# MAGNET – Mutable Arithmetic Expressions with Generators and Exceptions

# 1 INTRODUCTION

MAGNET is a toy language for the COSE212 course at Korea University. MAGNET stands for the **M**utable **A**rithmetic Expressions with **Gen**erators and **E**xceptions, and it supports the following features:

- undefined value (undefined):
- number values (0, 1, -1, 2, -2, 3, -3, ...)
- boolean values (true and false)
- arithmetic operators: negation (-), addition (+), subtraction (-), multiplication (\*), division (/), and modulo (%)
- logical operators: conjunction (&&), disjunction (||), and negation (!)
- **comparison operators**: equality (== and !=) and relational (<, >, <=, and >=)
- mutable variable definitions (var) and identifier lookup (x)
- variable assignment (=) and sequences (;)
- augmented assignment (+=, -=, \*=, /=, and %=) and increment/decrement (++ and --)
- conditionals (if-else), while loops (while), break (break), and continue (continue)
- first-class functions (=> or function)
- function applications and return (return)
- try-catch (try-catch) and throw (throw)
- generators (=>\* or function\*) and yield (yield)
- iterator next (\_.next) and iterator result accessors (\_.value and \_.done)
- for-of loops (for-of)

This document is the specification of MAGNET. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax with the desugaring rules. Then, Section 4 describes the small-step operational (reduction) semantics of MAGNET.

# **2 CONCRETE SYNTAX**

The concrete syntax of MAGNET is written in a variant of the extended Backus–Naur form (EBNF). The notation <nt> denotes a nonterminal, and "t" denotes a terminal. We use? to denote an optional element and + (or \*) to denote one or more (or zero or more) repetitions of the preceding element. We use butnot to denote a set difference to exclude some strings from a producible set of strings. We omit some obvious terminals using the ellipsis (...) notation.

```
// expressions
<expr> ::= "undefined" | <number> | "true" | "false"
        // unary and binary operators
        | <uop> <expr> | <expr> <bop> <expr>
        // parentheses
        | "(" <expr> ")" | "{" <expr> "}"
        // mutable variable definitions
        "var" <id> "=" <expr> ";" <expr> | <id>
        // variable (augmented) assignment and sequence
        | <id> <aop> <expr> | <expr> ";" <expr>
        // increment and decrement
        // conditionals and loops
        | "if" "(" <expr> ")" <expr> "else" <expr>
        | "while" "(" <expr> ")" <expr>
        // first-class functions
        | <params> "=>" <expr> | "function" <params> "{" <expr> "}" <expr>
        // function applications and returns
        | <expr> "(" <expr> ")" | "return" <expr>
        // try-catch and throw
        | "try" <expr> "catch" "(" <id> ")" <expr> | "throw" <expr>
        // generators and yields
        <params> "=>" "*" <expr>
        | "function" "*" <params> "{" <expr> "}" <expr> | "yield" <expr>
        // iterator next and iterator result accessors
        | <expr> "." "next" "(" <expr>? ")"
        | <expr> "." "value" | <expr> "." "done"
// operators
<aop> ::= "=" | "+=" | "-=" | "*=" | "/=" | "%="
<uop> ::= "-" | "!"
<bop> ::= "+" | "-" | "*" | "/" | "%" | "&&" | "||"
       | "==" | "!=" | "<" | "<=" | ">" | ">="
// parameters
<params> ::= "(" ")" | "(" <id> ")" | "(" <id> ["," <id>]* ")"
```

The precedence and associativity of operators are defined as follows:

Description	Operator	Precedence	Associativity
Postfix Unary	++,,next,value,done	1	left
Prefix Unary	-, !, ++,	2	right
Multiplicative	*, /, %	3	left
Additive	+, -	4	
Relational	<, <=, >, >=	5	
Equality	==, !=	6	
Logical Conjunction	&&	7	
Logical Disjunction		8	
Assignment	=, +=, -=, *=, /=, %=, var	9	right
Sequence	;	10	left

