Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

$$w = \quad \cdots \mid B \mid a \mid b \mid b \mid B \mid B \mid B \mid B \mid \cdots$$

$$f(w) = \quad \cdots \mid B \mid a \mid b \mid b \mid a \mid b \mid b \mid B \mid \cdots$$

1: **while** there are input symbols **do**
2:     Find and Replace a (or b) with Z
3:     Find and Fill the first blank with X (or Y) for a (or b)
4:     Roll back Z to the original a (or b)
5: Replace X's and Y's with a's and b's
6: Go to the first input symbol

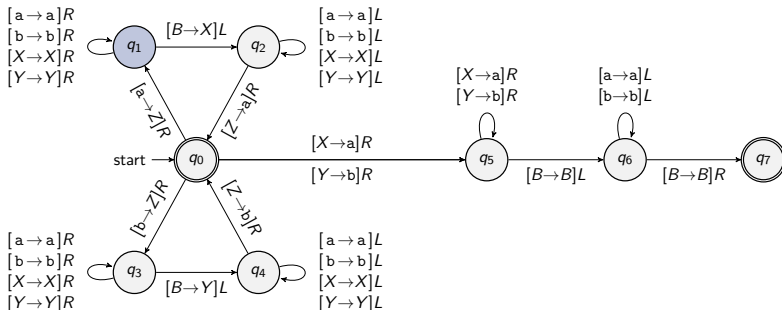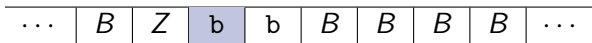Construct a **Turing machine** that **computes** the function:

$$f(w \in \{\mathtt{a}, \mathtt{b}\}^*) = ww$$

⋯	B	a	b	b	B	B	B	B	⋯
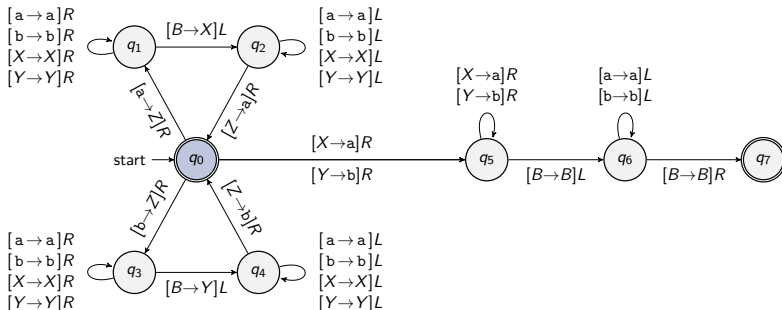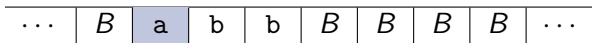
Find and Replace a (or b) with Z.

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{\mathtt{a}, \mathtt{b}\}^*) = ww$$

	B	Z	b	b	B	B	B	B	
$\cdots$									$\cdots$

q_1 transitions:
$[\mathtt{a} \to \mathtt{a}]R$
$[\mathtt{b} \to \mathtt{b}]R$
$[X \to X]R$
$[Y \to Y]R$

$[B \to X]L$

q_2 transitions:
$[\mathtt{a} \to \mathtt{a}]L$
$[\mathtt{b} \to \mathtt{b}]L$
$[X \to X]L$
$[Y \to Y]L$

$[a \to Z]R$

$[Z \to a]R$

$[X \to \mathtt{a}]R$
$[Y \to \mathtt{b}]R$

$[X \to \mathtt{a}]R$
$[Y \to \mathtt{b}]R$

$[\mathtt{a} \to \mathtt{a}]L$
$[\mathtt{b} \to \mathtt{b}]L$

start $\to q_0$

$[X \to \mathtt{a}]R$

$[Y \to \mathtt{b}]R$

q_5

$[B \to B]L$

q_6

$[B \to B]R$

q_7

$[b \to Z]R$

$[Z \to b]R$

q_3 transitions:
$[\mathtt{a} \to \mathtt{a}]R$
$[\mathtt{b} \to \mathtt{b}]R$
$[X \to X]R$
$[Y \to Y]R$

$[B \to Y]L$

q_4 transitions:
$[\mathtt{a} \to \mathtt{a}]L$
$[\mathtt{b} \to \mathtt{b}]L$
$[X \to X]L$
$[Y \to Y]L$

Find and Fill the first blank with X (or Y) for a (or 1).

