

MAGNET – Mutable Arithmetic Expressions with Generators and Exceptions

1 INTRODUCTION

MAGNET is a toy language for the [COSE212](#) course at Korea University. MAGNET stands for the **M**utable **A**rithmetic **E**xpressions with **G**enerators and **E**xceptions, and it supports the following features:

- **undefined value** (undefined):
- **number values** (0, 1, -1, 2, -2, 3, -3, ...)
- **boolean values** (true and false)
- **arithmetic operators**: negation (-), addition (+), subtraction (-), multiplication (*), division (/), and modulo (%)
- **logical operators**: conjunction (&&), disjunction (||), and negation (!)
- **comparison operators**: equality (== and !=) and relational (<, >, <=, and >=)
- **mutable variable definitions** (var) and **identifier lookup** (x)
- **variable assignment** (=) and **sequences** (;)
- **augmented assignment** (+=, -=, *=, /=, and %=) and **increment/decrement** (++ and --)
- **conditionals** (if-else), **while loops** (while), **break** (break), and **continue** (continue)
- **first-class functions** (=> or function)
- **function applications** and **return** (return)
- **try-catch** (try-catch) and **throw** (throw)
- **generators** (=>* or function*) and **yield** (yield)
- **iterator next** (_.next) and **iterator result accessors** (_.value and _.done)
- **for-of loops** (for-of)

This document is the specification of MAGNET. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax with the desugaring rules. Then, Section 4 describes the small-step operational (reduction) semantics of MAGNET.

2 CONCRETE SYNTAX

The concrete syntax of MAGNET is written in a variant of the extended Backus–Naur form (EBNF). The notation `<nt>` denotes a nonterminal, and `"t"` denotes a terminal. We use `?` to denote an optional element and `+` (or `*`) to denote one or more (or zero or more) repetitions of the preceding element. We use **butnot** to denote a set difference to exclude some strings from a producible set of strings. We omit some obvious terminals using the ellipsis (`...`) notation.

```
// basic elements
<digit>    ::= "0" | "1" | "2" | ... | "9"
<number>   ::= "-"? <digit>+
<alphabet> ::= "A" | "B" | "C" | ... | "Z" | "a" | "b" | "c" | ... | "z"
<idstart>  ::= <alphabet> | "_"
<idcont>   ::= <alphabet> | "_" | <digit>
<keyword>  ::= "break" | "catch" | "continue" | "else" | "false"
             | "for" | "function" | "if" | "of" | "return" | "throw"
             | "true" | "try" | "undefined" | "var" | "while" | "yield"
<id>       ::= <idstart> <idcont>* butnot <keyword>
```

```

// expressions
<expr> ::= "undefined" | <number> | "true" | "false"
        // unary/binary operators and parentheses
        | <uop> <expr> | <expr> <bop> <expr> | "(" <expr> ")" | "{" <expr> "}"
        // mutable variable definitions
        "var" <id> "=" <expr> ";" <expr> | <id>
        // variable (augmented) assignment and sequence
        | <id> <aop> <expr> | <expr> ";" <expr>
        // increment and decrement
        | "++" <id> | "--" <id> | <id> "++" | <id> "--"
        // conditionals and loops
        | "if" "(" <expr> ")" <expr> "else" <expr>
        | "while" "(" <expr> ")" <expr>
        // first-class functions
        | <id> "=>" <expr> | <params> "=>" <expr>
        | "function" <id> <params> "{" <expr> "}" <expr>
        // function applications and returns
        | <expr> "(" <expr> ")" | "return" <expr>
        // try-catch and throw
        | "try" <expr> "catch" "(" <id> ")" <expr> | "throw" <expr>
        // generators and yields
        | <id> "=>" "*" <expr> | <params> "=>" "*" <expr>
        | "function" "*" <id> <params> "{" <expr> "}" <expr> | "yield" <expr>
        // iterator next and iterator result accessors
        | <expr> "." "next" "(" <expr>? ")"
        | <expr> "." "value" | <expr> "." "done"

// operators
<aop> ::= "=" | "+=" | "-=" | "*=" | "/=" | "%="
<uop> ::= "-" | "!"
<bop> ::= "+" | "-" | "*" | "/" | "%" | "&&" | "||"
        | "==" | "!=" | "<" | "<=" | ">" | ">="

// parameters
<params> ::= "(" " " ")" | "(" <id> ")" | "(" <id> [" " <id>]* ")"

