



→ Forward Prop:

$$\rightarrow y_1^{[1]} = w_{11}^{[1]} x_1 + w_{12}^{[1]} x_2 + b_1^{[1]}$$

$$\rightarrow y_2^{[1]} = w_{21}^{[1]} x_1 + w_{22}^{[1]} x_2 + b_2^{[1]}$$

$$\rightarrow y_3^{[1]} = w_{31}^{[1]} x_1 + w_{32}^{[1]} x_2 + b_3^{[1]}$$

$$\rightarrow y_1^{[2]} = w_{11}^{[2]} y_1^{[1]} + w_{12}^{[2]} y_2^{[1]} + w_{13}^{[2]} y_3^{[1]} + b_1^{[2]}$$

$$\rightarrow \text{MSE} = \frac{1}{n} \sum_{i=1}^n (y_1^{[2]} - y)^2$$

Target variable

→ Back Prop:

$$\rightarrow \frac{dc}{dy^{[2]}} = \frac{2}{n} \sum_{i=1}^n (y_1^{[2]} - y) \quad (\text{Change in cost w.r.t } y_1^{[2]})$$

$$\rightarrow \frac{dc}{dw_{11}^{[2]}} = \frac{dy_1^{[2]}}{dw_{11}^{[2]}} \times \frac{dc}{dy^{[2]}} = \frac{2}{n} y_1^{[1]} \sum_{i=1}^n (y_1^{[2]} - y)$$

$$\rightarrow \frac{dc}{dw_{12}^{[2]}} = \frac{2}{n} y_1^{[1]} \leq (y_1^{[2]} - y)$$

$$\rightarrow \frac{dc}{dw_{13}^{[2]}} = \frac{2}{n} y_3^{[1]} \leq (y_1^{[2]} - y)$$

$$\rightarrow \frac{dc}{db_1^{[2]}} = \frac{2}{n} \leq (y_1^{[2]} - y)$$

$$\begin{aligned} \rightarrow \frac{dc}{dy_1^{[1]}} &= \frac{dy_1^{[2]}}{dy_1^{[1]}} * \frac{dc}{dy_1^{[2]}} \\ &= w_{11}^{[2]} * \frac{2}{n} \sum_{i=1}^n (y_{i1}^{[2]} - y) \end{aligned}$$

→ Similarly for $\frac{dc}{dy_2^{[1]}}$, $\frac{dc}{dy_3^{[1]}}$

$$\begin{aligned} \rightarrow \frac{dc}{dw_{11}^{[1]}} &= \frac{dy_1^{[1]}}{dw_{11}^{[1]}} * \frac{dc}{dy_1^{[1]}} \\ &= x_1 w_{11}^{[2]} * \frac{2}{n} \sum (y_{i1}^{[2]} - y) \end{aligned}$$

$$\rightarrow \frac{dc}{db_1^{[1]}} = w_{11}^{[2]} * \frac{2}{n} \sum (y_{i1}^{[2]} - y)$$

→ Similarly, we do chain derivation for other variables.

Follow the river and will reach the sea.

→ Updating variables: (α = learning rate)

$$\rightarrow w_{11}^{[2]} = w_{11}^{[1]} - \alpha \frac{dc}{dw_{11}^{[2]}}$$

$$\rightarrow w_{12}^{[2]} = w_{12}^{[1]} - \alpha \frac{dc}{dw_{12}^{[2]}}$$

$$\rightarrow b_1^{[2]} = b_1^{[1]} - \alpha \frac{dc}{db_1^{[2]}}$$

→ So to fast the operations for number of examples we do vectorisation of example:

$$\begin{bmatrix} 1 & 1 & 1 & \dots \\ x_1 & x_2 & x_3 & \dots \\ 1 & 1 & 1 & \dots \end{bmatrix} \rightarrow \text{This is our input to nn.}$$

first data pt 2nd data pt