[CS M51A Fall 14] Solution to Assignment 1

Due: 10/17/14

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Homework Problems (70 points total)

Problem 1 (10 points)

Find x and y such that the following conditions are satisfied and show all the steps of your work.

1. $(EC76)_{16} = (x)_8$

   $(EC76)_{16} = (1110110001110110)_2 = (166166)_8$

2. $(465)_7 + (383)_9 = (y)_{11}$

   $(465)_7 + (383)_9 = (243)_{10} + (318)_{10} = (561)_{10} = (470)_{11}$

Problem 2 (10 points)

Show that the following holds using the postulates of Boolean algebra.

1. $x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$

   Solution

$$
\begin{aligned}
& x'y'z' + x'y'z + x'yz + xy'z + xyz \\
= \;& x'y'(z' + z) + x'y'z + x'yz + xz(y' + y) \\
= \;& x'y' + x'yz + xz \\
= \;& x'(y' + yz) + xz \\
= \;& x'(y' + z) + xz \\
= \;& x'y' + x'z + xz \\
= \;& x'y' + z
\end{aligned}
$$

2. $xy' + xzw + yw = xy' + yw$

   Solution

$$
\begin{aligned}
xy' + xzw + yw &= xy' + x(y + y')zw + yw \\
&= xy' + xyzw + xy'zw + yw \\
&= xy'(1 + zw) + yw(xz + 1) \\
&= xy' + yw
\end{aligned}
$$

Problem 3 (10 points)

We would like to convert the given switching expression into the specified form.

$$E(x_3, x_2, x_1, x_0) = (((x_3 + x_2 + x_2'x_1')x_1 + x_0)' + x_3x_2')'$$

1. Convert the given expression into a simplified sum of products form.

   Solution

$$\begin{aligned}
& (((x_3 + x_2 + x_2'x_1')x_1 + x_0)' + x_3x_2')' \\
= \;& ((x_3 + x_2 + x_2'x_1')x_1 + x_0) \cdot (x_3x_2')' \\
= \;& ((x_3 + x_2 + x_2'x_1')x_1 + x_0)(x_3' + x_2) \\
= \;& (x_3x_1 + x_2x_1 + x_2'x_1'x_1 + x_0)(x_3' + x_2) \\
= \;& (x_3x_1 + x_2x_1 + x_0)(x_3' + x_2) \\
= \;& x_3x_3'x_1 + x_3'x_2x_1 + x_3'x_0 + x_3x_2x_1 + x_2x_1 + x_2x_0 \\
= \;& x_3'x_0 + x_2x_1(x_3' + x_3 + 1) + x_2x_0 \\
= \;& x_3'x_0 + x_2x_1 + x_2x_0
\end{aligned}$$

2. Convert the sum of products form into a sum of minterms form.

   Solution

$$\begin{aligned}
& x_3'x_0 + x_2x_1 + x_2x_0 \\
= \;& x_3'(x_2 + x_2')(x_1 + x_1')x_0 + (x_3 + x_3')x_2x_1(x_0 + x_0') + (x_3 + x_3')x_2(x_1 + x_1')x_0 \\
= \;& (m_7 + m_5 + m_3 + m_1) + (m_{15} + m_{14} + m_7 + m_6) + (m_{15} + m_{13} + m_7 + m_5) \\
= \;& \sum m(1, 3, 5, 6, 7, 13, 14, 15)
\end{aligned}$$

Problem 4 (10 points)

Convert the following truth table to a switching expression (Boolean Algebra) and simplify the expression as much as possible.

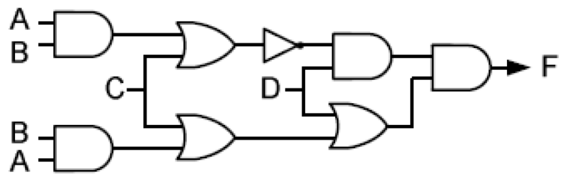
x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Solution

$$\begin{aligned}
& F = x'y'z' + x'y'z + x'yz' + xy'z \\
= \;& x'z'(y' + y) + y'z(x + x') \\
= \;& x'z' + y'z
\end{aligned}$$

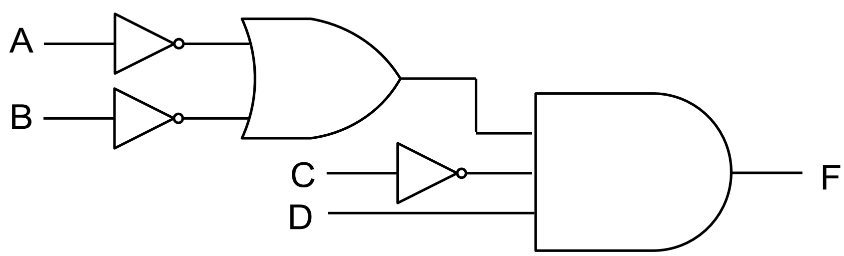
Problem 5 (10 points)

For the following diagrams, give the simplified POS expressions and show the corresponding two-level gate network.



