Lecture 19 – Closure Properties of
Context-Free Languages

COSE215: Theory of Computation

Jihyeok Park

APLRG

2025 Spring

- A **context-free language (CFL)** is defined in three different ways:

  - A **context free grammar (CFG)**

  - A **pushdown automaton (PDA)** with **final states**

  - A **pushdown automaton (PDA)** with **empty stacks**

- We have learned that the class of **regular languages** is **closed** under various operations. (**Closure Properties**)

- For which operations is the class of **CFLs closed**?

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Closure Properties of CFLs

Definition (Closure Properties)

The class of CFLs is **closed** under an *n*-ary operator op if and only if
op($L_1, \cdots, L_n$) is context-free for any CFLs $L_1, \cdots, L_n$. We say that such
properties are **closure properties** of CFLs.

The class of CFLs is closed under the following operations:

- Union
- Concatenation
- Kleene Star
- Reverse
- Homomorphism
- Inverse Homomorphism

Closure under Union

Theorem (Closure under Union)

If L_1 and L_2 are context-free languages, then so is $L_1 \cup L_2$.

Closure under Union

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If L_1 and L_2 are context-free languages, then so is $L_1 \cup L_2$.

Proof) For given two CFLs L_1 and L_2, we can always construct two CFGs:

$$G_1 = (V_1, \Sigma, S_1, R_1)$$
$$G_2 = (V_2, \Sigma, S_2, R_2)$$

such that $L_1 = L(G_1)$ and $L_2 = L(G_2)$.

Note that the variables of G_1 and G_2 should be disjoint. (i.e., $V_1 \cap V_2 = \varnothing$)

Then, $L_1 \cup L_2$ is accepted by the CFG $G = (V, \Sigma, S, R)$ where:

- $V = V_1 \cup V_2 \cup \{S\}$
- S is a new start variable (i.e., $S \notin V_1 \cup V_2$)
- $R = R_1 \cup R_2 \cup \{S \to S_1, S \to S_2\}$

□

For example, consider the following two CFLs:

$$L_1 = \{\mathtt{ab}^n \mid n \geq 0\} \qquad L_2 = \{\mathtt{ac}^n \mid n \geq 0\}$$

Then, L_1 is accepted by:

$$S_1 \to \mathtt{a}X \qquad X \to \mathtt{b}X \mid \epsilon$$

and L_2 is accepted by:

$$S_2 \to \mathtt{a}X \qquad X \to \mathtt{c}X \mid \epsilon$$

But, the same variable X is used in both grammars.
So, we need to rename it to different variables, such as B and C.

Closure under Union – Example

For example, consider the following two CFLs:

$$L_1 = \{\text{ab}^n \mid n \geq 0\} \qquad L_2 = \{\text{ac}^n \mid n \geq 0\}$$

Then, L_1 is accepted by:

$$S_1 \rightarrow \text{a}B \qquad B \rightarrow \text{b}B \mid \epsilon$$

and L_2 is accepted by:

$$S_2 \rightarrow \text{a}C \qquad C \rightarrow \text{c}C \mid \epsilon$$

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and L_2 is accepted by:

$$S_2 \to \mathtt{a}C \qquad C \to \mathtt{c}C \mid \epsilon$$

Then, $L_1 \cup L_2$ is accepted by the following CFG:

$$
\begin{aligned}
S &\to S_1 \mid S_2 \\
S_1 &\to \mathtt{a}B & B &\to \mathtt{b}B \mid \epsilon \\
S_2 &\to \mathtt{a}C & C &\to \mathtt{c}C \mid \epsilon
\end{aligned}
$$

Theorem (Closure under Concatenation)

If L_1 and L_2 are context-free languages, then so is $L_1 \cdot L_2$.