```

The precedence and associativity of operators are defined as follows:

Description	Operator	Precedence	Associativity
Postfix Unary	++, --, _.next, _.value, _.done	1	left
Prefix Unary	-, !, ++, --	2	right
Multiplicative	*, /, %	3	left
Additive	+, -	4	
Relational	<, <=, >, >=	5	
Equality	==, !=	6	
Logical Conjunction	&&	7	
Logical Disjunction		8	
Assignment	=, +=, -=, *=, /=, %=, var	9	right
Sequence	;	10	left

3 ABSTRACT SYNTAX

The abstract syntax of MAGNET is defined as follows:

Expressions	$\mathbb{E} \ni e ::= \text{undefined}$	(EUnDef)	$\text{if } (e) \ e \text{ else } e$	(EIf)
	$ n$	(ENum)	$\text{while } (e) \ e$	(EWhile)
	$ b$	(EBool)	break	(EBreak)
	$ e + e$	(Add)	continue	(EContinue)
	$ e * e$	(Mul)	$\lambda(x, \dots, x).e$	(EFun)
	$ e / e$	(EDiv)	$e(e, \dots, e)$	(EApp)
	$ e \% e$	(EMod)	$\text{return } e$	(EReturn)
	$ e == e$	(EEq)	$\text{try } e \text{ catch } (x) \ e$	(ETry)
	$ e < e$	(ELt)	$\text{throw } e$	(EThrow)
	$ \text{var } x = e; \ e$	(EVar)	$\lambda^*(x, \dots, x).e$	(EGen)
	$ x$	(EId)	$e.\text{next}(e^?)$	(EIterNext)
	$ x = e$	(EAssign)	$\text{yield } e$	(EYield)
	$ e; \ e$	(ESeq)	$e.\text{value}$	(EValueField)
			$e.\text{done}$	(EDoneField)

where $\left\{ \begin{array}{ll} \text{Identifier} & x \in \mathbb{X} \quad (\text{String}) \\ \text{Number} & n \in \mathbb{Z} \quad (\text{BigInt}) \end{array} \right.$ Boolean $\mathbb{B} \ni b ::= \text{true} \mid \text{false} \quad (\text{Boolean})$

The semantics of the remaining cases are defined with the following desugaring rules:

$$\begin{aligned}
 \mathcal{D}[-e] &= \mathcal{D}[e] * (-1) & \mathcal{D}[x += e] &= x = x + \mathcal{D}[e] \\
 \mathcal{D}[e_1 - e_2] &= \mathcal{D}[e_1] + \mathcal{D}[-e_2] & \mathcal{D}[x -= e] &= x = x - \mathcal{D}[e] \\
 \mathcal{D}[e_1 \ \&\& \ e_2] &= \text{if } (\mathcal{D}[e_1]) \ \mathcal{D}[e_2] \text{ else false} & \mathcal{D}[x *= e] &= x = x * \mathcal{D}[e] \\
 \mathcal{D}[e_1 \ || \ e_2] &= \text{if } (\mathcal{D}[e_1]) \ \text{true else } \mathcal{D}[e_2] & \mathcal{D}[x /= e] &= x = x / \mathcal{D}[e] \\
 \mathcal{D}[\text{! } e] &= \text{if } (\mathcal{D}[e]) \ \text{false else true} & \mathcal{D}[x \% e] &= x = x \% \mathcal{D}[e] \\
 \mathcal{D}[e_1 \ != \ e_2] &= \mathcal{D}[\text{! } (e_1 == e_2)] & \mathcal{D}[++x] &= \mathcal{D}[x += 1] \\
 \mathcal{D}[e_1 \ <= \ e_2] &= \text{var } \underline{x_1} = \mathcal{D}[e_1]; & \mathcal{D}[--x] &= \mathcal{D}[x -= 1] \\
 &\quad \text{var } \underline{x_2} = \mathcal{D}[e_2]; & \mathcal{D}[x++] &= \text{var } \underline{x_1} = x; \ \mathcal{D}[x += 1]; \ \underline{x_1} \\
 &\quad \mathcal{D}[(\underline{x_1} == \underline{x_2}) \ || \ (\underline{x_1} < \underline{x_2})] & \mathcal{D}[x--] &= \text{var } \underline{x_1} = x; \ \mathcal{D}[x -= 1]; \ \underline{x_1} \\
 \mathcal{D}[e_1 > e_2] &= \mathcal{D}[\text{! } (e_1 <= e_2)] \\
 \mathcal{D}[e_1 >= e_2] &= \mathcal{D}[\text{! } (e_1 < e_2)]
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{D}[\text{function } x \ (x_1, \dots, x_n) \{ e_1 \} e_2] &= \text{var } x = (x_1, \dots, x_n) \Rightarrow e_1; e_2 \\
 \mathcal{D}[\text{function}^* x \ (x_1, \dots, x_n) \{ e_1 \} e_2] &= \text{var } x = (x_1, \dots, x_n) \Rightarrow^* e_1; e_2
 \end{aligned}$$

