Lecture 17 – Compiling with Continuations

COSE212: Programming Languages

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PLRG

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Recall

- We will learn about **continuations** with the following topics:
  - **Continuations** (Lecture 14 & 15)
  - **First-Class Continuations** (Lecture 16)
  - **Compiling with continuations** (Lecture 17)

- A **continuation** represents the **rest of the computation**.
  - First-Class Continuations
  - KFAE – FAE with first-class continuations
  - Control Statements

- In this lecture, let's learn **compiling with continuations**.

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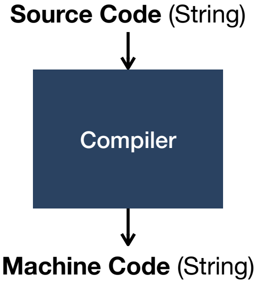
Contents

Compilers

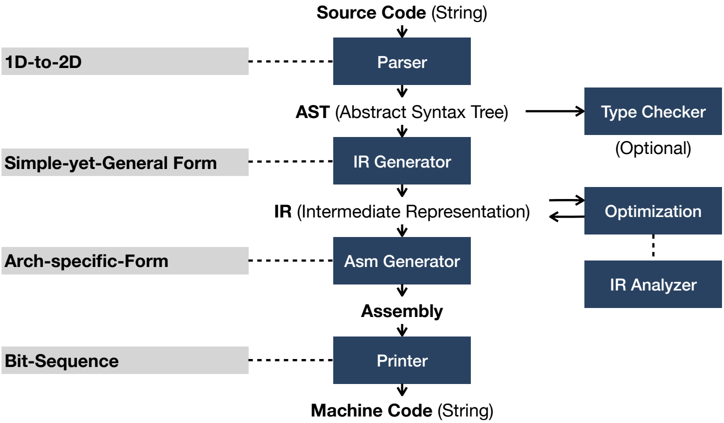
A **compiler** is a program that translates a program written in one language (the **source language**) into an equivalent program in another language (the **target language**).



Typically, the source language is a **high-level language** (e.g., Scala, Python, JavaScript, etc.) and the target language is a **low-level language** (e.g., JVM bytecode, LLVM IR, assembly, etc.).

Compilers

The following figure shows a typical compilation process:



Let's focus on the **IR Generator** to learn how to compile with functional languages with continuations into a **low-level IR**.

Compiling Functional Languages

How to compile our **functional languages** into a **low-level IR**?

```
/* FAE */
val twice = f => {
  a => f(f(a))
};
twice({
  b => b * 2 + 1
})(3) + 5
```

$\Longrightarrow$

```
/* IR */
F1:
    mov r4, r3
    jmp r2
F2:
    mov r4, F1
    jmp r2
F3:
    mov r1, F2
    jmp r2
F4:
    mul r1, r1, 2
    add r1, r1, 1
    jmp r4
```

```
F5:
    add r1, r1, 5
    jmp HALT
F6:
    mov r4, r1
    mov r1, 3
    mov r2, F4
    mov r3, F5
    jmp r4
START:
    mov r1, F4
    mov r2, F6
    jmp F3
HALT:
```

Let's learn how to compile with **continuations**!

Contents

Recall: Continuation-Passing Style (CPS)

We learned that **continuation-passing style (CPS)** is a style of programming that passes the continuation as an explicit parameter to a function and calls it to give the result to the continuation.

For example, consider the following Scala code written in **direct style**:

```scala
def sum(n: Int): Int =
  if (n <= 1) 1
  else sum(n - 1) + n
sum(3) * 5                // (1 + 2 + 3) * 5 = 30
```

We can rewrite it in **continuation-passing style** as follows:

```scala
type Cont = Int => Int
def sumCPS(n: Int, k: Cont): Int =
  if (n <= 1) k(1)
  else sumCPS(n - 1, x => k(x + n))
sumCPS(3, x => x * 5)      // (1 + 2 + 3) * 5 = 30
```

Continuation-Passing Style (CPS)

Let's apply the CPS transformation to our running example.

