

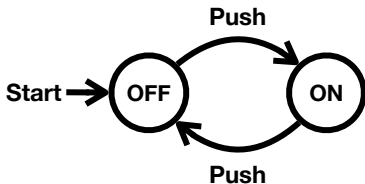
Lecture 3 – Deterministic Finite Automata (DFA)

COSE215: Theory of Computation

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① Mathematical Preliminaries

- Mathematical Notations
- Inductive Proofs
- Notations in Languages

② Basic Introduction of Scala

- Basic Features
- Object-Oriented Programming (OOP)
- Functional Programming (FP)
- Immutable Collections (Data Structures)

1. Deterministic Finite Automata (DFA)

- Definition

- Transition Diagram and Transition Table

- Extended Transition Function

- Acceptance of a Word

- Language of DFA (Regular Language)

- Examples

Definition (Deterministic Finite Automata (DFA))

A **deterministic finite automaton** (DFA) is a 5-tuple:

$$D = (Q, \Sigma, \delta, q_0, F)$$

- Q is a finite set of **states**
- Σ is a finite set of **symbols**
- $\delta : Q \times \Sigma \rightarrow Q$ is the **transition function**
- $q_0 \in Q$ is the **initial state**
- $F \subseteq Q$ is the set of **final states**

$$D_1 = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$$

$$\delta(q_0, a) = q_1$$

$$\delta(q_1, a) = q_2$$

$$\delta(q_2, a) = q_2$$

$$\delta(q_0, b) = q_0$$

$$\delta(q_1, b) = q_0$$

$$\delta(q_2, b) = q_0$$

```
// The type definitions of states and symbols
type State = Int
type Symbol = Char
// The definition of DFA
case class DFA(
  states: Set[State],
  symbols: Set[Symbol],
  trans: Map[(State, Symbol), State],
  initState: State,
  finalStates: Set[State],
)
```

```
// An example of DFA
val dfa1: DFA = DFA(
  states      = Set(0, 1, 2),
  symbols     = Set('a', 'b'),
  trans       = Map(
    (0, 'a') -> 1, (1, 'a') -> 2, (2, 'a') -> 2,
    (0, 'b') -> 0, (1, 'b') -> 0, (2, 'b') -> 0,
  ),
  initState   = 0,
  finalStates = Set(2),
)
```

$$D_1 = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$$

$$\delta(q_0, a) = q_1$$

$$\delta(q_1, a) = q_2$$

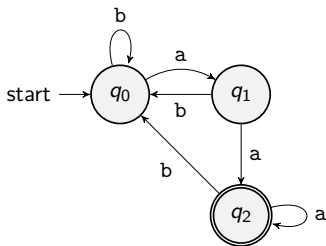
$$\delta(q_2, a) = q_2$$

$$\delta(q_0, b) = q_0$$

$$\delta(q_1, b) = q_0$$

$$\delta(q_2, b) = q_0$$

Transition Diagram



Transition Table

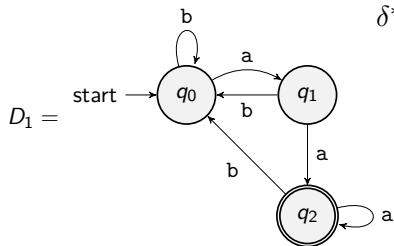
q	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_0
$*q_2$	q_2	q_0

Definition (Extended Transition Function)

For a given DFA $D = (Q, \Sigma, \delta, q_0, F)$, the **extended transition function** $\delta^* : Q \times \Sigma^* \rightarrow Q$ is defined as follows:

- **(Basis Case)** $\delta^*(q, \epsilon) = q$
- **(Induction Case)** $\delta^*(q, xw) = \delta^*(\delta(q, x), w)$

where $x \in \Sigma$, $w \in \Sigma^*$

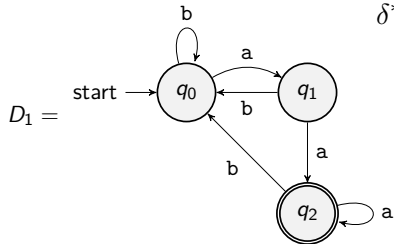


$$\begin{aligned}
 \delta^*(q_0, baa) &= \delta^*(\delta(q_0, b), aa) = \delta^*(q_0, aa) \\
 &= \delta^*(\delta(q_0, a), a) = \delta^*(q_1, a) \\
 &= \delta^*(\delta(q_1, a), \epsilon) = \delta^*(q_2, \epsilon) \\
 &= q_2
 \end{aligned}$$

Definition (Extended Transition Function)

For a given DFA $D = (Q, \Sigma, \delta, q_0, F)$, the **extended transition function** $\delta^* : Q \times \Sigma^* \rightarrow Q$ is defined as follows:

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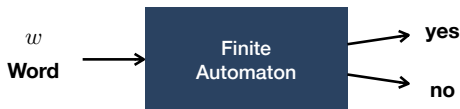
$$\begin{aligned}
 \delta^*(q_0, aba) &= \delta^*(\delta(q_0, a), ba) = \delta^*(q_1, ba) \\
 &= \delta^*(\delta(q_1, b), a) = \delta^*(q_0, a) \\
 &= \delta^*(\delta(q_0, a), \epsilon) = \delta^*(q_1, \epsilon) \\
 &= q_1
 \end{aligned}$$


```
// The type definition of words
type Word = String
case class DFA(...):
  ...
  // The extended transition function of DFA
  def extTrans(q: State, w: Word): State = w match
    case ""      => q
    case x <| w => extTrans(trans(q, x), w)

// An example transition for a word "baa"
dfa1.extTrans(0, "baa") // 2
// An example transition for a word "aba"
dfa1.extTrans(0, "aba") // 1
```

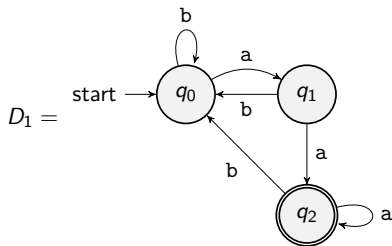
where `<|` is a helper function to extract the first symbol and the rest of the word but you do not need to understand the details of how it works.

```
// A helper function to extract first symbol and rest of word
object `<|` { def unapply(w: Word) = w.headOption.map((_, w.drop(1))) }
```



Definition (Acceptance of a Word)

For a given DFA $D = (Q, \Sigma, \delta, q_0, F)$, we say that D **accepts** a word $w \in \Sigma^*$ if and only if $\delta^*(q_0, w) \in F$

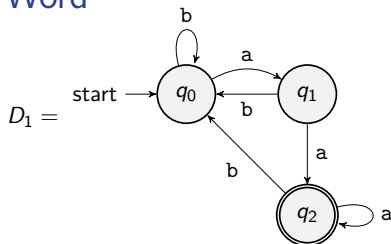


$$\delta^*(q_0, baa) = q_2 \in F$$

It means that D_1 **accepts** baa.

$$\delta^*(q_0, aba) = q_1 \notin F$$

It means that D_1 does **not accept** aba.



```
case class DFA(...):
  ...
  // The acceptance of a word by DFA
  def accept(w: Word): Boolean =
    finalStates.contains(extTrans(initState, w))

// An example acceptance of a word "baa"
dfa1.accept("baa") // true

// An example non-acceptance of a word "aba"
dfa1.accept("aba") // false
```

Definition (Language of DFA)

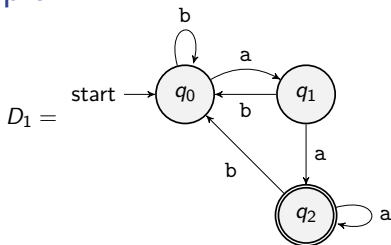
For a given DFA $D = (Q, \Sigma, \delta, q_0, F)$, the **language** of D is defined as:

$$L(D) = \{w \in \Sigma^* \mid D \text{ accepts } w\}$$

Definition (Regular Language)

A language L is **regular** if and only if there exists a DFA D such that $L(D) = L$

Example 1



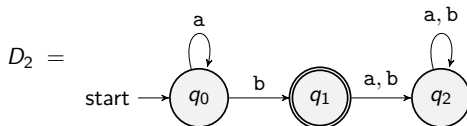
$\delta^*(q_0, baa) = q_2 \in F$
 $\Rightarrow D_1$ accepts baa
 $\Rightarrow baa \in L(D_1)$

$\epsilon, a, b, ab, ba, bb, aab, aba, abb, bab, \dots \notin L(D_1)$
 $aa, aaa, baa, aaaa, abaa, baaa, bbaa, \dots \in L(D_1)$

$$L(D_1) = \{waa \mid w \in \{a, b\}^*\}$$

- q_0 represents ϵ or any word ending with b
- q_1 represents any word ending with exactly one a
- q_2 represents any word ending with at least two a 's

Example 2



$\epsilon, a, aa, ba, bb, aaa, aba, abb, baa, bab, bba, \dots \notin L(D_2)$

$b, ab, aab, aaab, aaaab, aaaaab, aaaaaab, \dots \in L(D_2)$

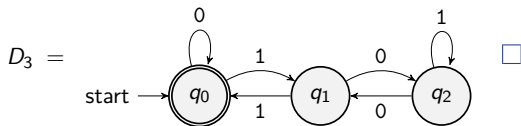
$$L(D_2) = \{a^n b \mid n \geq 0\}$$

- q_0 represents zero or more a's
- q_1 represents zero or more a's followed by b
- q_2 represents any other words

Theorem

The language $L = \{w \in \{0, 1\}^* \mid d(w) \equiv 0 \pmod{3}\}$ is regular ($d(w)$ is the natural number represented by w in binary).

Proof) You need to construct a DFA D_2 such that $L(D_2) = L$. Consider the following DFA D_2 :



- q_0 represents binary format of an integer n s.t. $n \equiv 0 \pmod{3}$
- q_1 represents binary format of an integer n s.t. $n \equiv 1 \pmod{3}$
- q_2 represents binary format of an integer n s.t. $n \equiv 2 \pmod{3}$

Theorem

The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.

You need to construct a DFA D such that $L(D) = L$. However, it is **impossible** because L is actually **not regular**.

Then, is it possible to prove that L is not regular?

Yes, it is possible BUT you will learn how to prove it (using Pumping Lemma) later in this course.

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- Nondeterministic Finite Automata (NFA)

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