AN ILLUSTRATIVE DIGITAL DESIGN

PROBLEM: DESIGN A DIGITAL SYSTEM TO ADD TWO INTEGERS IN THE RANGE [0, 1, .., 15] WITH CARRY-IN AND CARRY-OUT.

SPECIFICATION

a) HIGH LEVEL: INPUTS: $x, y \in \{0, \ldots, 15\}$

$c_{in} \in \{0, 1\}$

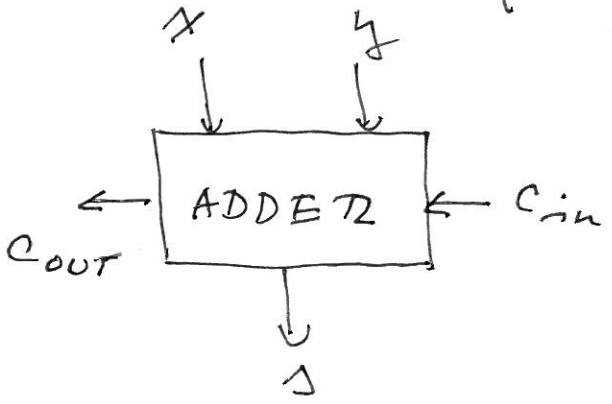
OUTPUTS: $S \in \{0, \ldots, 15\}$

$c_{out} \in \{0, 1\}$

FUNCTION:

$$S = (\underbrace{x + y + c_{in}}_{W}) \bmod 16$$

$$c_{out} = \begin{cases} 1 & \text{if } W \geq 16 \\ 0 & \text{OTHERWISE} \end{cases}$$



b) BINARY LEVEL:

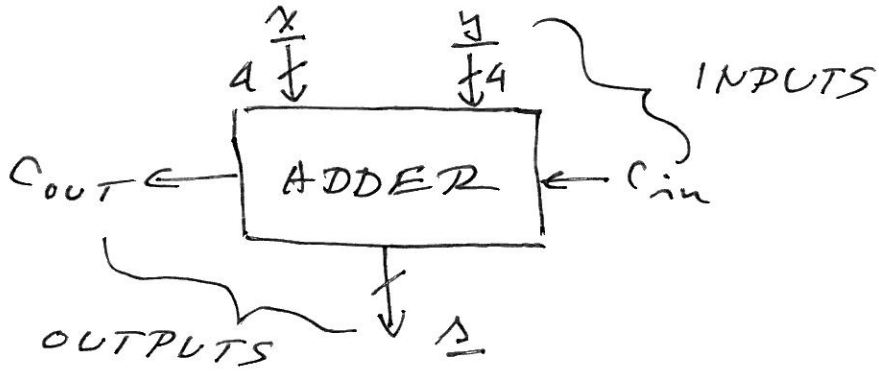
REPRESENT x, y, s AS BIT-VECTORS

$x \xrightarrow{\text{CODE}} \underline{x} = (x_3, x_2, x_1, x_0) \quad x_i \in \{0, 1\}$

$y \rightarrow \underline{y} \qquad s \rightarrow \underline{s}$

SUCH THAT $x = \sum_{i=0}^{3} x_i 2^i$ (BINARY CODE)

E.G. $13_{10} \rightarrow 1101_2$ etc.



FUNCTION:

$$s_0 = f_0(x_0, y_0, C_{in})$$
$$s_1 = f_1(x_1, y_1, x_0, y_0, C_{in})$$
$$s_2 = f_2(\ldots)$$
$$s_3 = f_3(\underline{x}, \underline{y}, C_{in})$$

