Construct a **Turing machine** that **computes** the function:
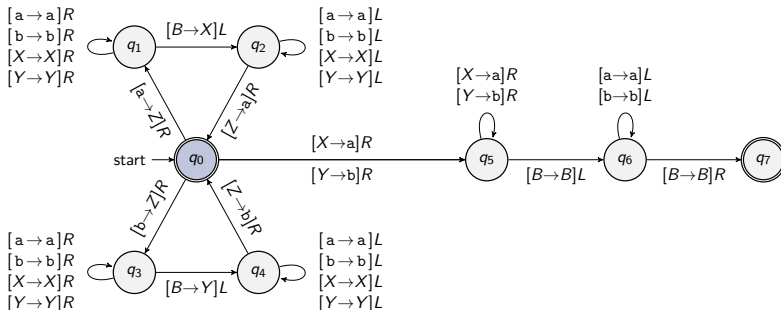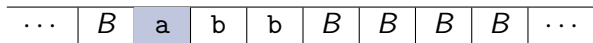
$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

$\cdots$	B	a	b	b	B	B	B	B	$\cdots$

1: **while** there are input symbols **do**
2:     Find and Replace a (or b) with Z
3:     Find and Fill the first blank with X (or Y) for a (or b)
4:     Roll back Z to the original a (or b)
5: Replace X's and Y's with a's and b's
6: Go to the first input symbol

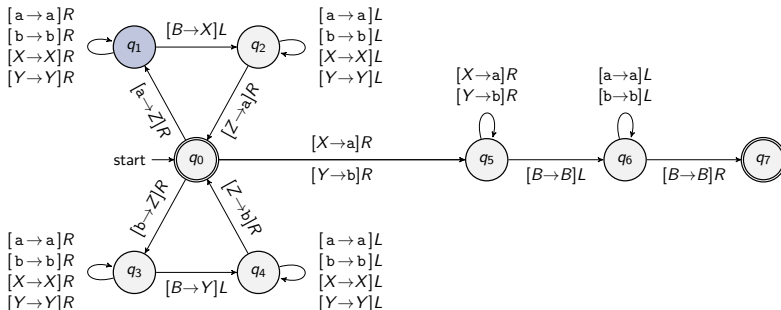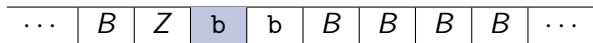Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

$\cdots$	B	a	b	b	B	B	B	B	$\cdots$

Find and Replace a (or b) with Z.

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

	$\cdots$	B	Z	b	b	B	B	B	B	$\cdots$

Find and Fill the first blank with X (or Y) for a (or 1).

