

Lecture 27 – Course Review

COSE215: Theory of Computation

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2024 Spring

Goal of This Course



- What is the *mathematical model* of computers?

- What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

- What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve every *problem* using computers?

- What is the *mathematical model* of computers?

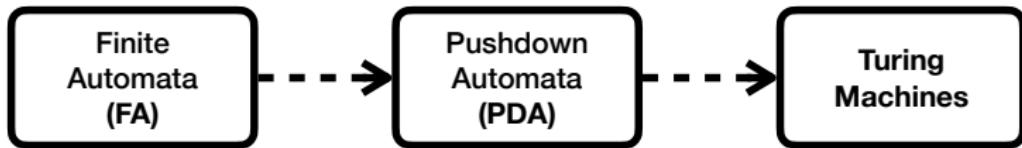
Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve every *problem* using computers?

No!

Let's learn **Undecidability** and **Intractability**



- **Finite Automata (FA)**
 - Regular Expressions and Languages
 - Applications: text search, etc.
- **Pushdown Automata (PDA)**
 - Context-Free Grammars (CFGs) and Languages (CFLs)
 - Applications: programming languages, natural language processing, etc.
- **Turing Machines (TMs)**
 - Extensions of Turing Machines
 - Undecidability and Intractability

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) NFA \longleftrightarrow DFA		(Lecture 3) RL
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala

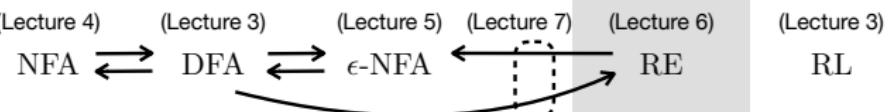
Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) NFA \leftrightarrow DFA \leftrightarrow ϵ -NFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) NFA \leftrightarrow DFA \leftrightarrow ϵ -NFA	(Lecture 6)	(Lecture 3) RE RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2)	Scala

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) (Lecture 3)</p> <p>NFA \leftrightarrow DFA \leftrightarrow ϵ-NFA $\xleftarrow{\quad}$ RE</p> 		RL
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p style="text-align: center;"> (Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) (Lecture 3) </p> <pre> graph LR NFA[NFA] <--> DFA[DFA] DFA <--> eNFA[epsilon-NFA] eNFA --> RE[RE] </pre>	<p style="text-align: center;"> RL ↓ Closure Properties (Lecture 8) </p>	
(Part 0) Basic Concepts	<p style="text-align: center;"> (Lecture 1) Mathematical Preliminaries </p>	<p style="text-align: center;"> (Lecture 2) Scala </p>	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6)</p> <pre> graph LR NFA[NFA] -- "Lecture 4" --> DFA[DFA] DFA -- "Lecture 3" --> eNFA[epsilon-NFA] eNFA -- "Lecture 5" --> RE[RE] RE -- "Lecture 6" --> eNFA NFA -- "Lecture 7" --> RE DFA -.-> eNFA RE -.-> Closure[Closure Properties (Lecture 8)] RE -.-> Pumping[Pumping Lemma (Lecture 9)] </pre>	<p>(Lecture 3)</p> <p>RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>	
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA \leftrightarrow DFA (Lecture 3) DFA \leftrightarrow ϵ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) ϵ -NFA \leftrightarrow RE (Lecture 7) RE \leftrightarrow RL (Lecture 6) RE \leftrightarrow Closure Properties (Lecture 8) (Lecture 3) RL \leftrightarrow Pumping Lemma (Lecture 9)	
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL
(Part 1) Finite Automata	(Lecture 4) NFA \leftrightarrow (Lecture 3) DFA \leftrightarrow (Lecture 5) ϵ -NFA $\xleftarrow{\text{RE}}$ (Lecture 7) RE 	(Lecture 3) RL Closure Properties (Lecture 8)	(Lecture 9) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2)	Scala