Construct a **Turing machine** that **computes** the function:
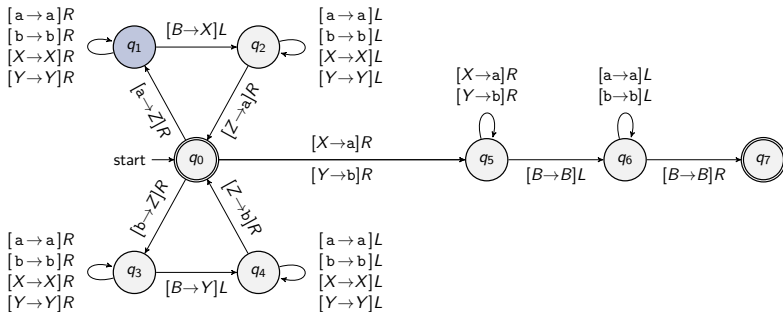
$$f(w \in \{a, b\}^*) = ww$$

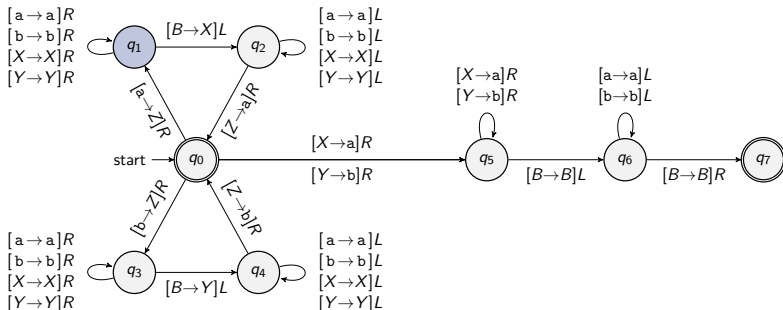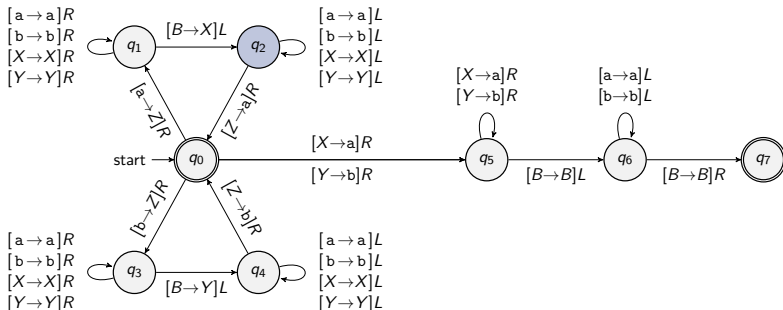| $\cdots$	B	Z	b	b	B	B	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

