MathBasic

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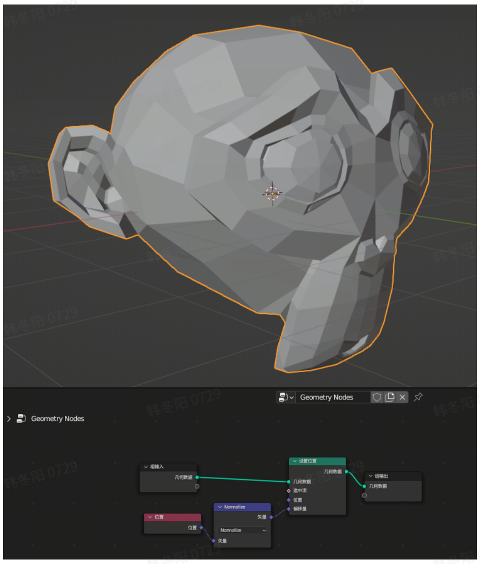
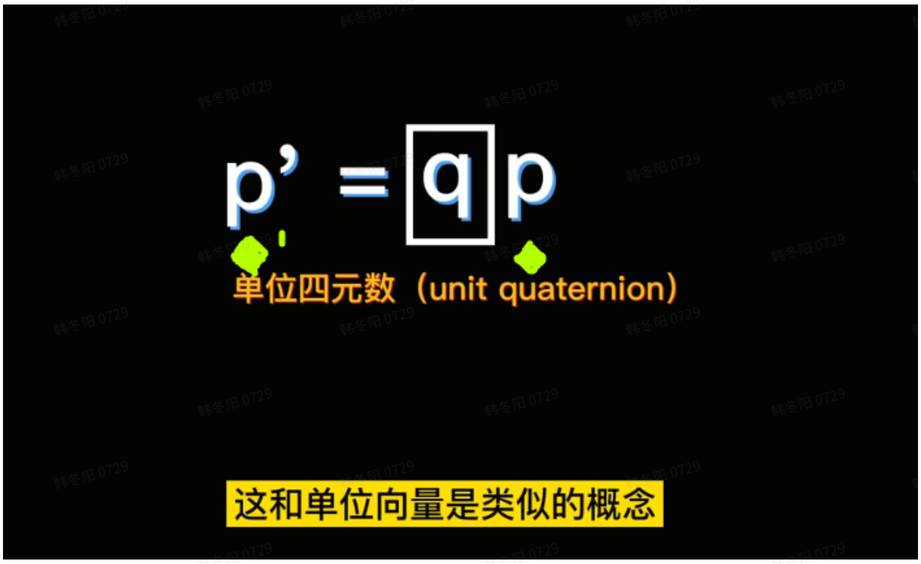
> 1. **世界坐标**：描述物体在三维空间中的绝对位置。
> 2. **视图坐标**：相对于相机位置的三维坐标。
> 3. **屏幕坐标**：二维像素坐标，表示在显示器上的位置。（左上角为原点，像素值）
> 4. **NDC坐标**：标准化设备坐标，范围为 [-1, 1]，用于统一表示在设备上的位置。（标准化，屏幕中心为原点）
>
> 5. **逆矩阵**：一个矩阵的反，*AB=I，则A为B的逆矩阵（倒数）*。逆矩阵常用于视图转换
> 6. **转置矩阵**：行列互换，A的转置为AT，常用于法线变换
> 7. **齐次坐标**：通过增加一个额外的维度来表示点或向量，使得平旋缩可以统一使用矩阵乘法处理。