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA \leftrightarrow DFA (Lecture 3) DFA \leftrightarrow ϵ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) ϵ -NFA \leftrightarrow RE (Lecture 7) RE \leftrightarrow RE (Lecture 6) RE	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) PDA _{FS} PDA _{ES}	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) ϵ -NFA ↔ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	PDA _{FS} $\xleftrightarrow{\text{(Lecture 14/15)}}$ PDA _{ES} $\xleftrightarrow{\text{(Lecture 16)}}$ CFG (Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity		
(Part 1) Finite Automata	NFA $\xleftrightarrow{\text{(Lecture 4)}}$ DFA $\xleftrightarrow{\text{(Lecture 3)}}$ ϵ -NFA $\xleftrightarrow{\text{(Lecture 5)}}$ RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RL Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)	(Lecture 3)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> $\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 14/15})} \text{PDA}_{\text{ES}}$ $\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 16})} \text{CFG}$ \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup $\text{DPDA}_{\text{ES}} \rightleftharpoons_{(\text{Lecture 17})} \text{RE}$ </p>		<p style="text-align: center;"> $\text{CFL} \rightleftharpoons_{(\text{Lecture 11})} \text{CFG}$ $\text{CFL} \rightleftharpoons_{(\text{Lecture 13})} \text{Parse Trees \& Ambiguity}$ </p>
(Part 1) Finite Automata	<p style="text-align: center;"> $\text{NFA} \rightleftharpoons_{(\text{Lecture 4})} \text{DFA} \rightleftharpoons_{(\text{Lecture 3})} \epsilon\text{-NFA} \rightleftharpoons_{(\text{Lecture 5})} \text{RE}$ $\text{Equivalence \& Minimization} \quad (\text{Lecture 10})$ </p>		<p style="text-align: center;"> $\text{RE} \rightleftharpoons_{(\text{Lecture 6})} \text{RL}$ $\text{RL} \rightleftharpoons_{(\text{Lecture 3})} \text{Closure Properties} \quad (\text{Lecture 8})$ $\text{Closure Properties} \rightleftharpoons_{(\text{Lecture 8})} \text{Pumping Lemma} \quad (\text{Lecture 9})$ </p>
(Part 0) Basic Concepts	<p style="text-align: center;"> $\text{Mathematical Preliminaries} \quad (\text{Lecture 1})$ $\text{Scala} \quad (\text{Lecture 2})$ </p>		

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	$\text{PDA}_{\text{FS}} \rightleftarrows \text{PDA}_{\text{ES}}$ \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup (Lecture 17) \bowtie	$\text{PDA}_{\text{FS}} \rightleftarrows \text{PDA}_{\text{ES}}$ $\rightleftarrows \text{CFG}$ $\vdash \text{Chomsky Normal Form}$ (Lecture 18)	(Lecture 11) $\text{CFL} \cdots$ (Lecture 13) $\text{Parse Trees \& Ambiguity}$
(Part 1) Finite Automata	$\text{NFA} \rightleftarrows \text{DFA}$ $\text{NFA} \rightleftarrows \epsilon\text{-NFA}$ $\epsilon\text{-NFA} \rightleftarrows \text{RE}$ $\text{Equivalence \& Minimization}$ (Lecture 10)		(Lecture 3) RL $\vdash \text{Closure Properties}$ (Lecture 8) $\dashv \text{Pumping Lemma}$ (Lecture 9)
(Part 0) Basic Concepts	$\text{Mathematical Preliminaries}$ (Lecture 1)		(Lecture 2) Scala

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> $\text{PDA}_{\text{FS}} \rightleftharpoons \text{PDA}_{\text{ES}}$ (Lecture 14/15) $\text{PDA}_{\text{FS}} \rightleftharpoons \text{DPDA}_{\text{ES}}$ (Lecture 17) $\text{PDA}_{\text{ES}} \rightleftharpoons \text{DPDA}_{\text{ES}}$ (Lecture 16) </p> <p style="text-align: center;"> \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup (Lecture 17) \curlyeqsucc </p>	<p style="text-align: center;"> $\text{CFG} \rightleftharpoons \text{Chomsky Normal Form}$ (Lecture 11/12, Lecture 18) </p> <p style="text-align: center;"> \vdash $\text{Closure Properties}$ (Lecture 19) </p>	<p style="text-align: center;"> $\text{CFL} \rightleftharpoons \text{Parse Trees \& Ambiguity}$ (Lecture 11, Lecture 13) </p> <p style="text-align: center;"> \vdash $\text{Closure Properties}$ (Lecture 19) </p>
(Part 1) Finite Automata	<p style="text-align: center;"> $\text{NFA} \rightleftharpoons \text{DFA}$ (Lecture 4, Lecture 3) $\text{Equivalence \& Minimization}$ (Lecture 10) $\text{DFA} \rightleftharpoons \epsilon\text{-NFA}$ (Lecture 5) $\epsilon\text{-NFA} \rightleftharpoons \text{RE}$ (Lecture 7, Lecture 6) </p>		<p style="text-align: center;"> $\text{RE} \rightleftharpoons \text{RL}$ (Lecture 3) \vdash $\text{Closure Properties}$ (Lecture 8) $\text{RL} \rightleftharpoons \text{Pumping Lemma}$ (Lecture 9) </p>
(Part 0) Basic Concepts		<p style="text-align: center;"> (Lecture 1) Mathematical Preliminaries </p>	<p style="text-align: center;"> (Lecture 2) Scala </p>

