Lecture 17 - More Dynamic Programming (Longest Common Subsequence)

Fall 2025, Korea University

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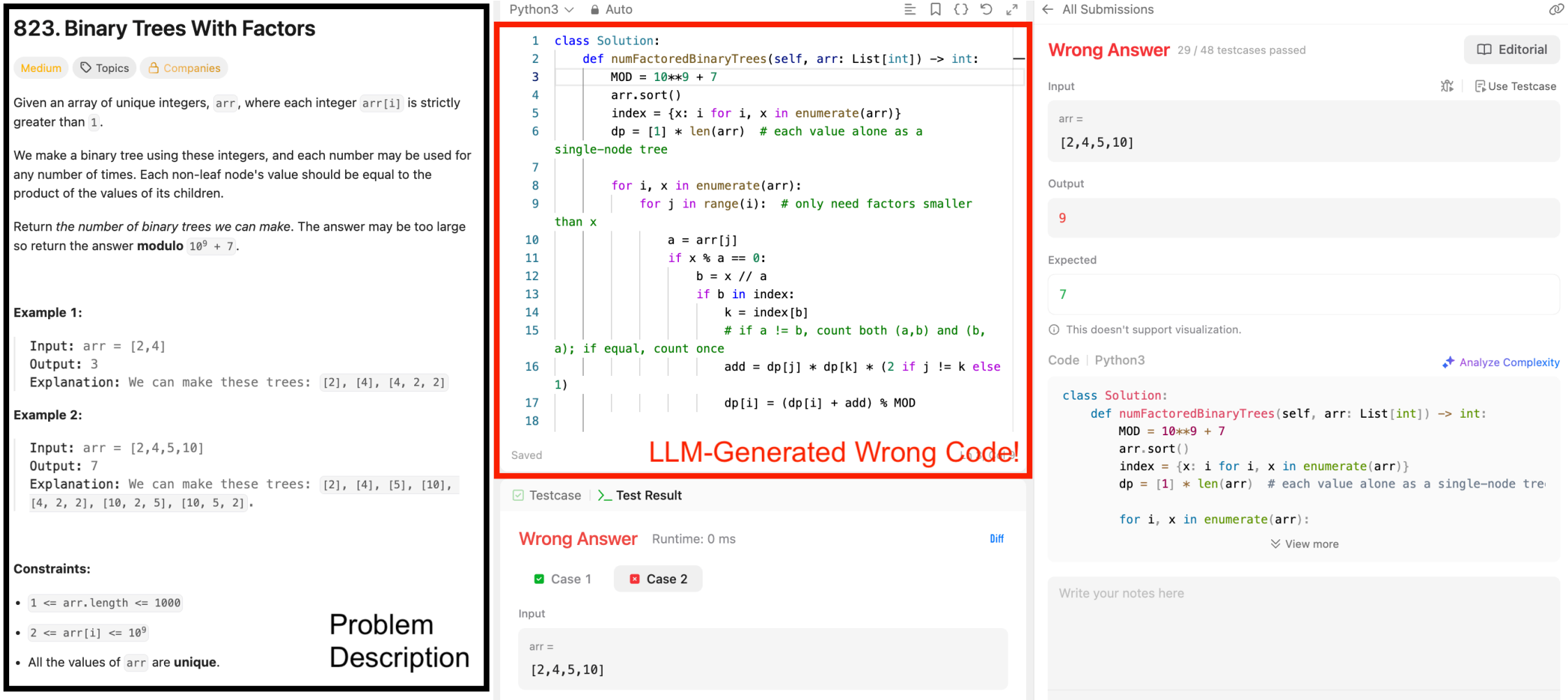
Bonus Assignment: You Code, I Fix (Due: Dec 8, 11:59 PM, KST)

Are you using LLMs when coding?

They are fast and convenient—but LLMs don't always produce correct code when you actually test it. In this assignment, you'll probe, catch, and correct LLM mistakes on real algorithm problems.


