

# Lecture 27 – Course Review

## COSE215: Theory of Computation

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2024 Spring

# Goal of This Course



- What is the *mathematical model* of computers?

- What is the *mathematical model* of computers?

## Turing Machine!

Let's learn **Turing Machine**

- What is the *mathematical model* of computers?

## Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve every *problem* using computers?

- What is the *mathematical model* of computers?

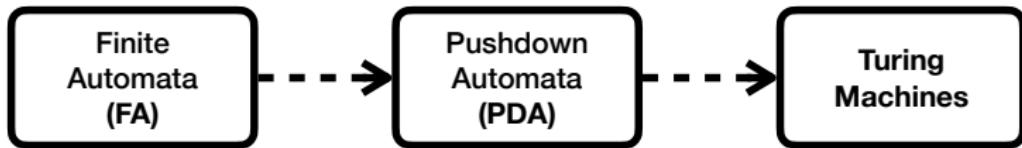
## Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve every *problem* using computers?

**No!**

Let's learn **Undecidability** and **Intractability**



- **Finite Automata (FA)**
  - Regular Expressions and Languages
  - Applications: text search, etc.
- **Pushdown Automata (PDA)**
  - Context-Free Grammars (CFGs) and Languages (CFLs)
  - Applications: programming languages, natural language processing, etc.
- **Turing Machines (TMs)**
  - Extensions of Turing Machines
  - Undecidability and Intractability

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) <b>Mathematical Preliminaries</b>	(Lecture 2) <b>Scala</b>	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3) NFA $\longleftrightarrow$ DFA		(Lecture 3) RL
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala

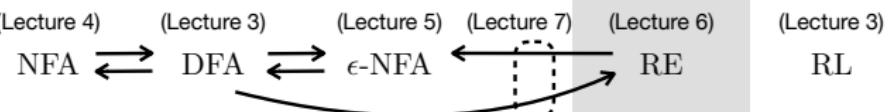
# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3)      (Lecture 5) NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3)      (Lecture 5) NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA	(Lecture 6)	(Lecture 3) RE      RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2)	Scala

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4)      (Lecture 3)      (Lecture 5)      (Lecture 7)      (Lecture 6)      (Lecture 3)</p> <p>NFA <math>\leftrightarrow</math> DFA <math>\leftrightarrow</math> <math>\epsilon</math>-NFA <math>\xleftarrow{\quad}</math> RE</p> 		RL
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p style="text-align: center;">         (Lecture 4)      (Lecture 3)      (Lecture 5)      (Lecture 7)      (Lecture 6)      (Lecture 3)       </p> <pre> graph LR     NFA[NFA] &lt;--&gt; DFA[DFA]     DFA &lt;--&gt; eNFA[epsilon-NFA]     eNFA --&gt; RE[RE]   </pre>	<p style="text-align: center;">         RL          ↓          Closure          Properties          (Lecture 8)       </p>	
(Part 0) Basic Concepts	<p style="text-align: center;">         (Lecture 1)  <b>Mathematical Preliminaries</b> </p>	<p style="text-align: center;">         (Lecture 2)  <b>Scala</b> </p>	

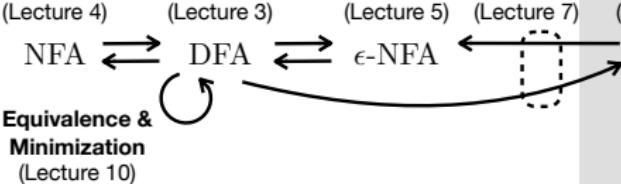
# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p style="text-align: center;">         (Lecture 4) <math>\leftrightarrow</math> (Lecture 3) <math>\leftrightarrow</math> (Lecture 5) <math>\leftrightarrow</math> (Lecture 7) <math>\leftrightarrow</math> (Lecture 6)       </p> <p style="text-align: center;">         NFA <math>\leftrightarrow</math> DFA <math>\leftrightarrow</math> <math>\epsilon</math>-NFA <math>\leftrightarrow</math> RE       </p>	<p style="text-align: center;">(Lecture 3)</p> <p style="text-align: center;">RL</p> <p style="text-align: center;">Closure Properties (Lecture 8)</p> <p style="text-align: center;">Pumping Lemma (Lecture 9)</p>	
(Part 0) Basic Concepts	<p style="text-align: center;">(Lecture 1)</p> <p style="text-align: center;">Mathematical Preliminaries</p>	<p style="text-align: center;">(Lecture 2)</p> <p style="text-align: center;">Scala</p>	

## Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA ↔ DFA (Lecture 3) DFA ↔ $\epsilon$ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) $\epsilon$ -NFA ← RE	(Lecture 6) RE (Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries (Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) $\epsilon$ -NFA ↔ (Lecture 7) RE  Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA $\leftrightarrow$ DFA (Lecture 3) DFA $\leftrightarrow$ $\epsilon$ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) $\epsilon$ -NFA $\leftrightarrow$ RE (Lecture 7) RE $\leftrightarrow$ RE (Lecture 6) RE	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) PDA <sub>FS</sub> PDA <sub>ES</sub>	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) $\epsilon$ -NFA  Equivalence & Minimization (Lecture 10)	(Lecture 7) RE ← ↔ → (Lecture 6)	(Lecture 3) RL Closure Properties (Lecture 8) ← ↔ → Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	PDA <sub>FS</sub> $\xleftrightarrow{\text{(Lecture 14/15)}}$ PDA <sub>ES</sub> $\xleftrightarrow{\text{(Lecture 16)}}$ CFG (Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity		
(Part 1) Finite Automata	NFA $\xleftrightarrow{\text{(Lecture 4)}}$ DFA $\xleftrightarrow{\text{(Lecture 3)}}$ $\epsilon$ -NFA $\xleftrightarrow{\text{(Lecture 7)}}$ RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RL Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)	(Lecture 3)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	$\text{PDA}_{\text{FS}} \xleftrightarrow{\text{(Lecture 14/15)}} \text{PDA}_{\text{ES}}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ $\text{(Lecture 17)}$ $\Leftarrow$	$\text{(Lecture 16)} \longleftrightarrow \text{CFG}$ $\text{(Lecture 11/12)}$	$\text{(Lecture 11)} \quad \text{(Lecture 13)}$ $\text{CFL} \cdots \text{Parse Trees & Ambiguity}$
(Part 1) Finite Automata	$\text{NFA} \xleftrightarrow{\text{(Lecture 4)}} \text{DFA} \xleftrightarrow{\text{(Lecture 3)}} \epsilon\text{-NFA} \xleftrightarrow{\text{(Lecture 5)}} \text{RE}$ $\text{Equivalence \& Minimization (Lecture 10)}$	$\text{(Lecture 7)}$ $\text{(Lecture 6)}$	$\text{(Lecture 3)}$ $\text{RL}$ $\text{Closure Properties (Lecture 8)}$ $\text{Pumping Lemma (Lecture 9)}$
(Part 0) Basic Concepts	$\text{(Lecture 1)}$ <b>Mathematical Preliminaries</b>	$\text{(Lecture 2)}$ <b>Scala</b>	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	$\text{PDA}_{\text{FS}} \leftrightarrow \text{PDA}_{\text{ES}}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ (Lecture 17) $\Leftrightarrow$	$\text{PDA}_{\text{FS}} \leftrightarrow \text{PDA}_{\text{ES}}$ $\leftrightarrow \text{CFG}$ $\vdash \text{Chomsky Normal Form}$ (Lecture 18)	(Lecture 11) $\text{CFL} \cdots$ (Lecture 13) $\text{Parse Trees \& Ambiguity}$
(Part 1) Finite Automata	$\text{NFA} \leftrightarrow \text{DFA}$ $\text{Equivalence \& Minimization}$ (Lecture 10)	$\text{NFA} \leftrightarrow \text{DFA} \leftrightarrow \epsilon\text{-NFA} \leftrightarrow \text{RE}$ $\text{RE} \leftrightarrow \text{RE}$ (dashed arrow)	(Lecture 3) $\text{RL}$ $\vdash \text{Closure Properties}$ (Lecture 8) $\text{Pumping Lemma}$ (Lecture 9)
(Part 0) Basic Concepts		(Lecture 1) $\text{Mathematical Preliminaries}$ (Lecture 2) $\text{Scala}$	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> <math>\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 14/15})} \text{PDA}_{\text{ES}}</math>      <math>\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 16})} \text{CFG}</math>  <math>\cup</math>  <math>\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}</math>  <math>\cup</math>      <math>\text{DPDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 17})} \text{DPDA}_{\text{ES}}</math> </p>	<p style="text-align: center;"> <math>\text{CFG} \rightleftharpoons_{(\text{Lecture 11/12})} \text{Chomsky Normal Form}</math>  <math>\vdash</math>      <math>\text{Chomsky Normal Form} \rightleftharpoons_{(\text{Lecture 18})} \text{Closure Properties}</math>  <math>\vdash</math>      <math>\text{Closure Properties} \rightleftharpoons_{(\text{Lecture 19})} \text{Parse Trees \&amp; Ambiguity}</math> </p>	<p style="text-align: center;"> <math>\text{CFL} \rightleftharpoons_{(\text{Lecture 11})} \text{Closure Properties}</math>  <math>\vdash</math>      <math>\text{Closure Properties} \rightleftharpoons_{(\text{Lecture 13})} \text{Parse Trees \&amp; Ambiguity}</math> </p>
(Part 1) Finite Automata	<p style="text-align: center;"> <math>\text{NFA} \rightleftharpoons_{(\text{Lecture 4})} \text{DFA} \rightleftharpoons_{(\text{Lecture 3})} \epsilon\text{-NFA} \rightleftharpoons_{(\text{Lecture 5})} \text{RE}</math>  <math>\text{Equivalence \&amp; Minimization} \rightleftharpoons_{(\text{Lecture 10})} \text{RE}</math> </p>		<p style="text-align: center;"> <math>\text{RE} \rightleftharpoons_{(\text{Lecture 6})} \text{RL}</math>  <math>\vdash</math>      <math>\text{RL} \rightleftharpoons_{(\text{Lecture 3})} \text{Closure Properties}</math>  <math>\vdash</math>      <math>\text{Closure Properties} \rightleftharpoons_{(\text{Lecture 8})} \text{Pumping Lemma}</math>  <math>\vdash</math>      <math>\text{Pumping Lemma} \rightleftharpoons_{(\text{Lecture 9})} \text{Pumping Lemma}</math> </p>
(Part 0) Basic Concepts	<p style="text-align: center;"> <math>\text{Mathematical Preliminaries} \rightleftharpoons_{(\text{Lecture 1})} \text{Scala}</math> </p>		<p style="text-align: center;"> <math>\text{Scala} \rightleftharpoons_{(\text{Lecture 2})} \text{Scala}</math> </p>

