TFAE – FAE with Type System

1 INTRODUCTION

TFAE is a toy language for the COSE212 course at Korea University. TFAE stands for an extension of the FAE language with a **type system**, and it supports the following features:

- **number (integer) values** (`0`, `1`, `-1`, `2`, `-2`, `3`, `-3`, ...)
- **arithmetic operators**: addition (`+`) and multiplication (`*`)
- **immutable variable definitions** (`val`)
- **first-class functions** (`=>`)
- **static type checking**

This document is the specification of TFAE. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax. Then, Section 4 describes the type system. Finally, Section 5 describes the big-step operational (natural) semantics of TFAE.

2 CONCRETE SYNTAX

The concrete syntax of TFAE is written in a variant of the extended Backus–Naur form (EBNF). The notation `<nt>` denotes a nonterminal, and `"t"` denotes a terminal. We use `?` to denote an optional element and `+` (or `*`) to denote one or more (or zero or more) repetitions of the preceding element. We use `butnot` to denote a set difference to exclude some strings from a producible set of strings. We omit some obvious terminals using the ellipsis (`...`) notation.

```
// basic elements
<digit>    ::= "0" | "1" | "2" | ... | "9"
<number>   ::= "-"? <digit>+
<alphabet> ::= "A" | "B" | "C" | ... | "Z" | "a" | "b" | "c" | ... | "z"
<idstart>  ::= <alphabet> | "_"
<idcont>   ::= <alphabet> | "_" | <digit>
<keyword>  ::= "val" | "Number"
<id>       ::= <idstart> <idcont>* butnot <keyword>

// expressions
<expr>     ::= <number> | <expr> "+" <expr> | <expr> "*" <expr>
             | "(" <expr> ")" | "{" <expr> "}"
             | "val" <id> "=" <expr> ";" <expr> | <id>
             | "(" <id> ":" <type> ")" "=>" <expr> | <expr> "(" <expr> ")"

// types
<type>     ::= "(" <type> ")" | "Number" | <type> "=>" <type>
```

For types, the arrow (`=>`) operator is right-associative. For expressions, the precedence and associativity of operators are defined as follows:

Operator	Associativity	Precedence
*	left	1
+	left	2

3 ABSTRACT SYNTAX

The abstract syntax of TFAE is defined as follows:

$$
\begin{array}{llll}
\text{Expressions} & \mathbb{E} \ni e ::= n & \texttt{(Num)} & \text{Numbers} \quad n \in \mathbb{Z} \quad \texttt{(BigInt)} \\
& \mid e + e & \texttt{(Add)} & \\
& \mid e * e & \texttt{(Mul)} & \text{Identifiers} \quad x \in \mathbb{X} \quad \texttt{(String)} \\
& \mid \texttt{val}\ x = e;\ e & \texttt{(Val)} & \\
& \mid x & \texttt{(Id)} & \text{Types} \quad \mathbb{T} \ni \tau ::= \texttt{num} \quad \texttt{(NumT)} \\
& \mid \lambda x{:}\tau.e & \texttt{(Fun)} & \qquad\qquad \mid \tau \to \tau \quad \texttt{(ArrowT)} \\
& \mid e(e) & \texttt{(App)} &
\end{array}
$$

4 TYPE SYSTEM

This section explains type system of TFAE, and we use the following notations:

$$\text{Type Environments} \quad \Gamma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{T} \quad \texttt{(TypeEnv)}$$

In the type system, type checking is defined with the following typing rules:

$$\boxed{\Gamma \vdash e : \tau}$$

$$\tau\text{–Num}\ \frac{}{\Gamma \vdash n : \texttt{num}}$$

$$\tau\text{–Add}\ \frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash e_1 + e_2 : \texttt{num}} \qquad \tau\text{–Mul}\ \frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash e_1 * e_2 : \texttt{num}}$$

$$\tau\text{–Val}\ \frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma[x : \tau_1] \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{val}\ x = e_1;\ e_2 : \tau_2} \qquad \tau\text{–Id}\ \frac{x \in \text{Domain}(\Gamma)}{\Gamma \vdash x : \Gamma(x)}$$

$$\tau\text{–Fun}\ \frac{\Gamma[x : \tau] \vdash e : \tau'}{\Gamma \vdash \lambda x{:}\tau.e : \tau \to \tau'} \qquad \tau\text{–App}\ \frac{\Gamma \vdash e_0 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_1 : \tau_1}{\Gamma \vdash e_0(e_1) : \tau_2}$$

5 SEMANTICS

We use the following notations in the semantics:

$$
\begin{array}{llll}
\text{Values} & \mathbb{V} \ni v ::= n & \texttt{(NumV)} & \text{Environments} \quad \sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{V} \quad \texttt{(Env)} \\
& \mid \langle \lambda x.e, \sigma \rangle & \texttt{(CloV)} &
\end{array}
$$

The big-step operational (natural) semantics of TFAE is defined as follows:

$$\boxed{\sigma \vdash e \Rightarrow v}$$

$$\texttt{Num}\ \frac{}{\sigma \vdash n \Rightarrow n} \qquad \texttt{Add}\ \frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma \vdash e_2 \Rightarrow n_2}{\sigma \vdash e_1 + e_2 \Rightarrow n_1 + n_2} \qquad \texttt{Mul}\ \frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma \vdash e_2 \Rightarrow n_2}{\sigma \vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$

$$\texttt{Val}\ \frac{\sigma \vdash e_1 \Rightarrow v_1 \qquad \sigma[x \mapsto v_1] \vdash e_2 \Rightarrow v_2}{\sigma \vdash \texttt{val}\ x = e_1;\ e_2 \Rightarrow v_2} \qquad \texttt{Fun}\ \frac{}{\sigma \vdash \lambda x{:}\tau.e \Rightarrow \langle \lambda x.e, \sigma \rangle}$$

$$\texttt{Id}\ \frac{x \in \text{Domain}(\sigma)}{\sigma \vdash x \Rightarrow \sigma(x)} \qquad \texttt{App}\ \frac{\sigma \vdash e_0 \Rightarrow \langle \lambda x.e_2, \sigma' \rangle \qquad \sigma \vdash e_1 \Rightarrow v_1 \qquad \sigma'[x \mapsto v_1] \vdash e_2 \Rightarrow v_2}{\sigma \vdash e_0(e_1) \Rightarrow v_2}$$