(Assume that FAE is extended with multiple parameters.)

```
/* FAE */
val twice = f => {
  a => f(f(a))
};
twice({
  b => b * 2 + 1
})(3) + 5
```

Continuation-Passing Style (CPS)

Let's apply the CPS transformation to our running example.

(Assume that FAE is extended with multiple parameters.)

```
/* FAE */
val HALT = x => x;
val twice = f => {
  a => f(f(a))
};
HALT(twice({
  b => b * 2 + 1
})(3) + 5)
```

Let's transform the `twice` function into CPS.

Continuation-Passing Style (CPS)

Let's apply the CPS transformation to our running example.

(Assume that FAE is extended with multiple parameters.)

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1(a => f(f(a)))
};
twice({
  b => b * 2 + 1
}, x1 => HALT(x1(3) + 5))
```

Let's transform the `a => f(f(a))` function into CPS.

Continuation-Passing Style (CPS)

Let's apply the CPS transformation to our running example.

(Assume that FAE is extended with multiple parameters.)

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => k2(f(f(a))))
};
twice({
  b => b * 2 + 1
}, x1 => x1(3, x2 => HALT(x2 + 5)))
```

Let's transform the body of `x2 => HALT(x2 + 5)` into CPS using the syntactic sugar for `val`.

Continuation-Passing Style (CPS)

Let's apply the CPS transformation to our running example.

(Assume that FAE is extended with multiple parameters.)

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => k2(f(f(a))))
};
twice({
  b => b * 2 + 1
}, x1 => x1(3, x2 => {
  val x3 = x2 + 5;
  HALT(x3)
})
```

Let's transform the `b => b * 2 + 1` function into CPS.

Continuation-Passing Style (CPS)

Let's apply the CPS transformation to our running example.

(Assume that FAE is extended with multiple parameters.)

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
twice({
  (b, k3) => k3(b * 2 + 1)
}, x1 => x1(3, x2 => {
  val x3 = x2 + 5;
  HALT(x3)
}))
```

Let's transform the body of `(b, k3) => k3(b * 2 + 1)` into CPS using the syntactic sugar for `val`.

Continuation-Passing Style (CPS)

Let's apply the CPS transformation to our running example.

(Assume that FAE is extended with multiple parameters.)

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
twice((b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
}, x1 => x1(3, x2 => {
  val x3 = x2 + 5;
  HALT(x3)
}))
```

This is the CPS version of our running example.

Lambda Lifting

A **lambda lifting** transformation lifts nested functions to top-level functions.

Let's apply the **lambda lifting** transformation to our running example.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
twice((b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
}, x1 => x1(3, x2 => {
  val x3 = x2 + 5;
  HALT(x3)
}))
```

Lambda Lifting

Let's apply the **lambda lifting** transformation to our running example.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
twice((b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
}, x1 => x1(3, x2 => {
  val x3 = x2 + 5;
  HALT(x3)
}))
```

First, let's lift the `(b, k3) => ...` function to top-level.

Lambda Lifting

Let's apply the **lambda lifting** transformation to our running example.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
twice(x7, x1 => x1(3, x2 => {
  val x3 = x2 + 5;
  HALT(x3)
}))
```

Lambda Lifting

Let's apply the **lambda lifting** transformation to our running example.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
twice(x7, x1 => x1(3, x2 => {
  val x3 = x2 + 5;
  HALT(x3)
}))
```

Next, let's lift the `x2 => ...` function to top-level.

Lambda Lifting

Let's apply the **lambda lifting** transformation to our running example.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
val C1 = x2 => {
  val x3 = x2 + 5;
  HALT(x3)
};
twice(x7, x1 => x1(3, C1))
```

We use the name `Ck` to denote that the function is a **continuation**.

