

# MAGNET – Mutable Arithmetic Expressions with Generators and Exceptions

## 1 INTRODUCTION

MAGNET is a toy language for the [COSE212](#) course at Korea University. MAGNET stands for the **M**utable **A**rithmetic **E**xpressions with **G**enerators and **E**xceptions, and it supports the following features:

- **undefined value** (undefined):
- **number (integer) values** (0, 1, -1, 2, -2, 3, -3, ...)
- **boolean values** (true and false)
- **arithmetic operators**: negation (-), addition (+), subtraction (-), multiplication (\*), division (/), and modulo (%)
- **logical operators**: conjunction (&&), disjunction (| |), and negation (!)
- **comparison operators**: equality (== and !=) and relational (<, >, <=, and >=)
- **mutable variable definitions** (var)
- **variable assignment** (=)
- **sequences** (;)
- **augmented assignment** (+=, -=, \*=, /=, and %=)
- **increment/decrement** (++ and --)
- **conditionals** (if-else)
- **while loops** (while)
- **loop controls** (break and continue)
- **first-class functions** (=> or function)
- **return** (return)
- **try-catch** (try-catch)
- **throw** (throw)
- **generators** (=>\* or function\*)
- **yield** (yield)
- **iterator operations** (\_.next, \_.value, and \_.done)
- **for-of loops** (for-of)

This document is the specification of MAGNET. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax with the desugaring rules. Then, Section 4 describes the small-step operational (reduction) semantics of MAGNET.

## 2 CONCRETE SYNTAX

The concrete syntax of MAGNET is written in a variant of the extended Backus–Naur form (EBNF). The notation `<nt>` denotes a nonterminal, and `"t"` denotes a terminal. We use `?` to denote an optional element and `+` (or `*`) to denote one or more (or zero or more) repetitions of the preceding element. We use **butnot** to denote a set difference to exclude some strings from a producible set of strings. We omit some obvious terminals using the ellipsis (`...`) notation.

```
// basic elements
<digit>      ::= "0" | "1" | "2" | ... | "9"
<number>     ::= "-"? <digit>+
<alphabet>  ::= "A" | "B" | "C" | ... | "Z" | "a" | "b" | "c" | ... | "z"
```

```

<idstart> ::= <alphabet> | "_"
<idcont>  ::= <alphabet> | "_" | <digit>
<keyword> ::= "break" | "catch" | "continue" | "else" | "false"
           | "for" | "function" | "if" | "of" | "return" | "throw"
           | "true" | "try" | "undefined" | "var" | "while" | "yield"
<id>      ::= <idstart> <idcont>* butnot <keyword>

```

```

// expressions
<expr> ::= "undefined" | <number> | "true" | "false"
        // unary/binary operators and parentheses
        | <uop> <expr> | <expr> <bop> <expr> | "(" <expr> ")" | "{" <expr> "}"
        // mutable variable definitions
        "var" <id> "=" <expr> ";" <expr> | <id>
        // variable (augmented) assignment and sequence
        | <id> <aop> <expr> | <expr> ";" <expr>?
        // increment and decrement
        | "++" <id> | "--" <id> | <id> "++" | <id> "--"
        // conditionals and loops
        | "if" "(" <expr> ")" <expr> "else" <expr>
        | "while" "(" <expr> ")" <expr>
        // first-class functions
        | <id> "=>" <expr> | <params> "=>" <expr>
        | "function" <id> <params> "{" <expr> "}" <expr>
        // function applications and returns
        | <expr> "(" <expr> ")" | "return" <expr>
        // try-catch and throw
        | "try" <expr> "catch" "(" <id> ")" <expr> | "throw" <expr>
        // generators and yields
        | <id> "=>" "*" <expr> | <params> "=>" "*" <expr>
        | "function" "*" <id> <params> "{" <expr> "}" <expr> | "yield" <expr>
        // iterator next and iterator result accessors
        | <expr> "." "next" "(" <expr>? ")"
        | <expr> "." "value" | <expr> "." "done"

// operators
<aop> ::= "=" | "+=" | "-=" | "*=" | "/=" | "%="
<uop> ::= "-" | "!"
<bop> ::= "+" | "-" | "*" | "/" | "%" | "&&" | "||"
        | "==" | "!=" | "<" | "<=" | ">" | ">="

// parameters
<params> ::= "(" ")" | "(" <id> ")" | "(" <id> [", " <id>]* ")"