欧拉旋转的数值是相对初始的物体坐标系的，和视图中的实时更新的物体局部坐标系不同

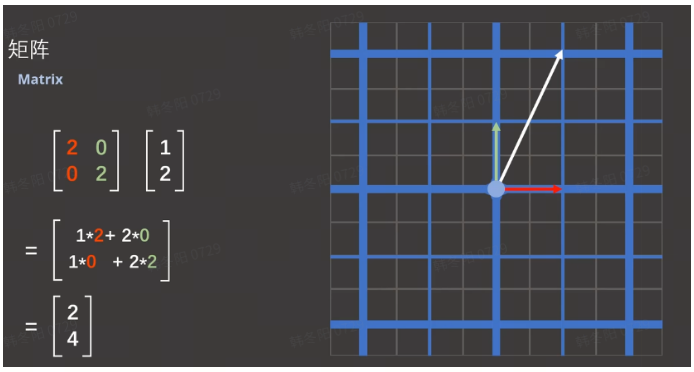


（三维向量xyz）规格化可以将矢量长度均变为单位三元数，故四元数向量也可以通过规格化成为单位四元数



这和单位向量是类似的概念

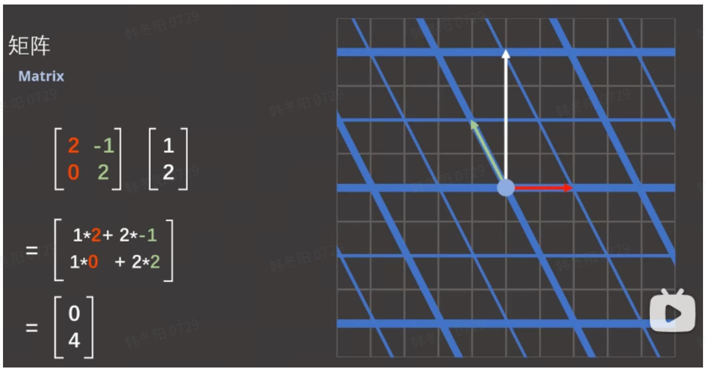
几何数据旋转的矩阵原理


矩阵
Matrix

$$\begin{bmatrix}2&0\\0&2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$=\begin{bmatrix}1*2+2*0\\1*0\ +2*2\end{bmatrix}$$

$$=\begin{bmatrix}2\\4\end{bmatrix}$$


矩阵
Matrix

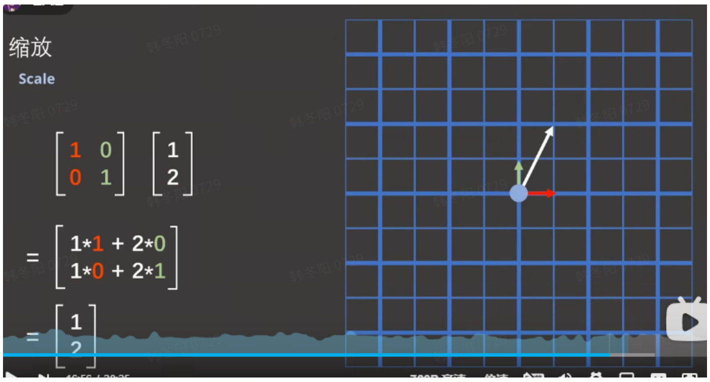
$$\begin{bmatrix}2&-1\\0&2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$=\begin{bmatrix}1*2+2*-1\\1*0\ +2*2\end{bmatrix}$$

$$=\begin{bmatrix}0\\4\end{bmatrix}$$

世界坐标系向量×矩阵坐标系=世界坐标系向量（变换后

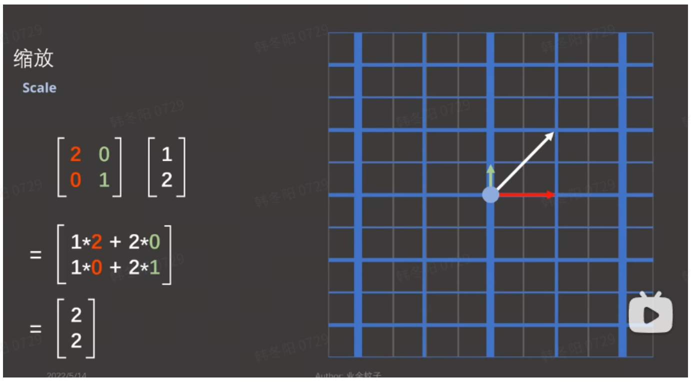
在红绿坐标系中，白色向量均为（1，2）但是在世界坐标系中却改变了


缩放
Scale

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$=\begin{bmatrix}1*1+2*0\\1*0+2*1\end{bmatrix}$$