Closure under Concatenation

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- $V = V_1 \cup V_2 \cup \{S\}$
- S is a new start variable (i.e., $S \notin V_1 \cup V_2$)
- $R = R_1 \cup R_2 \cup \{S \to S_1 S_2\}$

$\square$

Closure under Concatenation – Example

PLRG

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Then, L_1 is accepted by:

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and L_2 is accepted by:

$$S_2 \rightarrow \mathtt{a}C \qquad C \rightarrow \mathtt{c}C \mid \epsilon$$

Then, $L_1 \cdot L_2$ is accepted by the following CFG:

$$\begin{aligned}
S \;&\rightarrow\; S_1 S_2 \\
S_1 \;&\rightarrow\; \mathtt{a}B \qquad B \rightarrow \mathtt{b}B \mid \epsilon \\
S_2 \;&\rightarrow\; \mathtt{a}C \qquad C \rightarrow \mathtt{c}C \mid \epsilon
\end{aligned}$$

Closure under Kleene Star

Theorem (Closure under Kleene Star)

If L is a context-free language, then so is L^*.

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Proof) For a given CFL L, we can always construct a CFG:

$$G = (V, \Sigma, S, R)$$

such that $L = L(G)$.
Then, L^* is accepted by the CFG $G' = (V', \Sigma, S', R')$ where:

- $V' = V \cup \{S'\}$

- S' is a new start variable (i.e., $S' \notin V$)

- $R' = R \cup \{S' \to \epsilon, S' \to SS'\}$

□

Closure under Kleene Star – Example

PLRG

For example, consider the following CFL:

$$L = \{\mathtt{a}^n\mathtt{b}^n \mid n \geq 0\}$$

Then, L is accepted by:

$$S \to \epsilon \mid \mathtt{a}S\mathtt{b}$$

Then, L^* is accepted by the following CFG:

$$S' \to \epsilon \mid SS'$$
$$S \to \epsilon \mid \mathtt{a}S\mathtt{b}$$

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Closure under Reverse

Theorem (Closure under Reverse)

If L is a context-free language, then so is L^R.

Theorem (Closure under Reverse)

If L is a context-free language, then so is L^R.

Proof) For a given CFL L, we can always construct a CFG:

$$G = (V, \Sigma, S, R)$$

such that $L = L(G)$.
Then, L^R is accepted by the CFG $G' = (V, \Sigma, S, R')$ where:

- $R' = \{X \to \alpha^R \mid X \to \alpha \in R\}$

$\square$

Closure under Reverse – Example

For example, consider the following CFL:

$$L = \{(\mathtt{ab})^n \mathtt{c}^n \mathtt{d}^m \mid n, m \geq 0\}$$

Then, L is accepted by:

$$S \rightarrow X \mid S\mathtt{d}$$
$$X \rightarrow \epsilon \mid \mathtt{ab}X\mathtt{c}$$

Then, L^R is accepted by the following CFG:

$$S \rightarrow X \mid \mathtt{d}S$$
$$X \rightarrow \epsilon \mid \mathtt{c}X\mathtt{ba}$$

Closure under Homomorphism

Let's recall the definition of a **homomorphism**.

Definition (Homomorphism)

Suppose Σ_0 and Σ_1 are two finite sets of symbols. Then, a function

$$h : \Sigma_0 \to \Sigma_1^*$$

is called a **homomorphism**. For a given word $w = \mathtt{a}_1 \mathtt{a}_2 \cdots \mathtt{a}_n \in \Sigma_0^*$,

$$h(w) = h(\mathtt{a}_1)h(\mathtt{a}_2)\cdots h(\mathtt{a}_n)$$

For a language $L \subseteq \Sigma_0^*$,

$$h(L) = \{h(w) \mid w \in L\} \subseteq \Sigma_1^*$$

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For a language $L \subseteq \Sigma_0^*$,

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Example

Let $\Sigma_0 = \{0, 1\}$, $\Sigma_1 = \{\mathtt{a}, \mathtt{b}\}$, and $h(0) = \mathtt{ab}$, $h(1) = \mathtt{a}$. Then,

$$h(10) = \mathtt{aab} \qquad h(010) = \mathtt{abaab} \qquad h(1100) = \mathtt{aaabab}$$

Closure under Homomorphism

Theorem (Closure under Homomorphism)

If h is a homomorphism and L is a context-free language, then so is h(L).

