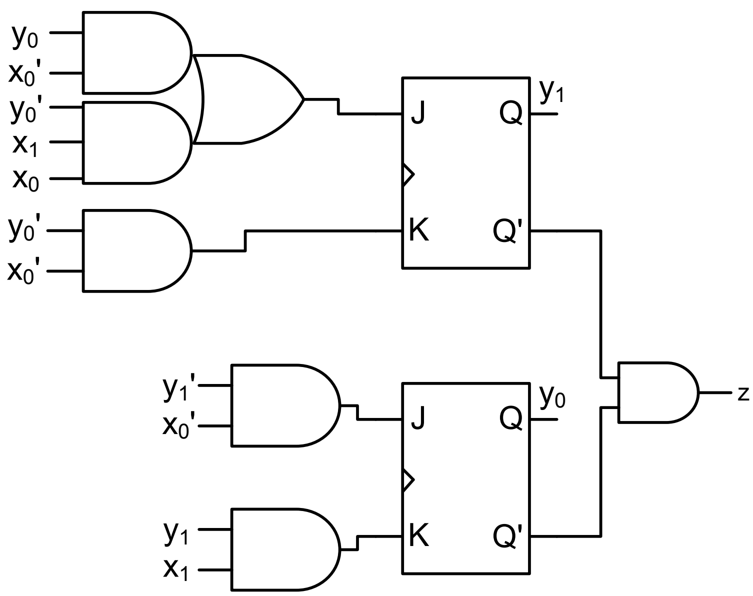
[CS M51A F14] Solution to Quiz 4

Date: 12/05/14

Quiz Problems (60 points total)

Problem 1 (20 points)

We would like to analyze the following sequential network. It has two input bits x_1 and x_0, with a single output bit z.



1. **(10 points)** Write the minimal sum of product expressions for z, $y_1(t+1)$ and $y_0(t+1)$. Assume that a literal without a time label is equal to its value at time t, i. e. y_0 is short for $y_0(t)$. To obtain the expressions for $y_1(t+1)$ and $y_0(t+1)$, use the JK flip-flop characteristic expression shown here:

$$Q(t+1) = Q(t)K'(t) + Q'(t)J(t)$$

Solution From the given circuit, we can directly write the following:

$$\begin{aligned} z(t) &= y_1'y_0' \\ J_1(t) &= y_0x_0' + y_0'x_1x_0 \\ K_1(t) &= y_0'x_0' \\ J_0(t) &= y_1'x_0' \\ K_0(t) &= y_1x_1 \end{aligned}$$

Using the JK characteristic expression, we can derive:

$$\begin{aligned} y_1(t+1) &= y_1K_1' + y_1'J_1 \\ &= y_1(y_0'x_0')' + y_1'(y_0x_0' + y_0'x_1x_0) \\ &= y_1(y_0 + x_0) + (y_1'y_0x_0' + y_1'y_0'x_1x_0) \\ &= (y_1y_0 + y_1x_0) + (y_1'y_0x_0' + y_1'y_0'x_1x_0) \\ &= y_0(y_1 + y_1'x_0') + x_0(y_1 + y_1'y_0'x_1) \\ &= y_0(y_1 + x_0') + x_0(y_1 + y_0'x_1) \\ &= y_1y_0 + y_0x_0' + y_1x_0 + y_0'x_1x_0 \\ &= y_1y_0(x_0 + x_0') + y_0x_0' + y_1x_0 + y_0'x_1x_0 \\ &= y_1y_0x_0 + y_1y_0x_0' + y_0x_0' + y_1x_0 + y_0'x_1x_0 \\ &= (y_1y_0x_0 + y_1x_0) + (y_1y_0x_0' + y_0x_0') + y_0'x_1x_0 \\ &= y_1x_0 + y_0x_0' + y_0'x_1x_0 \\ y_0(t+1) &= y_0K_0' + y_0'J_0 \\ &= y_0(y_1x_1)' + y_0'(y_1'x_0') \\ &= y_0(y_1' + x_1') + y_1'y_0'x_0' \\ &= y_1'y_0 + y_0x_1' + y_1'y_0'x_0' \\ &= y_1'(y_0 + y_0'x_0') + y_0x_1' \\ &= y_1'(y_0 + x_0') + y_0x_1' \\ &= y_1'y_0 + y_1'x_0' + y_0x_1' \end{aligned}$$

2. **(10 points)** Using the expressions, fill in the table below.

Solution From the expressions we can fill in the following:

PS	Input $x_1(t)x_0(t)$				Output
$y_1(t)y_0(t)$	00	01	10	11	z
00	01	00	01	10	1
01	11	01	11	01	0
10	00	10	00	10	0
11	11	11	10	10	0
	$y_1(t+1)y_0(t+1)$				
	NS				

The easiest approach to this is to find which locations each sum term turns to 1 and fill those with 1s, then any remaining slots are 0s. For example, from $y_1'y_0$ we can deduce that the whole row 01 is 1 for $y_0(t+1)$.