```

The precedence and associativity of operators are defined as follows:

| Description         | Operator                        | Precedence | Associativity |
|---------------------|---------------------------------|------------|---------------|
| Postfix Unary       | ++, --, _.next, _.value, _.done | 1          | left          |
| Prefix Unary        | -, !, ++, --                    | 2          | right         |
| Multiplicative      | *, /, %                         | 3          | left          |
| Additive            | +, -                            | 4          |               |
| Relational          | <, <=, >, >=                    | 5          |               |
| Equality            | ==, !=                          | 6          |               |
| Logical Conjunction | &&                              | 7          |               |
| Logical Disjunction |                                 | 8          |               |
| Assignment          | =, +=, -=, *=, /=, %=, var      | 9          | right         |
| Sequence            | ;                               | 10         | left          |

### 3 ABSTRACT SYNTAX

The abstract syntax of MAGNET is defined as follows:

|             |                        |                    |                                   |
|-------------|------------------------|--------------------|-----------------------------------|
| Expressions | $\mathbb{E} \ni e ::=$ | undefined (EUnDef) | if (e) e else e (EIf)             |
|             | n                      | (ENum)             | while (e) e (EWhile)              |
|             | b                      | (EBool)            | break (EBreak)                    |
|             | e + e                  | (Add)              | continue (EContinue)              |
|             | e * e                  | (Mul)              | $\lambda(x, \dots, x).e$ (EFun)   |
|             | e / e                  | (EDiv)             | e(e, ..., e) (EApp)               |
|             | e % e                  | (EMod)             | return e (EReturn)                |
|             | e == e                 | (EEq)              | try e catch (x) e (ETry)          |
|             | e < e                  | (ELt)              | throw e (EThrow)                  |
|             | var x = e; e           | (EVar)             | $\lambda^*(x, \dots, x).e$ (EGen) |
|             | x                      | (EId)              | e.next( $e^?$ ) (EIterNext)       |
|             | x = e                  | (EAssign)          | yield e (EYield)                  |
|             | e; e                   | (ESeq)             | e.value (EValueField)             |
|             |                        |                    | e.done (EDoneField)               |

where  $\left\{ \begin{array}{ll} \text{Identifier} & x \in \mathbb{X} \quad (\text{String}) \\ \text{Number} & n \in \mathbb{Z} \quad (\text{BigInt}) \end{array} \right.$  Boolean  $\mathbb{B} \ni b ::= \text{true} \mid \text{false} \quad (\text{Boolean})$

The semantics of the remaining cases are defined with the following desugaring rules:

|   |   |
|---|---|
| $\mathcal{D}[-e] = \mathcal{D}[e] * (-1)$   | $\mathcal{D}[x += e] = x = x + \mathcal{D}[e]$  |
| $\mathcal{D}[e_1 - e_2] = \mathcal{D}[e_1] + \mathcal{D}[-e_2]$                                 | $\mathcal{D}[x -= e] = x = x - \mathcal{D}[e]$  |
| $\mathcal{D}[e_1 \&\& e_2] = \text{if } (\mathcal{D}[e_1]) \mathcal{D}[e_2] \text{ else false}$ | $\mathcal{D}[x *= e] = x = x * \mathcal{D}[e]$  |
| $\mathcal{D}[e_1    e_2] = \text{if } (\mathcal{D}[e_1]) \text{ true else } \mathcal{D}[e_2]$   | $\mathcal{D}[x /= e] = x = x / \mathcal{D}[e]$  |
| $\mathcal{D}[\text{! } e] = \text{if } (\mathcal{D}[e]) \text{ false else true}$                | $\mathcal{D}[x \%= e] = x = x \% \mathcal{D}[e]$  |
| $\mathcal{D}[e_1 != e_2] = \mathcal{D}[\text{! } (e_1 == e_2)]$                                 | $\mathcal{D}[\text{++ } x] = \mathcal{D}[x += 1]$   |
| $\mathcal{D}[e_1 <= e_2] = \text{var } \underline{x_1} = \mathcal{D}[e_1];$                     | $\mathcal{D}[\text{-- } x] = \mathcal{D}[x -= 1]$   |
| $\text{var } \underline{x_2} = \mathcal{D}[e_2];$   | $\mathcal{D}[x ++] = \text{var } \underline{x_1} = x; \mathcal{D}[x += 1]; \underline{x_1}$ |
| $\mathcal{D}[(\underline{x_1} == \underline{x_2})    (\underline{x_1} < \underline{x_2})]$      | $\mathcal{D}[x --] = \text{var } \underline{x_1} = x; \mathcal{D}[x -= 1]; \underline{x_1}$ |
| $\mathcal{D}[e_1 > e_2] = \mathcal{D}[\text{! } (e_1 <= e_2)]$                                  |   |
| $\mathcal{D}[e_1 >= e_2] = \mathcal{D}[\text{! } (e_1 < e_2)]$                                  |   |

$\mathcal{D}[\text{function } x (x_1, \dots, x_n) \{ e_1 \} e_2] = \text{var } x = (x_1, \dots, x_n) \Rightarrow e_1; e_2$   
 $\mathcal{D}[\text{function* } x (x_1, \dots, x_n) \{ e_1 \} e_2] = \text{var } x = (x_1, \dots, x_n) \Rightarrow^* e_1; e_2$

$$\mathcal{D}[\text{for } (x \text{ of } e_1) e_2] = \mathcal{D} \left[ \begin{array}{l} \text{var } \underline{x_1} = e_1; \\ \text{var } \underline{x_2} = \underline{x_1}.\text{next}(); \\ \text{while } (! \underline{x_2}.\text{done}) \{ \\ \quad \text{var } x = \underline{x_2}.\text{value}; \\ \quad e_2; \underline{x_2} = \underline{x_1}.\text{next}() \\ \} \end{array} \right]$$

where  $\underline{x_k}$  denotes a fresh temporary variable. All the omitted cases recursively apply the desugaring rule to their sub-expressions. For example,  $\mathcal{D}[e_1 + e_2] = \mathcal{D}[e_1] + \mathcal{D}[e_2]$ .

#### 4 SEMANTICS

We use the following notations in the semantics:

|                     |   |            |  |
|---------------------|---|------------|--|
| States              | $\langle \kappa \parallel s \parallel H \parallel M \rangle \in \mathbb{K} \times \mathbb{S} \times \mathbb{H} \times \mathbb{M}$ (State) |            |  |
| Continuations       | $\kappa \in \mathbb{K}$   | (Cont)     |  |
|                     | $\kappa ::= \square \mid i :: \kappa$   |            |  |
| Instructions        | $i \in \mathbb{I}$  | (Inst)     |  |
|                     | $i ::= (\sigma \vdash e)$   | (IEval)    | jmp-if[ $\psi$ ] (IJmpIf)                                |
|                     | (+)   | (IAdd)     | jmp[ $c$ ] (IJmp)  |
|                     | (*)   | (IMul)     | call[ $n$ ] (ICall)                                      |
|                     | (/)   | (IDiv)     | return (IReturn)   |
|                     | (%)   | (IMod)     | next (INext)   |
|                     | (==)  | (IEq)      | yield (IYield)   |
|                     | (<)   | (ILt)      | value (IValueField)                                      |
|                     | def[ $x, \dots, x$ ][ $\sigma \vdash e$ ]   | (IDef)     | done (IDoneField)  |
|                     | write[ $a$ ]  | (IWrite)   | pop (IPop)   |
| Value Stacks        | $s \in \mathbb{S}$  | (Stack)    |  |
|                     | $s ::= \blacksquare \mid v :: s$  |            |  |
| Values              | $v \in \mathbb{V}$  | (Value)    |  |
|                     | $v ::= \text{undefined}$  | (UndefV)   | $\langle \kappa \parallel s \parallel H \rangle$ (ContV) |
|                     | $n$   | (NumV)     | $\langle \lambda*(x, \dots, x).e, \sigma \rangle$ (GenV) |
|                     | $b$   | (BoolV)    | iter[ $a$ ] (IterV)                                      |
|                     | $\langle \lambda(x, \dots, x).e, \sigma \rangle$  | (CloV)     | {value : $v$ , done : $b$ } (ResultV)                    |
| Control Handlers    | $H \in \mathbb{H} = \mathbb{C} \xrightarrow{\text{fin}} \Psi$   | (Handler)  |  |
| Control Operators   | $c \in \mathbb{C}$  | (Control)  |  |
|                     | $c ::= \text{return}$   | (Return)   | throw (Throw)  |
|                     | break   | (Break)    | finally (Finally)  |
|                     | continue  | (Continue) | yield (Yield)  |
| Continuation Values | $\psi, \langle \kappa \parallel s \parallel H \rangle \in \Psi = \mathbb{K} \times \mathbb{S} \times \mathbb{H}$                          | (KValue)   |  |
| Memories            | $M \in \mathbb{M} = \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}$   | (Mem)      |  |
| Environments        | $\sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{A}$   | (Env)      |  |
| Addresses           | $a \in \mathbb{A}$  | (Addr)     |  |