#### 3 ABSTRACT SYNTAX

The abstract syntax of MAGNET is defined as follows:

```
Expressions \mathbb{E} \ni e ::= \text{undefined}
                                                        | if (e) e else e
                                      (EUndef)
                                                                               (EIf)
                                      (ENum)
                                                        | while (e) e
                                                                               (EWhile)
                                      (EBool)
                                                        break
                                                                               (EBreak)
                                                                               (EContinue)
                                      (Add)
                                                        continue
                       | e * e
                                      (Mul)
                                                        |\lambda(x,\ldots,x).e|
                                                                               (EFun)
                       |e|/e
                                      (EDiv)
                                                        |e(e,\ldots,e)|
                                                                               (EApp)
                       l e % e
                                      (EMod)
                                                        | return e
                                                                               (EReturn)
                       |e| == e
                                                        | try e catch (x) e
                                      (EEq)
                                                                              (ETry)
                       |e| < e
                                      (ELt)
                                                        | throw e
                                                                               (EThrow)
                                                        |\lambda*(x,\ldots,x).e|
                       | var x=e; e |
                                      (EVar)
                                                                               (EGen)
                                                        |e.next(e^?)|
                                                                               (EIterNext)
                                      (EId)
                                                        | yield e
                       |x=e|
                                      (EAssign)
                                                                               (EYield)
                       | e; e
                                                        |e.value|
                                                                               (EValueField)
                                      (ESeq)
                                                        |e.done|
                                                                               (EDoneField)
```

$$\text{where } \left\{ \begin{array}{ll} \text{Identifier} & x \in \mathbb{X} & (\texttt{String}) \\ \text{Number} & n \in \mathbb{Z} & (\texttt{BigInt}) \end{array} \right. \text{Boolean} \quad \mathbb{B} \ni b ::= \mathsf{true} \mid \mathsf{false} \quad (\mathsf{Boolean})$$

The semantics of the remaining cases are defined with the following desugaring rules:

```
 \mathcal{D} \llbracket -e \rrbracket = \mathcal{D} \llbracket e_1 \rrbracket * (-1) \qquad \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket e_1 - e_2 \rrbracket = \mathcal{D} \llbracket e_1 \rrbracket + \mathcal{D} \llbracket - e_2 \rrbracket \qquad \mathcal{D} \llbracket x - = e \rrbracket = x = x - \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket e_1 \& \& e_2 \rrbracket = \mathrm{if} \left( \mathcal{D} \llbracket e_1 \rrbracket \right) \mathcal{D} \llbracket e_2 \rrbracket \text{ else false } \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket e_1 \& \& e_2 \rrbracket = \mathrm{if} \left( \mathcal{D} \llbracket e_1 \rrbracket \right) \text{ true else } \mathcal{D} \llbracket e_2 \rrbracket \qquad \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket e_1 \& \& e_2 \rrbracket = \mathrm{if} \left( \mathcal{D} \llbracket e_1 \rrbracket \right) \text{ true else } \mathcal{D} \llbracket e_2 \rrbracket \qquad \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + x = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + x = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + x = e \rrbracket = x = x + \mathcal{D} \llbracket e \rrbracket \\ \mathcal{D} \llbracket x + x = e \rrbracket = x = x + x + \mathcal{
```

$$\mathcal{D}[\![\![\!]\!]\!] \text{for } (x \text{ of } e_1) e_2]\!] = \mathcal{D}\begin{bmatrix} \text{var } \underline{x_1} = e_1; \\ \text{var } \underline{x_2} = \underline{x_1}.\mathsf{next()}; \\ \text{while } (! \ \underline{x_2}.\mathsf{done}) \{ \\ \text{var } x = \underline{x_2}.\mathsf{value}; \\ e_2; \ \underline{x_2} = \underline{x_1}.\mathsf{next()} \end{bmatrix}$$

where  $\underline{x_k}$  denotes a fresh temporary variable. All the omitted cases recursively apply the desugaring rule to their sub-expressions. For example,  $\mathcal{D}[\![e_1 + e_2]\!] = \mathcal{D}[\![e_1]\!] + \mathcal{D}[\![e_2]\!]$ .