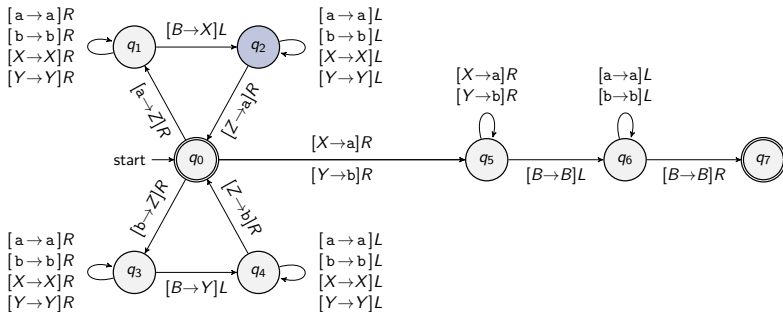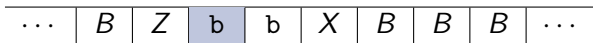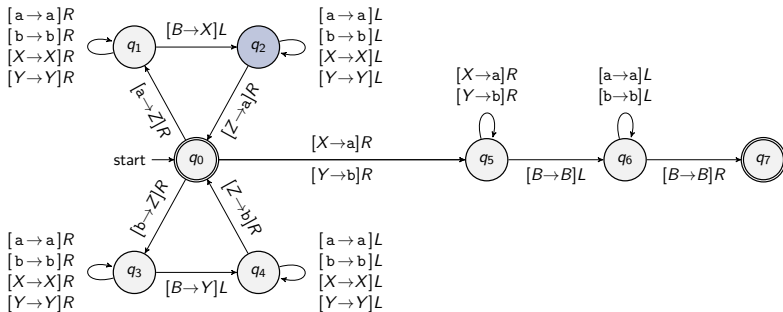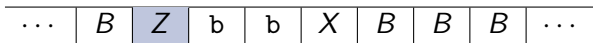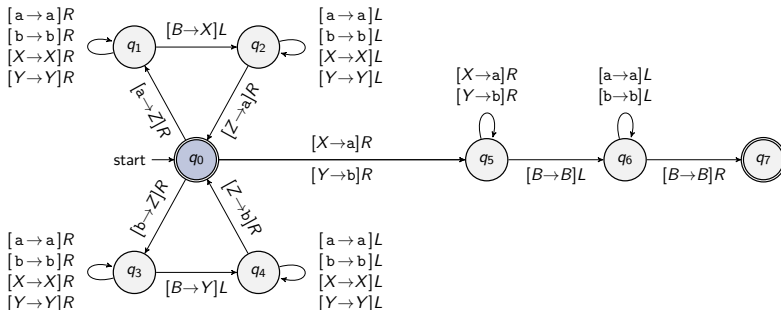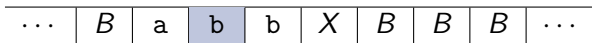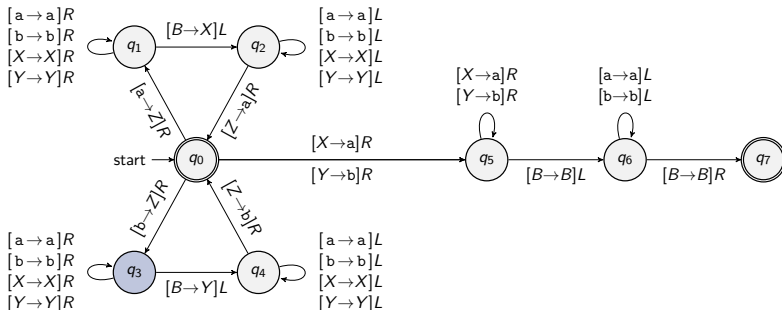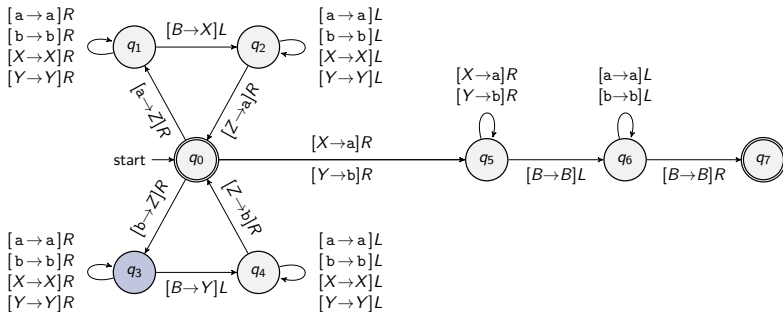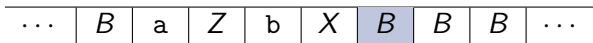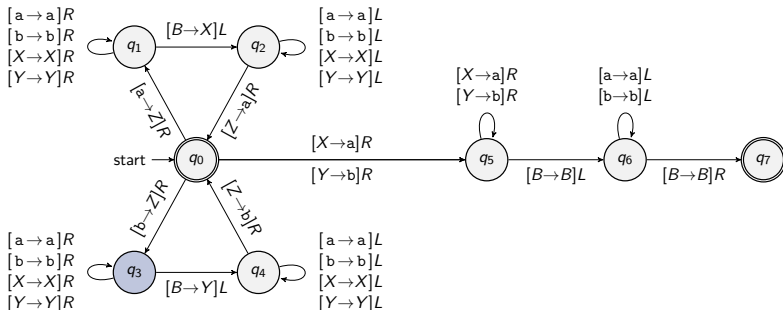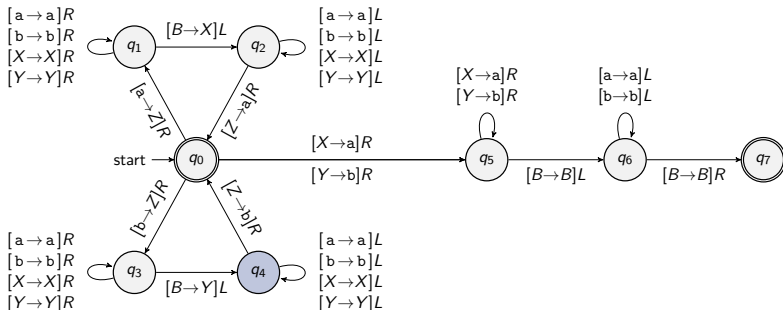Construct a **Turing machine** that **computes** the function:

$$f(w \in \{\mathtt{a}, \mathtt{b}\}^*) = ww$$

Roll back Z to the original a (or b).

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

$\cdots$	B	Z	b	b	X	B	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

| $\cdots$ | B | Z | b | b | X | B | B | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

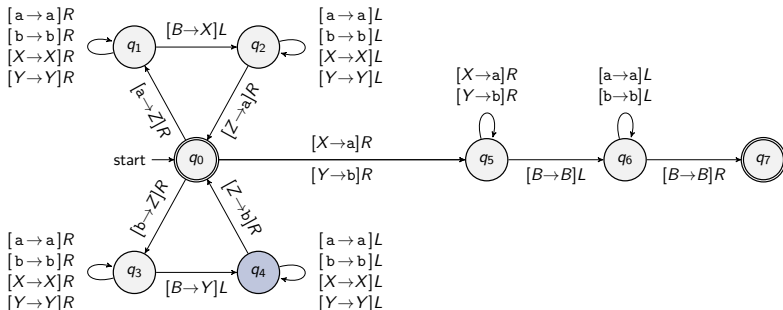| $\cdots$	B	a	b	b	X	B	B	B	$\cdots$

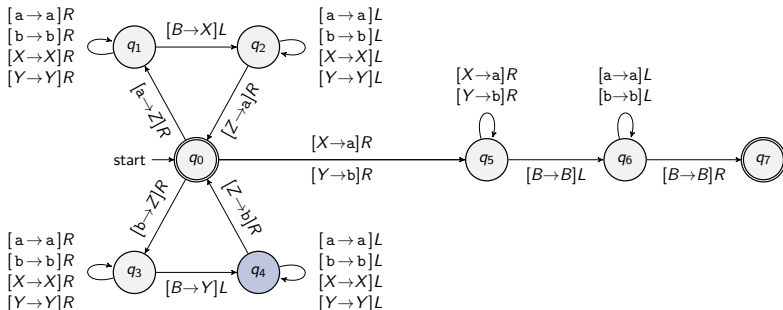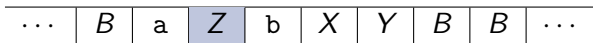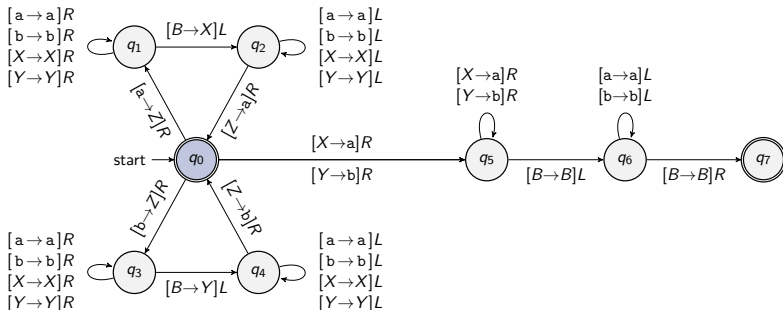Repeat until no input symbols left.

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:

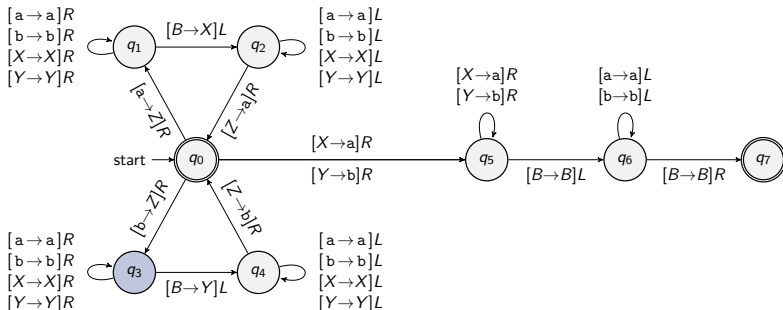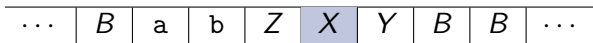$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:

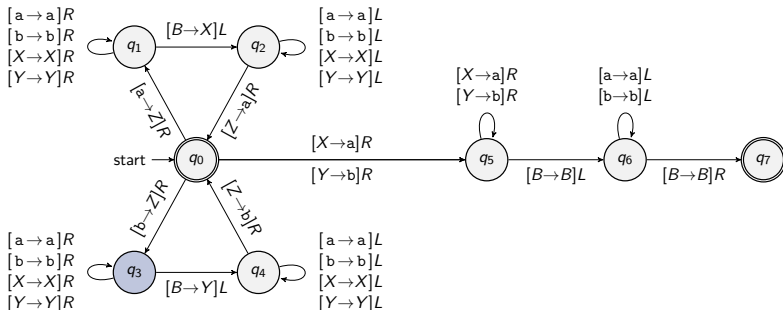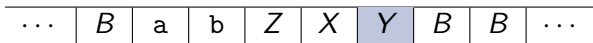$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

$\cdots$	B	a	b	b	X	Y	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:
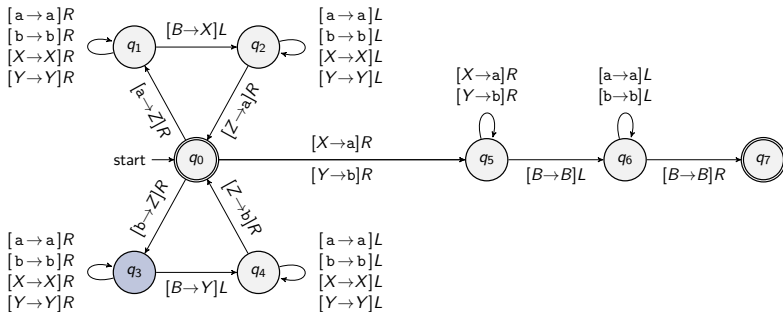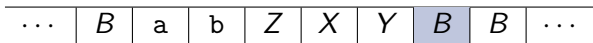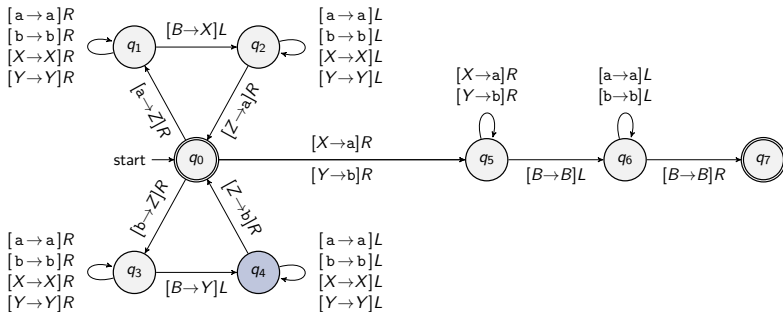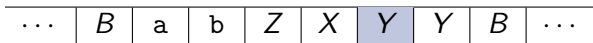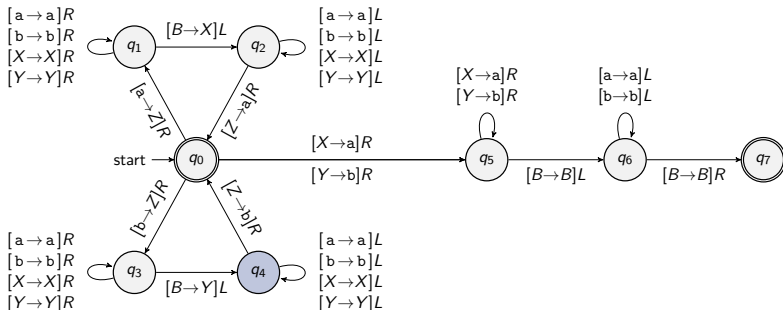
$$f(w \in \{a, b\}^*) = ww$$

| $\cdots$ | B | a | b | Z | X | Y | B | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{\mathtt{a}, \mathtt{b}\}^*) = ww$$