The small-step operational (reduction) semantics of MAGNET is defined in the following form of the reduction relation ( $\rightarrow$ ):

$$\langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow \langle \kappa \parallel s \parallel H \parallel M \rangle$$

#### 4.1 Reduction Relations for IEval

|         |  |  |
|---------|--|--|
| EUnDef  | $\langle (\sigma \vdash \text{undefined}) :: \kappa \parallel s \parallel H \parallel M \rangle$         | $\rightarrow \langle \kappa \parallel \text{undefined} :: s \parallel H \parallel M \rangle$   |
| ENum    | $\langle (\sigma \vdash n) :: \kappa \parallel s \parallel H \parallel M \rangle$                        | $\rightarrow \langle \kappa \parallel n :: s \parallel H \parallel M \rangle$  |
| EBool   | $\langle (\sigma \vdash b) :: \kappa \parallel s \parallel H \parallel M \rangle$                        | $\rightarrow \langle \kappa \parallel b :: s \parallel H \parallel M \rangle$  |
| EAdd    | $\langle (\sigma \vdash e_1 + e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$                | $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (+) :: \kappa \parallel s \parallel H \parallel M \rangle$        |
| EMul    | $\langle (\sigma \vdash e_1 * e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$                | $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (*) :: \kappa \parallel s \parallel H \parallel M \rangle$        |
| EDiv    | $\langle (\sigma \vdash e_1 / e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$                | $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (/) :: \kappa \parallel s \parallel H \parallel M \rangle$        |
| EMod    | $\langle (\sigma \vdash e_1 \% e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$               | $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (\%) :: \kappa \parallel s \parallel H \parallel M \rangle$       |
| EEq     | $\langle (\sigma \vdash e_1 == e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$               | $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (==) :: \kappa \parallel s \parallel H \parallel M \rangle$       |
| ELt     | $\langle (\sigma \vdash e_1 < e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$                | $\rightarrow \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (<) :: \kappa \parallel s \parallel H \parallel M \rangle$        |
| EVar    | $\langle (\sigma \vdash \text{var } x = e_1; e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$ | $\rightarrow \langle (\sigma \vdash e_1) :: \text{def}[x][\sigma \vdash e_2] :: \kappa \parallel s \parallel H \parallel M \rangle$  |
| EId     | $\langle (\sigma \vdash x) :: \kappa \parallel s \parallel H \parallel M \rangle$                        | $\rightarrow \langle \kappa \parallel M(\sigma(x)) :: s \parallel H \parallel M \rangle$   |
| EAssign | $\langle (\sigma \vdash x = e) :: \kappa \parallel s \parallel H \parallel M \rangle$                    | $\rightarrow \langle (\sigma \vdash e) :: \text{write}(\sigma(x)) :: \kappa \parallel s \parallel H \parallel M \rangle$             |
| ESeq    | $\langle (\sigma \vdash e_1; e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$                 | $\rightarrow \langle (\sigma \vdash e_1) :: \text{pop} :: (\sigma \vdash e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$ |

