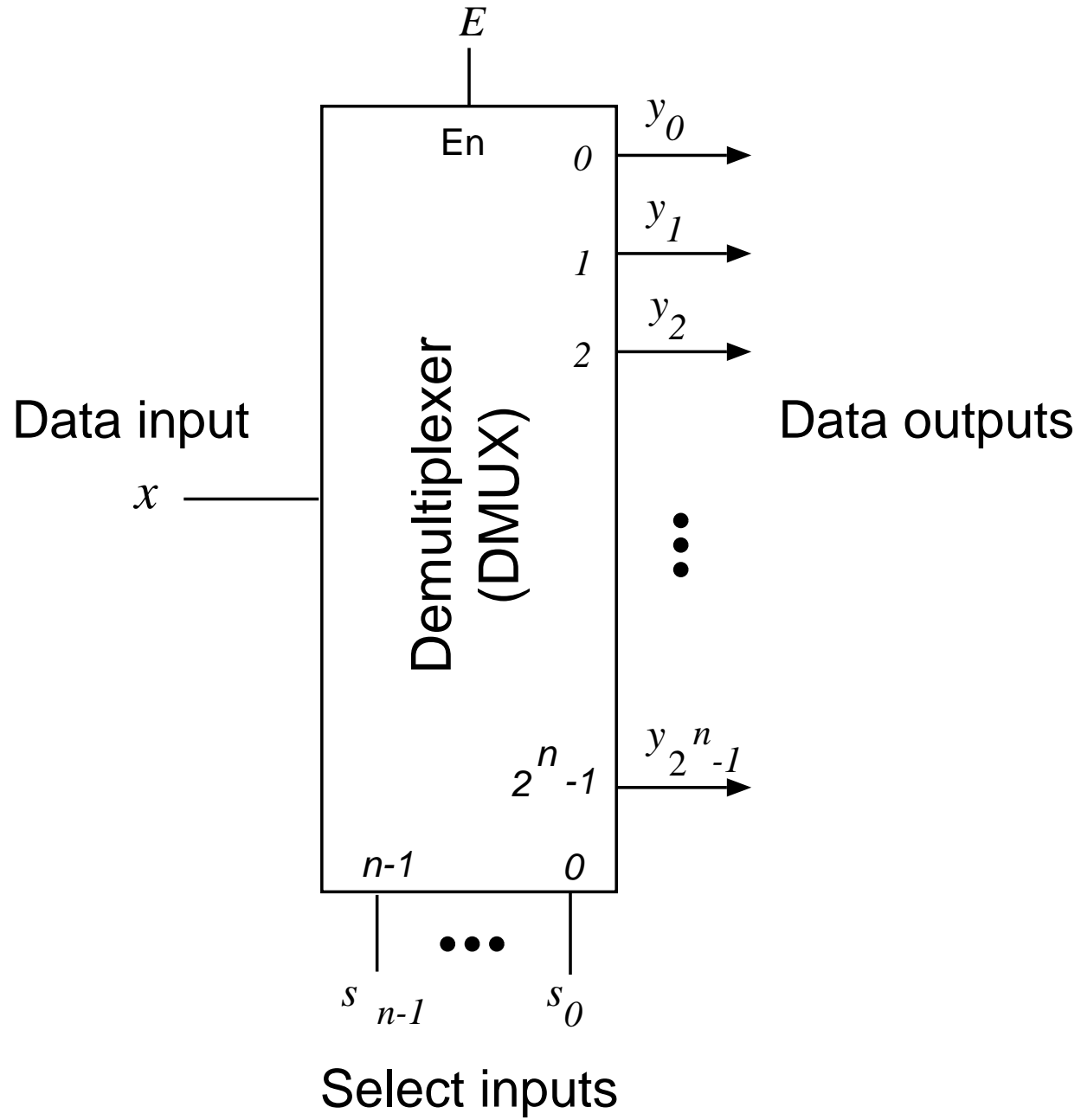
INPUTS: $x, E \in \{0, 1\}$

$$\underline{s} = (s_{n-1}, \dots, s_0) \quad , \quad s_j \in \{0, 1\}$$

OUTPUTS: $\underline{y} = (y_{2^n-1}, \dots, y_0) \quad , \quad y_i \in \{0, 1\}$

FUNCTION: $y_i = \begin{cases} x & \text{if } (i = s) \text{ and } (E = 1) \\ 0 & \text{if } (i \neq s) \text{ or } (E = 0) \end{cases}$

$$s = \sum_{j=0}^{n-1} s_j 2^j, \quad 0 \leq i \leq 2^n - 1$$

Figure 9.25: 2^n -OUTPUT DEMULTIPLEXER.

