

# Lecture 27 – Course Review

## COSE215: Theory of Computation

Jihyeok Park



2025 Spring

# Goal of This Course



What is the *mathematical model* of computers?

What is the *mathematical model* of computers?

## Turing Machine!

Let's learn **Turing Machine**

What is the *mathematical model* of computers?

## Turing Machine!

Let's learn **Turing Machine**

Is it possible to solve *every problem* using computers?

What is the *mathematical model* of computers?

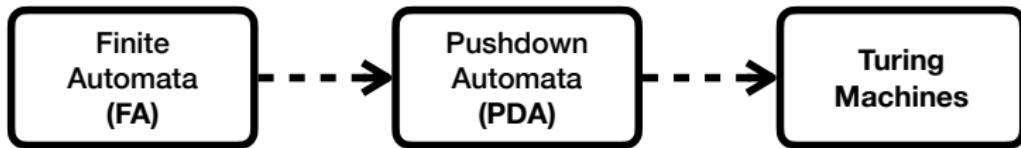
## Turing Machine!

Let's learn **Turing Machine**

Is it possible to solve *every problem* using computers?

**No!**

Let's learn **Undecidability** and **Intractability**



## Finite Automata (FA)

Regular Expressions and Languages

Applications: text search, etc.

## Pushdown Automata (PDA)

Context-Free Grammars (CFGs) and Languages (CFLs)

Applications: programming languages, natural language processing, etc.

## Turing Machines (TMs)

Extensions of Turing Machines

Undecidability and Intractability

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) <b>Mathematical Preliminaries</b>		

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1)  Mathematical Preliminaries	(Lecture 2)  Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3) NFA $\longleftrightarrow$ DFA		(Lecture 3) RL
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala

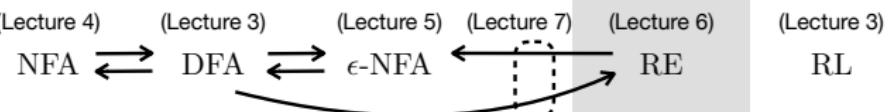
# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3)      (Lecture 5) NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3)      (Lecture 5) NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA	(Lecture 6)	(Lecture 3) RE      RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2)	Scala

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4)      (Lecture 3)      (Lecture 5)      (Lecture 7)      (Lecture 6)      (Lecture 3)</p> <p>NFA <math>\leftrightarrow</math> DFA <math>\leftrightarrow</math> <math>\epsilon</math>-NFA <math>\xleftarrow{\quad}</math> RE</p> 		RL
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4)      (Lecture 3)      (Lecture 5)      (Lecture 7)      (Lecture 6)</p> <p>NFA <math>\leftrightarrow</math> DFA <math>\leftrightarrow</math> <math>\epsilon</math>-NFA <math>\xleftarrow{\quad}</math> RE</p> <p>(Lecture 3)</p> <p>RL</p> <p>Closure Properties (Lecture 8)</p>		
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