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> <math>\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 14/15})} \text{PDA}_{\text{ES}}</math>      <math>\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 16})} \text{CFG}</math>  <math>\cup</math>  <math>\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}</math>  <math>\cup</math>      <math>\text{DPDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 17})} \text{DPDA}_{\text{ES}}</math> </p>	<p style="text-align: center;"> <math>\text{Chomsky Normal Form} (\text{Lecture 18})</math> </p>	<p style="text-align: center;"> <math>\text{CFL} \rightleftharpoons_{(\text{Lecture 11})} \text{Closure Properties} (\text{Lecture 19})</math>  <math>\rightleftharpoons_{(\text{Lecture 13})} \text{Parse Trees \&amp; Ambiguity}</math>  <math>\rightleftharpoons_{(\text{Lecture 20})} \text{Pumping Lemma}</math> </p>
(Part 1) Finite Automata	<p style="text-align: center;"> <math>\text{NFA} \rightleftharpoons_{(\text{Lecture 4})} \text{DFA} \rightleftharpoons_{(\text{Lecture 3})} \epsilon\text{-NFA} \rightleftharpoons_{(\text{Lecture 5})} \text{RE}</math>  <math>\text{Equivalence \&amp; Minimization} (\text{Lecture 10})</math> </p>		<p style="text-align: center;"> <math>\text{RL} \rightleftharpoons_{(\text{Lecture 3})} \text{Closure Properties} (\text{Lecture 8})</math>  <math>\rightleftharpoons_{(\text{Lecture 9})} \text{Pumping Lemma}</math> </p>
(Part 0) Basic Concepts		<p style="text-align: center;"> <math>\text{Mathematical Preliminaries}</math> </p>	<p style="text-align: center;"> <math>\text{Scala}</math> </p>

