PTFAE – TFAE with Parametric Polymorphism

1 INTRODUCTION

PTFAE is a toy language for the COSE212 course at Korea University. PTFAE stands for an extension of the TFAE language with **parametric polymorphism**, and it supports the following features:

- **number (integer) values** (`0`, `1`, `-1`, `2`, `-2`, `3`, `-3`, ...)
- **arithmetic operators**: addition (`+`) and multiplication (`*`)
- **immutable variable definitions** (`val`)
- **first-class functions** (`=>`)
- **parametric polymorphism** (`forall`)
- **static type checking**

This document is the specification of PTFAE. First, Section 2 describes the concrete syntax, and Section 3 describes the abstract syntax. Then, Section 4 describes the type system. Finally, Section 5 describes the big-step operational (natural) semantics of PTFAE.

2 CONCRETE SYNTAX

The concrete syntax of PTFAE is written in a variant of the extended Backus–Naur form (EBNF). The notation `<nt>` denotes a nonterminal, and `"t"` denotes a terminal. We use `?` to denote an optional element and `+` (or `*`) to denote one or more (or zero or more) repetitions of the preceding element. We use `butnot` to denote a set difference to exclude some strings from a producible set of strings. We omit some obvious terminals using the ellipsis (...) notation.

```
// basic elements
<digit>    ::= "0" | "1" | "2" | ... | "9"
<number>   ::= "-"? <digit>+
<alphabet> ::= "A" | "B" | "C" | ... | "Z" | "a" | "b" | "c" | ... | "z"
<idstart>  ::= <alphabet> | "_"
<idcont>   ::= <alphabet> | "_" | <digit>
<keyword>  ::= "val" | "Number" | "forall"
<id>       ::= <idstart> <idcont>* butnot <keyword>

// expressions
<expr>     ::= <number> | <expr> "+" <expr> | <expr> "*" <expr>
             | "(" <expr> ")" | "{" <expr> "}"
             | "val" <id> "=" <expr> ";"? <expr> | <id>
             | "(" <id> ":" <type> ")" "=>" <expr> | <expr> "(" <expr> ")"
             | "forall" "[" <id> "]" <expr> | <expr> "[" <type> "]"

// types
<type>     ::= "(" <type> ")" | "Number" | <type> "=>" <type>
             | <id> | "[" <id> "]" <type>
```

For types, the arrow (`=>`) operator is right-associative. For expressions, the precedence and associativity of operators are defined as follows:

Operator	Associativity	Precedence
*	left	2
+	left	1

3 ABSTRACT SYNTAX

The abstract syntax of PTFAE is defined as follows:

$$
\begin{array}{lll}
\text{Expressions} & \mathbb{E} \ni e ::= n & \texttt{(Num)} \\
& \mid e + e & \texttt{(Add)} \\
& \mid e * e & \texttt{(Mul)} \\
& \mid \texttt{val}\ x = e;\ e & \texttt{(Val)} \\
& \mid x & \texttt{(Id)} \\
& \mid \lambda x : \tau . e & \texttt{(Fun)} \\
& \mid e(e) & \texttt{(App)} \\
& \mid \forall \alpha . e & \texttt{(TypeAbs)} \\
& \mid e[\tau] & \texttt{(TypeApp)}
\end{array}
\qquad
\begin{array}{lll}
\text{Numbers} & n \in \mathbb{Z} & \texttt{(BigInt)} \\
\text{Identifiers} & x \in \mathbb{X} & \texttt{(String)} \\
\text{Type Variables} & \alpha \in \mathbb{X}_\alpha & \texttt{(String)} \\
 & & \\
\text{Types} & \mathbb{T} \ni \tau ::= \texttt{num} & \texttt{(NumT)} \\
 & \mid \tau \to \tau & \texttt{(ArrowT)} \\
 & \mid \alpha & \texttt{(VarT)} \\
 & \mid \forall \alpha . \tau & \texttt{(PolyT)}
\end{array}
$$

4 TYPE SYSTEM

This section explains type system of PTFAE, and we use the following notations:

$$\text{Type Environments} \quad \Gamma \in (\mathbb{X} \xrightarrow{\text{fin}} \mathbb{T}) \times \mathcal{P}(\mathbb{X}_\alpha) \quad \texttt{(TypeEnv)}$$