#### 4 SEMANTICS

We use the following notations in the semantics:

```
\langle \kappa \mid \mid s \mid \mid H \mid \mid M \rangle \in \mathbb{K} \times \mathbb{S} \times \mathbb{H} \times \mathbb{M} (State)
                   Continuations
                                                               \kappa \in \mathbb{K}
                                                                                                (Cont)
                                                               \kappa ::= \Box \mid i :: \kappa
Instructions i \in \mathbb{I}
                                                            (Inst)
                    i := (\sigma \vdash e)
                                                            (IEval)
                                                                                    |jmp-if[\psi]|
                                                                                                         (IJmpIf)
                         (+)
                                                            (IAdd)
                                                                                    | jmp[c]
                                                                                                          (IJmp)
                         |(*)
                                                                                    | call[n]
                                                                                                          (ICall)
                                                            (IMul)
                         +(/)
                                                            (IDiv)
                                                                                    return
                                                                                                          (IReturn)
                          (%)
                                                            (IMod)
                                                                                    next
                                                                                                          (INext)
                          | (==)
                                                            (IEq)
                                                                                    |yield
                                                                                                          (IYield)
                          (<)
                                                            (ILt)
                                                                                    | value
                                                                                                          (IValueField)
                          | def[x,...,x][\sigma \vdash e]
                                                            (IDef)
                                                                                    done
                                                                                                          (IDoneField)
                          |write[a]
                                                            (IWrite)
                                                                                    pop
                                                                                                          (IPop)
                                   Value Stacks s \in \mathbb{S}
                                                                                (Stack)
                                                        s := \blacksquare \mid v :: s
   Values v \in \mathbb{V}
                                                   (Value)
                v := undefined
                                                   (UndefV)
                                                                           |\langle \kappa || s || H \rangle
                                                                                                             (ContV)
                     \mid n \mid
                                                   (NumV)
                                                                           |\langle \lambda *(x,\ldots,x).e,\sigma \rangle|
                                                                                                             (GenV)
                                                                           | iter[a]
                                                   (BoolV)
                                                                                                             (IterV)
                      |\langle \lambda(x,\ldots,x).e,\sigma\rangle|
                                                                                                            (ResultV)
                                                  (CloV)
                                                                           | \{ value : v, done : b \} 
                           Control Handlers H \in \mathbb{H} = \mathbb{C} \xrightarrow{\text{fin}} \Psi (Handler)
        Control Operators
                                     c \in \mathbb{C}
                                                              (Control)
                                                              (Return)
                                     c := return
                                                                                          | throw
                                                                                                       (Throw)
                                                              (Break)
                                                                                          | throw
                                                                                                       (Finally)
                                           break
                                           | continue (Continue)
                                                                                          |yield
                Continuation Values \psi, \langle \kappa \mid \mid s \mid \mid H \rangle \in \Psi = \mathbb{K} \times \mathbb{S} \times \mathbb{H} (KValue)
                                                                M \in \mathbb{M} = \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}
                                                                                                  (Mem)
                Memories
                                                                 \sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{A}
                Environments
                                                                                                  (Env)
                Addresses
                                                                  a \in \mathbb{A}
                                                                                                  (Addr)
```

The small-step operational (reduction) semantics of MAGNET is defined in the following form of the reduction relation  $(\rightarrow)$ :

$$| \langle \kappa || s || H || M \rangle \rightarrow \langle \kappa || s || H || M \rangle$$

#### 4.1 Reduction Relations for IEval

```
EUndef \langle (\sigma \vdash \mathsf{undefined}) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle \rightarrow \langle \kappa \mid \mid \mathsf{undefined} :: s \mid \mid H \mid \mid M \rangle
                        \langle (\sigma \vdash n) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                        \rightarrow \langle \kappa \mid \mid n :: s \mid \mid H \mid \mid M \rangle
ENum
EBool
                        \langle (\sigma \vdash b) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                        \rightarrow \langle \kappa \mid \mid b :: s \mid \mid H \mid \mid M \rangle
EAdd
                        \langle (\sigma \vdash e_1 + e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                        \rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (+) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
EMul
                        \langle (\sigma \vdash e_1 * e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                        \rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (\star) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
EDiv
                        \langle (\sigma \vdash e_1 / e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                        \rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (/) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
EMod
                        \langle (\sigma \vdash e_1 \% e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                       \rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (\%) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                        \langle (\sigma \vdash e_1 == e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
EEa
                                                                                                                       \rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (==) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
ELt
                        \langle (\sigma \vdash e_1 \lessdot e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                       \rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (<) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
EVar
                        \langle (\sigma \vdash \mathsf{var} \ x = e_1; \ e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \rightarrow \langle (\sigma \vdash e_1) :: \mathsf{def}[x][\sigma \vdash e_2] :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
EId
                        \langle (\sigma \vdash x) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                                                                                                                      \rightarrow \langle \kappa \mid \mid M(\sigma(x)) :: s \mid \mid H \mid \mid M \rangle
EAssign \langle (\sigma \vdash x=e) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle
                                                                                                                        \rightarrow \langle (\sigma \vdash e) :: write(\sigma(x)) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle
                        \langle (\sigma \vdash e_1; e_2) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle
                                                                                                                    \rightarrow \langle (\sigma \vdash e_1) :: \mathsf{pop} :: (\sigma \vdash e_2) :: \kappa \mid | s \mid | H \mid | M \rangle
ESeq
```