##### 4.1.1 Conditionals and While Loops.

|               |  |
|---------------|--|
| EIF           | $\langle (\sigma \vdash \text{if } (e_1) e_2 \text{ else } e_3) :: \kappa \parallel s \parallel H \parallel M \rangle$   |
| $\rightarrow$ | $\langle (\sigma \vdash e_1) :: \text{jmp-if}[\langle (\sigma \vdash e_2) :: \kappa \parallel s \parallel H \parallel M \rangle] :: (\sigma \vdash e_3) :: \kappa \parallel s \parallel H \parallel M \rangle$ |

|               |  |
|---------------|--|
| EWhile        | $\langle (\sigma \vdash \text{while } (e_1) e_2) :: \kappa \parallel s \parallel H \parallel M \rangle$                                      |
| $\rightarrow$ | $\langle (\sigma \vdash e_1) :: \text{jmp-if}[\psi_{\text{body}}] :: \kappa \parallel \text{undefined} :: s \parallel H \parallel M \rangle$ |

$$\text{where } \begin{cases} \psi_{\text{body}} &= \langle (\sigma \vdash e_2) :: \text{jmp}[\text{continue}] :: \square \parallel s \parallel H_{\text{body}} \rangle \\ H_{\text{body}} &= H[\text{continue} \mapsto \psi_{\text{continue}}, \text{break} \mapsto \psi_{\text{break}}] \\ \psi_{\text{continue}} &= \langle \text{pop} :: (\sigma \vdash \text{while } (e_1) e_2) :: \kappa \parallel s \parallel H \rangle \\ \psi_{\text{break}} &= \langle \kappa \parallel s \parallel H \rangle \end{cases}$$

|               |   |
|---------------|---|
| EBreak        | $\langle (\sigma \vdash \text{break}) :: \kappa \parallel s \parallel H \parallel M \rangle$                  |
| $\rightarrow$ | $\langle \text{jmp}[\text{break}] :: \square \parallel \text{undefined} :: s \parallel H \parallel M \rangle$ |

|               |  |
|---------------|--|
| EContinue     | $\langle (\sigma \vdash \text{continue}) :: \kappa \parallel s \parallel H \parallel M \rangle$                  |
| $\rightarrow$ | $\langle \text{jmp}[\text{continue}] :: \square \parallel \text{undefined} :: s \parallel H \parallel M \rangle$ |

##### 4.1.2 Functions and Return.

|               |  |
|---------------|--|
| EFun          | $\langle (\sigma \vdash \lambda(x, \dots, x).e) :: \kappa \parallel s \parallel H \parallel M \rangle$   |
| $\rightarrow$ | $\langle \kappa \parallel \langle \lambda(x, \dots, x).e, \sigma \rangle :: s \parallel H \parallel M \rangle$   |
| EApp          | $\langle (\sigma \vdash e(e_1, \dots, e_n)) :: \kappa \parallel s \parallel H \parallel M \rangle$   |
| $\rightarrow$ | $\langle (\sigma \vdash e) :: (\sigma \vdash e_1) :: \dots :: (\sigma \vdash e_n) :: \text{call}[n] :: \kappa \parallel s \parallel H \parallel M \rangle$ |
| EReturn       | $\langle (\sigma \vdash \text{return } e) :: \kappa \parallel s \parallel H \parallel M \rangle$   |
| $\rightarrow$ | $\langle (\sigma \vdash e) :: \text{return} :: \kappa \parallel s \parallel H \parallel M \rangle$   |

