MFAE – FAE with Mutable Variables

1 INTRODUCTION

MFAE is a toy language for the COSE212 course at Korea University. MFAE stands for an extension of the FAE language with **mutable variables**, and it supports the following features:

- **number (integer) values** (`0`, `1`, `-1`, `2`, `-2`, `3`, `-3`, ...)
- **arithmetic operators**: addition (`+`) and multiplication (`*`)
- **mutable variable definitions** (`var`)
- **first-class functions** (`=>`)
- **assignment** (`=`)
- **augmented assignment**: additive (`+=`) and multiplicative (`*=`)
- **sequences** (`;`)

This document is the specification of MFAE. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax with the desugaring rules. Then, Section 4 describes the big-step operational (natural) semantics of MFAE.

2 CONCRETE SYNTAX

The concrete syntax of MFAE is written in a variant of the extended Backus–Naur form (EBNF). The notation `<nt>` denotes a nonterminal, and `"t"` denotes a terminal. We use `?` to denote an optional element and `+` (or `*`) to denote one or more (or zero or more) repetitions of the preceding element. We use `butnot` to denote a set difference to exclude some strings from a producible set of strings. We omit some obvious terminals using the ellipsis (...) notation.

```
// basic elements
<digit>    ::= "0" | "1" | "2" | ... | "9"
<number>   ::= "-"? <digit>+
<alphabet> ::= "A" | "B" | "C" | ... | "Z" | "a" | "b" | "c" | ... | "z"
<idstart>  ::= <alphabet> | "_"
<idcont>   ::= <alphabet> | "_" | <digit>
<keyword>  ::= "var"
<id>       ::= <idstart> <idcont>* butnot <keyword>

// expressions
<expr> ::= <number> | <expr> "+" <expr> | <expr> "*" <expr>
         | "(" <expr> ")" | "{" <expr> "}"
         | "var" <id> "=" <expr> ";" <expr> | <id>
         | <id> "=>" <expr> | <expr> "(" <expr> ")"
         | <id> "=" <expr> | <id> "+=" <expr> | <id> "*=" <expr>
         | <expr> ";" <expr>
```

The precedence and associativity of operators are defined as follows:

Description	Operator	Precedence	Associativity
Multiplicative	*	1	left
Additive	+	2	
Assignment	=, +=, *=	3	right

3 ABSTRACT SYNTAX

The abstract syntax of MFAE is defined as follows:

$$
\begin{array}{llllll}
\text{Expressions} & \mathbb{E} \ni e ::= n & \texttt{(Num)} & \mid \lambda x.e & \texttt{(Fun)} \\
& \mid e + e & \texttt{(Add)} & \mid e(e) & \texttt{(App)} \\
& \mid e * e & \texttt{(Mul)} & \mid x = e & \texttt{(Assign)} \\
& \mid \texttt{var } x = e;\ e & \texttt{(Var)} & \mid e;\ e & \texttt{(Seq)} \\
& \mid x & \texttt{(Id)} &
\end{array}
$$

where

$$\text{Numbers} \quad n \in \mathbb{Z} \quad \texttt{(BigInt)} \qquad \text{Identifiers} \quad x \in \mathbb{X} \quad \texttt{(String)}$$

The semantics of the remaining cases are defined with the following desugaring rules:

$$\mathcal{D}[\![x \mathrel{+}= e]\!] = (x = x + \mathcal{D}[\![e]\!]) \qquad \mathcal{D}[\![x \mathrel{*}= e]\!] = (x = x * \mathcal{D}[\![e]\!])$$

The omitted cases recursively apply the desugaring rule to sub-expressions.

4 SEMANTICS

We use the following notations in the semantics:

$$
\begin{array}{llllll}
\text{Environments} & \sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{A} & \texttt{(Env)} & \text{Memories} & M \in \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V} & \texttt{(Mem)} \\
\text{Values} & \mathbb{V} \ni v ::= n & \texttt{(NumV)} & \text{Addresses} & a \in \mathbb{A} & \texttt{(Addr)} \\
& \mid \langle \lambda x.e, \sigma \rangle & \texttt{(CloV)} &
\end{array}
$$

The big-step operational (natural) semantics of MFAE is defined as follows:

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\texttt{Num}\ \frac{}{\sigma, M \vdash n \Rightarrow n, M} \qquad \texttt{Add}\ \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M_1 \qquad \sigma, M_1 \vdash e_2 \Rightarrow n_2, M_2}{\sigma, M \vdash e_1 + e_2 \Rightarrow n_1 + n_2, M_2}$$

$$\texttt{Mul}\ \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M_1 \qquad \sigma, M_1 \vdash e_2 \Rightarrow n_2, M_2}{\sigma, M \vdash e_1 * e_2 \Rightarrow n_1 \times n_2, M_2} \qquad \texttt{Id}\ \frac{x \in \text{Domain}(\sigma)}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M}$$

$$\texttt{Var}\ \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M_1 \qquad a \notin \text{Domain}(M_1) \qquad \sigma[x \mapsto a], M_1[a \mapsto v_1] \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash \texttt{var } x = e_1;\ e_2 \Rightarrow v_2, M_2}$$

$$\texttt{Fun}\ \frac{}{\sigma, M \vdash \lambda x.e \Rightarrow \langle \lambda x.e, \sigma \rangle, M}$$

$$\texttt{App}\ \frac{\begin{array}{ll}\sigma, M \vdash e_1 \Rightarrow \langle \lambda x.e_3, \sigma' \rangle, M_1 & \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \\ a \notin \text{Domain}(M_2) & \sigma'[x \mapsto a], M_2[a \mapsto v_2] \vdash e_3 \Rightarrow v_3, M_3\end{array}}{\sigma, M \vdash e_1(e_2) \Rightarrow v_3, M_3}$$

$$\texttt{Assign}\ \frac{\sigma, M \vdash e \Rightarrow v, M' \qquad x \in \text{Domain}(\sigma)}{\sigma, M \vdash x = e \Rightarrow v, M'[\sigma(x) \mapsto v]}$$

$$\texttt{Seq}\ \frac{\sigma, M \vdash e_1 \Rightarrow \_, M_1 \qquad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1;\ e_2 \Rightarrow v_2, M_2}$$

4.1 Call-By-Reference (CBR) semantics

The above semantics is defined with **call-by-value (CBV)** evaluation strategy. We can augment it with **call-by-reference (CBR)** evaluation strategy by replacing the rule for function application with the following two rules:

$$\textsf{App}_x \ \frac{\sigma, M \vdash e_1 \Rightarrow \langle \lambda x'.e_2, \sigma' \rangle, M_1 \qquad x \in \text{Domain}(\sigma) \qquad \sigma'[x' \mapsto \sigma(x)], M_1 \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1(x) \Rightarrow v_2, M_2}$$

$$\textsf{App} \ \frac{\begin{array}{ll}\sigma, M \vdash e_1 \Rightarrow \langle \lambda x.e_3, \sigma' \rangle, M_1 & \forall x'.e_2 \neq x' \qquad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \\ a \notin \text{Domain}(M_2) & \sigma'[x \mapsto a], M_2[a \mapsto v_2] \vdash e_3 \Rightarrow v_3, M_3\end{array}}{\sigma, M \vdash e_1(e_2) \Rightarrow v_3, M_3}$$