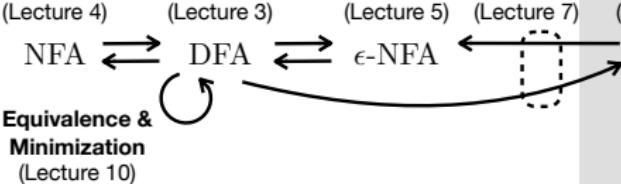
# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4)      (Lecture 3)      (Lecture 5)      (Lecture 7)      (Lecture 6)</p> <pre> graph LR     NFA[NFA] --&gt; DFA[DFA]     DFA --&gt; eNFA[epsilon-NFA]     eNFA --&gt; RE[RE]     RE --&gt; DFA     NFA -- curve --&gt; RE     NFA -.-&gt; eNFA     eNFA -.-&gt; DFA     RE -.-&gt; DFA   </pre>	<p>(Lecture 3)</p> <p>RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>	
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA $\leftrightarrow$ DFA (Lecture 3) DFA $\leftrightarrow$ $\epsilon$ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) $\epsilon$ -NFA $\leftrightarrow$ RE (Lecture 7) RE $\leftrightarrow$ RL (Lecture 6) RE $\leftrightarrow$ Closure Properties (Lecture 8) (Lecture 3) RL $\leftrightarrow$ Pumping Lemma (Lecture 9)	
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) $\epsilon$ -NFA ↔ (Lecture 7) RE 	(Lecture 3) RL Closure Properties (Lecture 8)	Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA $\leftrightarrow$ DFA (Lecture 3) DFA $\leftrightarrow$ $\epsilon$ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) $\epsilon$ -NFA $\leftrightarrow$ RE (Lecture 7) RE $\leftrightarrow$ RE (Lecture 6) RE	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) PDA <sub>FS</sub> PDA <sub>ES</sub>	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) $\epsilon$ -NFA  Equivalence & Minimization (Lecture 10)	(Lecture 7) RE ← ↔ → (Lecture 6)	(Lecture 3) RL Closure Properties (Lecture 8) ← ↔ → Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	PDA <sub>FS</sub> $\xleftrightarrow{\text{(Lecture 14/15)}}$ PDA <sub>ES</sub> $\xleftrightarrow{\text{(Lecture 16)}}$ CFG (Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity		
(Part 1) Finite Automata	NFA $\xleftrightarrow{\text{(Lecture 4)}}$ DFA $\xleftrightarrow{\text{(Lecture 3)}}$ $\epsilon$ -NFA $\xleftrightarrow{\text{(Lecture 5)}}$ RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RL Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)	(Lecture 3)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> <math>\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 14/15})} \text{PDA}_{\text{ES}}</math>      <math>\text{PDA}_{\text{FS}} \rightleftharpoons_{(\text{Lecture 16})} \text{CFG}</math>  <math>\cup</math>  <math>\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}</math>  <math>\cup</math>      <math>\text{DPDA}_{\text{ES}} \rightleftharpoons_{(\text{Lecture 17})} \text{RE}</math> </p>		<p style="text-align: center;"> <math>\text{CFL} \rightleftharpoons_{(\text{Lecture 11})} \text{CFG}</math>      <math>\text{CFL} \rightleftharpoons_{(\text{Lecture 13})} \text{Parse Trees \&amp; Ambiguity}</math> </p>
(Part 1) Finite Automata	<p style="text-align: center;"> <math>\text{NFA} \rightleftharpoons_{(\text{Lecture 4})} \text{DFA} \rightleftharpoons_{(\text{Lecture 3})} \epsilon\text{-NFA} \rightleftharpoons_{(\text{Lecture 5})} \text{RE}</math>  <math>\text{Equivalence \&amp; Minimization} \quad (\text{Lecture 10})</math> </p>		<p style="text-align: center;"> <math>\text{RE} \rightleftharpoons_{(\text{Lecture 6})} \text{RL}</math>  <math>\text{RL} \rightleftharpoons_{(\text{Lecture 3})} \text{Closure Properties} \quad (\text{Lecture 8})</math>  <math>\text{Closure Properties} \rightleftharpoons_{(\text{Lecture 8})} \text{Pumping Lemma} \quad (\text{Lecture 9})</math> </p>
(Part 0) Basic Concepts	<p style="text-align: center;"> <math>\text{Mathematical Preliminaries} \rightleftharpoons_{(\text{Lecture 1})} \text{Scala}</math>  <math>\text{Scala} \rightleftharpoons_{(\text{Lecture 2})} \text{Mathematical Preliminaries}</math> </p>		

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	$\text{PDA}_{\text{FS}} \leftrightarrow \text{PDA}_{\text{ES}}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ (Lecture 17) $\Leftrightarrow$	$\text{PDA}_{\text{FS}} \leftrightarrow \text{PDA}_{\text{ES}}$ $\leftrightarrow \text{CFG}$ $\vdash \text{Chomsky Normal Form}$ (Lecture 18)	(Lecture 11) $\text{CFL} \cdots$ (Lecture 13) $\text{Parse Trees \& Ambiguity}$
(Part 1) Finite Automata	$\text{NFA} \leftrightarrow \text{DFA}$ $\leftrightarrow \epsilon\text{-NFA}$ $\text{Equivalence \& Minimization}$ (Lecture 10)	$\text{NFA} \leftrightarrow \text{DFA} \leftrightarrow \epsilon\text{-NFA}$ $\leftrightarrow \text{RE}$	(Lecture 3) $\text{RL}$ $\vdash \text{Closure Properties}$ (Lecture 8) $\text{Pumping Lemma}$ (Lecture 9)
(Part 0) Basic Concepts		(Lecture 1) $\text{Mathematical Preliminaries}$ (Lecture 2) $\text{Scala}$	