### 4.1.3 Exceptions.

$$\begin{array}{lcl}
\text{ETry} & \langle (\sigma \vdash \text{try } e_1 \text{ catch } (x) e_2) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e_1) :: \text{jmp}[\text{finally}] :: \square \parallel s \parallel H_{\text{body}} \parallel M \rangle \\
\\
& \text{where } \begin{cases} H_{\text{body}} = H[\text{throw} \mapsto \psi_{\text{throw}}, \text{finally} \mapsto \psi_{\text{finally}}] \\ \psi_{\text{throw}} = \langle \text{def}[x][\sigma \vdash e_2] :: \kappa \parallel s \parallel H \rangle \\ \psi_{\text{finally}} = \langle \kappa \parallel s \parallel H \rangle \end{cases} \\
\\
\text{EThrow} & \langle (\sigma \vdash \text{throw } e) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{jmp}[\text{throw}] :: \square \parallel s \parallel H \parallel M \rangle
\end{array}$$

### 4.1.4 Generators.

$$\begin{array}{lcl}
\text{EGen} & \langle (\sigma \vdash \lambda*(x_1, \dots, x_n).e) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle \kappa \parallel \langle \lambda*(x_1, \dots, x_n).e, \sigma \rangle :: s \parallel H \parallel M \rangle \\
\\
\text{EIterNext}_1 & \langle (\sigma \vdash e_1.\text{next}()) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e_1) :: (\sigma \vdash \text{undefined}) :: \text{next} :: \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{EIterNext}_2 & \langle (\sigma \vdash e_1.\text{next}(e_2)) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: \text{next} :: \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{EYield} & \langle (\sigma \vdash \text{yield } e) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{yield} :: \square \parallel \text{false} :: \psi_{\text{next}} :: s \parallel H \parallel M \rangle \\
\\
& \text{where } \psi_{\text{next}} = \langle \kappa \parallel s \parallel H \rangle \\
\\
\text{EValueField} & \langle (\sigma \vdash e.\text{value}) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{value} :: \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{EDoneField} & \langle (\sigma \vdash e.\text{done}) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{done} :: \kappa \parallel s \parallel H \parallel M \rangle
\end{array}$$

## 4.2 Reduction Relations for Other Instructions

$$\begin{array}{lcl}
\text{IAdd} & \langle (+) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow \langle \kappa \parallel (n_1 + n_2) :: s \parallel H \parallel M \rangle \\
\text{IMul} & \langle (*) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow \langle \kappa \parallel (n_1 \times n_2) :: s \parallel H \parallel M \rangle \\
\text{IDiv} & \langle (/) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow \langle \kappa \parallel (n_1 \div n_2) :: s \parallel H \parallel M \rangle & \text{if } n_2 \neq 0 \\
\text{IMod} & \langle (\%) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow \langle \kappa \parallel (n_1 \% n_2) :: s \parallel H \parallel M \rangle & \text{if } n_2 \neq 0 \\
\text{IEq} & \langle (==) :: \kappa \parallel v_2 :: v_1 :: s \parallel H \parallel M \rangle & \rightarrow \langle \kappa \parallel \text{eq}(v_1, v_2) :: s \parallel H \parallel M \rangle \\
\text{ILt} & \langle (<) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow \langle \kappa \parallel (n_1 < n_2) :: s \parallel H \parallel M \rangle
\end{array}$$

$$\text{where } \text{eq}(v_1, v_2) = \begin{cases} \text{true} & \text{if } v_1 = v_2 = \text{iter}[a] \\ \text{true} & \text{if } v_1 = v_2 = n \\ \text{true} & \text{if } v_1 = v_2 = b \end{cases} \quad \begin{cases} \text{true} & \text{if } v_1 = v_2 = \text{undefined} \\ \text{true} & \text{if } v_1 = v_2 = \{\text{value} : v, \text{done} : b\} \\ \text{false} & \text{otherwise} \end{cases}$$

$$\begin{aligned} \text{IDef} \quad & \langle \text{def}[x_1, \dots, x_n][\sigma \vdash e] :: \kappa \parallel v_n :: \dots :: v_1 :: s \parallel H \parallel M \rangle \\ \rightarrow \quad & \langle (\sigma[x_1 \mapsto a_1, \dots, x_n \mapsto a_n] \vdash e) :: \kappa \parallel s \parallel H \parallel M[a_1 \mapsto v_1, \dots, a_n \mapsto v_n] \rangle \end{aligned}$$