$$\mathcal{D}[\text{for } (x \text{ of } e_1) \ e_2] = \mathcal{D} \left[\begin{array}{l} \text{var } \underline{x_1} = e_1; \\ \text{var } \underline{x_2} = \underline{x_1}.\text{next}(); \\ \text{while } (\text{! } \underline{x_2}.\text{done}) \{ \\ \quad \text{var } x = \underline{x_2}.\text{value}; \\ \quad e_2; \ \underline{x_2} = \underline{x_1}.\text{next}() \\ \} \end{array} \right]$$

where $\underline{x_k}$ denotes a fresh temporary variable. All the omitted cases recursively apply the desugaring rule to their sub-expressions. For example, $\mathcal{D}[e_1 + e_2] = \mathcal{D}[e_1] + \mathcal{D}[e_2]$.

4 SEMANTICS

We use the following notations in the semantics:

States	$\langle \kappa \parallel s \parallel H \parallel M \rangle \in \mathbb{K} \times \mathbb{S} \times \mathbb{H} \times \mathbb{M}$ (State)		
Continuations	$\kappa \in \mathbb{K}$	(Cont)	
	$\kappa ::= \square \mid i :: \kappa$		
Instructions	$i \in \mathbb{I}$	(Inst)	
	$i ::= (\sigma \vdash e)$	(IEval)	$\mid \text{jmp-if}[\psi]$ (IJmpIf)
	$\mid (+)$	(IAdd)	$\mid \text{jmp}[c]$ (IJmp)
	$\mid (*)$	(IMul)	$\mid \text{call}[n]$ (ICall)
	$\mid (/)$	(IDiv)	$\mid \text{return}$ (IReturn)
	$\mid (\%)$	(IMod)	$\mid \text{next}$ (INext)
	$\mid (==)$	(IEq)	$\mid \text{yield}$ (IYield)
	$\mid (<)$	(ILt)	$\mid \text{value}$ (IValueField)
	$\mid \text{def}[x, \dots, x][\sigma \vdash e]$	(IDef)	$\mid \text{done}$ (IDoneField)
	$\mid \text{write}[a]$	(IWrite)	$\mid \text{pop}$ (IPop)
Value Stacks	$s \in \mathbb{S}$	(Stack)	
	$s ::= \blacksquare \mid v :: s$		
Values	$v \in \mathbb{V}$	(Value)	
	$v ::= \text{undefined}$	(UndefV)	$\mid \langle \kappa \parallel s \parallel H \rangle$ (ContV)
	$\mid n$	(NumV)	$\mid \langle \lambda^*(x, \dots, x).e, \sigma \rangle$ (GenV)
	$\mid b$	(BoolV)	$\mid \text{iter}[a]$ (IterV)
	$\mid \langle \lambda(x, \dots, x).e, \sigma \rangle$	(CloV)	$\mid \{\text{value} : v, \text{done} : b\}$ (ResultV)
Control Handlers	$H \in \mathbb{H} = \mathbb{C} \xrightarrow{\text{fin}} \Psi$	(Handler)	
Control Operators	$c \in \mathbb{C}$	(Control)	
	$c ::= \text{return}$	(Return)	$\mid \text{throw}$ (Throw)
	$\mid \text{break}$	(Break)	$\mid \text{finally}$ (Finally)
	$\mid \text{continue}$	(Continue)	$\mid \text{yield}$ (Yield)
Continuation Values	$\psi, \langle \kappa \parallel s \parallel H \rangle \in \Psi = \mathbb{K} \times \mathbb{S} \times \mathbb{H}$	(KValue)	
Memories	$M \in \mathbb{M} = \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}$	(Mem)	
Environments	$\sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{A}$	(Env)	
Addresses	$a \in \mathbb{A}$	(Addr)	

