Lecture 10 – Mutable Data Structures

COSE212: Programming Languages

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2025 Fall

Recall



- Recursion
 - Recursion in F1VAE and FVAE
 - mkRec helper function
 - RFAE FAE with recursion and conditionals

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- In this lecture, we will learn mutable data structures (boxes)





- Recursion
 - Recursion in F1VAE and FVAE
 - mkRec helper function
 - RFAE FAE with recursion and conditionals
- In this lecture, we will learn mutable data structures (boxes)
- BFAE FAE with mutable boxes
 - Concrete and Abstract Syntax
 - Interpreter and Natural Semantics

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1. Mutable Data Structures

2. BFAE - FAE with Mutable Boxes

Concrete Syntax Abstract Syntax

3. Interpreter and Natural Semantics for BFAE

Evaluation with Memories

Interpreter and Natural Semantics

Addition

Box Creation

Box Content Getter

Box Content Setter

Sequence

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- All functions are **pure** (no side effects)
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Mutation makes it possible to **change the state** of a program by **updating the contents** of a data structure or a variable after its creation.

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- Mutable variables (e.g., var in Scala)



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In this lecture, we will learn mutable data structures.



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We can define our own **mutable data structure** – a **Box**:

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Now, let's extend FAE into BFAE to support mutable boxes.

(We support variable definitions (val) as syntactic sugar.)

BFAE - FAE with Mutable Boxes



Now, let's extend FAE into BFAE to support **mutable boxes**.

(We support variable definitions (val) as syntactic sugar.)

For BFAE, we need to extend expressions of FAE with

- 1 box creation (Box)
- 2 box operations: content getter (get) and setter (set)
- 3 sequence of expressions

Concrete Syntax



```
// expressions
<expr> ::= ...
| "Box" "(" <expr> ")"
| <expr> "." "get"
| <expr> "." "set" "(" <expr> ")"
| <expr> ";" <expr>
```

For BFAE, we need to extend expressions of FAE with

- box creation
- 2 box operations: get and set
- **3 sequence** of expressions

Abstract Syntax



Let's define the abstract syntax of BFAE in BNF:

```
enum Expr:
...
// box creation
case NewBox(expr: Expr)
// box content getter
case GetBox(box: Expr)
// box content setter
case SetBox(box: Expr, expr: Expr)
// sequence
case Seq(left: Expr, right: Expr)
```

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How to evaluate the following BFAE expression?

```
/* BFAE */
val box = Box(5);
box.get; // 5
box.set(8);
box.get // 8
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Let's evaluate it with a **memory** M, which is a finite **mapping** from addresses to their values.

$$M\in\mathbb{A}\xrightarrow{\mathrm{fin}}\mathbb{V}$$

- box creation allocates a memory cell and stores the value
- box content getter reads the value from the memory cell
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/* BFAE */
val a = Box(1);
val f = x => x + a.get;
f(5);

a.set(2);
f(5);

val b = Box(a);
b.get.set(3);
f(5);
```





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```

```
\sigma = [ \mathbf{a} \mapsto a_0 ] \mathbb{A} \quad : \quad a_0 \quad a_1 \quad a_2 \quad \dots M \quad = \boxed{1} \quad \boxed{\dots}
```





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\begin{split} \sigma &= [\\ \mathbf{a} &\mapsto a_0\\ \mathbf{f} &\mapsto \langle \lambda \mathbf{x}. (\mathbf{x} + \mathbf{a}. \mathtt{get}), [\mathbf{a} \mapsto a_0] \rangle \end{split}
```





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/* BFAE */
val a = Box(1);
val f = x => x + a.get;
f(5);    /* 5 + 1 = 6 */ *

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```
/* BFAE */
val a = Box(1);
val f = x => x + a.get;
f(5);   /* 5 + 1 = 6 */
a.set(2);
f(5);   /* 5 + 2 = 7 */ *

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b.get.set(3);
f(5);
```

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```
/* BFAE */
val a = Box(1);
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val f = x => x + a.get;
f(5);   /* 5 + 1 = 6 */
a.set(2);
f(5);   /* 5 + 2 = 7 */
val b = Box(a);
b.get.set(3);
f(5);   /* 5 + 3 = 8 */ *
```

