

Department of Mathematical Sciences, Kannur University
Bridge Course - Algebra
Problem Sheet 1

1 Problems

1. Let D_4 be the dihedral group of symmetries of a square.
 - (a) How many elements does D_4 have?
 - (b) How many of these are rotations?
 - (c) How many are reflections?
2. Find all subgroups of D_4 . Which of them are cyclic?
3. Let S_3 be the symmetric group on 3 elements. Let $\sigma = (1\ 2)$ and $\tau = (1\ 2\ 3)$.
 - (a) Compute $\sigma\tau$ and $\tau\sigma$ and compare.
 - (b) List all subgroups of S_3 . Identify which are cyclic and which are not.
4. Prove that set of all rotations in D_n form a subgroup of D_n .
5. Let G be a group and H_1 and H_2 be two subgroups of G .
 - (a) Prove that $H_1 \cap H_2$ is a subgroup and also $\bigcap H_\alpha$, $\alpha \in I$ where I is an indexing set
 - (b) what about $H_1 \cup H_2$?
6. Verify that $GL_n(\mathbb{R})$ satisfies all four group axioms under matrix multiplication. Define the special linear group $SL_2(\mathbb{R})$ as the set of 2×2 real matrices with determinant 1.
 - (a) Show that $SL_2(\mathbb{R})$ is a subgroup of $GL_2(\mathbb{R})$.
 - (b) Verify whether the matrix $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ belongs to $SL_2(\mathbb{R})$.
7. Let \mathbb{Z} be the group of integers and let $k, l \in \mathbb{Z}$. What is the subgroup generated by k and l ?