The small-step operational (reduction) semantics of MAGNET is defined in the following form of the reduction relation (\rightarrow):

$$\boxed{\langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow \langle \kappa \parallel s \parallel H \parallel M \rangle}$$

4.1 Reduction Relations for IEval

EUnDef	$\langle(\sigma \vdash \text{undefined}) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle\kappa \parallel \text{undefined} :: s \parallel H \parallel M\rangle$
ENum	$\langle(\sigma \vdash n) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle\kappa \parallel n :: s \parallel H \parallel M\rangle$
EBool	$\langle(\sigma \vdash b) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle\kappa \parallel b :: s \parallel H \parallel M\rangle$
EAdd	$\langle(\sigma \vdash e_1 + e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (+) :: \kappa \parallel s \parallel H \parallel M\rangle$
EMul	$\langle(\sigma \vdash e_1 * e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (*) :: \kappa \parallel s \parallel H \parallel M\rangle$
EDiv	$\langle(\sigma \vdash e_1 / e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (/) :: \kappa \parallel s \parallel H \parallel M\rangle$
EMod	$\langle(\sigma \vdash e_1 \% e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (\%) :: \kappa \parallel s \parallel H \parallel M\rangle$
EEq	$\langle(\sigma \vdash e_1 == e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (==) :: \kappa \parallel s \parallel H \parallel M\rangle$
ELt	$\langle(\sigma \vdash e_1 < e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: (\sigma \vdash e_2) :: (<) :: \kappa \parallel s \parallel H \parallel M\rangle$
EVar	$\langle(\sigma \vdash \text{var } x = e_1; e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: \text{def}[x][\sigma \vdash e_2] :: \kappa \parallel s \parallel H \parallel M\rangle$
EId	$\langle(\sigma \vdash x) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle\kappa \parallel M(\sigma(x)) :: s \parallel H \parallel M\rangle$
EAssign	$\langle(\sigma \vdash x = e) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e) :: \text{write}(\sigma(x)) :: \kappa \parallel s \parallel H \parallel M\rangle$
ESeq	$\langle(\sigma \vdash e_1; e_2) :: \kappa \parallel s \parallel H \parallel M\rangle \rightarrow \langle(\sigma \vdash e_1) :: \text{pop} :: (\sigma \vdash e_2) :: \kappa \parallel s \parallel H \parallel M\rangle$

4.1.1 Conditionals and While Loops.

EIF	$\langle(\sigma \vdash \text{if } (e_1) e_2 \text{ else } e_3) :: \kappa \parallel s \parallel H \parallel M\rangle$
\rightarrow	$\langle(\sigma \vdash e_1) :: \text{jmp-if}[\langle(\sigma \vdash e_2) :: \kappa \parallel s \parallel H\rangle] :: (\sigma \vdash e_3) :: \kappa \parallel s \parallel H \parallel M\rangle$
EWhile	$\langle(\sigma \vdash \text{while } (e_1) e_2) :: \kappa \parallel s \parallel H \parallel M\rangle$
\rightarrow	$\langle(\sigma \vdash e_1) :: \text{jmp-if}[\psi_{\text{body}}] :: \kappa \parallel \text{undefined} :: s \parallel H \parallel M\rangle$
where	$\begin{cases} \psi_{\text{body}} &= \langle(\sigma \vdash e_2) :: \text{jmp}[\text{continue}] :: \square \parallel s \parallel H_{\text{body}}\rangle \\ H_{\text{body}} &= H[\text{continue} \mapsto \psi_{\text{continue}}, \text{break} \mapsto \psi_{\text{break}}] \\ \psi_{\text{continue}} &= \langle\text{pop} :: (\sigma \vdash \text{while } (e_1) e_2) :: \kappa \parallel s \parallel H\rangle \\ \psi_{\text{break}} &= \langle\kappa \parallel s \parallel H\rangle \end{cases}$
EBreak	$\langle(\sigma \vdash \text{break}) :: \kappa \parallel s \parallel H \parallel M\rangle$
\rightarrow	$\langle\text{jmp}[\text{break}] :: \square \parallel \text{undefined} :: s \parallel H \parallel M\rangle$
EContinue	$\langle(\sigma \vdash \text{continue}) :: \kappa \parallel s \parallel H \parallel M\rangle$
\rightarrow	$\langle\text{jmp}[\text{continue}] :: \square \parallel \text{undefined} :: s \parallel H \parallel M\rangle$