$$=\begin{bmatrix}1\\2\end{bmatrix}$$

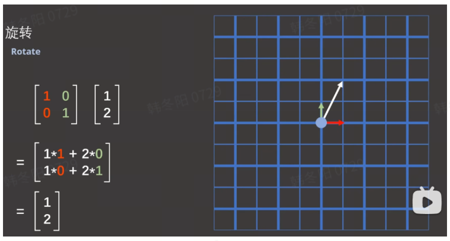

缩放
Scale

$$\begin{bmatrix}2&0\\0&1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$=\begin{bmatrix}1*2+2*0\\1*0+2*1\end{bmatrix}$$

$$=\begin{bmatrix}2\\2\end{bmatrix}$$

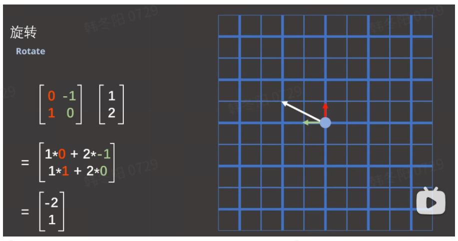
以矩阵理解缩放，就是给了向量一个新空间相乘


旋转
Rotate

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$=\begin{bmatrix}1*1+2*0\\1*0+2*1\end{bmatrix}$$

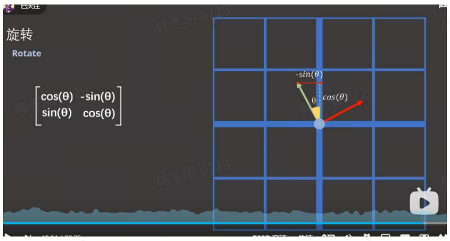
$$=\begin{bmatrix}1\\2\end{bmatrix}$$


旋转
Rotate

$$\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

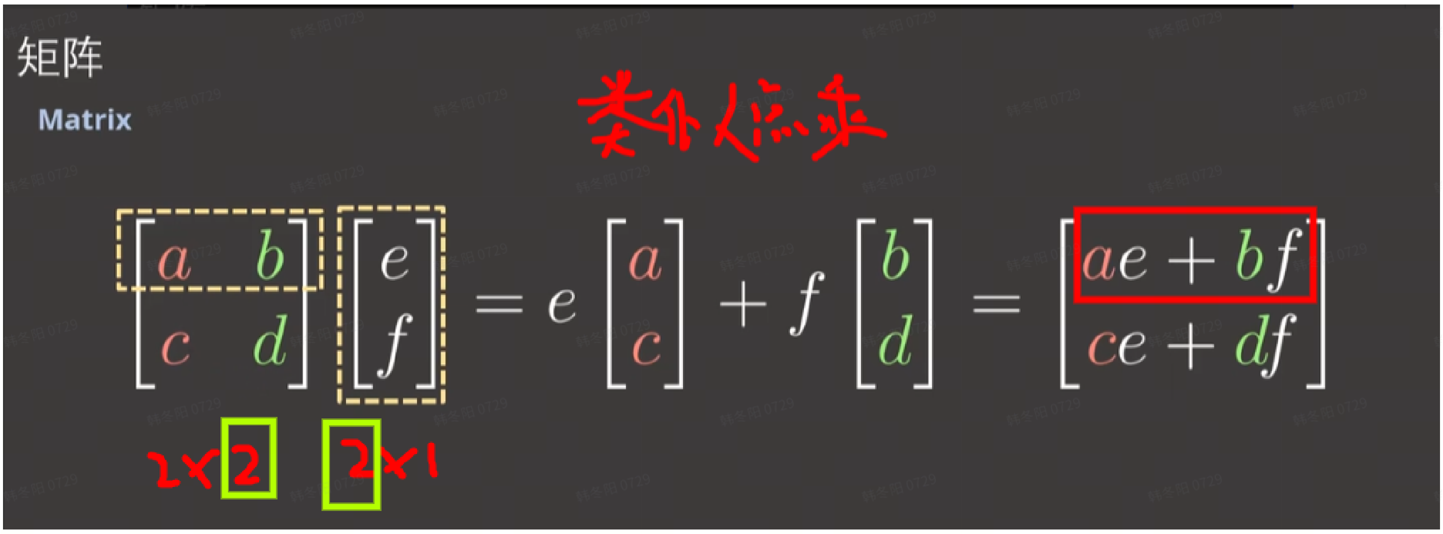
$$=\begin{bmatrix}1*0+2*-1\\1*1+2*0\end{bmatrix}$$

