# AE – Arithmetic Expressions

## 1 INTRODUCTION

AE is a toy language for the COSE212 course at Korea University. AE stands for a language of **a**rithmetic **e**xpressions that supports the following features:

- **number (integer) values** (0, 1, -1, 2, -2, 3, -3, . . .)
- **arithmetic operators**: addition (+) and multiplication (*)

This document is the specification of AE. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax. Then, Section 4 describes the big-step operational (natural) semantics of AE.

## 2 CONCRETE SYNTAX

The concrete syntax of AE is written in a variant of the extended Backus–Naur form (EBNF). The notation `<nt>` denotes a nonterminal, and `"t"` denotes a terminal. We use ? to denote an optional element and + (or *) to denote one or more (or zero or more) repetitions of the preceding element. We omit some obvious terminals using the ellipsis (. . .) notation.

```
// basic elements
<digit>  ::= "0" | "1" | "2" | ... | "9"
<number> ::= "-"? <digit>+

// expressions
<expr>   ::= <number> | <expr> "+" <expr> | <expr> "*" <expr> | "(" <expr> ")"
```

The precedence and associativity of operators are defined as follows:

| Operator | Associativity | Precedence |
|:--------:|:-------------:|:----------:|
| *        | left          | 2          |
| +        | left          | 1          |

## 3 ABSTRACT SYNTAX

The abstract syntax of AE is defined as follows:

$$\text{Expressions} \quad \mathbb{E} \ni e ::= n \quad (\text{Num}) \\ \mid e + e \quad (\text{Add}) \quad \text{where} \quad \text{Numbers} \quad n \in \mathbb{Z} \quad (\text{BigInt}) \\ \mid e * e \quad (\text{Mul})$$

## 4 SEMANTICS

The big-step operational (natural) semantics of AE is defined as follows:

$$\boxed{\vdash e \Rightarrow n}$$

$$\text{Num} \frac{}{\vdash n \Rightarrow n} \qquad \text{Add} \frac{\vdash e_1 \Rightarrow n_1 \qquad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2} \qquad \text{Mul} \frac{\vdash e_1 \Rightarrow n_1 \qquad \vdash e_2 \Rightarrow n_2}{\vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$

The equivalent small-step operational (reduction) semantics of AE is defined as follows:

$$\frac{e_1 \rightarrow e_1'}{e_1 + e_2 \rightarrow e_1' + e_2} \qquad \frac{e_2 \rightarrow e_2'}{n_1 + e_2 \rightarrow n_1 + e_2'} \qquad \frac{}{n_1 + n_2 \rightarrow n_1 + n_2}$$

$$\frac{e_1 \rightarrow e_1'}{e_1 * e_2 \rightarrow e_1' * e_2} \qquad \frac{e_2 \rightarrow e_2'}{n_1 * e_2 \rightarrow n_1 * e_2'} \qquad \frac{}{n_1 * n_2 \rightarrow n_1 \times n_2}$$