Lecture 13 – Parse Trees and Ambiguity

COSE215: Theory of Computation

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PLRG

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Recall

- A **context-free grammar (CFG)**:

$$G = (V, \Sigma, S, R)$$

- The **language** of a CFG G:

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow^* w\}$$

- A language L is a **context-free language (CFL)**:

$$\exists \text{ CFG } G.\ L(G) = L$$

- For a given word $w \in L(G)$, a **derivation** for w is $S \Rightarrow^* w$
- A sequence $\alpha \in (V \cup \Sigma)^*$ is a **sentential form** if $S \Rightarrow^* \alpha$.

Contents

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Parse Trees

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$$S \to \epsilon \mid (S) \mid SS$$

There are two different derivations for the sentential form $(S)(S)$:

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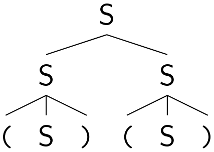
However, **parse trees** focus on the structure of the derivations instead of considering the order of the derivation steps.

For example, the above two derivations have the same parse tree:



Parse Trees

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Definition (Parse Trees)

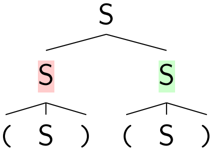
For a given CFG $G = (V, \Sigma, S, R)$, **parse trees** are trees satisfying:

1. The **root node** is labeled with the **start variable** S.
2. Each **internal node** is labeled with a **variable** $A \in V$.
   If its children are labeled with:
   $$X_1, X_2, \cdots, X_k$$
   from the left to the right, then $A \to X_1 X_2 \cdots X_k \in R$.
3. Each **leaf node** is labeled with a variable, symbol, or ϵ. However, if a leaf node is labeled with ϵ, it must be the only child of its parent.

Parse Trees – Example 1: Balanced Parentheses

$$S \to \epsilon \mid (S) \mid SS$$

A parse tree for $(S)(S)$:



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Parse Trees – Example 2: Even Palindromes

$$S \to \epsilon \mid \texttt{a}S\texttt{a} \mid \texttt{b}S\texttt{b}$$

A parse tree for `abba`:

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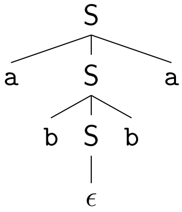
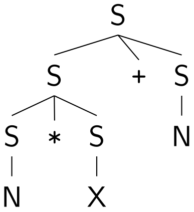
A parse tree for abba:



Parse Trees – Example 3: Arithmetic Expressions

$$
\begin{aligned}
S &\rightarrow N \mid X \mid S\texttt{+}S \mid S\texttt{*}S \mid \texttt{(}S\texttt{)} \\
N &\rightarrow \texttt{0} \mid \cdots \mid \texttt{9} \mid \texttt{0}N \mid \cdots \mid \texttt{9}N \\
X &\rightarrow \texttt{a} \mid \cdots \mid \texttt{z}
\end{aligned}
$$

A parse tree for $N\texttt{*}X\texttt{+}N$:



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The sequence obtained by concatenating the labels (without ϵ) of the leaf nodes of a parse tree from left to right is called the **yield** of the parse tree.

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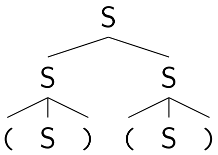

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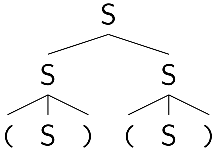

Its yield is $(S)(S)$.

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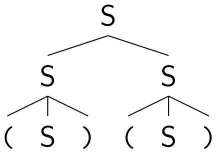

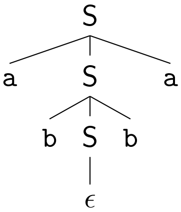

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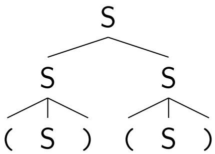
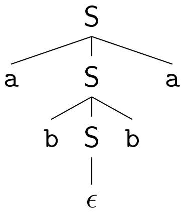


Its yield is $\texttt{(}S\texttt{)}\texttt{(}S\texttt{)}$.



Its yield is $\texttt{abba}$.