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

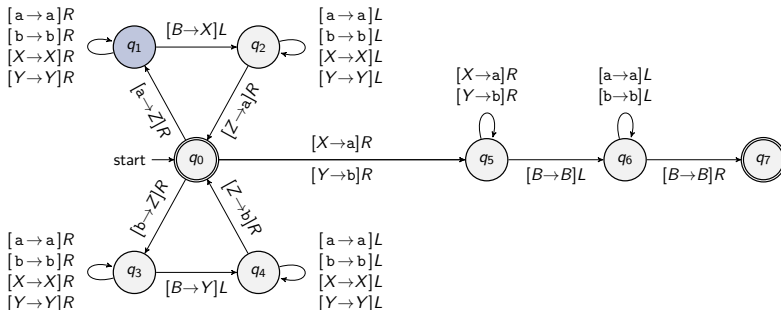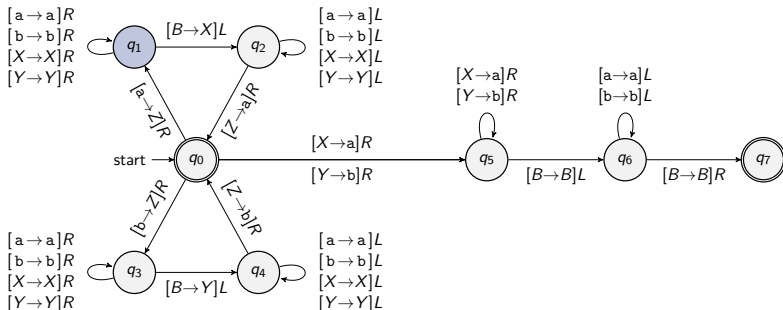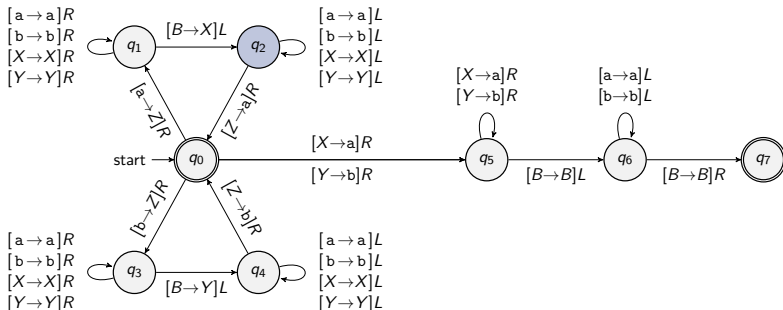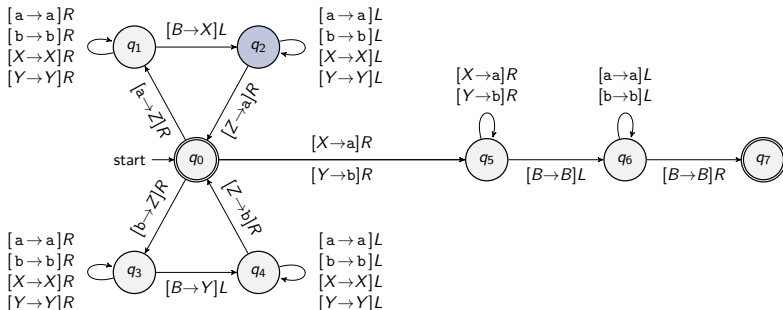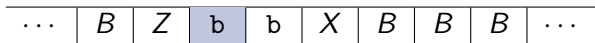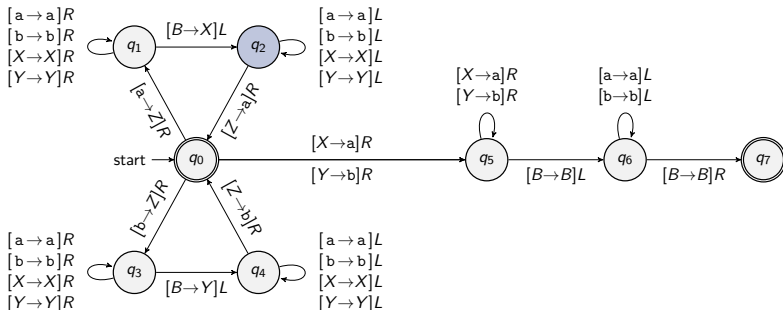Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

$\cdots$	B	Z	b	b	B	B	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

| $\cdots$ | B | Z | b | b | X | B | B | B | $\cdots$ |

Roll back Z to the original a (or b).

Construct a **Turing machine** that **computes** the function:
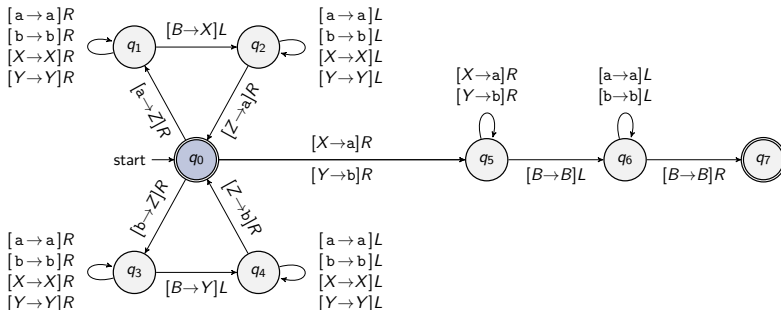
$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

| $\cdots$ | B | Z | $\mathtt{b}$ | $\mathtt{b}$ | X | B | B | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