Problem 2 (10 points)

We wish to create an SR flip-flop using a T flip-flop. The transition table for the SR flip-flop is:

$PS = Q(t)$	$S(t)R(t)$			
	00	01	10	11
0	0	0	1	-
1	1	0	1	-
	$NS = Q(t+1)$			

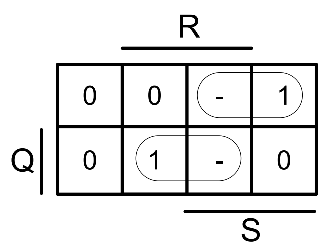
1. **(5 points)** Fill in the table below.

   Solution From the given transition table, we can write:

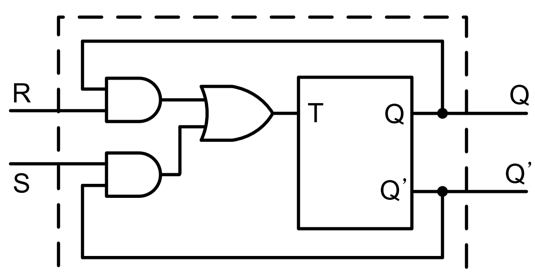
$PS = Q(t)$	$S(t)R(t)$				$S(t)R(t)$			
	00	01	10	11	00	01	10	11
0	0	0	1	-	0	0	1	-
1	1	0	1	-	0	1	0	-
	$NS = Q(t+1)$				T			

2. **(5 points)** Using the table, obtain the expression for T and draw the final circuit.

   Solution From the completed table, we can get the following K-map:



and from this we get $T = QR + Q'S$.
The final circuit looks like:



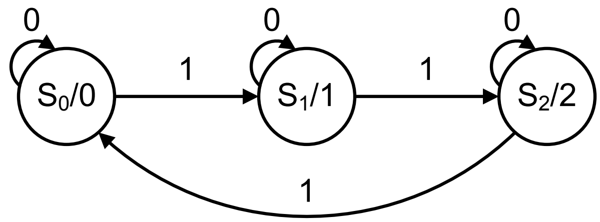
Problem 3 (30 points)

Design a modulo-3 counter using JK flip-flops. Use a Moore machine. The counter specifications are as shown here:

Input:	$x(t) \in \{0, 1\}$
Output:	$z(t) \in \{0, 1, 2\}$
State:	$s(t) \in \{S_0, S_1, S_2\}$
Initial state:	$s(0) = S_0$
Function:	In modulo-3, the system counts the number of 1's in the input sequence $x(0, t-1)$

1. **(10 points)** Draw the state transition diagram for the counter. Clearly show **ALL** transitions from each state. Show the output of each state.

   Solution The state transition diagram is shown.



2. **(10 points)** Using the following **unconventional** encoding for the states, complete the table below.

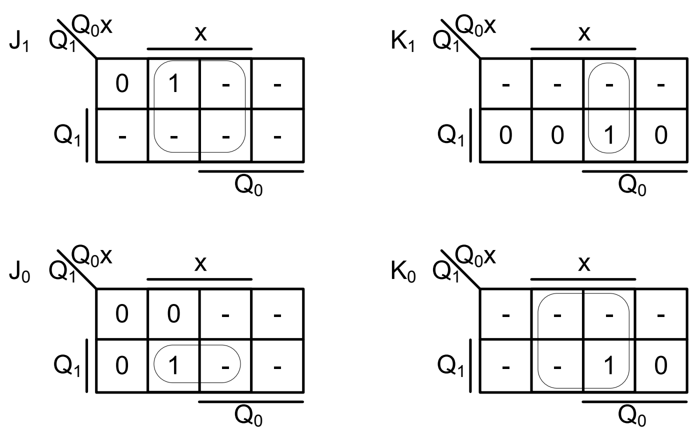
	Q_1	Q_0
S_0	0	0
S_1	1	0
S_2	1	1

   Solution

Q_1Q_0	$x=0$	$x=1$	$x=0$	$x=1$
00	00	10	0– 0–	1– 0–
01	– –	– –	– – – –	– – – –
10	10	11	–0 0–	–0 1–
11	11	00	–0 –0	–1 –1
	NS		$J_1K_1J_0K_0$	

3. **(10 points)** Complete the K-maps and obtain the minimal expressions for J_1, K_1, J_0 and K_0.

   Solution



From these we get:

$$
\begin{aligned}
J_1 &= x \\
K_1 &= Q_0x \\
J_0 &= Q_1x \\
K_0 &= x
\end{aligned}
$$