Closure under Homomorphism

Theorem (Closure under Homomorphism)

If h is a homomorphism and L is a context-free language, then so is $h(L)$.

Proof) For a given CFL L, we can always construct a CFG:

$$G = (V, \Sigma_0, S, R)$$

such that $L = L(G)$.

Then, for a given homomorphism $h : \Sigma_0 \to \Sigma_1^*$, $h(L)$ is accepted by the CFG $G' = (V', \Sigma_1, S, R')$ where:

- $V' = V \cup \{X_a \mid a \in \Sigma_0\}$

- $R' = \{Y \to Y_1' \cdots Y_n' \mid Y \to Y_1 \cdots Y_n \in R\} \cup \{X_a \to h(a) \mid a \in \Sigma_0\}$

  where $\forall 1 \leq i \leq n$. $Y_i' = \begin{cases} Y_i & \text{if } Y_i \in V \\ X_a & \text{if } Y_i = a \in \Sigma_0 \end{cases}$

$\square$

Closure under Homomorphism – Example

For example, consider the following CFL:

$$L = \{ww^R \mid w \in \{0, 1\}^*\}$$

Then, L is accepted by:

$$S \to \epsilon \mid 0S0 \mid 1S1$$

If a homomorphism $h : \{0, 1\} \to \{a, b\}^*$ is defined as follows:

$$h(0) = ab \qquad h(1) = a$$

Then, $h(L)$ is accepted by the following CFG:

$$
\begin{aligned}
S &\to \epsilon \mid X_0 S X_0 \mid X_1 S X_1 \\
X_0 &\to ab \\
X_1 &\to a
\end{aligned}
$$

Closure under Inverse Homomorphism

Let's recall the definition of an **inverse homomorphism**.

Definition (Inverse Homomorphism)

Suppose Σ_0 and Σ_1 are two finite sets of symbols. For a given language $L \subseteq \Sigma_1^*$ and a homomorphism $h : \Sigma_0 \to \Sigma_1^*$,

$$h^{-1}(L) = \{w \in \Sigma_0^* \mid h(w) \in L\} \subseteq \Sigma_0^*$$

Closure under Inverse Homomorphism

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Example

Let $\Sigma_0 = \{0, 1\}$, $\Sigma_1 = \{a, b\}$, and $h(0) = aa$, $h(1) = b$. Consider the following language $L \subseteq \Sigma_1^*$:

$$L = \{ a^n b^n \mid n \geq 0 \}$$

Then, $011 \in h^{-1}(L)$ because $h(011) = aabb \in L$.

However, $10 \notin h^{-1}(L)$ because $h(10) = baa \notin L$.

Theorem (Closure under Inverse Homomorphism)

If $h : \Sigma_0 \to \Sigma_1^*$ is a homomorphism and $L \subseteq \Sigma_1^*$ is a context-free language, then so is $h^{-1}(L)$.

Closure under Inverse Homomorphism

Theorem (Closure under Inverse Homomorphism)

If $h : \Sigma_0 \to \Sigma_1^*$ is a homomorphism and $L \subseteq \Sigma_1^*$ is a context-free language, then so is $h^{-1}(L)$.

Proof) Consider a PDA $P = (Q, \Sigma_1, \Gamma, \delta, q_0, Z, F)$ for L by final states.

The key idea is to construct a new PDA P' that simulates P with pairs of
1) states and 2) remaining symbols of Σ_1 as new states.

Closure under Inverse Homomorphism

Theorem (Closure under Inverse Homomorphism)

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The key idea is to construct a new PDA P' that simulates P with pairs of
1) states and 2) remaining symbols of Σ_1 as new states.