EXAMPLE 9.13: 4-OUTPUT DEMULTIPLEXER

| E | s_1 | s_0 | s | y_3 | y_2 | y_1 | y_0 |
|-----|-------|-------|-----|-------|-------|-------|-------|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | x |
| 1 | 0 | 1 | 1 | 0 | 0 | x | 0 |
| 1 | 1 | 0 | 2 | 0 | x | 0 | 0 |
| 1 | 1 | 1 | 3 | x | 0 | 0 | 0 |
| 0 | - | - | - | 0 | 0 | 0 | 0 |

$$y_i = E \cdot x \cdot m_i(\underline{s}), \quad 0 \leq i \leq 2^n - 1$$

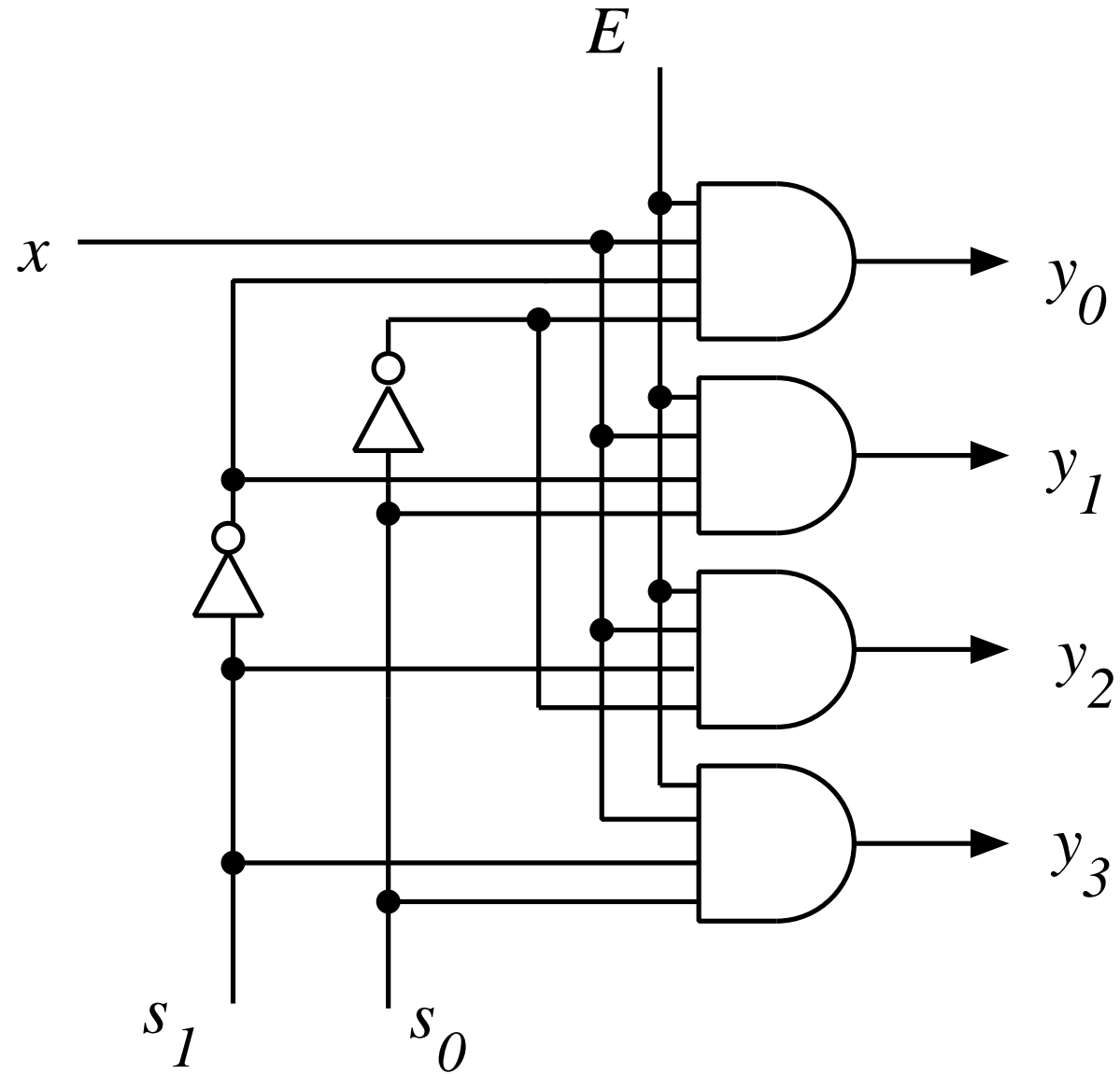


Figure 9.26: GATE NETWORK IMPLEMENTATION OF A 4-OUTPUT DEMULTIPLEXER.