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> $\text{PDA}_{\text{FS}} \rightleftarrows_{(\text{Lecture 14/15})} \text{PDA}_{\text{ES}}$ $\text{PDA}_{\text{FS}} \rightleftarrows_{(\text{Lecture 16})} \text{CFG}$ \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup $\text{DPDA}_{\text{FS}} \rightleftarrows_{(\text{Lecture 17})} \text{DPDA}_{\text{ES}}$ </p>	<p style="text-align: center;"> $\text{Chomsky Normal Form} (\text{Lecture 18})$ </p>	<p style="text-align: center;"> $\text{CFL} \rightleftarrows_{(\text{Lecture 11})} \text{Closure Properties} (\text{Lecture 19})$ $\text{Parse Trees \& Ambiguity} \rightleftarrows_{(\text{Lecture 13})} \text{Pumping Lemma} (\text{Lecture 20})$ </p>
(Part 1) Finite Automata	<p style="text-align: center;"> $\text{NFA} \rightleftarrows_{(\text{Lecture 4})} \text{DFA} \rightleftarrows_{(\text{Lecture 3})} \epsilon\text{-NFA} \rightleftarrows_{(\text{Lecture 5})} \text{RE}$ $\text{Equivalence \& Minimization} (\text{Lecture 10})$ </p>		<p style="text-align: center;"> $\text{RL} \rightleftarrows_{(\text{Lecture 3})} \text{Closure Properties} (\text{Lecture 8})$ $\text{Pumping Lemma} (\text{Lecture 9})$ </p>
(Part 0) Basic Concepts		<p style="text-align: center;"> $\text{Mathematical Preliminaries} (\text{Lecture 1})$ </p>	<p style="text-align: center;"> $\text{Scala} (\text{Lecture 2})$ </p>

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $\xrightarrow{\quad}$ PDA _{FS} $\xleftarrow{\quad}$ PDA _{ES} $\xleftarrow{\quad}$ (Lecture 16) $\xrightarrow{\quad}$ CFG \cup DPDA _{FS} \supset DPDA _{ES} \cup (Lecture 17) \curvearrowright	Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL \cdots Closure Properties (Lecture 19) \curvearrowright (Lecture 13) Parse Trees & Ambiguity Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA $\xleftrightarrow{\quad}$ (Lecture 3) DFA $\xleftrightarrow{\quad}$ (Lecture 5) ϵ -NFA $\xleftarrow{\quad}$ (Lecture 7) RE \curvearrowright Equivalence & Minimization (Lecture 10)		(Lecture 3) RL \cdots Closure Properties (Lecture 8) \curvearrowright (Lecture 9) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		(Lecture 2) Scala

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) (Lecture 21/22) ETM \leftrightarrow TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) (Lecture 16) (Lecture 11/12) PDA _{FS} \leftrightarrow PDA _{ES} \leftrightarrow CFG Chomsky Normal Form (Lecture 18) \cup DPDA _{FS} \supset DPDA _{ES} \cup (Lecture 17) \leftrightarrow		(Lecture 11) (Lecture 13) CFL Parse Trees & Closure Properties Ambiguity \vdash (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) NFA \leftrightarrow DFA \leftrightarrow ϵ -NFA \leftrightarrow RE \vdash Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	ETM \leftrightarrow TM \leftrightarrow LC (Lecture 23) (Lecture 21/22) (Lecture 24)		(Lecture 21) REL
(Part 2) Pushdown Automata	PDA _{FS} \leftrightarrow PDA _{ES} \leftrightarrow CFG (Lecture 14/15) (Lecture 16) (Lecture 11/12) DPDA _{FS} ⊂ DPDA _{ES} (Lecture 17)	Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL Closure Properties (Lecture 19) Parse Trees & Ambiguity Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	NFA \leftrightarrow DFA \leftrightarrow ϵ -NFA \leftrightarrow RE (Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	Mathematical Preliminaries (Lecture 1)		(Lecture 2) Scala

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			(Lecture 21) REL \cup DL (Lecture 25)
(Part 2) Pushdown Automata		Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL Parse Trees & Ambiguity Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata		(Lecture 6)	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts			(Lecture 2) Scala

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	ETM \leftrightarrow TM \leftrightarrow LC (Lecture 23) (Lecture 21/22) (Lecture 24)		(Lecture 21) REL \cup DL \supset NP $\stackrel{?}{=}$ P (Lecture 26) (Lecture 25)
(Part 2) Pushdown Automata	PDA _{FS} \leftrightarrow PDA _{ES} \leftrightarrow CFG (Lecture 14/15) (Lecture 16) (Lecture 11/12) DPDA _{FS} \supset DPDA _{ES} (Lecture 17)	Chomsky Normal Form (Lecture 18)	(Lecture 11) (Lecture 13) CFL ... Parse Trees & Ambiguity Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	NFA \leftrightarrow DFA \leftrightarrow ϵ -NFA \leftrightarrow RE (Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	Mathematical Preliminaries (Lecture 1)		(Lecture 2) Scala

- The final exam will be given in class.
- **Date:** 13:30-14:45 (1 hour 15 minutes), June 19 (Wed.).
- **Location:** 604, Woojung Hall of Informatics (우정정보관 604호)
- **Coverage:** Lectures 14 – 26
- **Format:** 7–9 questions with closed book and closed notes
 - Filling blanks in some tables, sentences, or expressions.
 - Construction of automata or grammars for given languages.
 - Proofs of given statements related to automata or grammars.
 - Yes/No questions about concepts in the theory of computation.
 - etc.
- Note that there is **no class** on **June 17 (Mon.)**.
- Please refer to the **previous exams** in the course website:

<https://plrg.korea.ac.kr/courses/cose215/>

- I hope you enjoyed the class!

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