Then, a PDA $P' = (Q \times h(\Sigma_0)_{\succeq}, \Sigma_0, \Gamma, \delta', (q_0, \epsilon), Z, F \times \{\epsilon\})$ accepts $h^{-1}(L)$ by final states where:

- $A_{\succeq} = \{x \in \Sigma_1^* \mid x \text{ is a suffix of } w \in A\}$ for any $A \subseteq \Sigma_1^*$
- For all $a \in \Sigma_0$, $q \in Q$, and $X \in \Sigma_1$,

$$\delta'((q, \epsilon), a, X) = \{((q, h(a)), X)\}$$

- For all $b \in \Sigma_1 \cup \{\epsilon\}$, $bx \in h(\Sigma_0)_{\succeq}$, $q \in Q$, and $X \in \Sigma_1$,

$$\delta'((q, bx), \epsilon, X) = \{((p, x), \gamma) \mid (p, \gamma) \in \delta(q, b, X)\}$$

Closure under Inverse Homomorphism – Example ◆PLRG
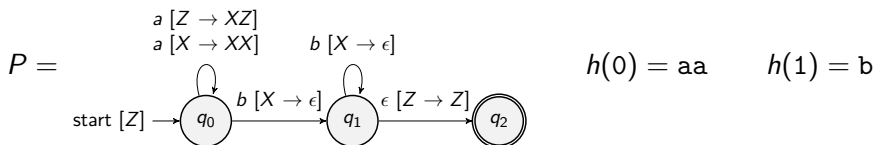
For example, consider the following PDA:

$$P = $$

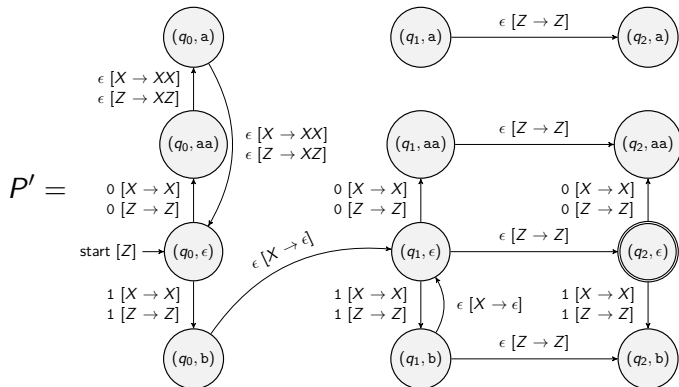that accepts $L = \{a^n b^n \mid n \geq 0\}$ by final states.

If a homomorphism $h : \{0, 1\} \rightarrow \{a, b\}^*$ is defined as follows:

$$h(0) = \texttt{aa} \qquad h(1) = \texttt{b}$$

Closure under Inverse Homomorphism – Example ◭PLRG

$$P = \quad \begin{array}{c} a\ [Z \to XZ] \\ a\ [X \to XX] \end{array} \qquad b\ [X \to \epsilon]$$

start $[Z] \longrightarrow (q_0) \xrightarrow{b\ [X \to \epsilon]} (q_1) \xrightarrow{\epsilon\ [Z \to Z]} ((q_2))$

$h(0) = \text{aa} \qquad h(1) = \text{b}$

Then, the following PDA accepts $h^{-1}(L)$ by final states:

$P' =$

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Definition (Closure Properties)

The class of CFLs is **closed** under an *n*-ary operator op if and only if
$\text{op}(L_1, \cdots, L_n)$ is context-free for any CFLs $L_1, \cdots, L_n$. We say that such
properties are **closure properties** of CFLs.

The class of CFLs is **NOT** closed under the following operations:

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- Complement
- Difference

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We will prove it by using the fact that the following language is not a CFL:

$$L = \{a^n b^n c^n \mid n \geq 0\}$$

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- Intersection
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We will prove it by using the fact that the following language is not a CFL:

$$L = \{a^n b^n c^n \mid n \geq 0\}$$

We will learn how to prove that L is not a CFL in the next lecture (Pumping Lemma for CFLs).