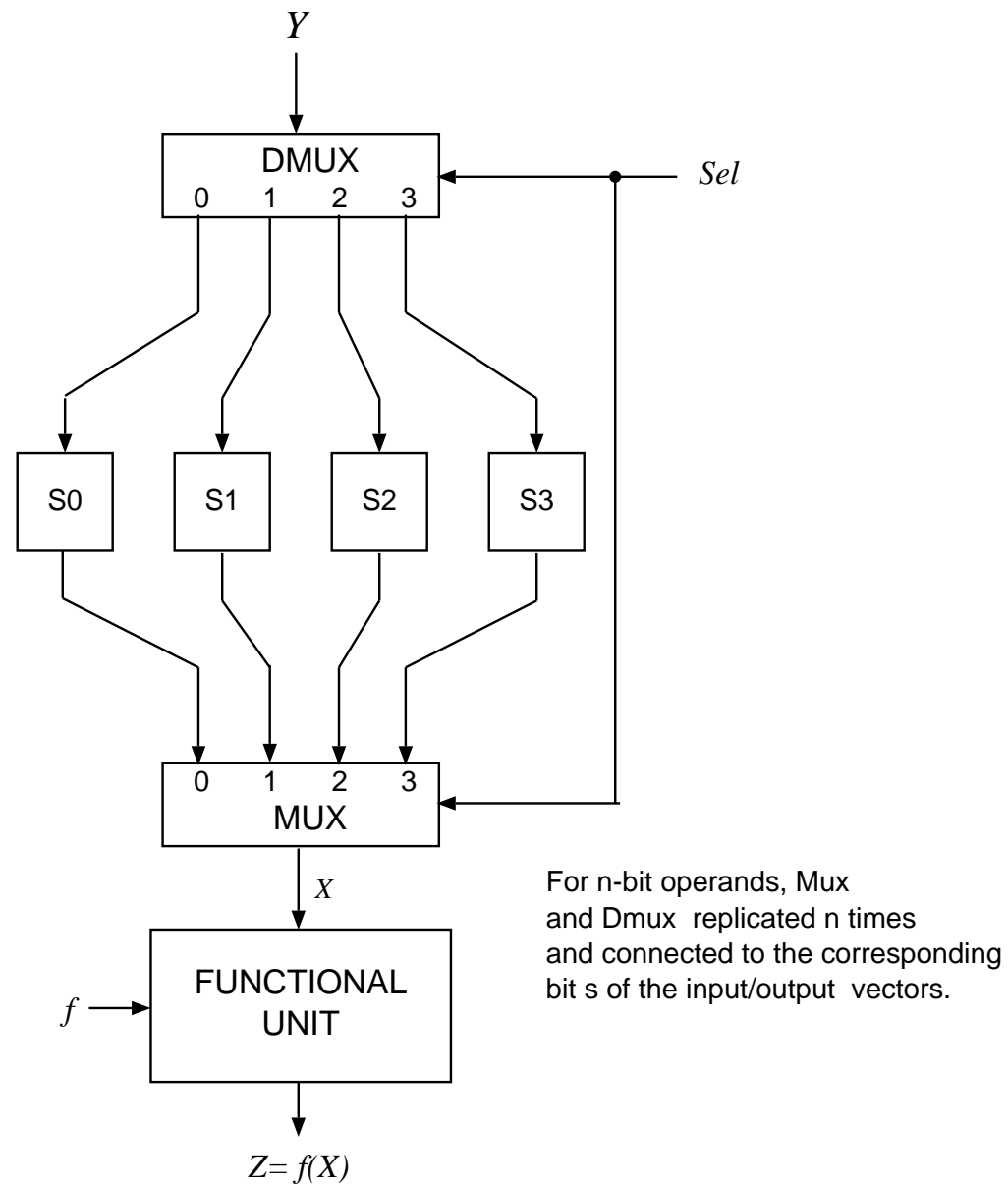


Figure 9.27: DEMULTIPLEXER: example of use.

SIMPLE SHIFTER

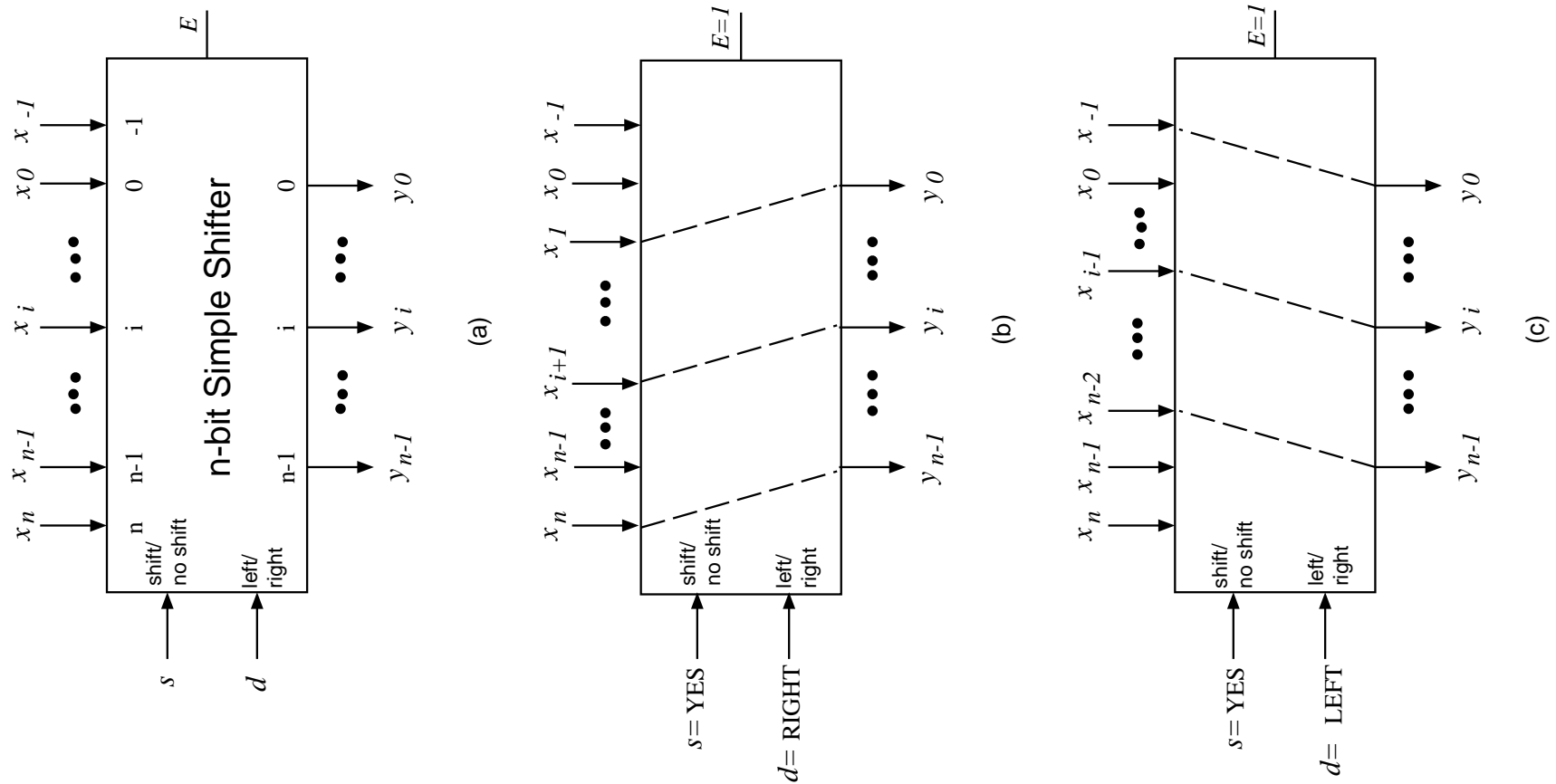


Figure 9.28: n -BIT SIMPLE SHIFTER: a) BLOCK DIAGRAM; b) RIGHT SHIFT; c) LEFT SHIFT.