$\cdots$	B	a	b	Z	X	Y	B	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

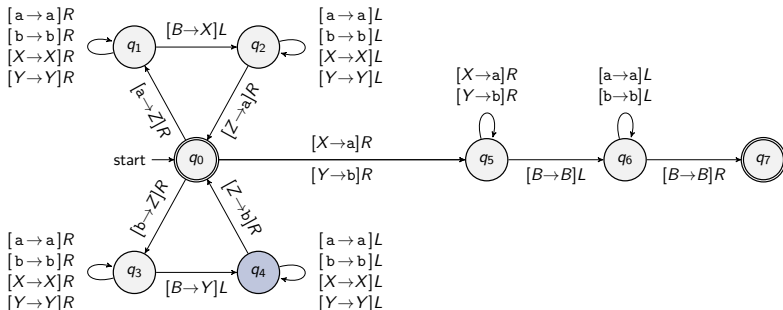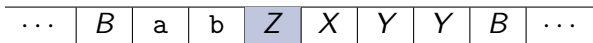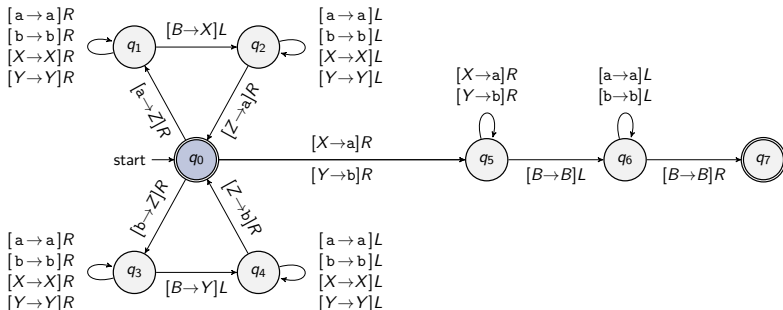| $\cdots$	B	a	b	Z	X	Y	Y	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

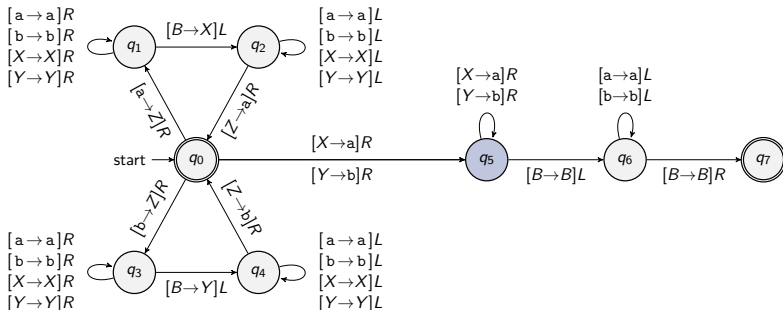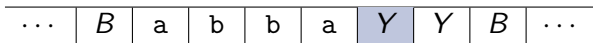Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

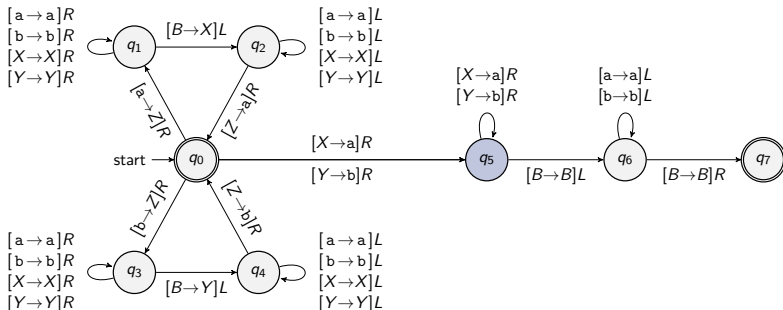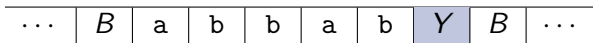| $\cdots$	B	a	b	Z	X	Y	Y	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

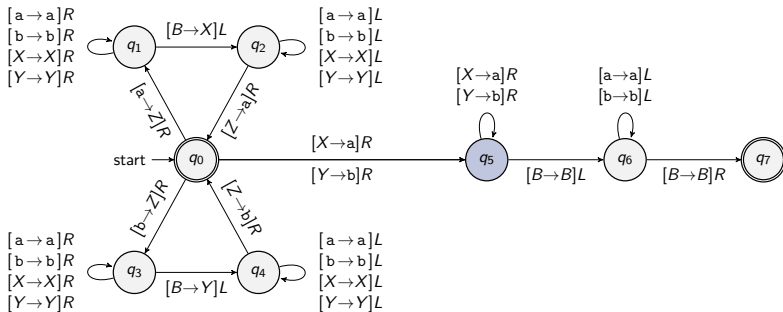|									
$\cdots$	B	a	b	b	X	Y	Y	B	$\cdots$

Replace X's and Y's with a's and b's.