823. Binary Trees With Factors
Medium
Topics
Companies
Given an array of unique integers, arr, where each integer arr[i] is strictly greater than 1.
We make a binary tree using these integers, and each number may be used for any number of times. Each non-leaf node's value should be equal to the product of the values of its children.
Return the number of binary trees we can make. The answer may be too large so return the answer modulo 10^9 + 7.
Example 1:
Input: arr = [2,4]
Output: 3
Explanation: We can make these trees: [2], [4], [4, 2, 2]
Example 2:
Input: arr = [2,4,5,10]
Output: 7
Explanation: We can make these trees: [2], [4], [5], [10], [4, 2, 2], [10, 2, 5], [10, 5, 2].
Constraints:
1 <= arr.length <= 1000
2 <= arr[i] <= 10^9
All the values of arr are unique.
Problem Description
Python3
Auto
class Solution:
def numFactoredBinaryTrees(self, arr: List[int]) -> int:
MOD = 10**9 + 7
arr.sort()
index = {x: i for i, x in enumerate(arr)}
dp = [1] * len(arr) # each value alone as a single-node tree
for i, x in enumerate(arr):
for j in range(i): # only need factors smaller than x
a = arr[j]
if x % a == 0:
b = x // a
if b in index:
k = index[b]
# if a != b, count both (a,b) and (b, a); if equal, count once
add = dp[j] * dp[k] * (2 if j != k else 1)
dp[i] = (dp[i] + add) % MOD
Saved
LLM-Generated Wrong Code!
Testcase
Test Result
Wrong Answer
Runtime: 0 ms
Diff
Case 1
Case 2
Input
arr =
[2,4,5,10]
All Submissions
Wrong Answer
29 / 48 testcases passed
Editorial
Input
Use Testcase
arr =
[2,4,5,10]
Output
9
Expected
7
This doesn't support visualization.
Code
Python3
Analyze Complexity
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for i, x in enumerate(arr):
View more
Write your notes here

Task Overview

Find **Two** LeetCode problems (pick at most 1 from each list):

- Divide and Conquer – https://leetcode.com/problem-list/divide-and-conquer/
- Sorting – https://leetcode.com/problem-list/sorting/
- Tree – https://leetcode.com/problem-list/tree/
- Dynamic Programming – https://leetcode.com/problem-list/dynamic-programming/

What You Must Do (Per Problem)

1. Copy & Paste the official problem description into an LLM chat (ChatGPT, Gemini, Claude, etc) and ask the LLM: "*Write a correct and efficient solution in Python*"
2. Evaluate the generated code by submitting it directly to LeetCode
3. If the LLM's solution does not pass all tests, **debug and fix the code yourself until it passes all tests**.
4. Create **Hallucination Reports!** (The template will be uploaded to LMS)
    - Submit a `zip` file containing two `pdf` files:
        - `COSE214_HallucinationReport_[YourStudentID]_[ProblemID_1].pdf`
        - `COSE214_HallucinationReport_[YourStudentID]_[ProblemID_2].pdf`

Purpose of This Activity

1. To Understand LLMs' Limitations
2. To Build Critical Thinking & Debugging Skills
3. To Learn How to Collaborate with AI
4. To Observe Hallucination Patterns

Grading Rubric (5 points per problem, total 10 points)

Criteria	Description	Points
Problem Selection	Selected from the specified lists (max 1 per category).	0.5
LLM-Generated Code & Evidence	Clearly shows the LLM's original code and submission result (**failed test cases, screenshots**).	1.0
Bug Diagnosis	Provides a logical and insightful explanation of **why the LLM's code failed**.	1.5
Corrected Solution	Presents a working, fully accepted version written and debugged by the student (not copied).	1.0
Comparison & Reflection	Clearly explains what was changed, how it fixes the issue, and what this reveals about LLM limitations.	1.0

Additional Notes about the Grading

- **Insight Bonus (+0.5 pts):**
  Exceptional observation about LLM behavior (e.g., overgeneralization, recursive depth misjudgment, complexity hallucination).
- **Penalty (0 pts):**
  Submitting others' accepted code or no debugging evidence.

Academic Integrity & Tool Policy

- You must not copy someone else's accepted solution from LeetCode, GitHub, or online forums. The "fixed" solution must be your own debugging and reasoning work, not another model's or person's answer.

Overview

- Last time:
  - Bellman-Ford (SSSP)
  - Floyd-Warshall (APSP)
- More **Dynamic Programming** for other problems!
  - **Today:** Longest Common Subsequence (LCS) 🧬
  - **Next time:** Knapsack 🎒

Motivation

How do we measure **similarity between sequences**?

- DNA sequence alignment 🧬
- Spell checking / autocorrect ✍️
- `git diff` / file comparison 💻

Longest Common Subsequence (LCS) is a core building block for these applications.

Subsequence vs Substring

- **Substring** = contiguous block
  - "**abc**" is a substring of "z**abc**t"
- **Subsequence** = not necessarily contiguous
  - "**abt**" is a subsequence of "z**ab**c**t**"

💡 Subsequence = delete some characters, keep order.