# 4.1.1 Conditionals and While Loops.

EIf 
$$\langle (\sigma \vdash if (e_1) e_2 \text{ else } e_3) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle$$
  $\langle (\sigma \vdash e_1) :: jmp-if[\langle (\sigma \vdash e_2) :: \kappa \mid\mid s \mid\mid H \rangle] :: (\sigma \vdash e_3) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle$  EWhile  $\langle (\sigma \vdash while (e_1) e_2) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle$   $\langle (\sigma \vdash e_1) :: jmp-if[\psi_{body}] :: \kappa \mid\mid undefined :: s \mid\mid H \mid\mid M \rangle$  
$$\begin{cases} \psi_{body} &= \langle (\sigma \vdash e_2) :: jmp[continue] :: \Box \mid\mid s \mid\mid H_{body} \rangle \\ H_{body} &= H[continue \mapsto \psi_{continue}, break \mapsto \psi_{break}] \\ \psi_{continue} &= \langle pop :: (\sigma \vdash while (e_1) e_2) :: \kappa \mid\mid s \mid\mid H \rangle \end{cases} \\ \text{EBreak} & \langle (\sigma \vdash break) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \\ \rightarrow & \langle jmp[break] :: \Box \mid\mid undefined :: s \mid\mid H \mid\mid M \rangle \end{cases}$$

$$EContinue & \langle (\sigma \vdash continue) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \\ \downarrow_{jmp}[continue] :: \Box \mid\mid undefined :: s \mid\mid H \mid\mid M \rangle \end{cases}$$

#### 4.1.2 Functions and Return.

$$\begin{split} & \mathsf{EFun} \qquad \langle (\sigma \vdash \lambda(x,\ldots,x).e) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \\ & \to \qquad \langle \kappa \mid\mid \langle \lambda(x,\ldots,x).e,\sigma \rangle :: s \mid\mid H \mid\mid M \rangle \\ & \mathsf{EApp} \qquad \langle (\sigma \vdash e(e_1,\ldots,e_n)) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \\ & \to \qquad \langle (\sigma \vdash e) :: (\sigma \vdash e_1) :: \ldots :: (\sigma \vdash e_n) :: \mathsf{call}[n] :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \\ & \to \qquad \langle (\sigma \vdash e) :: \mathsf{return} \: e) :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \\ & \to \qquad \langle (\sigma \vdash e) :: \mathsf{return} :: \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \end{aligned}$$

# 4.1.3 Exceptions.

$$\begin{split} \mathsf{ETry} & \quad \langle (\sigma \vdash \mathsf{try} \ e_1 \ \mathsf{catch} \ (x) \ e_2) \ :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle \\ \to & \quad \langle (\sigma \vdash e_1) \ :: \ \mathsf{jmp[finally]} \ :: \ \square \mid \mid s \mid \mid H_{\mathsf{body}} \mid \mid M \rangle \\ & \quad \mathsf{where} \left\{ \begin{array}{l} H_{\mathsf{body}} \ = \ H[\mathsf{throw} \mapsto \psi_{\mathsf{throw}}, \mathsf{finally} \mapsto \psi_{\mathsf{finally}}] \\ \psi_{\mathsf{throw}} \ = \ \langle \mathsf{def[}x\mathsf{]} [\sigma \vdash e_2] \ :: \kappa \mid \mid s \mid \mid H \rangle \\ \psi_{\mathsf{finally}} \ = \ \langle \kappa \mid \mid s \mid \mid H \rangle \\ \end{array} \right. \\ & \quad \mathsf{EThrow} \quad \langle (\sigma \vdash \mathsf{throw} \ e) \ :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle \\ \to & \quad \langle (\sigma \vdash e) \ :: \ \mathsf{jmp[throw]} \ :: \ \square \mid s \mid \mid H \mid \mid M \rangle \\ \end{split}$$