4.1.2 Functions and Return.

EFun	$\langle(\sigma \vdash \lambda(x, \dots, x).e) :: \kappa \parallel s \parallel H \parallel M\rangle$
\rightarrow	$\langle\kappa \parallel \langle\lambda(x, \dots, x).e, \sigma\rangle :: s \parallel H \parallel M\rangle$
EApp	$\langle(\sigma \vdash e(e_1, \dots, e_n)) :: \kappa \parallel s \parallel H \parallel M\rangle$
\rightarrow	$\langle(\sigma \vdash e) :: (\sigma \vdash e_1) :: \dots :: (\sigma \vdash e_n) :: \text{call}[n] :: \kappa \parallel s \parallel H \parallel M\rangle$
EReturn	$\langle(\sigma \vdash \text{return } e) :: \kappa \parallel s \parallel H \parallel M\rangle$
\rightarrow	$\langle(\sigma \vdash e) :: \text{return} :: \kappa \parallel s \parallel H \parallel M\rangle$

4.1.3 Exceptions.

$$\begin{array}{lcl}
\text{ETry} & \langle (\sigma \vdash \text{try } e_1 \text{ catch } (x) e_2) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e_1) :: \text{jmp}[\text{finally}] :: \square \parallel s \parallel H_{\text{body}} \parallel M \rangle \\
\\
& \text{where } \begin{cases} H_{\text{body}} = H[\text{throw} \mapsto \psi_{\text{throw}}, \text{finally} \mapsto \psi_{\text{finally}}] \\ \psi_{\text{throw}} = \langle \text{def}[x][\sigma \vdash e_2] :: \kappa \parallel s \parallel H \rangle \\ \psi_{\text{finally}} = \langle \kappa \parallel s \parallel H \rangle \end{cases} \\
\\
\text{EThrow} & \langle (\sigma \vdash \text{throw } e) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{jmp}[\text{throw}] :: \square \parallel s \parallel H \parallel M \rangle
\end{array}$$

4.1.4 Generators.

$$\begin{array}{lcl}
\text{EGen} & \langle (\sigma \vdash \lambda*(x_1, \dots, x_n).e) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle \kappa \parallel \langle \lambda*(x_1, \dots, x_n).e, \sigma \rangle :: s \parallel H \parallel M \rangle \\
\\
\text{EIterNext}_1 & \langle (\sigma \vdash e_1.\text{next}()) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e_1) :: (\sigma \vdash \text{undefined}) :: \text{next} :: \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{EIterNext}_2 & \langle (\sigma \vdash e_1.\text{next}(e_2)) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e_1) :: (\sigma \vdash e_2) :: \text{next} :: \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{EYield} & \langle (\sigma \vdash \text{yield } e) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{yield} :: \square \parallel \text{false} :: \psi_{\text{next}} :: s \parallel H \parallel M \rangle \\
\\
& \text{where } \psi_{\text{next}} = \langle \kappa \parallel s \parallel H \rangle \\
\\
\text{EValueField} & \langle (\sigma \vdash e.\text{value}) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{value} :: \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{EDoneField} & \langle (\sigma \vdash e.\text{done}) :: \kappa \parallel s \parallel H \parallel M \rangle \\
\rightarrow & \langle (\sigma \vdash e) :: \text{done} :: \kappa \parallel s \parallel H \parallel M \rangle
\end{array}$$

4.2 Reduction Relations for Other Instructions

$$\begin{array}{llcl}
\text{IAdd} & \langle (+) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow & \langle \kappa \parallel (n_1 + n_2) :: s \parallel H \parallel M \rangle \\
\text{IMul} & \langle (*) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow & \langle \kappa \parallel (n_1 * n_2) :: s \parallel H \parallel M \rangle \\
\text{IDiv} & \langle (/) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow & \langle \kappa \parallel (n_1 / n_2) :: s \parallel H \parallel M \rangle & \text{if } n_2 \neq 0 \\
\text{IMod} & \langle (\%) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow & \langle \kappa \parallel (n_1 \% n_2) :: s \parallel H \parallel M \rangle & \text{if } n_2 \neq 0 \\
\text{IEq} & \langle (==) :: \kappa \parallel v_2 :: v_1 :: s \parallel H \parallel M \rangle & \rightarrow & \langle \kappa \parallel \text{eq}(v_1, v_2) :: s \parallel H \parallel M \rangle \\
\text{ILt} & \langle (<) :: \kappa \parallel n_2 :: n_1 :: s \parallel H \parallel M \rangle & \rightarrow & \langle \kappa \parallel (n_1 < n_2) :: s \parallel H \parallel M \rangle
\end{array}$$