Solution

$$
\begin{aligned}
F &= (AB+C)'D(AB+C+D) \\
&= (AB)'C'D(AB+C+D) \\
&= (A'+B')C'D(AB+C+D) \\
&= (A'+B')(ABC'D+CC'D+C'D) \\
&= (A'+B')(ABC'D+C'D) \\
&= (A'+B')C'D(AB+1) \\
&= (A'+B')C'D
\end{aligned}
$$



Problem 6 (20 points)

Your goal is to design a module which adds two digits x and y belonging to the set $\{-1, 0, 1\}$ to produce an output in the set $\{-2, -1, 0, 1, 2\}$. The inputs x and y are encoded via two bits, $x_p x_n$ and $y_p y_n$ with values given by $x_p - x_n$ (i.e. setting $x_p = 0$ and $x_n = 1$ encodes -1, with zero having two possible encodings 00 and 11). The outputs are encoded via three bits z_s, z_1, z_0, whose value is interpreted via $(-1)^{z_s}(2\,z_1 + z_0)$.

1. Write the switching functions for the three output bits z_s, z_1, z_0 in tabular form.

   Solution The table of switching functions is as shown. The three bits are the values of z_s, z_1, z_0, in this order.

j	$x_p x_n y_p y_n$	z_s	z_1	z_0
0	0000	-	0	0
1	0001	1	0	1
2	0010	0	0	1
3	0011	-	0	0
4	0100	1	0	1
5	0101	1	1	0
6	0110	-	0	0
7	0111	1	0	1
8	1000	0	0	1
9	1001	-	0	0
10	1010	0	1	0
11	1011	0	0	1
12	1100	-	0	0
13	1101	1	0	1
14	1110	0	0	1
15	1111	-	0	0

2. Obtain the minterm expressions (in m-notation) of z_s, z_1 and z_0 respectively.

   Solution From the table of the previous section, we can write the following.

$$\begin{aligned} z_s &= \sum m(1,4,5,7,13) \\ z_1 &= \sum m(5,10) \\ z_0 &= \sum m(1,2,4,7,8,11,13,14) \end{aligned}$$

3. Obtain the maxterm expressions (in M-notation) of z_s, z_1 and z_0 respectively.

   Solution Similarly, from the table we can write the following.

$$\begin{aligned} z_s &= \prod M(2,8,10,11,14) \\ z_1 &= \prod M(0,1,2,3,4,6,7,8,9,11,12,13,14,15) \\ z_0 &= \prod M(0,3,5,6,9,10,12,15) \end{aligned}$$

4. Does any of the switching functions have a dc-set? If so, which one?

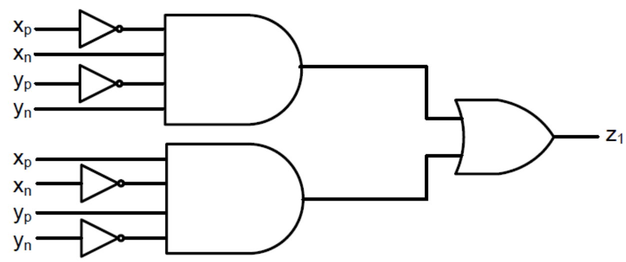
   Solution The output z_s has a don't care set of $dc - set(0,3,6,9,12,15)$, since for zero it does not matter what the sign value is.

5. Implement z_1 as a two-level AND-OR gate network. Note that NOT gates do not count as the two levels, so include them as needed.

   Solution Using the answer of 2., we get

$$\begin{aligned} z_1 &= \sum m(5,10) \\ &= x_p' x_n y_p' y_n + x_p x_n' y_p y_n' \end{aligned}$$

   Therefore, using the 2 level AND-OR network, the implementation would be


x_p
x_n
y_p
y_n
x_p
x_n
y_p
y_n
z_1