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> <math>\text{PDA}_{\text{FS}} \rightleftharpoons \text{PDA}_{\text{ES}}</math> (Lecture 14/15)      <math>\text{PDA}_{\text{FS}} \rightleftharpoons \text{DPDA}_{\text{ES}}</math> (Lecture 17)      <math>\text{PDA}_{\text{ES}} \rightleftharpoons \text{DPDA}_{\text{ES}}</math> (Lecture 16)         </p> <p style="text-align: center;"> <math>\cup</math>  <math>\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}</math>  <math>\cup</math> (Lecture 17)      <math>\curlyeqsucc</math> </p>	<p style="text-align: center;"> <math>\text{CFG} \rightleftharpoons \text{Chomsky Normal Form}</math> (Lecture 11/12, Lecture 18)         </p> <p style="text-align: center;"> <math>\vdash</math>  <math>\text{Closure Properties}</math> (Lecture 19)         </p>	<p style="text-align: center;"> <math>\text{CFL} \dashv \text{Parse Trees &amp; Ambiguity}</math> (Lecture 11, Lecture 13)         </p> <p style="text-align: center;"> <math>\vdash</math>  <math>\text{Closure Properties}</math> (Lecture 19)         </p>
(Part 1) Finite Automata	<p style="text-align: center;"> <math>\text{NFA} \rightleftharpoons \text{DFA}</math> (Lecture 4, Lecture 3)  <math>\text{NFA} \rightleftharpoons \epsilon\text{-NFA}</math> (Lecture 5, Lecture 7)  <math>\text{DFA} \rightleftharpoons \epsilon\text{-NFA}</math> (Lecture 6)  <math>\epsilon\text{-NFA} \rightleftharpoons \text{RE}</math> (Lecture 7, Lecture 6)         </p> <p style="text-align: center;"> <math>\curvearrowleft</math>  <b>Equivalence &amp; Minimization</b> (Lecture 10)         </p>		<p style="text-align: center;"> <math>\text{RE} \rightleftharpoons \text{RL}</math> (Lecture 3)  <math>\vdash</math>  <b>Closure Properties</b> (Lecture 8)  <math>\dashv</math>  <b>Pumping Lemma</b> (Lecture 9)         </p>
(Part 0) Basic Concepts		<p style="text-align: center;"> <math>\text{Mathematical Preliminaries}</math> (Lecture 1)         </p>	<p style="text-align: center;"> <math>\text{Scala}</math> (Lecture 2)         </p>

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p style="text-align: center;"> <math>\text{PDA}_{\text{FS}} \rightleftarrows_{(\text{Lecture 14/15})} \text{PDA}_{\text{ES}}</math>      <math>\text{PDA}_{\text{FS}} \rightleftarrows_{(\text{Lecture 16})} \text{CFG}</math>  <math>\cup</math>  <math>\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}</math>  <math>\cup</math>      <math>\text{DPDA}_{\text{FS}} \rightleftarrows_{(\text{Lecture 17})} \text{DPDA}_{\text{ES}}</math> </p>	<p style="text-align: center;"> <math>\text{Chomsky Normal Form} (\text{Lecture 18})</math> </p>	<p style="text-align: center;"> <math>\text{CFL} \rightleftarrows_{(\text{Lecture 11})} \text{Closure Properties} (\text{Lecture 19})</math>      <math>\text{Parse Trees \&amp; Ambiguity} \rightleftarrows_{(\text{Lecture 13})} \text{Pumping Lemma} (\text{Lecture 20})</math> </p>
(Part 1) Finite Automata	<p style="text-align: center;"> <math>\text{NFA} \rightleftarrows_{(\text{Lecture 4})} \text{DFA} \rightleftarrows_{(\text{Lecture 3})} \epsilon\text{-NFA} \rightleftarrows_{(\text{Lecture 5})} \text{RE}</math>  <math>\text{Equivalence \&amp; Minimization} (\text{Lecture 10})</math> </p>		<p style="text-align: center;"> <math>\text{RL} \rightleftarrows_{(\text{Lecture 3})} \text{Closure Properties} (\text{Lecture 8})</math>      <math>\text{Pumping Lemma} (\text{Lecture 9})</math> </p>
(Part 0) Basic Concepts		<p style="text-align: center;"> <math>\text{Mathematical Preliminaries} (\text{Lecture 1})</math> </p>	<p style="text-align: center;"> <math>\text{Scala} (\text{Lecture 2})</math> </p>