$$=\begin{bmatrix}-2\\1\end{bmatrix}$$


旋转
Rotate

$$\begin{bmatrix}\cos(\theta)&-\sin(\theta)\\\sin(\theta)&\cos(\theta)\end{bmatrix}$$

以矩阵理解旋转（可以是四元数，可以是欧拉。。）（θ为软件中我们输入的值）



矩阵

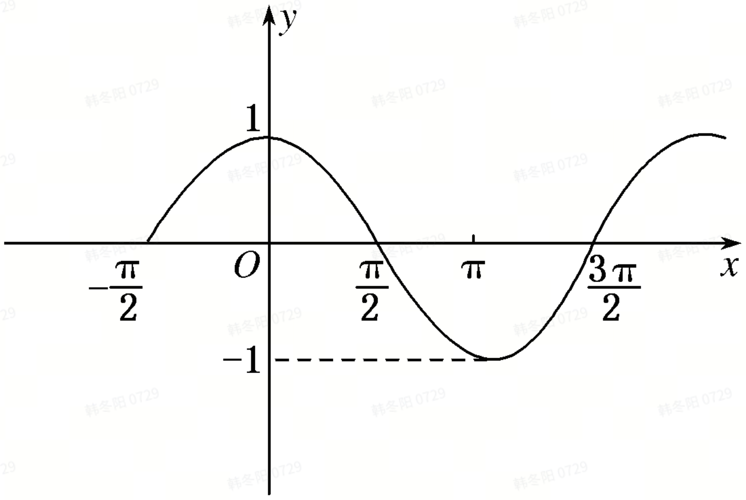
Matrix

类似点乘

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e \\ f \end{bmatrix} = e\begin{bmatrix} a \\ c \end{bmatrix} + f\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} \boxed{ae+bf} \\ ce+df \end{bmatrix}$$

$2\times\boxed{2}$ $\boxed{2}\times 1$

旋转缩放矩阵原理

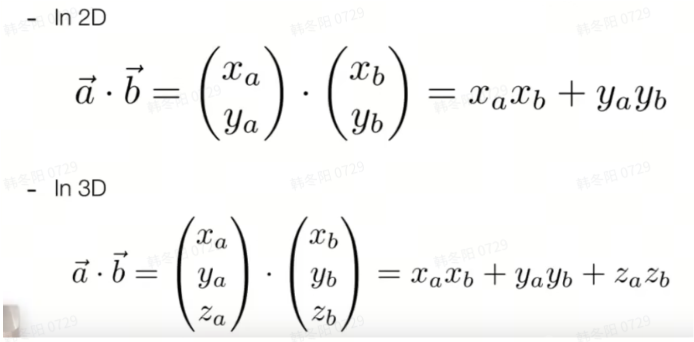


补充：cos

矩阵相乘（需要满足绿框内数字相等才可相乘）

向量点乘叉乘

点乘原理



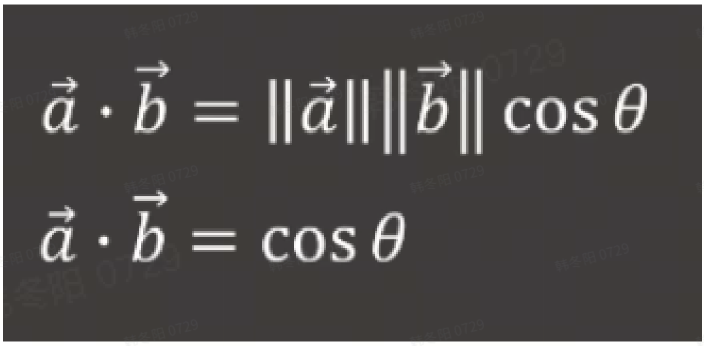
- In 2D

$$\vec{a}\cdot\vec{b}=\begin{pmatrix}x_a\\y_a\end{pmatrix}\cdot\begin{pmatrix}x_b\\y_b\end{pmatrix}=x_ax_b+y_ay_b$$