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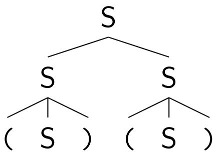

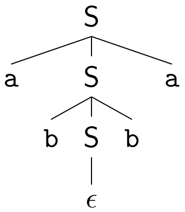

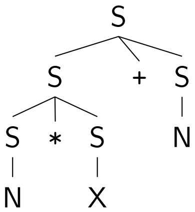

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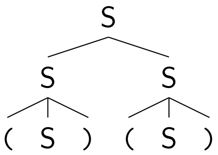
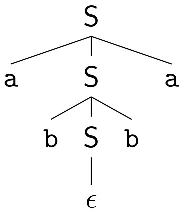


Its yield is `(S)(S)`.



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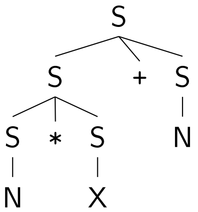

Its yield is $N*X+N$.

Relationship between Parse Trees and Derivations

Theorem (Parse Trees and Derivations)

For a given CFG $G = (V, \Sigma, S, R)$*, for any sequence* $\alpha \in (V \cup \Sigma)^*$*:*

$$S \Rightarrow^* \alpha \iff \exists \textit{ parse tree } T. \textit{ s.t. } T \textit{ yields } \alpha$$

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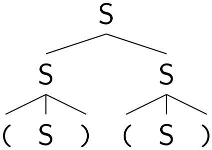
For example, consider the sequence $(S)(S)$:

$$S \Rightarrow SS \Rightarrow (S)S \Rightarrow (S)(S)$$

$$\Updownarrow$$



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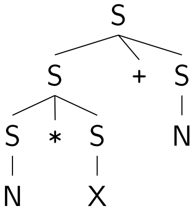

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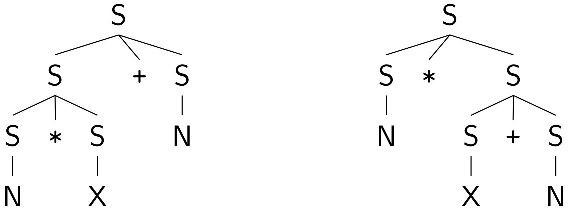

Actually, there are **two** parse trees for $N\texttt{*}X\texttt{+}N$.

Ambiguous Grammars

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Definition (Ambiguous Grammar)

A context-free grammar $G = (V, \Sigma, S, R)$ is **ambiguous** if there exist a word $w \in \Sigma^*$ and two distinct parse trees for w. If not, G is **unambiguous**.

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Theorem

Let $G = (V, \Sigma, S, R)$ be a CFG. Then, the following numbers are equal for any sequence of variables or symbols $w \in (V \cup \Sigma)^*$:

1. *The number of parse trees whose yields are w.*
2. *The number of left-most derivations for w.*
3. *The number of right-most derivations for w.*

Ambiguous Grammars – Example

$$
\begin{aligned}
S &\rightarrow N \mid X \mid S\texttt{+}S \mid S\texttt{*}S \mid \texttt{(}S\texttt{)} \\
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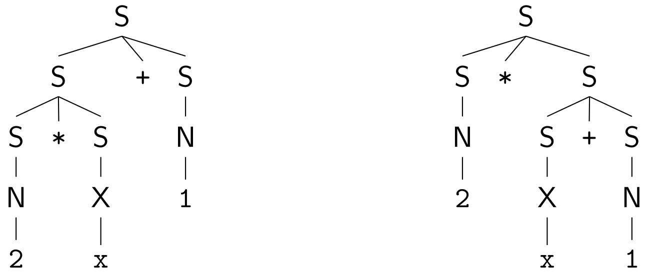
This grammar is **ambiguous** because there are **two** parse trees for the word `2 * x + 1`:



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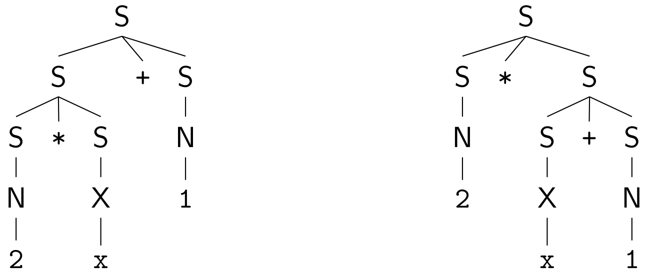

Note that it means that there are **two** left-most (or right-most) derivations for `2 * x + 1` by the previous theorem.