#### 4.1.4 Generators.

EGen 
$$\langle (\sigma \vdash \lambda*(x_1,\ldots,x_n).e) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$$
 $\rightarrow \langle \kappa \mid \mid \langle \lambda*(x_1,\ldots,x_n).e,\sigma \rangle :: s \mid \mid H \mid \mid M \rangle$ 

EIterNext<sub>1</sub>  $\langle (\sigma \vdash e_1.\operatorname{next}()) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 
 $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash \operatorname{undefined}) :: \operatorname{next} :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 

EIterNext<sub>2</sub>  $\langle (\sigma \vdash e_1.\operatorname{next}(e_2)) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 
 $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: \operatorname{next} :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 

EYield  $\langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: \operatorname{next} :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 
 $\rightarrow \langle (\sigma \vdash e) :: \operatorname{yield} :: \Box \mid \mid \operatorname{false} :: \psi_{\operatorname{next}} :: s \mid \mid H \mid \mid M \rangle$ 

where  $\psi_{\operatorname{next}} = \langle \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 

EValueField  $\langle (\sigma \vdash e.\operatorname{value}) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 

EValueField  $\langle (\sigma \vdash e.\operatorname{value}) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 

EDoneField  $\langle (\sigma \vdash e.\operatorname{done}) :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 
 $\rightarrow \langle (\sigma \vdash e) :: \operatorname{done} :: \kappa \mid \mid s \mid \mid H \mid \mid M \rangle$ 

#### 4.2 Reduction Relations for Other Instructions

$$\begin{array}{llll} \text{IAdd} & \langle (+) :: \kappa \mid \mid n_2 :: n_1 :: s \mid \mid H \mid \mid M \rangle & \rightarrow & \langle \kappa \mid \mid (n_1 + n_2) :: s \mid \mid H \mid \mid M \rangle \\ \text{IMul} & \langle (\star) :: \kappa \mid \mid n_2 :: n_1 :: s \mid \mid H \mid \mid M \rangle & \rightarrow & \langle \kappa \mid \mid (n_1 \star n_2) :: s \mid \mid H \mid \mid M \rangle \\ \text{IDiv} & \langle (/) :: \kappa \mid \mid n_2 :: n_1 :: s \mid \mid H \mid \mid M \rangle & \rightarrow & \langle \kappa \mid \mid (n_1 \star n_2) :: s \mid \mid H \mid \mid M \rangle \\ \text{IMod} & \langle (\%) :: \kappa \mid \mid n_2 :: n_1 :: s \mid \mid H \mid \mid M \rangle & \rightarrow & \langle \kappa \mid \mid (n_1 \star n_2) :: s \mid \mid H \mid \mid M \rangle \\ \text{IEq} & \langle (==) :: \kappa \mid \mid v_2 :: v_1 :: s \mid \mid H \mid \mid M \rangle & \rightarrow & \langle \kappa \mid \mid (n_1 \star n_2) :: s \mid \mid H \mid \mid M \rangle \\ \text{ILt} & \langle (<) :: \kappa \mid \mid n_2 :: n_1 :: s \mid \mid H \mid \mid M \rangle & \rightarrow & \langle \kappa \mid \mid (n_1 \star n_2) :: s \mid \mid H \mid \mid M \rangle \end{array}$$

$$\text{where eq}(v_1,v_2) = \left\{ \begin{array}{ll} \text{true} & \text{if } v_1 = v_2 = a \\ \text{true} & \text{if } v_1 = v_2 = a \\ \text{true} & \text{if } v_1 = v_2 = n \\ \text{true} & \text{if } v_1 = v_2 = b \end{array} \right. \qquad \text{true} \quad \begin{array}{ll} \text{if } v_1 = v_2 = \text{undefined} \\ \text{true} & \text{if } v_1 = v_2 = \{\text{value}: v, \text{done}: b\} \\ \text{false} & \text{otherwise} \end{array}$$

IDef 
$$\langle \operatorname{def}[x_1,\ldots,x_n] [\sigma \vdash e] :: \kappa \mid\mid v_n :: \ldots :: v_1 :: s \mid\mid H \mid\mid M \rangle$$
 $\rightarrow \langle \sigma[x_1 \mapsto a_1,\ldots,x_n \mapsto a_n] \vdash e :: \kappa \mid\mid s \mid\mid H \mid\mid M[a_1 \mapsto v_1,\ldots,a_n \mapsto v_n] \rangle$ 

where  $\forall 1 \leq p \leq n. \ a_p \notin \operatorname{Domain}(M) \land (\forall 1 \leq q < p. \ a_q \neq a_p)$ 