LCS Definition

- Given two sequences **X** (length = m) and **Y** (length = n)
- Find the **longest sequence Z** such that:
  - Z is a subsequence of X
  - Z is a subsequence of Y

Example:

- X = `abracadabra`
- Y = `bxqrabry`
- LCS = `brabr` (length = 5)

Brute Force Idea

- Generate **all subsequences** of X → 2^m possibilities 😱
- Check which ones are also subsequences of Y
- Keep the longest

Brute Force Idea - Example

- X = `abc`
- Y = `bxyeksc`

1. Generate **all subsequences** of X
   `['', 'a', 'b', 'c', 'ab', 'bc', 'ac', 'abc']` ($2^3 = 8$ possibilities)
2. Check which ones are also subsequences of Y
   `['', 'b', 'c', 'bc']`
3. Keep the longest
   Answer: `bc`

👉 Exponential time (**Too Slow!**) → impossible for large inputs.

Can We Apply Dynamic Programming?

Optimal Substructure: The optimal LCS is built from optimal LCS of prefixes

Look at the **last characters** of X and Y.

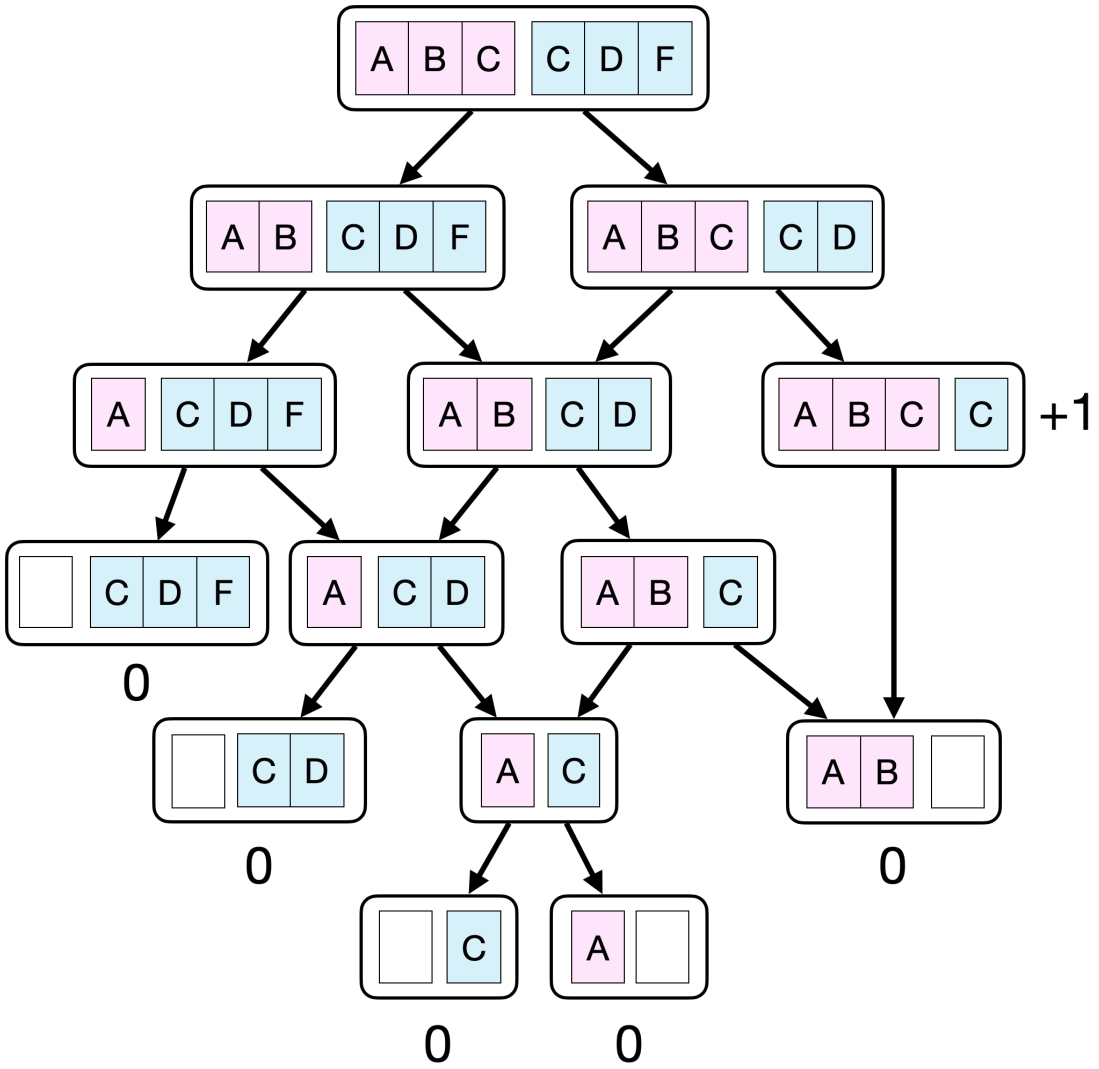
Case 1: $x_m = y_n$

- Then this character **must** be part of LCS.
- Problem reduces to LCS of prefixes $\mathrm{LCS}(X[1:m-1], Y[1:n-1])$.
- Example: `LCS('ABC', 'ADC') = LCS('AB', 'AD') + 'C'`

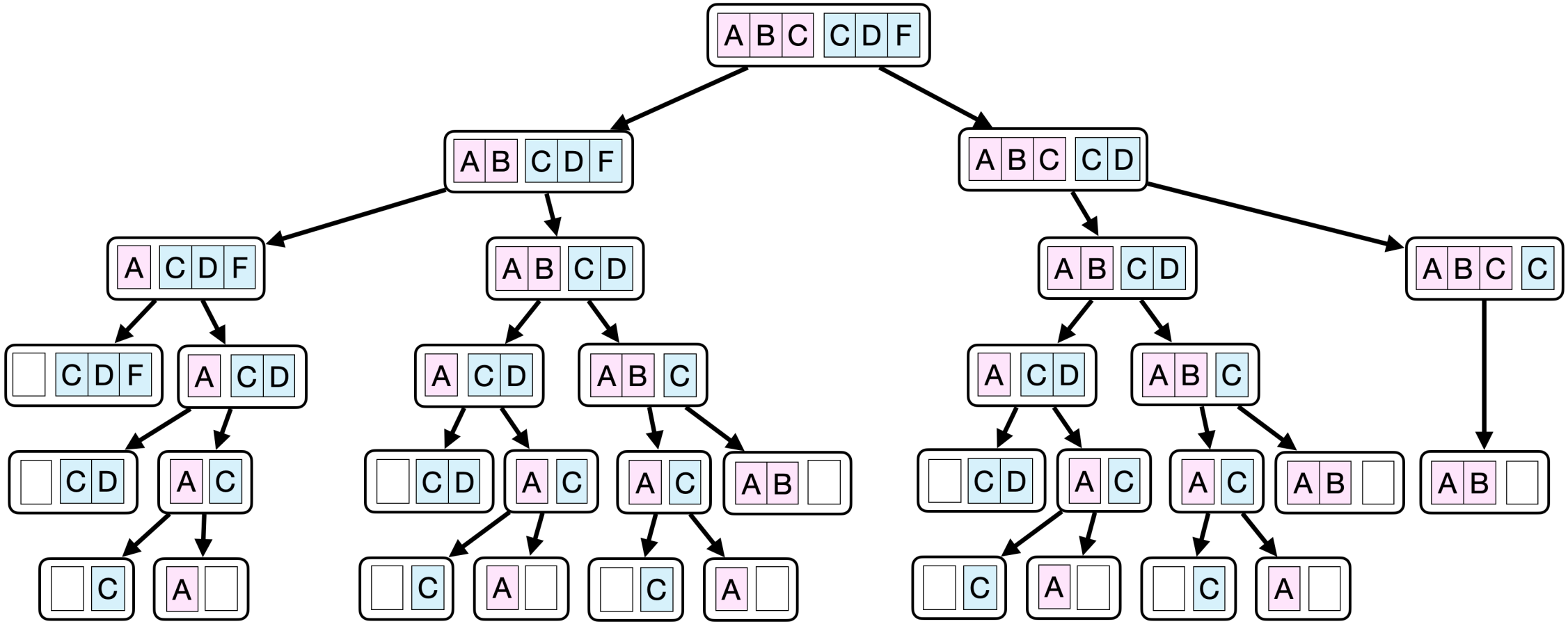
Case 2: $x_m \neq y_n$

- Drop one character and try both options:
  - $\mathrm{LCS}(X[1:m-1], Y)$ and $\mathrm{LCS}(X, Y[1:n-1])$
- Take the longer one.
- Example: `LCS('AB', 'AD') = longer(LCS('A', 'AD'), LCS('AB', 'A'))`