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	$\text{PDA}_{\text{FS}} \leftrightarrow \text{PDA}_{\text{ES}}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ (Lecture 17) $\Leftarrow$	$\text{PDA}_{\text{FS}} \leftrightarrow \text{PDA}_{\text{ES}} \leftrightarrow \text{CFG}$ $\vdash$ $\text{Chomsky Normal Form}$ (Lecture 18)	$\text{PDA}_{\text{ES}} \leftrightarrow \text{CFG}$ $\vdash$ $\text{CFL} \cdots \text{Parse Trees & Ambiguity}$ $\vdash$ $\text{Closure Properties}$ (Lecture 19) $\Leftarrow$ $\vdash$ $\text{Pumping Lemma}$ (Lecture 20)
(Part 1) Finite Automata	$\text{NFA} \leftrightarrow \text{DFA}$ $\vdash$ $\text{Equivalence \& Minimization}$ (Lecture 10)	$\text{NFA} \leftrightarrow \text{DFA} \leftrightarrow \epsilon\text{-NFA} \leftrightarrow \text{RE}$ $\vdash$ $\text{RE} \leftrightarrow \text{NFA}$ $\vdash$	$\text{NFA} \leftrightarrow \text{DFA} \leftrightarrow \epsilon\text{-NFA} \leftrightarrow \text{RE}$ $\vdash$ $\text{RE} \leftrightarrow \text{NFA}$ $\vdash$ $\text{RL} \cdots \text{Closure Properties}$ (Lecture 8) $\Leftarrow$ $\vdash$ $\text{Pumping Lemma}$ (Lecture 9)
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23)      (Lecture 21/22) ETM $\leftrightarrow$ TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15)      (Lecture 16)      (Lecture 11/12) PDA <sub>FS</sub> $\leftrightarrow$ PDA <sub>ES</sub> $\leftrightarrow$ CFG Chomsky Normal Form (Lecture 18)  $\cup$ DPDA <sub>FS</sub> $\supset$ DPDA <sub>ES</sub> $\cup$ (Lecture 17) $\leftrightarrow$		(Lecture 11)      (Lecture 13) CFL      Parse Trees & Closure Properties      Ambiguity  $\vdash$ (Lecture 19)      Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3)      (Lecture 5)      (Lecture 7)      (Lecture 6) NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA $\leftrightarrow$ RE  $\vdash$ Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8)      Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala

# Summary

	Automata	Grammars	Languages
<b>(Part 3) Turing Machines</b>	ETM $\leftrightarrow$ TM $\leftrightarrow$ LC (Lecture 23) (Lecture 21/22) (Lecture 24)		(Lecture 21) REL
<b>(Part 2) Pushdown Automata</b>	PDA <sub>FS</sub> $\leftrightarrow$ PDA <sub>ES</sub> $\leftrightarrow$ CFG (Lecture 14/15) (Lecture 16) (Lecture 11/12) DPDA <sub>FS</sub> ⊂ DPDA <sub>ES</sub> (Lecture 17)	Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL Closure Properties (Lecture 19) Parse Trees & Ambiguity Pumping Lemma (Lecture 20)
<b>(Part 1) Finite Automata</b>	NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA $\leftrightarrow$ RE (Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
<b>(Part 0) Basic Concepts</b>	Mathematical Preliminaries (Lecture 1)		(Lecture 2) Scala

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			(Lecture 21) REL $\cup$ DL (Lecture 25)
(Part 2) Pushdown Automata		(Lecture 11) CFL $\vdash$ Chomsky Normal Form (Lecture 18)	(Lecture 13) Parse Trees & Ambiguity Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata		(Lecture 6)	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts		(Lecture 2)	Scala

# Summary

	Automata	Grammars	Languages
<b>(Part 3) Turing Machines</b>	ETM $\leftrightarrow$ TM $\leftrightarrow$ LC (Lecture 23) (Lecture 21/22) (Lecture 24)		(Lecture 21) REL $\cup$ DL $\supset$ NP $\stackrel{?}{=}$ P (Lecture 26) (Lecture 25)
<b>(Part 2) Pushdown Automata</b>	PDA <sub>FS</sub> $\leftrightarrow$ PDA <sub>ES</sub> $\leftrightarrow$ CFG (Lecture 14/15) (Lecture 16) (Lecture 11/12) DPDA <sub>FS</sub> $\supset$ DPDA <sub>ES</sub> (Lecture 17)	Chomsky Normal Form (Lecture 18)	(Lecture 11) (Lecture 13) CFL ... Parse Trees & Ambiguity Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
<b>(Part 1) Finite Automata</b>	NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA $\leftrightarrow$ RE (Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
<b>(Part 0) Basic Concepts</b>	Mathematical Preliminaries (Lecture 1)		(Lecture 2) Scala

- I hope you enjoyed the class!

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