- In 3D

$$\vec{a}\cdot\vec{b}=\begin{pmatrix}x_a\\y_a\\z_a\end{pmatrix}\cdot\begin{pmatrix}x_b\\y_b\\z_b\end{pmatrix}=x_ax_b+y_ay_b+z_az_b$$

点乘原理



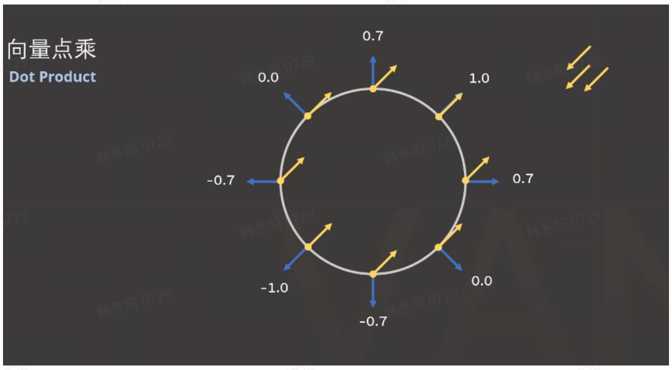
$$\vec{a}\cdot\vec{b}=\|\vec{a}\|\|\vec{b}\|\cos\theta$$

$$\vec{a}\cdot\vec{b}=\cos\theta$$

把向量之间夹角转化为系数值（0-1）

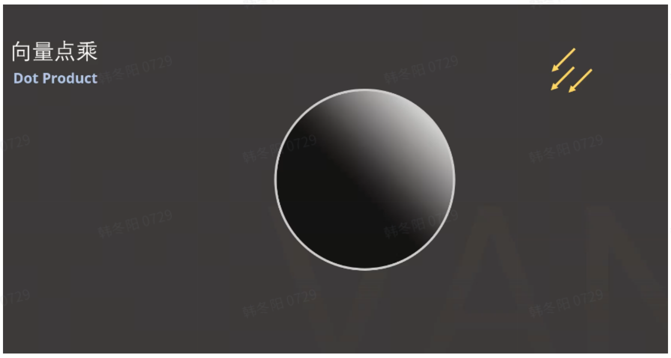
点乘用途

-



向量点乘

Dot Product

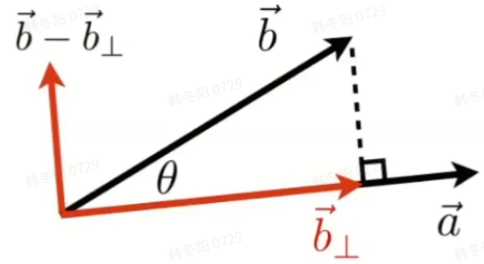


向量点乘

Dot Product

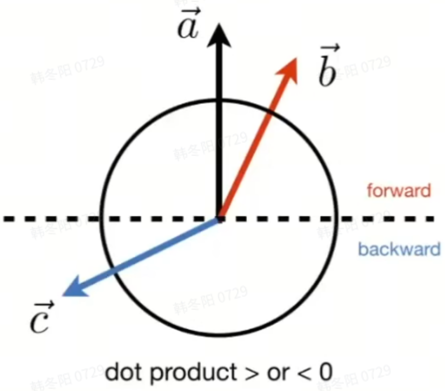
渲染的底层原理!