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

| $\cdots$ | B | a | b | b | a | Y | Y | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

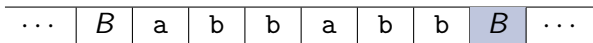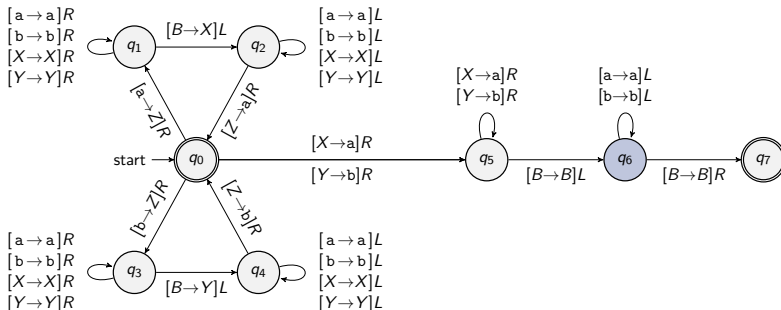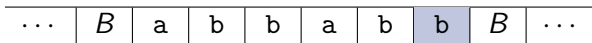| $\cdots$	B	a	b	b	a	b	Y	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

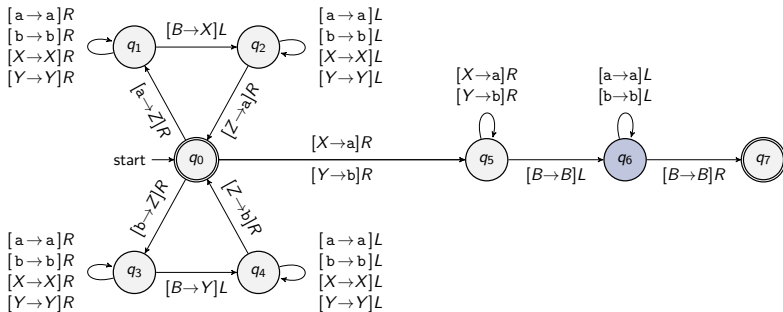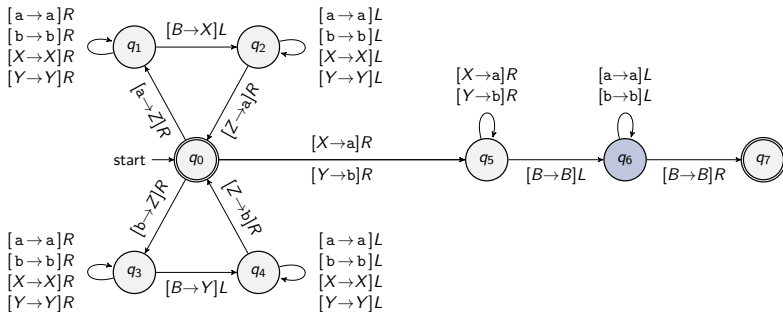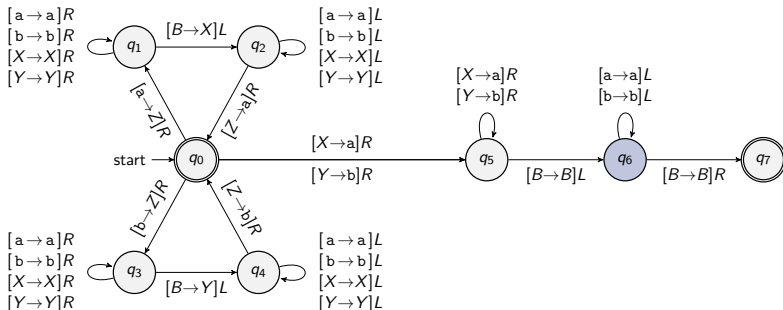Construct a **Turing machine** that **computes** the function:

$$f(w \in \{\mathtt{a}, \mathtt{b}\}^*) = ww$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Go to the first input symbol.