BINARY VAR

9 BINARY VARIABLES

f_3: DOMAIN $\rightarrow$ RANGE

$\{2^9$ 9-TUPLES$\} \rightarrow \{0, 1\}$

$\Rightarrow f_3$ IS A SWITCHING FUNCTION

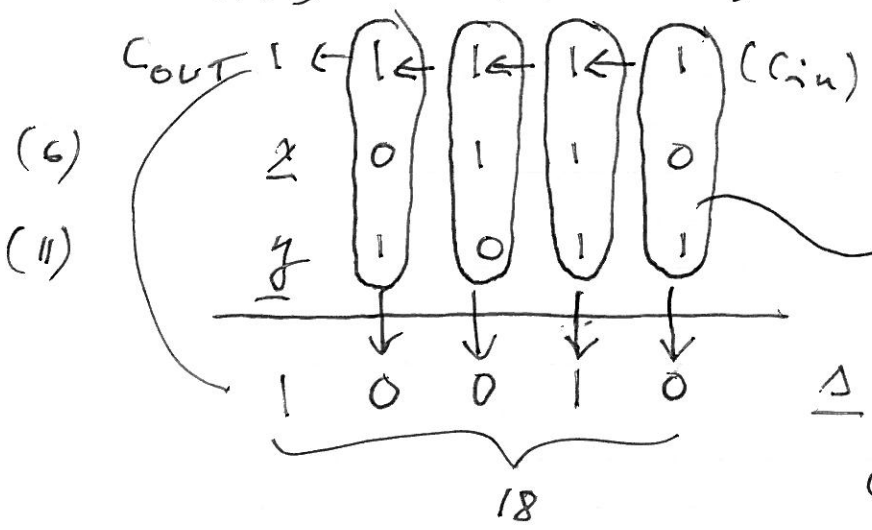
A CENTRAL CONCEPT

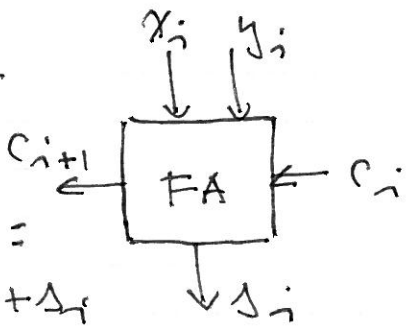
$$C_{OUT} = f_{OUT}(\underline{x}, \underline{y}, C_{in})$$

DIFFERENT <u>IMPLEMENTATIONS</u> POSSIBLE!

DIFFER IN COST, DELAY, POWER (AREA)

CONSIDER A COMMON "PAPER & PENCIL" METHOD OF ADDITION:





DEFINING ARITH. EXPRESSION:

$$x_i + y_i + c_i = 2c_{i+1} + s_i$$

↓ TABLE

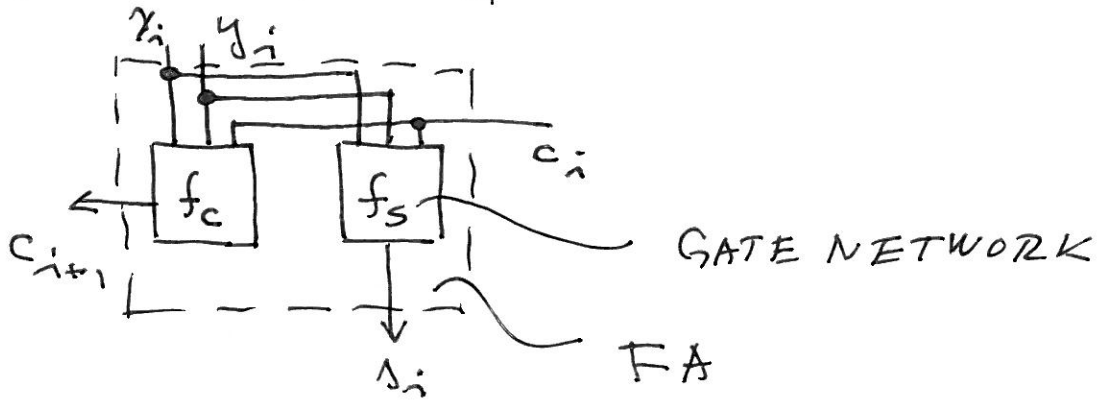
Σ	x_i	y_i	c_i	c_{i+1}	s_i
	0	0	0	0	0
	0	0	1	0	1
	0	1	0	0	1
	0	1	1	1	0
	1	0	0	0	1
	1	0	1	1	0
	1	1	0	1	0
	1	1	1	1	1