SIMPLE SHIFTER: HIGH-LEVEL DESCRIPTION

INPUTS: $\underline{x} = (x_n, x_{n-1}, \dots, x_0, x_{-1})$, $x_j \in \{0, 1\}$
 $d \in \{RIGHT, LEFT\}$
 $s \in \{YES, NO\}$
 $E \in \{0, 1\}$

OUTPUTS: $\underline{y} = (y_{n-1}, \dots, y_0)$, $y_j \in \{0, 1\}$

FUNCTION:

$$y_i = \begin{cases} x_{i-1} & \text{if } (d = LEFT) \text{ and } (s = YES) \text{ and } (E = 1) \\ x_{i+1} & \text{if } (d = RIGHT) \text{ and } (s = YES) \text{ and } (E = 1) \\ x_i & \text{if } (s = NO) \text{ and } (E = 1) \\ 0 & \text{if } (E = 0) \end{cases}$$

for $0 \leq i \leq n - 1$.

$$x_{-1} = \begin{cases} 0 & \text{left shift with 0 insert} \\ 1 & \text{left shift with 1 insert} \\ x_{n-1} & \text{left rotate} \end{cases}$$

$$x_n = \begin{cases} 0 & \text{right shift with 0 insert} \\ 1 & \text{right shift with 1 insert} \\ x_0 & \text{right rotate} \end{cases}$$

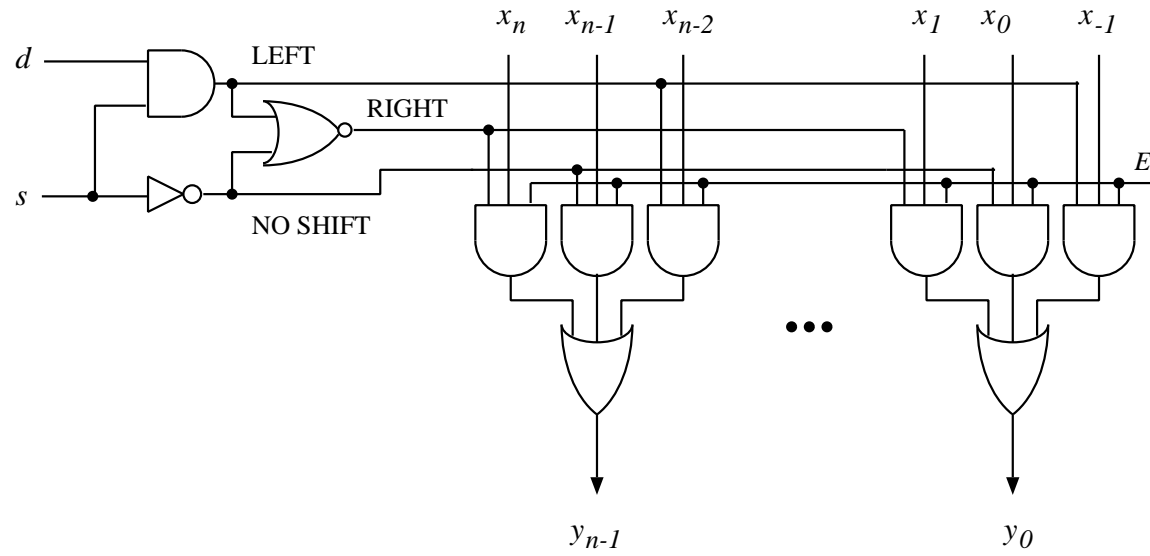
EXAMPLE 9.14: 4-INPUT SHIFTER

| | Control | | Data | | | | | |
|-------------|---------|-------|-------|-------|-------|-------|-------|----------|
| | s | d | x_4 | x_3 | x_2 | x_1 | x_0 | x_{-1} |
| | | | 1 | 0 | 0 | 1 | 1 | 0 |
| No shift | NO | – | | 0 | 0 | 1 | 1 | |
| Right shift | YES | RIGHT | 1 | 0 | 0 | 1 | | |
| Left shift | YES | LEFT | 0 | 1 | 1 | 0 | | |
| | | | y_3 | y_2 | y_1 | y_0 | | |

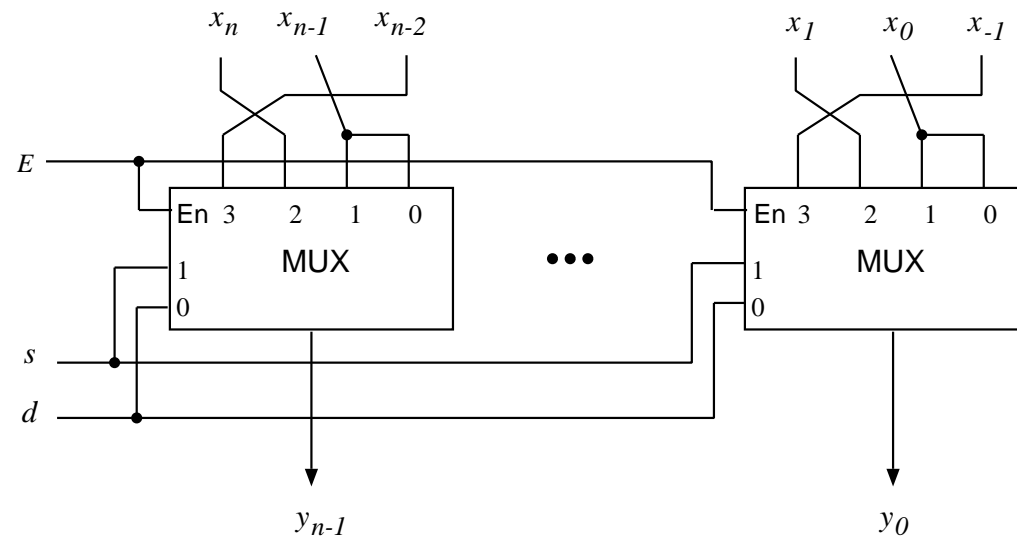
Coding:

| s | |
|-----|-----|
| 0 | NO |
| 1 | YES |

| d | |
|-----|-------|
| 0 | RIGHT |
| 1 | LEFT |



(a)



(b)

Figure 9.29: IMPLEMENTATION OF A SIMPLE SHIFTER: a) WITH GATES; b) WITH MULTIPLEXERS.

p -SHIFTER: HIGH-LEVEL DESCRIPTION

INPUTS: $\underline{x} = (x_{n+p-1}, \dots, x_n, x_{n-1}, \dots, x_0, x_{-1}, \dots, x_{-p})$, x_j
 $s \in \{0, 1, \dots, p\}$
 $d \in \{LEFT, RIGHT\}$
 $E \in \{0, 1\}$

OUTPUTS: $\underline{y} = (y_{n-1}, \dots, y_0)$, $y_j \in \{0, 1\}$

FUNCTION:

$$y_i = \begin{cases} x_{i-s} & \text{if } (d = LEFT) \text{ and } (E = 1) \\ x_{i+s} & \text{if } (d = RIGHT) \text{ and } (E = 1) \\ 0 & \text{if } (E = 0) \end{cases}$$

$$0 \leq i \leq n - 1$$

p -SHIFTER

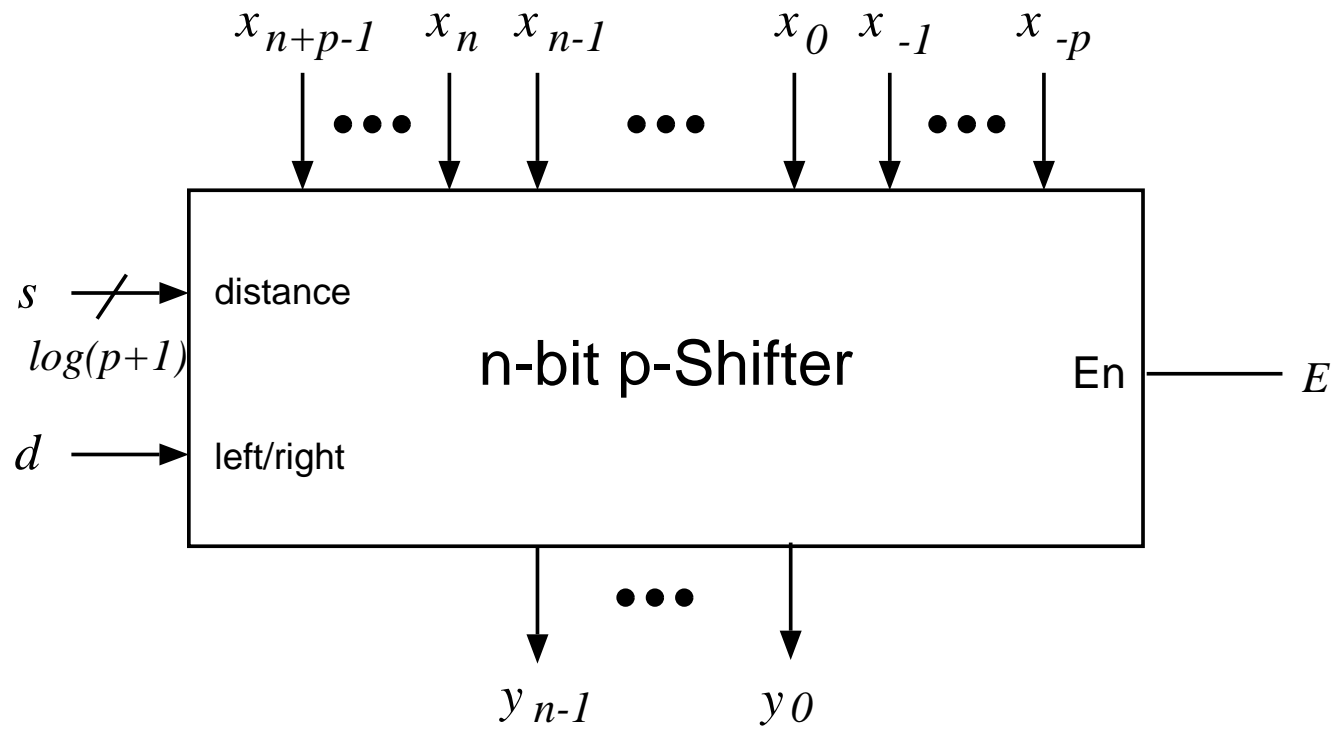


Figure 9.30: n -BIT p -SHIFTER.