$\cdots$	B	Z	$\mathtt{b}$	$\mathtt{b}$	X	B	B	B	$\cdots$
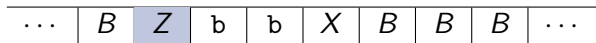
Construct a **Turing machine** that **computes** the function:

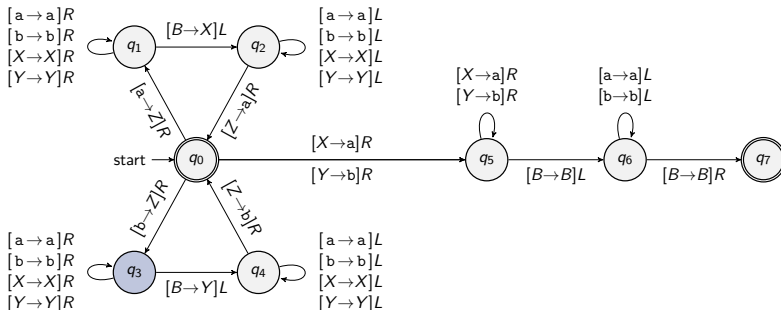$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

| $\cdots$ | B | a | b | b | X | B | B | B | $\cdots$ |

Repeat until no input symbols left.

Construct a **Turing machine** that **computes** the function:
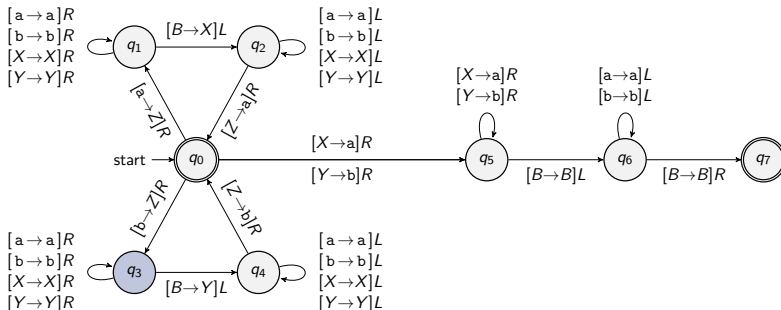
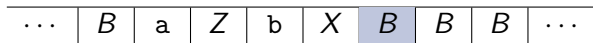$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

| $\cdots$ | B | a | Z | b | X | B | B | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

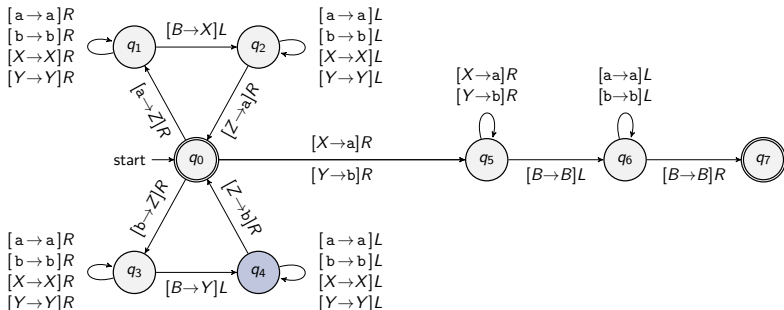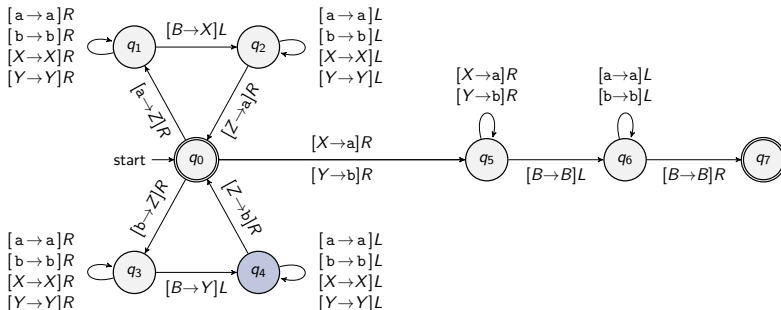| $\cdots$	B	a	Z	b	X	B	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