$$\text{where } \text{eq}(v_1, v_2) = \begin{cases} \text{true} & \text{if } v_1 = v_2 = \text{iter}[a] \\ \text{true} & \text{if } v_1 = v_2 = n \\ \text{true} & \text{if } v_1 = v_2 = b \end{cases} \quad \begin{cases} \text{true} & \text{if } v_1 = v_2 = \text{undefined} \\ \text{true} & \text{if } v_1 = v_2 = \{\text{value} : v, \text{done} : b\} \\ \text{false} & \text{otherwise} \end{cases}$$

$$\begin{array}{ll}
\text{IDef} & \langle \text{def}[x_1, \dots, x_n][\sigma \vdash e] :: \kappa \parallel v_n :: \dots :: v_1 :: s \parallel H \parallel M \rangle \\
\rightarrow & \langle \sigma[x_1 \mapsto a_1, \dots, x_n \mapsto a_n] \vdash e :: \kappa \parallel s \parallel H \parallel M[a_1 \mapsto v_1, \dots, a_n \mapsto v_n] \rangle \\
& \text{where } \forall 1 \leq p \leq n. a_p \notin \text{Domain}(M) \wedge (\forall 1 \leq q < p. a_q \neq a_p) \\
\\
\text{IWrite} & \langle \text{write}[a] :: \kappa \parallel v :: s \parallel H \parallel M \rangle \\
\rightarrow & \langle \kappa \parallel v :: s \parallel H \parallel M[a \mapsto v] \rangle \\
\\
\text{IPop} & \langle \text{pop} :: \kappa \parallel v :: s \parallel H \parallel M \rangle \\
\rightarrow & \langle \kappa \parallel s \parallel H \parallel M \rangle
\end{array}$$

4.2.1 Control Flow Instructions.

$$\begin{array}{ll}
\text{IJumpIf}_{\text{true}} & \langle \text{jmp-if}[\langle \kappa \parallel s \parallel H \rangle] :: _ \parallel \text{true} :: _ \parallel _ \parallel M \rangle \rightarrow \langle \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{IJumpIf}_{\text{false}} & \langle \text{jmp-if}[_] :: \kappa \parallel \text{false} :: s \parallel H \parallel M \rangle \rightarrow \langle \kappa \parallel s \parallel H \parallel M \rangle \\
\\
\text{IJump} & \langle \text{jmp}[c] :: \kappa \parallel v :: s \parallel H \parallel M \rangle \rightarrow \langle \kappa' \parallel v :: s' \parallel H'' \parallel M \rangle \\
& \text{where } \begin{cases} H(c) = \langle \kappa' \parallel s' \parallel H' \rangle \\ H'' = \begin{cases} H'[\text{yield} \mapsto H(\text{yield})] & \text{if } \text{yield} \in \text{Domain}(H) \\ H' & \text{otherwise} \end{cases} \end{cases}
\end{array}$$

4.2.2 Function Call/Return Instructions.

$$\begin{array}{ll}
\text{ICall}_{\lambda} & \langle \text{call}[n] :: \kappa \parallel v_n :: \dots :: v_1 :: \langle \lambda(x_1, \dots, x_m).e, \sigma' \rangle :: s \parallel H \parallel M \rangle \\
\rightarrow & \langle \text{def}[x_1, \dots, x_m][\sigma' \vdash \text{return } e] :: \square \parallel s_{\text{body}} \parallel H_{\text{body}} \parallel M \rangle \\
& \text{where } \begin{cases} s_{\text{body}} = \begin{cases} v_m :: \dots :: v_1 :: \blacksquare & \text{if } n \geq m \\ \text{undefined} :: \dots :: \text{undefined} :: v_n :: \dots :: v_1 :: \blacksquare & \text{otherwise} \end{cases} \\ H_{\text{body}} = H[\text{return} \mapsto \psi_{\text{return}}] \setminus \{\text{break, continue, yield}\} \\ \psi_{\text{return}} = \langle \kappa \parallel s \parallel H \rangle \end{cases}
\end{array}$$

