Department of Mathematical Sciences, Kannur University
Bridge Course -Linear Algebra
Problem Sheet 1

# 1 Problems

1. **Vector Space Identification:** For each of the following, determine whether the given set $V$ forms a vector space over the specified field $F$ with the defined operations.

   (a) $V = \{A \in M_{2\times 2}(\mathbb{R}) \mid \det(A) \neq 0\}$ (all $2 \times 2$ invertible matrices with real entries)

   (b) $V = P_n(\mathbb{R})$ (all polynomials with real coefficients of degree *exactly* $n$, where $n \geq 1$)

   (c) $V = \{(x, y, z) \in \mathbb{R}^3 \mid y = 2x \text{ and } z = 3x\}$ (the set of vectors in $\mathbb{R}^3$ that lie on the line passing through the origin defined by $y = 2x$ and $z = 3x$)

2. 
   - **Statement P:** $\forall v \in V, \exists v' \in V$ such that $v + v' = \theta$.
   - **Statement Q:** $\exists v' \in V$ such that $\forall v \in V, v + v' = \theta$.

   Is statement **P** and **Q** are equivalent?

3. Prove the following

   (a) Additive inverse is unique.

   (b) Scalar zero times any vector is zero vector.$(0 * v = \theta)$

   (c) Zero vector is unique.

   (d) Scalar times the zero vector is Zero vector.$(\alpha * \theta = \theta)$

   (e) $(-1) * v = v'$

   (f) $V + V = 2V$.

   (g) $\alpha v = \theta$ if and only if $\alpha = 0$ or $v = \theta$.

— End of Problem Sheet —