where  $\forall 1 \leq p \leq n. a_p \notin \text{Domain}(M) \wedge (\forall 1 \leq q < p. a_q \neq a_p)$

$$\begin{aligned} \text{IWrite} \quad & \langle \text{write}[a] :: \kappa \parallel v :: s \parallel H \parallel M \rangle \\ \rightarrow \quad & \langle \kappa \parallel v :: s \parallel H \parallel M[a \mapsto v] \rangle \end{aligned}$$

$$\begin{aligned} \text{IPop} \quad & \langle \text{pop} :: \kappa \parallel v :: s \parallel H \parallel M \rangle \\ \rightarrow \quad & \langle \kappa \parallel s \parallel H \parallel M \rangle \end{aligned}$$

#### 4.2.1 Control Flow Instructions.

$$\text{IJumpIf}_{\text{true}} \langle \text{jmp-if}[\langle \kappa \parallel s \parallel H \rangle] :: \_ \parallel \text{true} :: \_ \parallel \_ \parallel M \rangle \rightarrow \langle \kappa \parallel s \parallel H \parallel M \rangle$$

$$\text{IJumpIf}_{\text{false}} \langle \text{jmp-if}[\_] :: \kappa \parallel \text{false} :: s \parallel H \parallel M \rangle \rightarrow \langle \kappa \parallel s \parallel H \parallel M \rangle$$

$$\text{IJump} \quad \langle \text{jmp}[c] :: \kappa \parallel v :: s \parallel H \parallel M \rangle \rightarrow \langle \kappa' \parallel v :: s' \parallel H'' \parallel M \rangle$$

$$\text{where } \begin{cases} H(c) = \langle \kappa' \parallel s' \parallel H' \rangle \\ H'' = \begin{cases} H'[\text{yield} \mapsto H(\text{yield})] & \text{if } \text{yield} \in \text{Domain}(H) \\ H' & \text{otherwise} \end{cases} \end{cases}$$

#### 4.2.2 Function Call/Return Instructions.

$$\begin{aligned} \text{ICall}_{\lambda} \quad & \langle \text{call}[n] :: \kappa \parallel v_n :: \dots :: v_1 :: \langle \lambda(x_1, \dots, x_m).e, \sigma' \rangle :: s \parallel H \parallel M \rangle \\ \rightarrow \quad & \langle \text{def}[x_1, \dots, x_m][\sigma' \vdash \text{return } e] :: \square \parallel s_{\text{body}} \parallel H_{\text{body}} \parallel M \rangle \end{aligned}$$

$$\text{where } \begin{cases} s_{\text{body}} = \begin{cases} v_m :: \dots :: v_1 :: \blacksquare & \text{if } n \geq m \\ \text{undefined} :: \dots :: \text{undefined} :: v_n :: \dots :: v_1 :: \blacksquare & \text{otherwise} \end{cases} \\ H_{\text{body}} = H[\text{return} \mapsto \psi_{\text{return}}] \setminus \{\text{break, continue, yield}\} \\ \psi_{\text{return}} = \langle \kappa \parallel s \parallel H \rangle \end{cases}$$

(Note that  $A \setminus B$  denotes the removal of  $B$ 's elements from  $A$ )

$$\begin{aligned} \text{ICall}_{\lambda*} \quad & \langle \text{call}[n] :: \kappa \parallel v_n :: \dots :: v_1 :: \langle \lambda^*(x_1, \dots, x_m).e, \sigma' \rangle :: s \parallel H \parallel M \rangle \\ \rightarrow \quad & \langle \kappa \parallel \text{iter}[a] :: s \parallel H \parallel M[a \mapsto \psi_{\text{body}}] \rangle \end{aligned}$$