BARREL SHIFTER

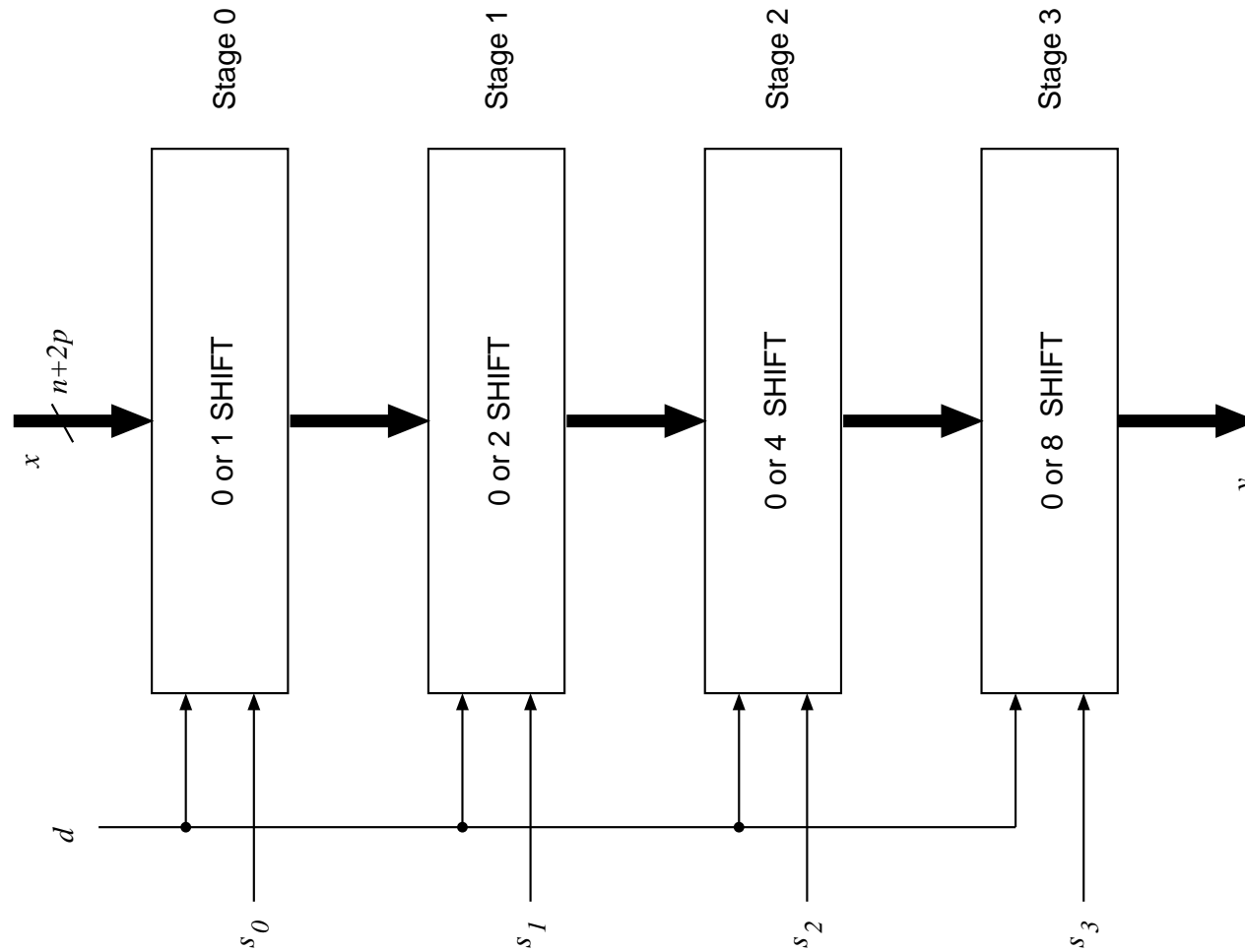


Figure 9.31: BARREL SHIFTER FOR $p = 15$.

UNIDIRECTIONAL SHIFTERS

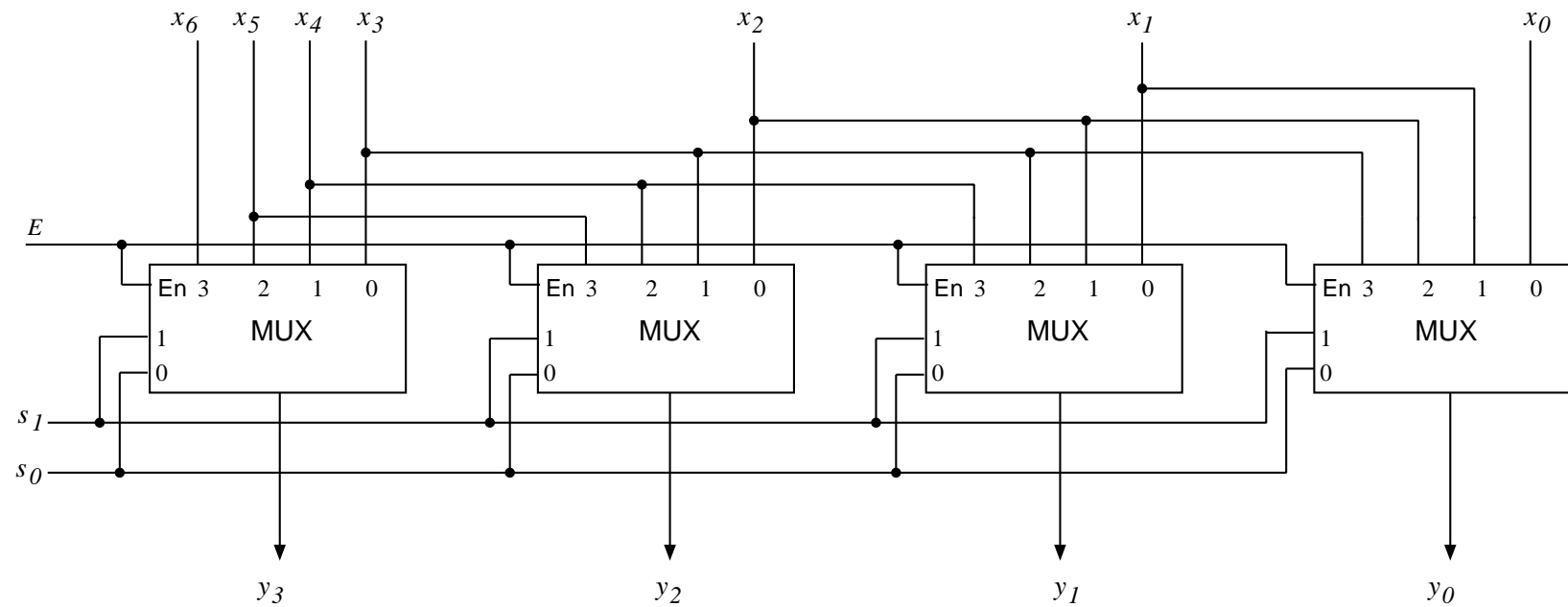


Figure 9.32: MULTIPLEXER IMPLEMENTATION OF A 4-BIT RIGHT 3-SHIFTER.

TYPICAL USES OF SHIFTERS

- ALIGNMENT OF A BIT-VECTOR
- REMOVAL OF THE LEADING (or trailing) BITS OF A VECTOR
- PERFORMING MULTIPLICATION OR DIVISION BY A POWER OF TWO
- EXTRACTING A SUBVECTOR from a bit-vector, using a shifter instead of a selector