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $\xrightarrow{\quad}$ PDA <sub>FS</sub> $\xleftarrow{\quad}$ PDA <sub>ES</sub> $\xleftarrow{\quad}$ (Lecture 16) $\xrightarrow{\quad}$ CFG $\cup$ DPDA <sub>FS</sub> $\supset$ DPDA <sub>ES</sub> $\cup$ (Lecture 17) $\curvearrowright$	Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL $\cdots$ Closure Properties (Lecture 19) $\curvearrowright$ Parse Trees & Ambiguity Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA $\xleftrightarrow{\quad}$ (Lecture 3) DFA $\xleftrightarrow{\quad}$ (Lecture 5) $\epsilon$ -NFA $\xleftarrow{\quad}$ (Lecture 7) RE $\curvearrowright$ Equivalence & Minimization (Lecture 10)		(Lecture 3) RL $\cdots$ Closure Properties (Lecture 8) $\curvearrowright$ Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		(Lecture 2) Scala

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23)      (Lecture 21/22) ETM $\leftrightarrow$ TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15)      (Lecture 16)      (Lecture 11/12) PDA <sub>FS</sub> $\leftrightarrow$ PDA <sub>ES</sub> $\leftrightarrow$ CFG Chomsky Normal Form (Lecture 18)  $\cup$ DPDA <sub>FS</sub> $\supset$ DPDA <sub>ES</sub> $\cup$ (Lecture 17) $\leftrightarrow$		(Lecture 11)      (Lecture 13) CFL      Parse Trees & Closure Properties      Ambiguity  (Lecture 19)      Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4)      (Lecture 3)      (Lecture 5)      (Lecture 7)      (Lecture 6) NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA $\leftrightarrow$ RE  Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8)      Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts		(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala

# Summary

	Automata	Grammars	Languages
<b>(Part 3) Turing Machines</b>	ETM $\leftrightarrow$ TM $\leftrightarrow$ LC (Lecture 23) (Lecture 21/22) (Lecture 24)		(Lecture 21) REL
<b>(Part 2) Pushdown Automata</b>	PDA <sub>FS</sub> $\leftrightarrow$ PDA <sub>ES</sub> $\leftrightarrow$ CFG (Lecture 14/15) (Lecture 16) (Lecture 11/12) DPDA <sub>FS</sub> ⊂ DPDA <sub>ES</sub> (Lecture 17)	Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL Closure Properties (Lecture 19) Parse Trees & Ambiguity Pumping Lemma (Lecture 20)
<b>(Part 1) Finite Automata</b>	NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA $\leftrightarrow$ RE (Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
<b>(Part 0) Basic Concepts</b>	Mathematical Preliminaries (Lecture 1)		(Lecture 2) Scala

# Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			(Lecture 21) REL $\cup$ DL (Lecture 25)
(Part 2) Pushdown Automata		Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL $\cdots$ Parse Trees & Ambiguity  Closure Properties (Lecture 19)
(Part 1) Finite Automata			(Lecture 3) RL $\cdots$ Closure Properties (Lecture 8)
(Part 0) Basic Concepts		(Lecture 2) Scala	Pumping Lemma (Lecture 20)

# Summary

	Automata	Grammars	Languages
<b>(Part 3) Turing Machines</b>	ETM $\leftrightarrow$ TM $\leftrightarrow$ LC (Lecture 23) (Lecture 21/22) (Lecture 24)		(Lecture 21) REL $\cup$ DL $\supset$ NP $\stackrel{?}{=}$ P (Lecture 26) (Lecture 25)
<b>(Part 2) Pushdown Automata</b>	PDA <sub>FS</sub> $\leftrightarrow$ PDA <sub>ES</sub> $\leftrightarrow$ CFG (Lecture 14/15) (Lecture 16) (Lecture 11/12) DPDA <sub>FS</sub> $\supset$ DPDA <sub>ES</sub> (Lecture 17)	Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ... Parse Trees & Ambiguity Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
<b>(Part 1) Finite Automata</b>	NFA $\leftrightarrow$ DFA $\leftrightarrow$ $\epsilon$ -NFA $\leftrightarrow$ RE (Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
<b>(Part 0) Basic Concepts</b>	Mathematical Preliminaries (Lecture 1)		(Lecture 2) Scala

The final exam will be given in class.

**Date:** 13:30-14:45 (1 hour 15 minutes), June 23 (Mon.).

**Location:** 301, Aegineung (애기능생활관 301호)

**Coverage:** Lectures 14 – 26

**Format:** 8 questions with closed book and closed notes

Yes/No questions about concepts in the theory of computation.

Construction of automata or grammars for given languages.

Proofs of given statements related to automata or grammars.

etc.

Note that there is **no class** on **June 18 (Wed.)**.

Please refer to the **previous exams** in the course website:

<https://plrg.korea.ac.kr/courses/cose215/>

I hope you enjoyed the class!

Jihyeok Park

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<https://plrg.korea.ac.kr>