$\cdots$	B	a	Z	b	X	B	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\texttt{a}, \texttt{b}\}^*$$

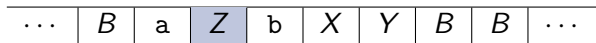| $\cdots$ | B | $\texttt{a}$ | Z | $\texttt{b}$ | X | Y | B | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

Construct a **Turing machine** that **computes** the function:
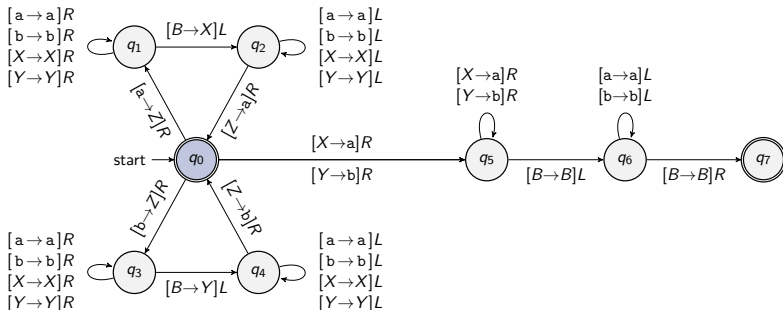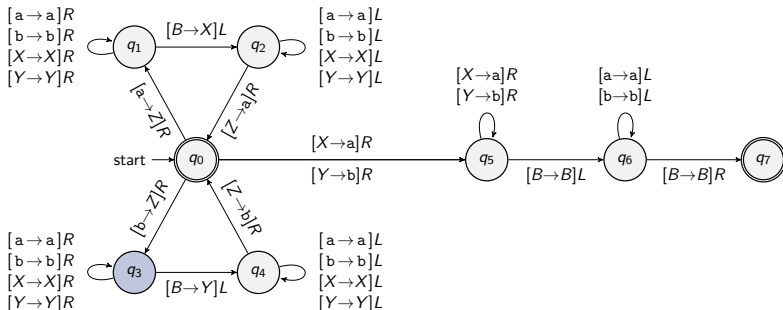
$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

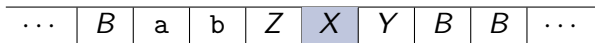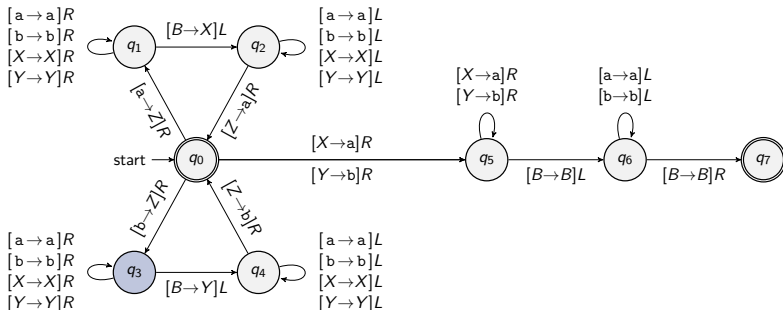| $\cdots$	B	a	Z	b	X	Y	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

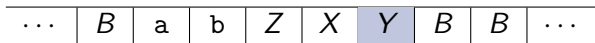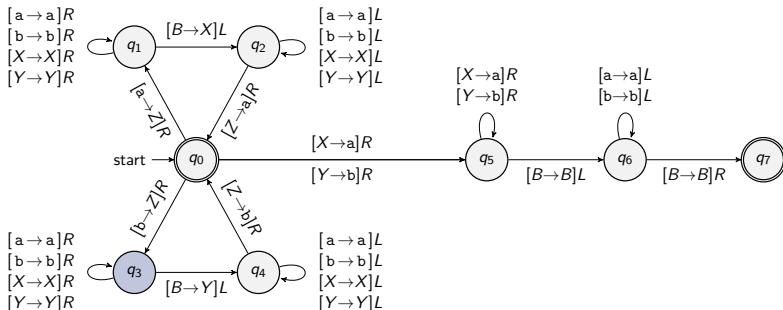| $\cdots$	B	a	b	b	X	Y	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

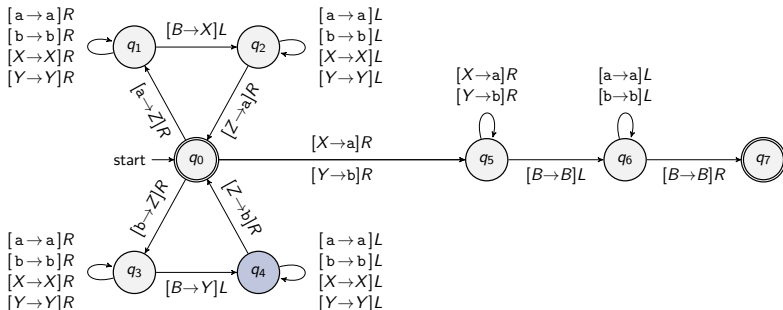| $\cdots$	B	a	b	Z	X	Y	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\text{a}, \text{b}\}^*$$

$\cdots$	B	a	b	Z	X	Y	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\texttt{a}, \texttt{b}\}^*$$

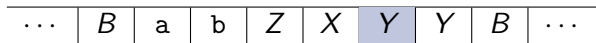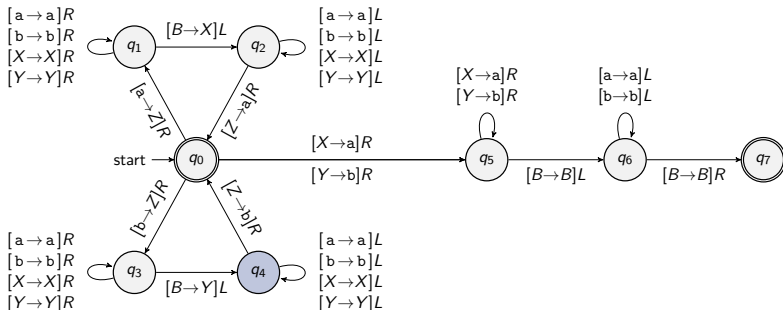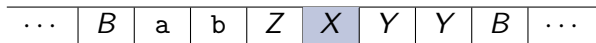| $\cdots$	B	a	b	Z	X	Y	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

$\cdots$	B	a	b	Z	X	Y	Y	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

$\cdots$	B	a	b	Z	X	Y	Y	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

| $\cdots$ | B | a | b | Z | X | Y | Y | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:
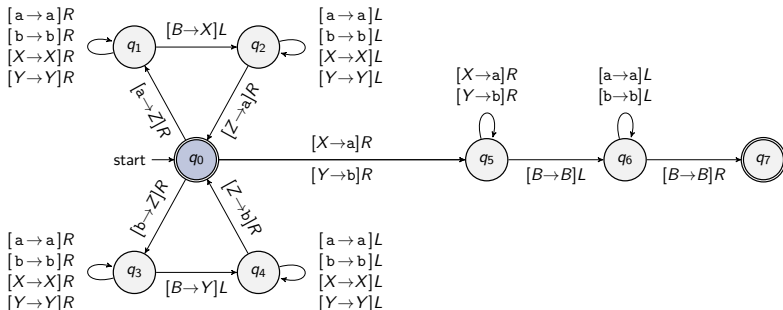
$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

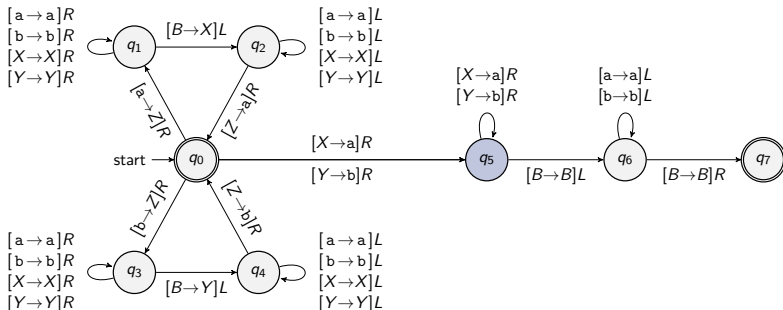| $\cdots$	B	a	b	b	X	Y	Y	B	$\cdots$

Replace X's and Y's with a's and b's.

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

$\cdots$	B	a	b	b	a	Y	Y	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

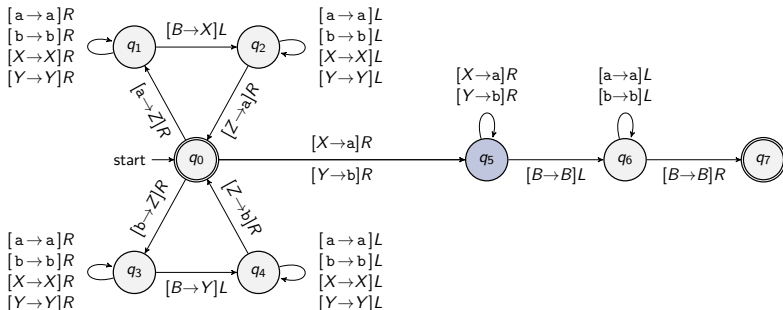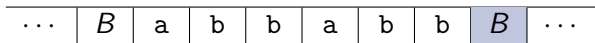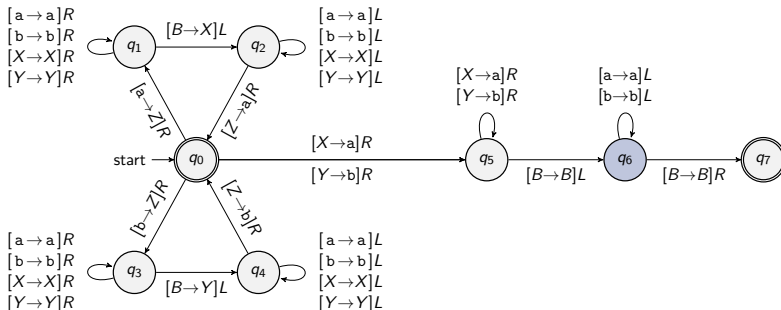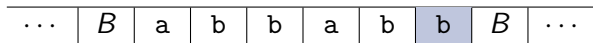| $\cdots$	B	a	b	b	a	b	Y	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

| $\cdots$ | B | a | b | b | a | b | b | B | $\cdots$ |

Go to the first input symbol.