Ambiguous Grammars – Example

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$$
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S &\to N \mid X \mid S\texttt{+}S \mid S\texttt{*}S \mid \texttt{(}S\texttt{)} \\
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\end{aligned}
$$

There are **two** left-most derivations for `2 * x + 1`:

1. Applying the production rule $S \to S\texttt{+}S$ first:

$$
\begin{aligned}
S &\Rightarrow_L S\texttt{+}S \Rightarrow_L S\texttt{*}S\texttt{+}S \Rightarrow_L N\texttt{*}S\texttt{+}S \Rightarrow_L \texttt{2*}S\texttt{+}S \\
&\Rightarrow_L \texttt{2*}X\texttt{+}S \Rightarrow_L \texttt{2*x+}S \Rightarrow_L \texttt{2*x+}N \Rightarrow_L \texttt{2*x+1}
\end{aligned}
$$

2. Applying the production rule $S \to S\texttt{*}S$ first:

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Eliminating Ambiguity

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Fortunately, there are well-known techniques to manually **eliminate** the ambiguity in a given grammar commonly used in programming languages.

$$\begin{array}{lcl} S & \rightarrow & N \mid X \mid S+S \mid S*S \mid (S) \\ N & \rightarrow & 0 \mid \cdots \mid 9 \mid 0N \mid \cdots \mid 9N \\ X & \rightarrow & \texttt{a} \mid \cdots \mid \texttt{z} \end{array}$$

For example, an equivalent but unambiguous grammar is:

$$\begin{array}{lcl} S & \rightarrow & T \mid S+T \\ T & \rightarrow & F \mid T*F \\ F & \rightarrow & N \mid X \mid (S) \\ N & \rightarrow & 0 \mid \cdots \mid 9 \mid 0N \mid \cdots \mid 9N \\ X & \rightarrow & \texttt{a} \mid \cdots \mid \texttt{z} \end{array}$$

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Now, the unique parse tree for `2 * x + 1` is:

$$
\begin{aligned}
S &\rightarrow T \mid S\texttt{+}T \\
T &\rightarrow F \mid T\texttt{*}F \\
F &\rightarrow N \mid X \mid \texttt{(}S\texttt{)} \\
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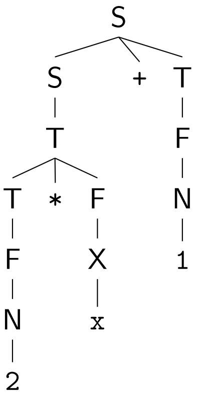

Eliminating Ambiguity

First, analyze why the original grammar is ambiguous.

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- The **precedence** of `+` and `*` is not specified.
  - For example, two parse trees for `1 * 2 + 3` interpreted as:

    `1 * (2 + 3)` and `(1 * 2) + 3`

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  - Let's give `*` higher precedence than `+` to interpret it as `(1 * 2) + 3`.
- The **associativity** of `+` (or `*`) is not specified.
  - For example, two parse trees for `1 + 2 + 3` interpreted as:

    `1 + (2 + 3)` and `(1 + 2) + 3`

  - Let's give the left-associativity to `+` to interpret it as `(1 + 2) + 3`.

Eliminating Ambiguity – Precedence

To enforce the **precedence**, define new variables F for factors and T for terms:

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- A **factor** is a number, a variable, or a parenthesized expression:

  $$\texttt{42}, \quad \texttt{x}, \quad \texttt{(1 + 2)}, \quad \cdots$$

  In the grammar, F is defined as:

  $$F \rightarrow N \mid X \mid (S)$$

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- A **term** is the multiplication of one or more factors:

  $$\texttt{42}, \quad \texttt{2 * x}, \quad \texttt{2 * (1 + 2)}, \quad \texttt{1 * (x * y) * z}, \quad \cdots$$

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  In the grammar, T is defined as:

  $$T \to F \mid T{*}F$$

- An **expression** is the addition of one or more terms:

  `42,   1 + 2,   1 + 2 * 3,   (1 + 2) * 3 + 4),   ···`

  In the grammar, S is defined as:

  $$S \to T \mid S{+}T$$

Eliminating Ambiguity – Associativity

The unambiguous grammar is:

$$
\begin{aligned}
S &\to T \mid S\texttt{+}T \\
T &\to F \mid T\texttt{*}F \\
F &\to N \mid X \mid \texttt{(}S\texttt{)} \\
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\end{aligned}
$$