$$\text{where } \begin{cases} a \notin \text{Domain}(M) \\ \psi_{\text{body}} = \langle \kappa_{\text{body}} \parallel s_{\text{body}} \parallel \emptyset \rangle \\ \kappa_{\text{body}} = \text{pop} :: \text{def}[x_1, \dots, x_m][\sigma' \vdash \text{return } (\text{try } e \text{ catch } (x) x)] :: \square \\ s_{\text{body}} = \begin{cases} v_m :: \dots :: v_1 :: \blacksquare & \text{if } n \geq m \\ \text{undefined} :: \dots :: \text{undefined} :: v_n :: \dots :: v_1 :: \blacksquare & \text{otherwise} \end{cases} \\ x \text{ could be any identifier.} \end{cases}$$

$$\begin{aligned} \text{IReturn} \quad & \langle \text{return} :: \kappa \parallel v :: s \parallel H \parallel M \rangle \\ \rightarrow \quad & \begin{cases} \langle \text{yield} :: \square \parallel v :: \text{true} :: \psi_{\text{done}} :: s \parallel H \parallel M \rangle & \text{if } \text{yield} \in \text{Domain}(H) \\ \langle \text{jmp}[\text{return}] :: \square \parallel v :: \blacksquare \parallel H \parallel M \rangle & \text{otherwise} \end{cases} \\ \text{where } \quad & \psi_{\text{done}} = \langle \text{return} :: \square \parallel \blacksquare \parallel \emptyset \rangle \end{aligned}$$

### 4.2.3 Generator Instructions.

|               |  |
|---------------|--|
| INext         | $\langle \text{next} :: \kappa \parallel v :: \text{iter}[a] :: s \parallel H \parallel M \rangle$   |
| $\rightarrow$ | $\langle \kappa' \parallel v :: s' \parallel H_{\text{body}} \parallel M \rangle$  |
|               | where $\begin{cases} M(a) = \langle \kappa' \parallel s' \parallel H' \rangle \\ H_{\text{body}} = H'[\text{yield} \mapsto \psi, \text{return} \mapsto \psi] \\ \psi = \langle \kappa \parallel \text{iter}[a] :: s \parallel H \rangle \end{cases}$ |
| IYield        | $\langle \text{yield} :: \_ \parallel v :: b :: v' :: \_ \parallel H \parallel M \rangle$  |
| $\rightarrow$ | $\langle \kappa' \parallel \{\text{value} : v, \text{done} : b\} :: s' \parallel H' \parallel M[a \mapsto v'] \rangle$   |
|               | where $H(\text{yield}) = \langle \kappa' \parallel \text{iter}[a] :: s' \parallel H' \rangle$  |
| IValueField   | $\langle \text{value} :: \kappa \parallel \{\text{value} : v, \text{done} : \_ \} :: s \parallel H \parallel M \rangle$  |
| $\rightarrow$ | $\langle \kappa \parallel v :: s \parallel H \parallel M \rangle$  |
| IDoneField    | $\langle \text{done} :: \kappa \parallel \{\text{value} : \_, \text{done} : b\} :: s \parallel H \parallel M \rangle$  |
| $\rightarrow$ | $\langle \kappa \parallel b :: s \parallel H \parallel M \rangle$  |

And  $\rightarrow^*$  is the reflexive-transitive closure of  $\rightarrow$  and denotes the repeated reduction:

$$\begin{aligned} & \langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow^* \langle \kappa' \parallel s' \parallel H' \parallel M' \rangle \\ & \frac{\langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow \langle \kappa' \parallel s' \parallel H' \parallel M' \rangle \quad \langle \kappa' \parallel s' \parallel H' \parallel M' \rangle \rightarrow^* \langle \kappa'' \parallel s'' \parallel H'' \parallel M'' \rangle}{\langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow^* \langle \kappa'' \parallel s'' \parallel H'' \parallel M'' \rangle} \end{aligned}$$

The evaluation result of an expression  $e$  is the value  $v$  if

$$\langle (\emptyset \vdash e) :: \square \parallel \blacksquare \parallel \emptyset \parallel \emptyset \rangle \rightarrow^* \langle \square \parallel v :: \blacksquare \parallel \_ \parallel \_ \rangle$$