→ SUM OF INPUTS ≡ SUM OF OUTPUTS

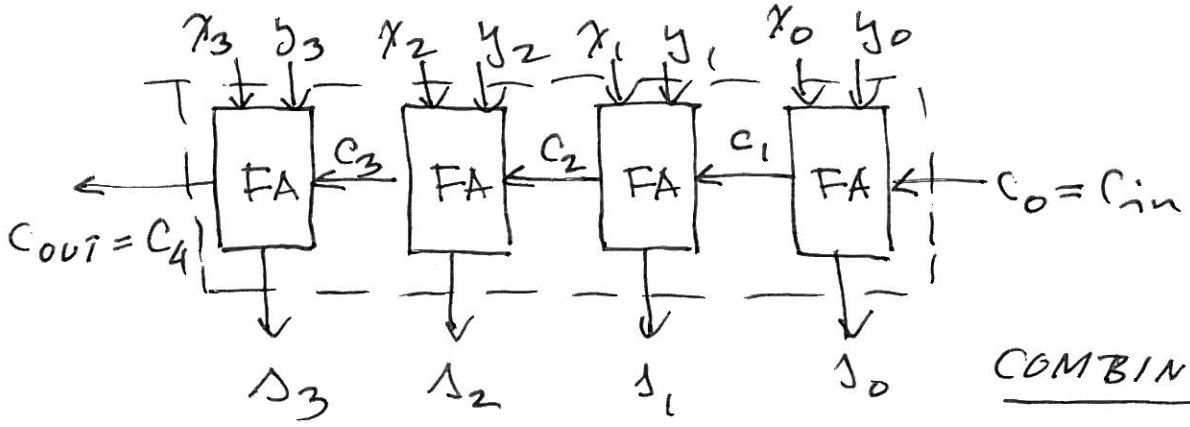
$$s_i = f_s(x_i, y_i, c_i)$$

$$c_{i+1} = f_c(x_i, y_i, c_i)$$

→ SIMPLE, FAST



FINAL DESIGN OF 4-BIT ADDER



COMBINATIONAL SYSTEM

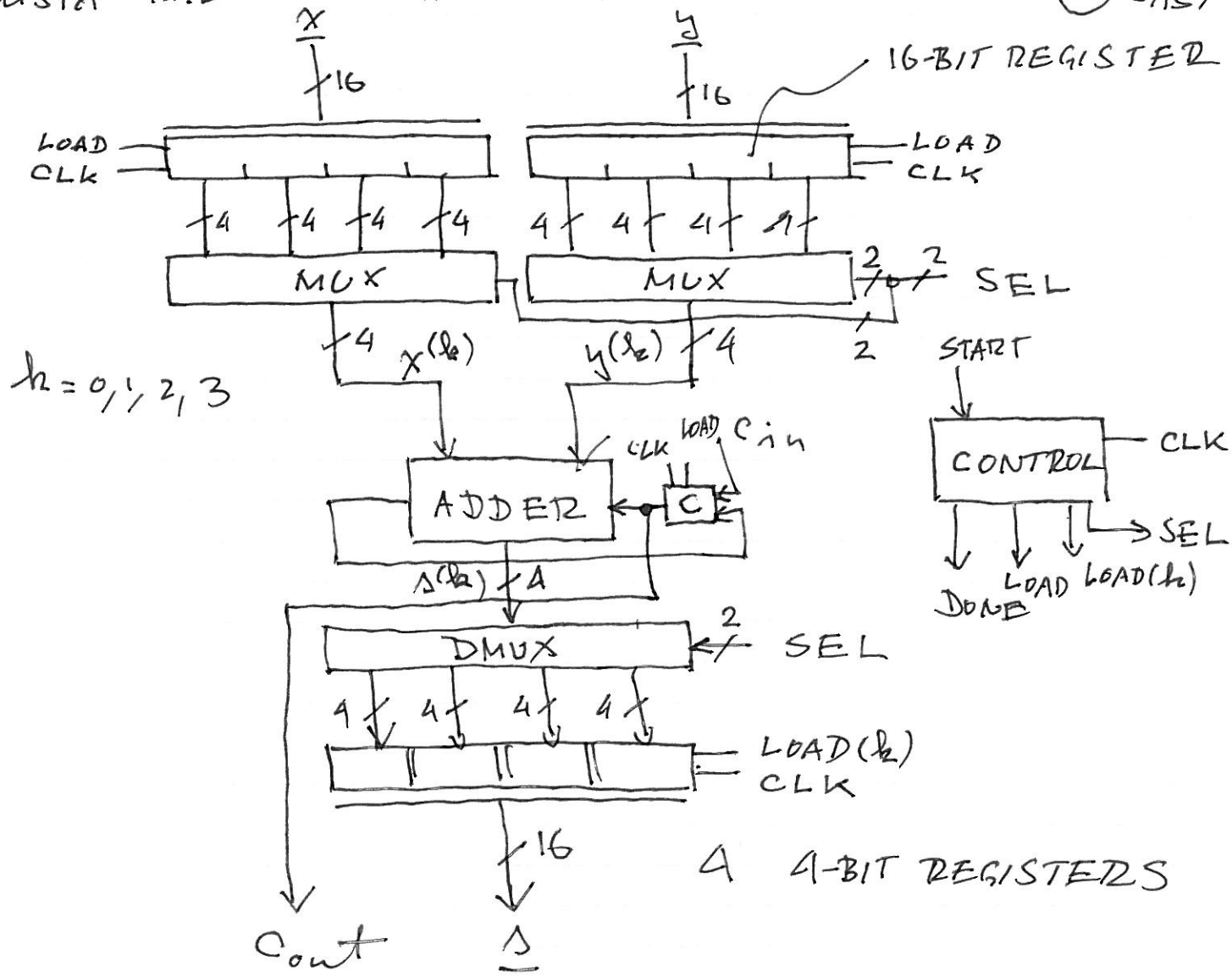
ESTIMATE: COST

DELAY

REPEAT FOR n-BIT ADDER

SUPPOS YOU NEED TO ADD 16-BIT INTEGERS HAVING ONLY A SINGLE 4-BIT ADDER, REGISTERS, ...

→ DESIGN A SEQUENTIAL SYSTEM



TIME BEHAVIOR: