VAE – AE with Variables

1 INTRODUCTION

VAE is a toy language for the COSE212 course at Korea University. VAE stands for an extension of the AE language with **variables**, and it supports the following features:

- **number (integer) values** (`0`, `1`, `-1`, `2`, `-2`, `3`, `-3`, ...)
- **arithmetic operators**: addition (`+`) and multiplication (`*`)
- **immutable variable definitions** (`val`)

This document is the specification of VAE. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax. Then, Section 4 describes the big-step operational (natural) semantics of VAE.

2 CONCRETE SYNTAX

The concrete syntax of VAE is written in a variant of the extended Backus–Naur form (EBNF). The notation `<nt>` denotes a nonterminal, and `"t"` denotes a terminal. We use `?` to denote an optional element and `+` (or `*`) to denote one or more (or zero or more) repetitions of the preceding element. We use `butnot` to denote a set difference to exclude some strings from a producible set of strings. We omit some obvious terminals using the ellipsis (...) notation.

```
// basic elements
<digit>    ::= "0" | "1" | "2" | ... | "9"
<number>   ::= "-"? <digit>+
<alphabet> ::= "A" | "B" | "C" | ... | "Z" | "a" | "b" | "c" | ... | "z"
<idstart>  ::= <alphabet> | "_"
<idcont>   ::= <alphabet> | "_" | <digit>
<keyword>  ::= "val"
<id>       ::= <idstart> <idcont>* butnot <keyword>

// expressions
<expr>     ::= <number> | <expr> "+" <expr> | <expr> "*" <expr>
             | "(" <expr> ")" | "{" <expr> "}"
             | "val" <id> "=" <expr> ";" <expr> | <id>
```

The precedence and associativity of operators are defined as follows:

Operator	Associativity	Precedence
*	left	2
+	left	1

3 ABSTRACT SYNTAX

The abstract syntax of VAE is defined as follows:

Expressions $\mathbb{E} \ni e ::= n$ (Num)
$\mid e + e$ (Add)
$\mid e * e$ (Mul)
$\mid \texttt{val}\ x = e;\ e$ (Val)
$\mid x$ (Id)

where

Numbers $n \in \mathbb{Z}$ (BigInt)
Identifiers $x \in \mathbb{X}$ (String)

4 SEMANTICS

We use the following notations in the semantics:

$$\text{Environments} \quad \sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{Z} \quad (\texttt{Env})$$

The big-step operational (natural) semantics of VAE is defined as follows:

$$\boxed{\sigma \vdash e \Rightarrow n}$$

$$\texttt{Num}\ \frac{}{\vdash n \Rightarrow n} \qquad \texttt{Add}\ \frac{\vdash e_1 \Rightarrow n_1 \qquad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2} \qquad \texttt{Mul}\ \frac{\vdash e_1 \Rightarrow n_1 \qquad \vdash e_2 \Rightarrow n_2}{\vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$

$$\texttt{Val}\ \frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma[x \mapsto n_1] \vdash e_2 \Rightarrow n_2}{\sigma \vdash \texttt{val}\ x = e_1;\ e_2 \Rightarrow n_2} \qquad \texttt{Id}\ \frac{x \in \text{Domain}(\sigma)}{\sigma \vdash x \Rightarrow \sigma(x)}$$