In the type system, type checking is defined with the following typing rules:

$$\boxed{\Gamma \vdash e : \tau}$$

$$\tau\texttt{–Num}\ \frac{}{\Gamma \vdash n : \texttt{num}} \qquad \tau\texttt{–Add}\ \frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash e_1 + e_2 : \texttt{num}} \qquad \tau\texttt{–Mul}\ \frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash e_1 * e_2 : \texttt{num}}$$

$$\tau\texttt{–Val}\ \frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma[x : \tau_1] \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{val}\ x = e_1;\ e_2 : \tau_2} \qquad \tau\texttt{–Id}\ \frac{x \in \text{Domain}(\Gamma)}{\Gamma \vdash x : \Gamma(x)}$$

$$\tau\texttt{–Fun}\ \frac{\Gamma \vdash \tau \qquad \Gamma[x : \tau] \vdash e : \tau'}{\Gamma \vdash \lambda x : \tau . e : \tau \to \tau'} \qquad \tau\texttt{–App}\ \frac{\Gamma \vdash e_0 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_1 : \tau_3 \qquad \tau_1 \equiv \tau_3}{\Gamma \vdash e_0(e_1) : \tau_2}$$

$$\tau\texttt{–TypeAbs}\ \frac{\alpha \notin \text{Domain}(\Gamma) \qquad \Gamma[\alpha] \vdash e : \tau}{\Gamma \vdash \forall \alpha . e : \forall \alpha . \tau} \qquad \tau\texttt{–TypeApp}\ \frac{\Gamma \vdash \tau \qquad \Gamma \vdash e : \forall \alpha . \tau'}{\Gamma \vdash e[\tau] : \tau'[\alpha \leftarrow \tau]}$$

the following rules for well-formedness of types:

$$\boxed{\Gamma \vdash \tau}$$

$$\frac{}{\Gamma \vdash \texttt{num}} \qquad \frac{\Gamma \vdash \tau \qquad \Gamma \vdash \tau'}{\Gamma \vdash \tau \to \tau'} \qquad \frac{\alpha \in \text{Domain}(\Gamma)}{\Gamma \vdash \alpha} \qquad \frac{\Gamma[\alpha] \vdash \tau}{\Gamma \vdash \forall \alpha . \tau}$$

and the following rules for type equivalence:

$$\boxed{\tau \equiv \tau}$$

$$\frac{}{\texttt{num} \equiv \texttt{num}} \qquad \frac{\tau_1 \equiv \tau_1' \qquad \tau_2 \equiv \tau_2'}{(\tau_1 \to \tau_2) \equiv (\tau_1' \to \tau_2')} \qquad \frac{}{\alpha \equiv \alpha} \qquad \frac{\tau \equiv \tau'[\alpha' \leftarrow \alpha]}{\forall \alpha . \tau \equiv \forall \alpha' . \tau'}$$

5 SEMANTICS

We use the following notations in the semantics:

$$\begin{array}{lll} \text{Values} \quad \mathbb{V} \ni v ::= n & (\texttt{NumV}) & \qquad \text{Environments} \quad \sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{V} \quad (\texttt{Env}) \\ \quad\quad | \ \langle \lambda x.e, \sigma \rangle & (\texttt{CloV}) & \\ \quad\quad | \ \langle \forall \alpha.e, \sigma \rangle & (\texttt{TypeAbsV}) & \end{array}$$

The big-step operational (natural) semantics of PTFAE is defined as follows:

$$\boxed{\sigma \vdash e \Rightarrow v}$$

$$\texttt{Num} \ \frac{}{\sigma \vdash n \Rightarrow n} \qquad \texttt{Add} \ \frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma \vdash e_2 \Rightarrow n_2}{\sigma \vdash e_1 + e_2 \Rightarrow n_1 + n_2} \qquad \texttt{Mul} \ \frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma \vdash e_2 \Rightarrow n_2}{\sigma \vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$

$$\texttt{Val} \ \frac{\sigma \vdash e_1 \Rightarrow v_1 \qquad \sigma[x \mapsto v_1] \vdash e_2 \Rightarrow v_2}{\sigma \vdash \texttt{val}\ x = e_1;\ e_2 \Rightarrow v_2} \qquad \texttt{Fun} \ \frac{}{\sigma \vdash \lambda x{:}\tau.e \Rightarrow \langle \lambda x.e, \sigma \rangle}$$

$$\texttt{Id} \ \frac{x \in \text{Domain}(\sigma)}{\sigma \vdash x \Rightarrow \sigma(x)} \qquad \texttt{App} \ \frac{\sigma \vdash e_0 \Rightarrow \langle \lambda x.e_2, \sigma' \rangle \qquad \sigma \vdash e_1 \Rightarrow v_1 \qquad \sigma'[x \mapsto v_1] \vdash e_2 \Rightarrow v_2}{\sigma \vdash e_0(e_1) \Rightarrow v_2}$$

$$\texttt{TypeAbs} \ \frac{}{\sigma \vdash \forall \alpha.e \Rightarrow \langle \forall \alpha.e, \sigma \rangle} \qquad \texttt{TypeApp} \ \frac{\sigma \vdash e \Rightarrow \langle \forall \alpha.e', \sigma' \rangle \qquad \sigma' \vdash e' \Rightarrow v}{\sigma \vdash e[\tau] : v}$$