Eliminating Ambiguity – Associativity

The unambiguous grammar is:

$$
\begin{array}{lcl}
S & \rightarrow & T \mid S\texttt{+}T \\
T & \rightarrow & F \mid T\texttt{*}F \\
F & \rightarrow & N \mid X \mid \texttt{(}S\texttt{)} \\
N & \rightarrow & \texttt{0} \mid \cdots \mid \texttt{9} \mid \texttt{0}N \mid \cdots \mid \texttt{9}N \\
X & \rightarrow & \texttt{a} \mid \cdots \mid \texttt{z}
\end{array}
$$

- This grammar supports the **left-associativity** of + and *. Why?

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- This grammar supports the **left-associativity** of + and ∗. Why?
  - $S \to S\texttt{+}T$ and $T \to T\texttt{*}F$ are **left-recursive**.

Eliminating Ambiguity – Associativity

The unambiguous grammar is:

$$\begin{array}{rcl} S & \rightarrow & T \mid \boxed{S+T} \\ T & \rightarrow & F \mid \boxed{T*F} \\ F & \rightarrow & N \mid X \mid (S) \\ N & \rightarrow & 0 \mid \cdots \mid 9 \mid 0N \mid \cdots \mid 9N \\ X & \rightarrow & \mathtt{a} \mid \cdots \mid \mathtt{z} \end{array}$$

- This grammar supports the **left-associativity** of + and $*$. Why?
  - $S \rightarrow S+T$ and $T \rightarrow T*F$ are **left-recursive**.
- Then, how to support the **right-associativity** of + and $*$?

Eliminating Ambiguity – Associativity

PLRG

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X &\rightarrow \texttt{a} \mid \cdots \mid \texttt{z}
\end{aligned}
$$

- This grammar supports the **left-associativity** of + and $*$. Why?
  - $S \rightarrow S\texttt{+}T$ and $T \rightarrow T\texttt{*}F$ are **left-recursive**.
- Then, how to support the **right-associativity** of + and $*$?
  - Replace the **left-recursive** rules with **right-recursive** rules!

$$
\begin{aligned}
S &\rightarrow T \mid \boxed{T\texttt{+}S} \\
T &\rightarrow F \mid \boxed{F\texttt{*}T} \\
&\cdots
\end{aligned}
$$

Inherent Ambiguity

So far, we have discussed the **ambiguity** for grammars.
We will now discuss the **inherent ambiguity** for languages.

Definition (Inherent Ambiguity)

A language L is **inherently ambiguous** if all CFGs whose languages are L are ambiguous. (i.e. there is no unambiguous grammar for L)

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For example, the following language is **inherently ambiguous**:

$$L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i, j, k \geq 0 \wedge (i = j \vee j = k)\}$$

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For example, the following language is **inherently ambiguous**:

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An example of ambiguous grammar for L is:

$$\begin{array}{rcl} S & \rightarrow & L \mid R \\ L & \rightarrow & A \mid L\mathtt{c} \\ A & \rightarrow & \epsilon \mid \mathtt{a}A\mathtt{b} \\ R & \rightarrow & B \mid \mathtt{a}R \\ B & \rightarrow & \epsilon \mid \mathtt{b}B\mathtt{c} \end{array}$$

Summary

1. Parse Trees
    - Definition
    - Yields
    - Relationship between Parse Trees and Derivations

2. Ambiguity
    - Ambiguous Grammars
    - Eliminating Ambiguity
    - Inherent Ambiguity

Midterm Exam

- The midterm exam will be given in class.
- **Date:** 13:30-14:45 (1 hour 15 minutes), April 24 (Wed.).
- **Location:** 604, Woojung Hall of Informatics (우정정보관 604호)
- **Coverage:** Lectures 1 – 13
- **Format:** 7–9 questions with closed book and closed notes
  - Filling blanks in some tables, sentences, or expressions.
  - Construction of automata or grammars for given languages.
  - Proofs of given statements related to automata or grammars.
  - Yes/No questions about concepts in the theory of computation.
  - etc.
- Note that there is **no class** on **April 22 (Mon.)**.
- Please refer to the **previous exams** in the course website:

https://plrg.korea.ac.kr/courses/cose215/

Next Lecture

- Pushdown Automata (PDA)

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