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Theorem (Non-Closure under Intersection)

*The class of CFLs is **NOT** closed under intersection.*

Theorem (Non-Closure under Intersection)

*The class of CFLs is **NOT** closed under intersection.*

Proof) Consider the following two languages:

$$L_1 = \{\texttt{a}^n\texttt{b}^n\texttt{c}^m \mid n, m \geq 0\} \qquad L_2 = \{\texttt{a}^m\texttt{b}^n\texttt{c}^n \mid n, m \geq 0\}$$

Then, L_1 is accepted by:

$$S_1 \to X \mid S_1\texttt{c} \qquad X \to \epsilon \mid \texttt{a}X\texttt{b}$$

and L_2 is accepted by:

$$S_2 \to Y \mid \texttt{a}S_2 \qquad Y \to \epsilon \mid \texttt{b}Y\texttt{c}$$

Thus, they are both CFLs. However, their intersection is not a CFL:

$$L_1 \cap L_2 = \{\texttt{a}^n\texttt{b}^n\texttt{c}^n \mid n \geq 0\}$$

□

Theorem (Non-Closure under Complement)

*The class of CFLs is **NOT** closed under complement.*

Theorem (Non-Closure under Complement)

*The class of CFLs is **NOT** closed under complement.*

Proof) Assume that the class of CFLs is closed under complement. Then, for any two CFLs L_1 and L_2, $L_1 \cap L_2$ is also a CFL:

$$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$$

However, we have already proved that the class of CFLs is not closed under intersection. Thus, the class of CFLs is not closed under complement. $\qquad\square$

Theorem (Non-Closure under Complement)

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However, we have already proved that the class of CFLs is not closed under intersection. Thus, the class of CFLs is not closed under complement. $\square$

Theorem (Non-Closure under Difference)

*The class of CFLs is **NOT** closed under difference.*

Theorem (Non-Closure under Complement)

The class of CFLs is **NOT** *closed under complement.*

Proof) Assume that the class of CFLs is closed under complement. Then, for any two CFLs L_1 and L_2, $L_1 \cap L_2$ is also a CFL:

$$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$$

However, we have already proved that the class of CFLs is not closed under intersection. Thus, the class of CFLs is not closed under complement. $\square$

Theorem (Non-Closure under Difference)

The class of CFLs is **NOT** *closed under difference.*

Proof) Similarly, we can prove it using the following fact:

$$L_1 \cap L_2 = L_1 \setminus (L_1 \setminus L_2)$$

$\square$

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Closure Properties of CFLs with Regular Languages ◆PLRG

Definition (Closure Properties)

The class of CFLs is **closed** under an *n*-ary operator op if and only if
op($L_1, \cdots, L_n$) is context-free for any CFLs $L_1, \cdots, L_n$. We say that such
properties are **closure properties** of CFLs.

The class of CFLs is closed under the following operations with RLs:
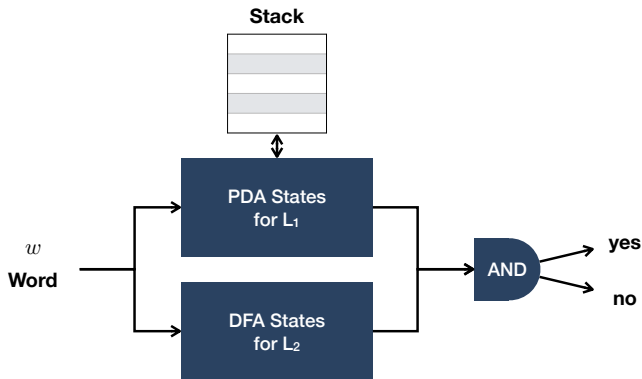
- Intersection
- Difference

Theorem (Closure under Intersection with RLs)

If L_1 Is a CFL and L_2 is a RL, then $L_1 \cap L_2$ is a CFL.

Theorem (Closure under Intersection with RLs)

If L_1 Is a CFL and L_2 is a RL, then $L_1 \cap L_2$ is a CFL.

There exists a PDA P that accepts L_1 by final states and a DFA D that accepts L_2. We will construct a PDA P' that accepts $L_1 \cap L_2$ as follows:

Theorem (Closure under Intersection with RLs)

If L_1 Is a CFL and L_2 is a RL, then $L_1 \cap L_2$ is a CFL.