Lambda Lifting

Let's apply the **lambda lifting** transformation to our running example.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
val C1 = x2 => {
  val x3 = x2 + 5;
  HALT(x3)
};
twice(x7, x1 => x1(3, C1))
```

Let's lift the `x1 => ...` function to top-level.

Lambda Lifting

Let's apply the **lambda lifting** transformation to our running example.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
val C1 = x2 => {
  val x3 = x2 + 5;
  HALT(x3)
};
val C2 = x1 => x1(3, C1);
twice(x7, C2)
```

Lambda Lifting

We cannot lift the `(a, k2) => ...` and `x4 => ...` functions because `f` is their **captured variable** from the `twice` function.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
val C1 = x2 => {
  val x3 = x2 + 5;
  HALT(x3)
};
val C2 = x1 => x1(3, C1);
twice(x7, C2)
```

Lambda Lifting

Similarly, `k2` in the `x4 => ...` function is also a **captured variable** from the `(a, k2) => ...` function.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
val C1 = x2 => {
  val x3 = x2 + 5;
  HALT(x3)
};
val C2 = x1 => x1(3, C1);
twice(x7, C2)
```

Closure Conversion

To resolve this problem, we need to perform **closure conversion** by passing the captured variables as arguments to the function.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, k2) => f(a, x4 => f(x4, k2)))
};
val x7 = (b, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6)
};
val C1 = x2 => {
  val x3 = x2 + 5;
  HALT(x3)
};
val C2 = x1 => x1(3, C1);
twice(x7, C2)
```

Closure Conversion

There are diverse **closure conversion** algorithms, but we skip their details in this course. If we perform one of them, the result is as follows.

```
/* FAE */
val HALT = x => x;
val twice = (f, k1) => {
  k1((a, f1, k2) => f1(a, f1, k2, (x4, f2, k4) => f2(x4, f2, k4, k4)))
};
val x7 = (b, f3, k5, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6, f3, k5)
};
val C1 = (x2, f4, k6) => {
  val x3 = x2 + 5;
  HALT(x3)
};
val C2 = x1 => x1(3, x7, C1);
twice(x7, C2)
```

Closure Conversion

Finally, we can perform **lambda lifting** transformation for remaining functions as follows:

```
/* FAE */
val HALT = x => x;
val C3 = (x4, f2, k4) => {
  f2(x4, f2, k4, k4)
};
val C4 = (a, f1, k2) => {
  f1(a, f1, k2, C3)
};
val twice = (f, k1) => {
  k1(C4)
};
```

```
val x7 = (b, f3, k5, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6, f3, k5)
};
val C1 = (x2, f4, k6) => {
  val x3 = x2 + 5;
  HALT(x3)
};
val C2 = x1 => {
  x1(3, x7, C1)
};
twice(x7, C2)
```

Closure Conversion

Now, our transformed code satisfies the following conditions.

1. Every function is in the **top-level scope**.
2. Every function call is in **tail position**.
3. Every function always **ends with function call**.

```
/* FAE */
val HALT = x => x;
val C3 = (x4, f2, k4) => {
  f2(x4, f2, k4, k4)
};
val C4 = (a, f1, k2) => {
  f1(a, f1, k2, C3)
};
val twice = (f, k1) => {
  k1(C4)
};
```

```
val x7 = (b, f3, k5, k3) => {
  val x5 = b * 2;
  val x6 = x5 + 1;
  k3(x6, f3, k5)
};
val C1 = (x2, f4, k6) => {
  val x3 = x2 + 5;
  HALT(x3)
};
val C2 = x1 => {
  x1(3, x7, C1)
};
twice(x7, C2)
```

Alpha Renaming

To easily convert the code into the **low-level IR**, we need to perform **alpha renaming** to make every variable name unique and in a consistent manner (`Fk`: k-th function, `xk`: k-th parameter).