IWrite  $\langle \operatorname{write}[a] :: \kappa \mid\mid v :: s \mid\mid H \mid\mid M \rangle$ 
 $\rightarrow \langle \kappa \mid\mid v :: s \mid\mid H \mid\mid M \rangle$ 

IPop  $\langle \operatorname{pop} :: \kappa \mid\mid v :: s \mid\mid H \mid\mid M \rangle$ 
 $\rightarrow \langle \kappa \mid\mid s \mid\mid H \mid\mid M \rangle$ 

# 4.2.1 Control Flow Instructions.

$$\begin{split} \operatorname{IJmpIf}_{\operatorname{true}} \ \langle \operatorname{jmp-if[}\langle \kappa \mid \mid s \mid \mid H \rangle ] &:: \_ \mid \mid \operatorname{true} :: \_ \mid \mid \_ \mid \mid M \rangle \to \langle \kappa \mid \mid s \mid \mid H \mid \mid M \rangle \\ \operatorname{IJmpIf}_{\operatorname{false}} \ \langle \operatorname{jmp-if[}\_] &:: \kappa \mid \mid \operatorname{false} :: s \mid \mid H \mid \mid M \rangle \\ \operatorname{IJmp} \ \ \langle \operatorname{jmp[}c] &:: \kappa \mid \mid v :: s \mid \mid H \mid \mid M \rangle \\ \\ \operatorname{Where} \left\{ \begin{array}{l} H(c) = \langle \kappa' \mid \mid s' \mid \mid H' \rangle \\ H'' = \begin{cases} H'[\operatorname{yield} \mapsto H(\operatorname{yield})] & \text{if yield} \in \operatorname{Domain}(H) \\ H' & \text{otherwise} \end{cases} \right. \end{split}$$

# 4.2.2 Function Call/Return Instructions.

$$\begin{array}{ll} \operatorname{ICall}_{\lambda} & \langle \operatorname{call}[n] :: \kappa \mid \mid v_n :: \ldots :: v_1 :: \langle \lambda(x_1, \ldots, x_m).e, \sigma' \rangle :: s \mid \mid H \mid \mid M \rangle \\ & \rightarrow & \langle \operatorname{def}[x_1, \ldots, x_m][\sigma' \vdash \operatorname{return} e] :: \square \mid \mid s_{\operatorname{body}} \mid \mid H_{\operatorname{body}} \mid \mid M \rangle \\ & \\ & \text{where} \left\{ \begin{array}{ll} s_{\operatorname{body}} &= \left\{ \begin{array}{ll} v_m :: \ldots :: v_1 :: \blacksquare & \text{if } n >= m \\ \text{undefined} :: \ldots :: \text{undefined} :: v_n :: \ldots :: v_1 :: \blacksquare & \text{otherwise} \end{array} \right. \\ & \left\{ \begin{array}{ll} H_{\operatorname{body}} &= H[\operatorname{return} \mapsto \psi_{\operatorname{return}}] \setminus \{\operatorname{break}, \operatorname{continue}, \operatorname{yield} \} \\ \psi_{\operatorname{return}} &= \langle \kappa \mid \mid s \mid \mid H \rangle \end{array} \right. \\ & \text{ICall}_{\lambda^*} & \langle \operatorname{call}[n] :: \kappa \mid \mid v_n :: \ldots :: v_1 :: \langle \lambda^*(x_1, \ldots, x_m).e, \sigma' \rangle :: s \mid \mid H \mid \mid M \rangle \\ & \rightarrow & \langle \kappa \mid | \operatorname{iter}[a] :: s \mid \mid H \mid \mid M[a \mapsto \psi_{\operatorname{body}}] \rangle \end{array} \\ & \text{where} & \left\{ \begin{array}{ll} a \not \in \operatorname{Domain}(M) \\ \psi_{\operatorname{body}} &= \langle \kappa_{\operatorname{body}} \mid \mid s_{\operatorname{body}} \mid \mid \varnothing \rangle \\ \kappa_{\operatorname{body}} &= \operatorname{pop} :: \operatorname{def}[x_1, \ldots, x_m][\sigma' \vdash \operatorname{return}(\operatorname{try} e \operatorname{catch}(x) x)] :: \square \\ s_{\operatorname{body}} &= \left\{ \begin{array}{ll} v_m :: \ldots :: v_1 :: \blacksquare & \operatorname{if} n >= m \\ \operatorname{undefined} :: \ldots :: \operatorname{undefined} :: v_n :: \ldots :: v_1 :: \blacksquare & \operatorname{otherwise} \\ x \operatorname{could} \operatorname{be} \operatorname{any} \operatorname{identifier}. \end{array} \right. \end{array} \right.$$