(Note that $A \setminus B$ denotes the removal of B 's elements from A)

$$\begin{array}{ll}
\text{ICall}_{\lambda*} & \langle \text{call}[n] :: \kappa \parallel v_n :: \dots :: v_1 :: \langle \lambda^*(x_1, \dots, x_m).e, \sigma' \rangle :: s \parallel H \parallel M \rangle \\
\rightarrow & \langle \kappa \parallel \text{iter}[a] :: s \parallel H \parallel M[a \mapsto \psi_{\text{body}}] \rangle \\
& \text{where } \begin{cases} a \notin \text{Domain}(M) \\ \psi_{\text{body}} = \langle \kappa_{\text{body}} \parallel s_{\text{body}} \parallel \emptyset \rangle \\ \kappa_{\text{body}} = \text{pop} :: \text{def}[x_1, \dots, x_m][\sigma' \vdash \text{return } (\text{try } e \text{ catch } (x) x)] :: \square \\ s_{\text{body}} = \begin{cases} v_m :: \dots :: v_1 :: \blacksquare & \text{if } n \geq m \\ \text{undefined} :: \dots :: \text{undefined} :: v_n :: \dots :: v_1 :: \blacksquare & \text{otherwise} \end{cases} \\ x \text{ could be any identifier.} \end{cases}
\end{array}$$

$$\begin{array}{ll}
\text{IReturn} & \langle \text{return} :: \kappa \parallel v :: s \parallel H \parallel M \rangle \\
\rightarrow & \begin{cases} \langle \text{yield} :: \square \parallel v :: \text{true} :: \psi_{\text{done}} :: s \parallel H \parallel M \rangle & \text{if } \text{yield} \in \text{Domain}(H) \\ \langle \text{jmp}[\text{return}] :: \square \parallel v :: \blacksquare \parallel H \parallel M \rangle & \text{otherwise} \end{cases} \\
& \text{where } \psi_{\text{done}} = \langle \text{return} :: \square \parallel \blacksquare \parallel \emptyset \rangle
\end{array}$$

4.2.3 Generator Instructions.

INext	$\langle \text{next} :: \kappa \parallel v :: \text{iter}[a] :: s \parallel H \parallel M \rangle$
\rightarrow	$\langle \kappa' \parallel v :: s' \parallel H_{\text{body}} \parallel M \rangle$
	where $\begin{cases} M(a) = \langle \kappa' \parallel s' \parallel H' \rangle \\ H_{\text{body}} = H'[\text{yield} \mapsto \psi, \text{return} \mapsto \psi] \\ \psi = \langle \kappa \parallel \text{iter}[a] :: s \parallel H \rangle \end{cases}$
IYield	$\langle \text{yield} :: _ \parallel v :: b :: v' :: _ \parallel H \parallel M \rangle$
\rightarrow	$\langle \kappa' \parallel \{\text{value} : v, \text{done} : b\} :: s' \parallel H' \parallel M[a \mapsto v'] \rangle$
	where $H(\text{yield}) = \langle \kappa' \parallel \text{iter}[a] :: s' \parallel H' \rangle$
IValueField	$\langle \text{value} :: \kappa \parallel \{\text{value} : v, \text{done} : _ \} :: s \parallel H \parallel M \rangle$
\rightarrow	$\langle \kappa \parallel v :: s \parallel H \parallel M \rangle$
IDoneField	$\langle \text{done} :: \kappa \parallel \{\text{value} : _, \text{done} : b\} :: s \parallel H \parallel M \rangle$
\rightarrow	$\langle \kappa \parallel b :: s \parallel H \parallel M \rangle$

And \rightarrow^* is the reflexive-transitive closure of \rightarrow and denotes the repeated reduction:

$$\begin{aligned} & \langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow^* \langle \kappa' \parallel s' \parallel H' \parallel M' \rangle \\ & \frac{\langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow \langle \kappa' \parallel s' \parallel H' \parallel M' \rangle \quad \langle \kappa' \parallel s' \parallel H' \parallel M' \rangle \rightarrow^* \langle \kappa'' \parallel s'' \parallel H'' \parallel M'' \rangle}{\langle \kappa \parallel s \parallel H \parallel M \rangle \rightarrow^* \langle \kappa'' \parallel s'' \parallel H'' \parallel M'' \rangle} \end{aligned}$$

The evaluation result of an expression e is the value v if

$$\langle (\emptyset \vdash e) :: \square \parallel \blacksquare \parallel \emptyset \parallel \emptyset \rangle \rightarrow^* \langle \square \parallel v :: \blacksquare \parallel _ \parallel _ \rangle$$