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

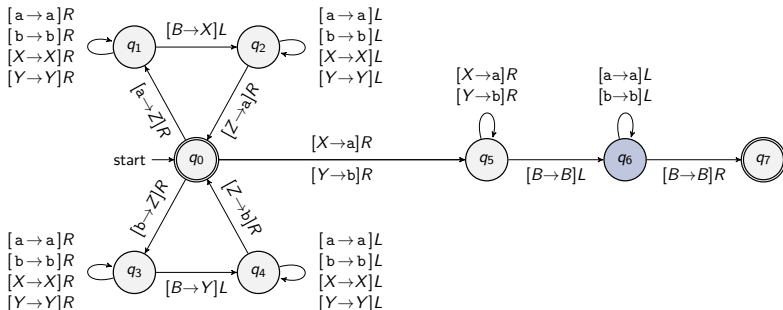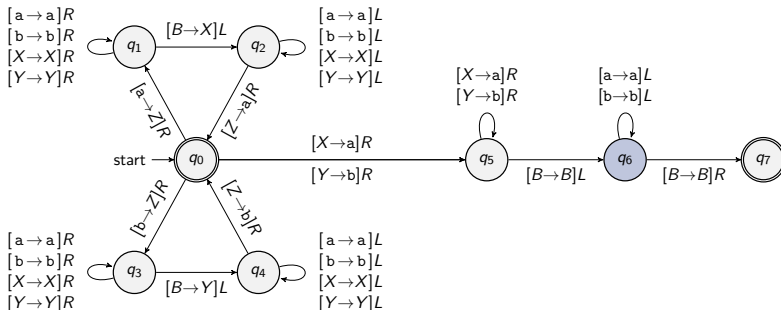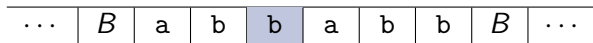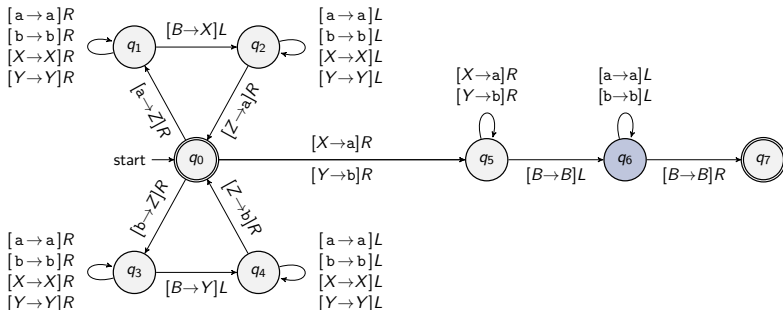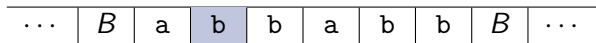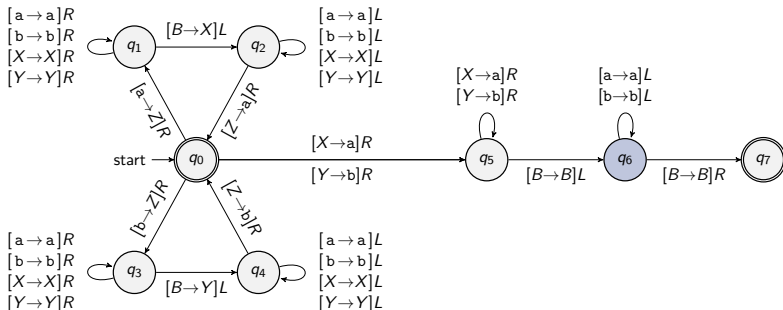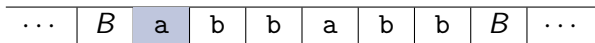| $\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

Construct a **Turing machine** that **computes** the function:
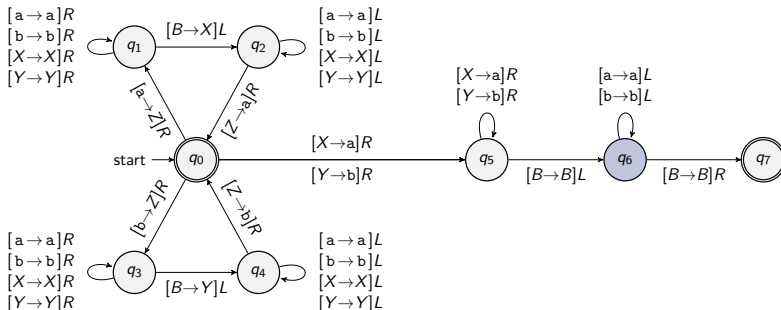
$$f(w) = ww \qquad \text{where} \qquad w \in \{a, b\}^*$$

	B	a	b	b	a	b	b	B	
$\cdots$									$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\mathtt{a}, \mathtt{b}\}^*$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{\text{a}, \text{b}\}^*$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Computed! $f(\text{abb}) = \text{abbabb}$