Overlapping Subproblems: Subproblems recur many times



If you don’t reuse the previously stored results and instead recompute everything from scratch…



Step 1: Recursive Formula

Define a table:

$$C[i, j] = \text{length of } \mathrm{LCS}(X[1 : i], Y[1 : j])$$

Recurrence:

$$C[i, j] = \begin{cases} 0, & i = 0 \text{ or } j = 0 \\ C[i-1, j-1] + 1, & X[i] = Y[j] \\ \max(C[i-1, j],\, C[i, j-1]), & X[i] \neq Y[j] \end{cases}$$

Step 2: DP Algorithm

We build table C $((m+1) \times (n+1))$ **bottom-up**:

1. **Initialize** first row/column with 0
2. **Fill row by row**, left → right
3. Answer stored at $C[m][n]$

```
def lenLCS(X, Y):
    m, n = len(X), len(Y)
    C = [[0] * (n+1) for _ in range(m+1)]
    for i in range(1, m+1):
        for j in range(1, n+1):
            if X[i-1] == Y[j-1]:
                C[i][j] = C[i-1][j-1] + 1
            else:
                C[i][j] = max(C[i-1][j], C[i][j-1])
    return C[m][n]
```

Example Table

- X = `ABCD` (length = 4)
- Y = `BDE` (length = 3)
- C = 5×4 matrix

	-	B	D	E
-	0	0	0	0
A	0			
B	0			
C	0			
D	0			

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A	0	0	0	0
B	0	1	1	1
C	0			
D	0			

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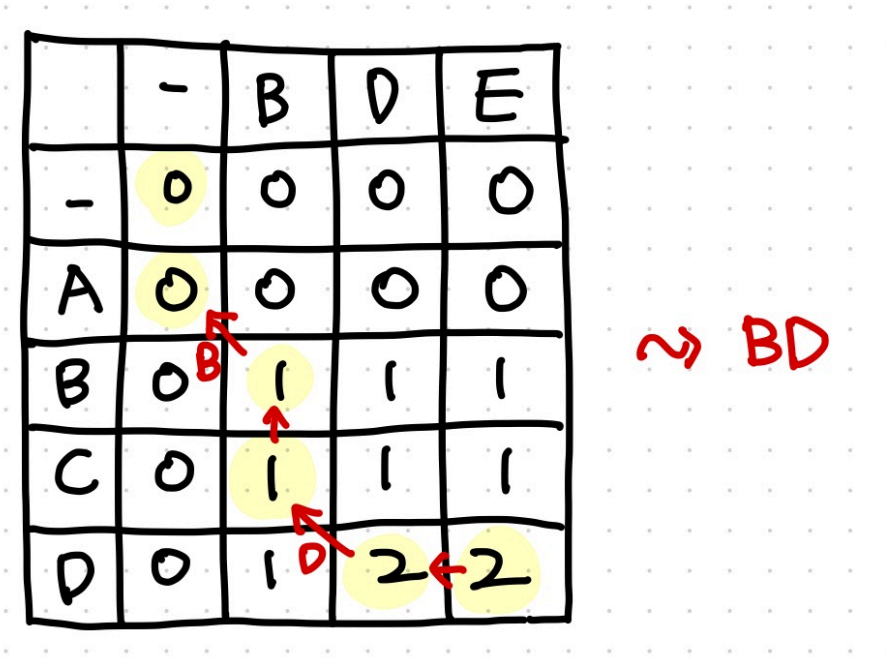
A	0	0	0	0
B	0	1	1	1
C	0	1	1	1
D	0	1	2	2

Step 3: Recovering Actual LCS

- Start at bottom-right cell (m, n)
- While $i > 0, j > 0$:
  - If $X[i] = Y[j]$: Add to LCS, move diagonally ↖
  - Else: move to the neighbor with larger value (← or ↑)



Complexity & Remarks

- Time: $O(mn)$
- Space: $O(mn)$
- LCS is a classic Dynamic Programming problem.
- LCS has no known faster (polynomial) algorithm.
- Steps: substructure → recursion → DP table → recover solution

Next Time: The Knapsack Problem 🎒

Credits & Resources

Lecture materials adapted from:

- Stanford CS161 slides and lecture notes
  - https://stanford-cs161.github.io/winter2025/
- *Algorithms Illuminated* by Tim Roughgarden
  - https://algorithmsilluminated.com/