Proof) Consider a PDA $P = (Q_P, \Sigma, \Gamma, \delta_P, q_P, Z, F_P)$ and a DFA $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$ such that:

$$L_F(P) = L_1 \qquad L(D) = L_2$$

Then, $L_1 \cap L_2$ is accepted by the PDA $P' = (Q, \Sigma, \Gamma, \delta, q_0, Z, F)$ by final states, where:

- $Q = Q_P \times Q_D$
- $\delta((p, q), \epsilon, X) = \{((p', q), \alpha) \mid (p', \alpha) \in \delta_P(p, \epsilon, X)\}$
- $\delta((p, q), a, X) = \{((p', q'), \alpha) \mid (p', \alpha) \in \delta_P(p, a, X) \land q' = \delta_D(q, a)\}$
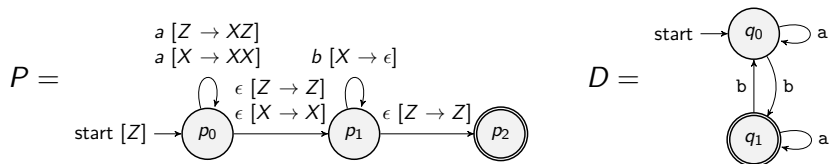- $q_0 = (q_P, q_D)$
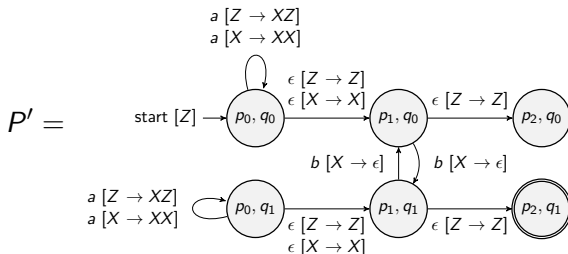- $F = F_P \times F_D$

$\square$

For example, consider the following PDA P and DFA D:
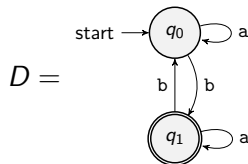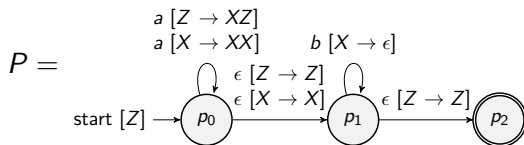
For example, consider the following PDA P and DFA D:

Then, a PDA P' that accepts $L_F(P) \cap L(D)$ by the final states can be constructed as follows:

Theorem (Closure under Difference with RLs)

If L_1 Is a CFL and L_2 is a RL, then $L_1 \setminus L_2$ is a CFL.

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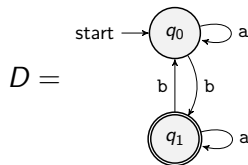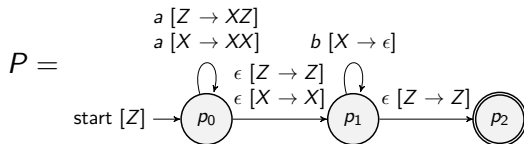
Proof) We know the following fact:

$$L_1 \setminus L_2 = L_1 \cap \overline{L_2}$$

Since the class of RLs is closed under complement, $\overline{L_2}$ is a RL. In addition, we know that the class of CFLs is closed under intersection with RLs. Thus, $L_1 \setminus L_2$ is a CFL. $\qquad \square$
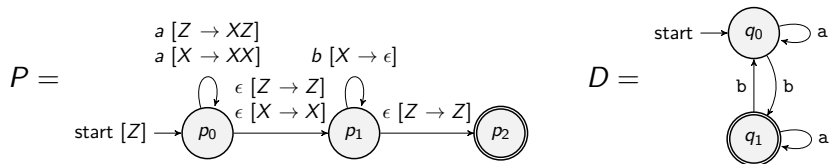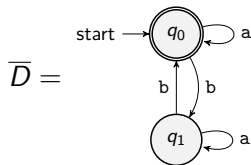
For example, consider the following PDA P and DFA D:

For example, consider the following PDA P and DFA D:

$$P =$$

$$a\ [Z \to XZ]$$
$$a\ [X \to XX] \qquad b\ [X \to \epsilon]$$
$$\epsilon\ [Z \to Z]$$
$$\epsilon\ [X \to X]$$
start $[Z] \to p_0 \xrightarrow{} p_1 \xrightarrow{\epsilon\ [Z \to Z]} p_2$

$$D =$$
start $\to q_0$ ⟲ a
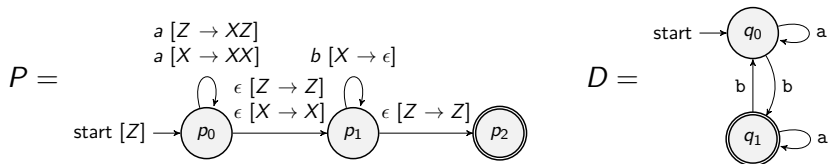q_0 ⟷ q_1 (b, b)
q_1 ⟲ a

Then, a DFA $\overline{D}$ that accepts $\overline{L(D)}$ and a PDA P' that accepts $L_F(P) \setminus L(D) = L_F(P) \cap \overline{L(D)}$ can be constructed as follows:

$$\overline{D} =$$
start $\to q_0$ ⟲ a
q_0 ⟷ q_1 (b, b)
q_1 ⟲ a

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For example, consider the following PDA P and DFA D:

$$P = \quad \text{start } [Z] \longrightarrow \begin{array}{c} a\ [Z \to XZ] \\ a\ [X \to XX] \\ \epsilon\ [Z \to Z] \\ \epsilon\ [X \to X] \end{array} \quad p_0 \quad \xrightarrow{} \quad p_1 \quad \xrightarrow{\epsilon\ [Z \to Z]} \quad p_2$$

$$D = \quad \text{start} \longrightarrow q_0 \circlearrowleft a \quad \begin{array}{c} b \\ \end{array} \quad q_1 \circlearrowleft a$$

Then, a DFA $\overline{D}$ that accepts $\overline{L(D)}$ and a PDA P' that accepts
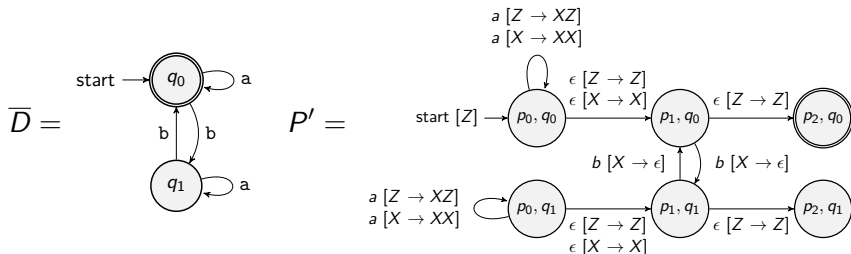$L_F(P) \setminus L(D) = L_F(P) \cap \overline{L(D)}$ can be constructed as follows:

$$\overline{D} = \quad \text{start} \longrightarrow q_0 \circlearrowleft a \quad q_1 \circlearrowleft a$$

$$P' =$$

1. Closure Properties of Context-Free Languages

Union
Concatenation
Kleene Star
Reversal
Homomorphism
Inverse Homomorphism

2. Non-Closure Properties of Context-Free Languages

Intersection
Complement and Difference

3. Closure Properties of CFLs with Regular Languages

Intersection with Regular Languages
Difference with Regular Languages

- Please see this document on GitHub:

https://github.com/ku-plrg-classroom/docs/tree/main/cose215/equiv-cfg-pda

- The due date is 23:59 on Jun. 2 (Mon.).

- Please only submit `Implementation.scala` file to **LMS**.

- The Pumping Lemma for Context-Free Languages

Jihyeok Park
jihyeok_park@korea.ac.kr
https://plrg.korea.ac.kr