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

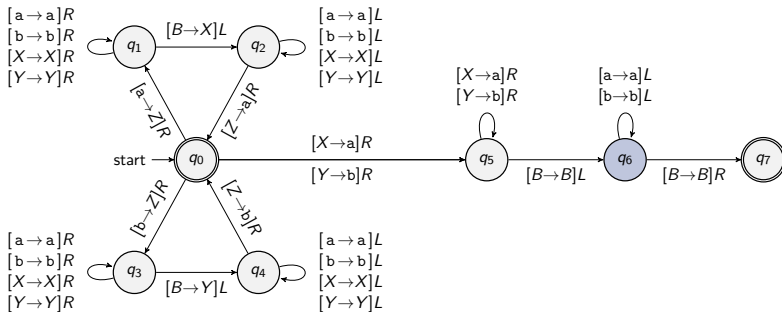| $\cdots$ | B | a | b | b | a | b | b | B | $\cdots$ |

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

Construct a **Turing machine** that **computes** the function:
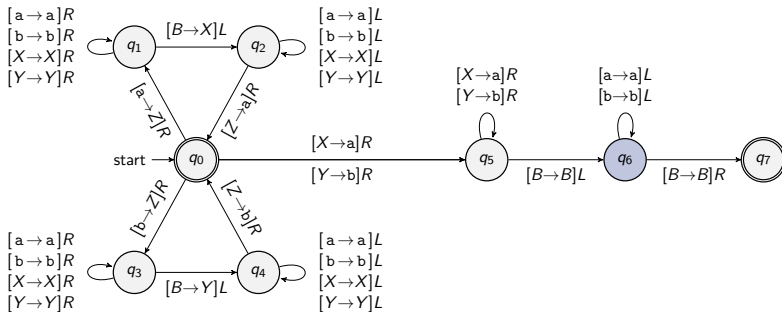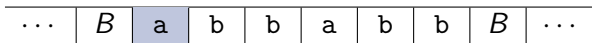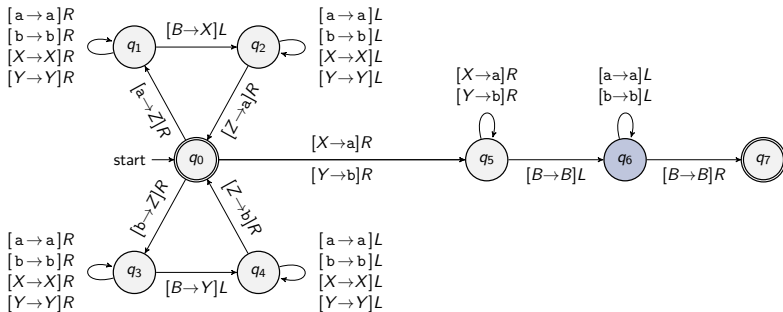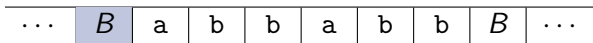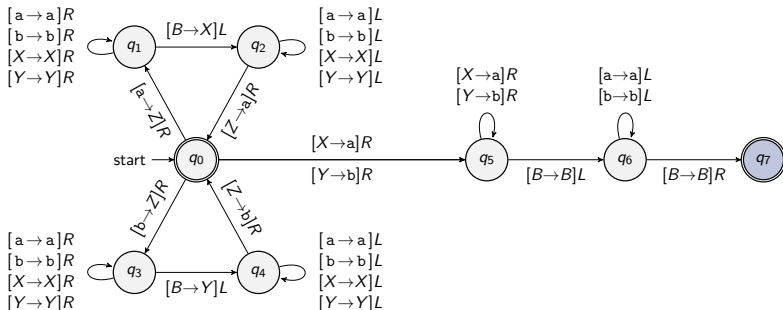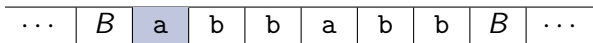
$$f(w \in \{a, b\}^*) = ww$$

$\cdots$	B	a	b	b	a	b	b	B	$\cdots$

Construct a **Turing machine** that **computes** the function:

$$f(w \in \{a, b\}^*) = ww$$

| $\cdots$ | B | a | b | b | a | b | b | B | $\cdots$ |

Computed! $f(\text{abb}) = \text{abbabb}$