$$\begin{array}{ll} \text{IReturn} & \langle \text{return} :: \kappa \mid \mid v :: s \mid \mid H \mid \mid M \rangle \\ \\ \rightarrow & \left\{ \begin{array}{ll} \langle \text{yield} :: \square \mid \mid v :: \text{true} :: \psi_{\text{done}} :: s \mid \mid H \mid \mid M \rangle & \text{if yield} \in \text{Domain}(H) \\ \\ \langle \text{jmp[return]} :: \square \mid \mid v :: s \mid \mid H \mid \mid M \rangle & \text{otherwise} \end{array} \right.$$

#### 4.2.3 Generator Instructions.

INext 
$$\langle \operatorname{next} :: \kappa \mid \mid v :: \operatorname{iter}[a] :: s \mid \mid H \mid \mid M \rangle$$

$$\langle \kappa' \mid \mid v :: s' \mid \mid H_{\operatorname{body}} \mid \mid M \rangle$$

$$\operatorname{where} \left\{ \begin{array}{l} M(a) = \langle \kappa' \mid \mid s' \mid \mid H' \rangle \\ H_{\operatorname{body}} = H'[\operatorname{yield} \mapsto \psi, \operatorname{return} \mapsto \psi] \\ \psi = \langle \kappa \mid \mid \operatorname{ter}[a] :: s \mid \mid H \rangle \end{array} \right.$$

$$\operatorname{IYield} \qquad \langle \operatorname{yield} :: \_ \mid \mid v :: b :: v' :: \_ \mid \mid H \mid \mid M \rangle \\ \langle \kappa' \mid \mid \{ \operatorname{value} : v, \operatorname{done} :: b \} :: s' \mid \mid H' \mid \mid M[a \mapsto v'] \rangle$$

$$\operatorname{where} \quad H(\operatorname{yield}) = \langle \kappa' \mid \mid \operatorname{iter}[a] :: s' \mid \mid H' \rangle$$

$$\operatorname{IValueField} \quad \langle \operatorname{value} :: \kappa \mid \mid \{ \operatorname{value} : v, \operatorname{done} :\_ \} :: s \mid \mid H \mid \mid M \rangle$$

$$\langle \kappa \mid \mid v :: s \mid \mid H \mid \mid M \rangle$$

$$\operatorname{IDoneField} \quad \langle \operatorname{done} :: \kappa \mid \mid \{ \operatorname{value} :\_, \operatorname{done} : b \} :: s \mid \mid H \mid \mid M \rangle$$

$$\langle \kappa \mid \mid b :: s \mid \mid H \mid \mid M \rangle$$

And  $\rightarrow^*$  is the reflexive-transitive closure of  $\rightarrow$  and denotes the repeated reduction:

$$\langle \kappa \mid \mid s \mid \mid H \mid \mid M \rangle \rightarrow^* \langle \kappa' \mid \mid s' \mid \mid H' \mid \mid M' \rangle$$

$$\frac{\langle \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \rightarrow^* \langle \kappa' \mid\mid s' \mid\mid H' \mid\mid M' \rangle \qquad \langle \kappa' \mid\mid s' \mid\mid H' \mid\mid M' \rangle \rightarrow^* \langle \kappa'' \mid\mid s'' \mid\mid H'' \mid\mid M'' \rangle}{\langle \kappa \mid\mid s \mid\mid H \mid\mid M \rangle \rightarrow^* \langle \kappa'' \mid\mid s'' \mid\mid H'' \mid\mid M'' \rangle}$$

The evaluation result of an expression e is the value v if

$$\langle (\varnothing \vdash e) :: \Box \mid \mid \blacksquare \mid \mid \varnothing \mid \mid \varnothing \rangle \rightarrow^* \langle \Box \mid \mid v :: \blacksquare \mid \mid \_ \mid \mid \_ \rangle$$