```
/* FAE */
val HALT = x => x;
val F1 = (x1, x2, x3) => {
  x2(x1, x2, x3, x3)
};
val F2 = (x1, x2, x3) => {
  x2(x1, x2, x3, F1)
};
val F3 = (x1, x2) => {
  x2(F2)
};
```

```
val F4 = (x1, x2, x3, x4) => {
  val x5 = x1 * 2;
  val x6 = x5 + 1;
  x4(x6, x2, x3)
};
val F5 = (x1, x2, x3) => {
  val x4 = x1 + 5;
  HALT(x4)
};
val F6 = x1 => {
  x1(3, F4, F5)
};
F3(F4, F6)
```

Transformation to Low-level IR

Now, we can easily convert the code into the **low-level IR**.

```
F1:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  mov a1, x1
  mov a2, x2
  mov a3, x3
  mov a4, x3
  jmp x2
F2:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  mov a1, x1
  mov a2, x2
  mov a3, x3
  mov a4, F1
  jmp x2
```

```
F3:
  mov x1, a1
  mov x2, a2
  mov a1, F2
  jmp x2
F4:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  mov x4, a4
  mul x5, x1, 2
  add x6, x5, 1
  mov a1, x6
  mov a2, x2
  mov a3, x3
  jmp x4
```

```
F5:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  add x4, x1, 5
  mov a1, x4
  jmp HALT
F6:
  mov x1, a1
  mov a1, 3
  mov a2, F4
  mov a3, F5
  jmp x1
START:
  mov a1, F4
  mov a2, F6
  jmp F3
HALT:
```

Optimization of Low-level IR

The following lines of code are actually **unnecessary**:

```
F1:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  mov a1, x1
  mov a2, x2
  mov a3, x3
  mov a4, x3
  jmp x2
F2:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  mov a1, x1
  mov a2, x2
  mov a3, x3
  mov a4, F1
  jmp x2
```

```
F3:
  mov x1, a1
  mov x2, a2
  mov a1, F2
  jmp x2
F4:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  mov x4, a4
  mul x5, x1, 2
  add x6, x5, 1
  mov a1, x6
  mov a2, x2
  mov a3, x3
  jmp x4
```

```
F5:
  mov x1, a1
  mov x2, a2
  mov x3, a3
  add x4, x1, 5
  mov a1, x4
  jmp HALT
F6:
  mov x1, a1
  mov a1, 3
  mov a2, F4
  mov a3, F5
  jmp x1
START:
  mov a1, F4
  mov a2, F6
  jmp F3
HALT:
```

Optimization of Low-level IR

After removing all unnecessary lines of code and assign registers based on the **graph coloring** algorithm, we get the following code:

```
/* IR */
F1:
    mov r4, r3
    jmp r2
F2:
    mov r4, F1
    jmp r2
F3:
    mov r1, F2
    jmp r2
F4:
    mul r1, r1, 2
    add r1, r1, 1
    jmp r4
```

```
F5:
    add r1, r1, 5
    jmp HALT
F6:
    mov r4, r1
    mov r1, 3
    mov r2, F4
    mov r3, F5
    jmp r4
START:
    mov r1, F4
    mov r2, F6
    jmp F3
HALT:
```

Summary

1. Compilers

2. Compiling with Continuations
    - Continuation Passing Style
    - Lambda Lifting
    - Closure Conversion
    - Alpha Renaming
    - Transformation to Low-level IR
    - Optimization of Low-level IR

Homework #3

PLRG

https://github.com/ku-plrg-classroom/docs/tree/main/cose212/mini-python

- Please see above document on GitHub:
  1. Implement `reduce` function.
  2. Implement `locals` function.

- The due date is 23:59 on Nov. 19 (Wed.).

- Please only submit **`Implementation.scala`** file to **LMS**.

- The given test cases are **not sufficient**. Please write your own test cases to test your implementation. We will use **randomly generated** test cases for grading.

Next Lecture

- Type Systems

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