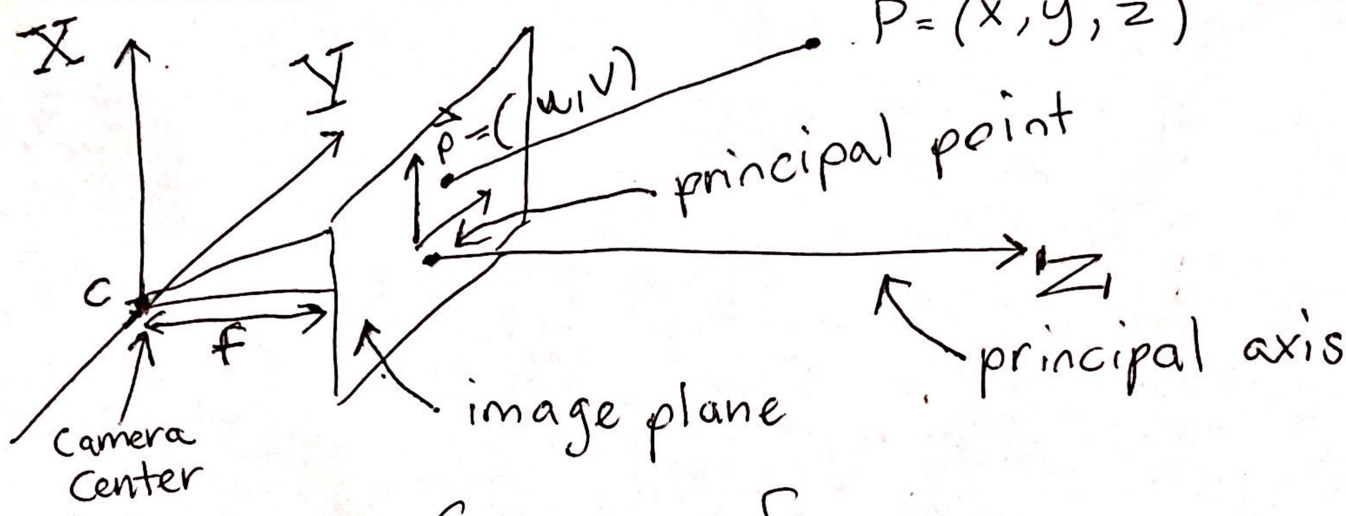
Recall pinhole camera model



$$u = \frac{f}{z} x \quad v = \frac{f}{z} y$$

Homogeneous coordinates:

$$(u, v) \Rightarrow \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \qquad (x, y, z) = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

image scene

Converting from homogeneous coords.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \Rightarrow \left(\frac{u}{w}, \frac{v}{w}\right) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)$$

$$(x, y, z) \Longrightarrow \left(\frac{f}{z} x, \frac{f}{z} y\right)$$

world/scene image

Let's represent this w/ homogeneous coords.

$$\begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

image point   matrix   world point

Confirm?

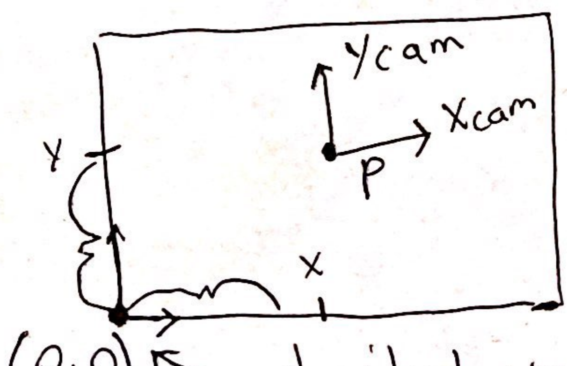
$$\begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} \checkmark \Rightarrow \frac{f}{Z}x, \frac{f}{Z}y$$

Principal point p

↳ point where principal axis intersects the image plane, it is ~~[illegible]~~ize the origin of a normalized coord system.

↳ principal axis is Z

Let's look at this in 2D



digital image may have origin at corner

p: principal point

principal point offset

Scanned with CamScanner

principal point offset (p_x, p_y)
↳ location of principal point relative to the origin of the image

$$(X, Y, Z) \rightarrow \left(\frac{f}{Z}X + p_x, \frac{f}{Z}Y + p_y\right)$$

$$\begin{bmatrix} fX + p_x Z \\ fY + p_y Z \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Final aspect is skew factor

$$K = \begin{bmatrix} f & s & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

* Pinhole has no skew, but real camera might *

$\boxed{\text{Intrinsic camera calibration matrix}}$

Points in the image plane are represented by physical measurements (mm), but points on the digital image are in pixels
↳ introduce pixel size $\underbrace{m_x \text{ and } m_y}_{\text{pixels per mm}}$

$$K = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & s & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} m_x f & m_x s & m_x p_x \\ 0 & m_y f & m_y p_y \\ 0 & 0 & 1 \end{bmatrix}$$

$\alpha = m_x f$ $\quad u_0 = m_x p_x$

$\beta = m_y f$ $\quad v_0 = m_y p_y$

$\theta = m_x s$

$$K = \begin{bmatrix} \alpha & \theta & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

"Intrinsic" camera calibration matrix

add back in the col for homogeneous coords.

$K[I \mid 0]$

3x3

identity 3x3

column of zeroes

What does it mean if $\alpha \approx \beta$?

↳ pixels are square

We can interpret u_0, v_0

↳ center of the image

What about extrinsic?

↳ also going to be a matrix

translate the camera center to the world coordinate and also rotate.

$$\vec{X}_{cam} = R(\vec{X}_w - C)$$

$\vec{X}_{cam}$: coords of camera frame (homogeneous)

R: rotation matrix

$\vec{X}_w$: Point in world Space (homogeneous)

C: coords of camera center in world coordinates.

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} \overbrace{R}^{3\times3 \text{ matrix}} & -RC \\ \hline 0 & 1 \end{array}\right]}_{4\times4 \text{ matrix}} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Let $t = -RC$

$$\underbrace{\left[\begin{array}{c|c} R & t \\ \hline 0 & 1 \end{array}\right]}_{\text{extrinsic matrix}} \Rightarrow [R \mid t]$$

Final mapping between world coords and pixels of the image:

$$P = \underbrace{K}_{\substack{(3\times4)\\ \text{matrix}\\ \text{intrinsic}}}\underbrace{[R \mid t]}_{\substack{(4\times4)\\ \text{matrix}\\ \text{extrinsic}}} \Rightarrow \underbrace{x}_{\substack{\text{pixel}\\ \text{coords}\\ (2D)}} = P\underbrace{X}_{\substack{\text{world}\\ \text{coords}\\ (3D)}}$$

How to estimate the matrix P?

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

We need correspondences

Consider a world coord. $\underline{X} = (X, Y, Z)$ which maps to an image coord (x, y)

$$P \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} P_{11}X + P_{12}Y + P_{13}Z + P_{14} \\ P_{21}X + P_{22}Y + P_{23}Z + P_{24} \\ P_{31}X + P_{32}Y + P_{33}Z + P_{34} \end{bmatrix}$$

this is w

therefore $x = \dfrac{P_{11}X + P_{12}Y + P_{13}Z + P_{14}}{w}$

$y =$ similarly

Set up a linear system with pairs of known world points and image points.

$-u_1$

$$Ap = 0 \qquad A = \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & \vdots & & & & & & \end{bmatrix}$$

knowns

vector of unknowns

Homogeneous linear system

↳ solved by singular value decomposition

* Example *