```
\begin{split} \sigma &= [\\ \mathbf{a} &\mapsto a_0\\ \mathbf{f} &\mapsto \langle \lambda \mathbf{x}. (\mathbf{x} + \mathbf{a}. \mathbf{get}), [\mathbf{a} \mapsto a_0] \rangle\\ \mathbf{b} &\mapsto a_1 \end{split} \\ ] \\ & \qquad \qquad \mathbb{A} \quad : \quad a_0 \quad a_1 \quad a_2 \quad \dots\\ M \quad = \quad \boxed{3} \quad \boxed{a_0} \quad \boxed{\dots} \end{split}
```





For BFAE, we need to 1) implement the **interpreter** with environments and **memories** by passing the updated memory in the result:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = ???

type Addr = Int
type Mem = Map[Addr, Value]
enum Value:
...
case BoxV(addr: Addr)
```





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type Mem = Map[Addr, Value]
enum Value:
    ...
case BoxV(addr: Addr)
```

and 2) define the **natural semantics** with environments and **memories** by passing the updated memory in the result:

$$\begin{array}{c|cccc} \sigma, M \vdash e \Rightarrow v, M \\ \hline \text{Addresses} & a \in \mathbb{A} & (\texttt{Addr}) \\ \hline \text{Memories} & M \in \mathbb{A} \xrightarrow{\mathsf{fin}} \mathbb{V} & (\texttt{Mem}) \\ \hline \text{Values} & \mathbb{V} \ni v ::= \dots \mid a & (\texttt{BoxV}) \\ \hline \end{array}$$

Addition



```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
  case Add(1, r) =>
    val (1v, lmem) = interp(1, env, mem)
    val (rv, rmem) = interp(r, env, lmem)
    (numAdd(1v, rv), rmem)
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\text{Add } \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M_1 \qquad \sigma, M_1 \vdash e_2 \Rightarrow n_2, M_2}{\sigma, M \vdash e_1 + e_2 \Rightarrow n_1 + n_2, M_2}$$

Addition



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```
/* BFAE */
val x = Box(5);
{ x.set(8); 2 } + x.get; // 2 + 8 = 10 -- NOT 2 + 5 = 7
```

Box Creation



```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
    ...
    case NewBox(c) =>
    val (cv, cmem) = interp(c, env, mem)
    val addr = malloc(cmem)
    (BoxV(addr), cmem + (addr -> cv))
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\operatorname{NewBox} \frac{\sigma, M \vdash e \Rightarrow v, M_1 \qquad a \notin \operatorname{Domain}(M_1)}{\sigma, M \vdash \operatorname{Box}(e) \Rightarrow a, M_1[a \mapsto v]}$$

Box Creation



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One way to implement malloc is to find the maximum address in the memory and increment it by one, 0 if the memory is empty:

```
def malloc(mem: Mem): Addr = mem.keySet.maxOption.fold(0)(_ + 1)
```

Box Content Getter



```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
   ...
   case GetBox(b) =>
    val (bv, bmem) = interp(b, env, mem)
   bv match
    case BoxV(addr) => (bmem(addr), bmem)
    case _ => error(s"not a box: ${bv.str}")
```

$$\sigma, M \vdash e \Rightarrow v, M$$

GetBox
$$\frac{\sigma, M \vdash e \Rightarrow a, M_1}{\sigma, M \vdash e. \mathtt{get} \Rightarrow M_1(a), M_1}$$

Box Content Setter



```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
  case SetBox(b, c) =>
    val (bv, bmem) = interp(b, env, mem)
    bv match
    case BoxV(addr) =>
        val (cv, cmem) = interp(c, env, bmem)
        (cv, cmem + (addr -> cv))
    case _ =>
        error(s"not a box: ${bv.str}")
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\texttt{SetBox} \ \frac{\sigma, M \vdash e_1 \Rightarrow a, M_1 \qquad \sigma, M_1 \vdash e_2 \Rightarrow v, M_2}{\sigma, M \vdash e_1.\mathtt{set}(e_2) \Rightarrow v, M_2[a \mapsto v]}$$

Sequence



```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
    ...
    case Seq(1, r) =>
      val (_, lmem) = interp(1, env, mem)
      interp(r, env, lmem)
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\mathrm{Seq} \ \frac{\sigma, M \vdash e_1 \Rightarrow _, M_1 \qquad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1; \ e_2 \Rightarrow v_2, M_2}$$

Summary



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2. BFAE - FAE with Mutable Boxes

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Exercise #6



https://github.com/ku-plrg-classroom/docs/tree/main/cose212/bfae

- Please see above document on GitHub:
 - Implement interp function.
- It is just an exercise, and you don't need to submit anything.
- However, some exam questions might be related to this exercise.

Next Lecture



Mutable Variables

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