-

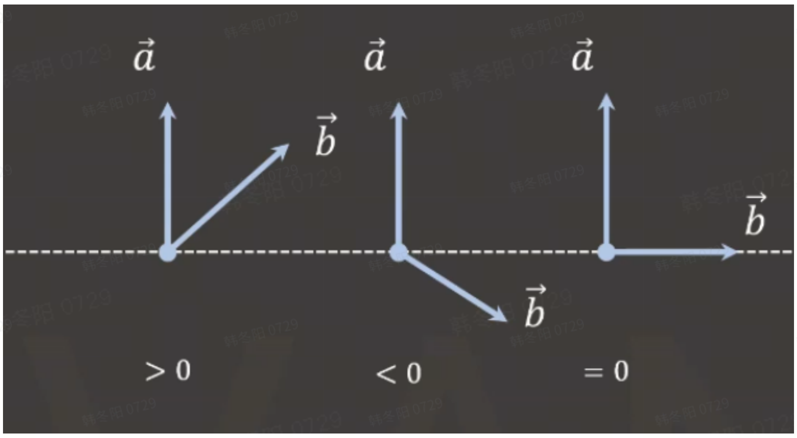


分解向量

-

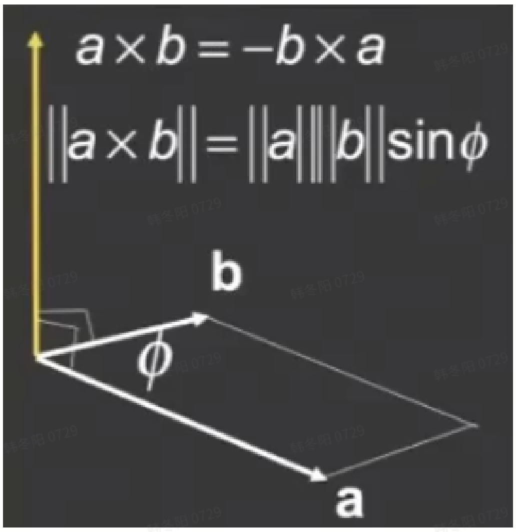


先后顺序



判断夹角

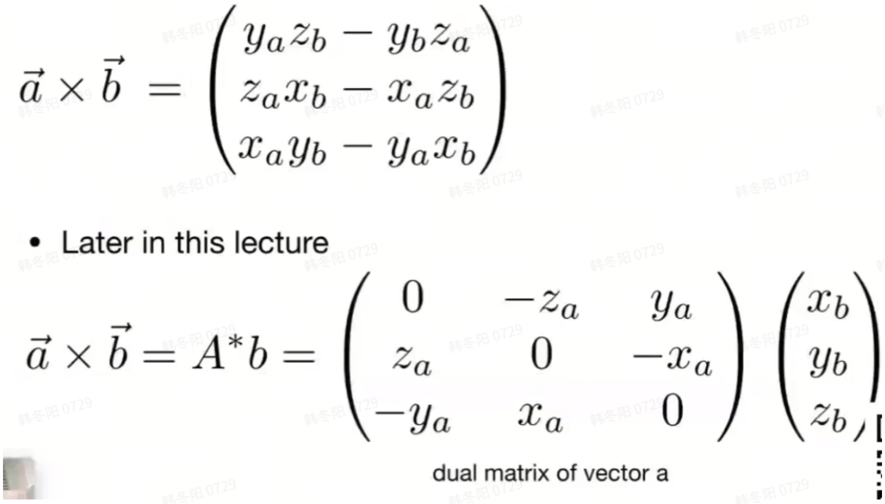
叉乘原理



$$a \times b = -b \times a$$

$$\|a \times b\| = \|a\|\|b\| \sin\phi$$

叉乘原理



$$\vec{a} \times \vec{b} = \begin{pmatrix} y_a z_b - y_b z_a \\ z_a x_b - x_a z_b \\ x_a y_b - y_a x_b \end{pmatrix}$$

- Later in this lecture

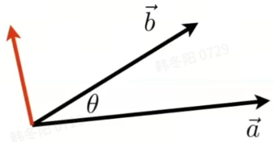
$$\vec{a} \times \vec{b} = A^* b = \begin{pmatrix} 0 & -z_a & y_a \\ z_a & 0 & -x_a \\ -y_a & x_a & 0 \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

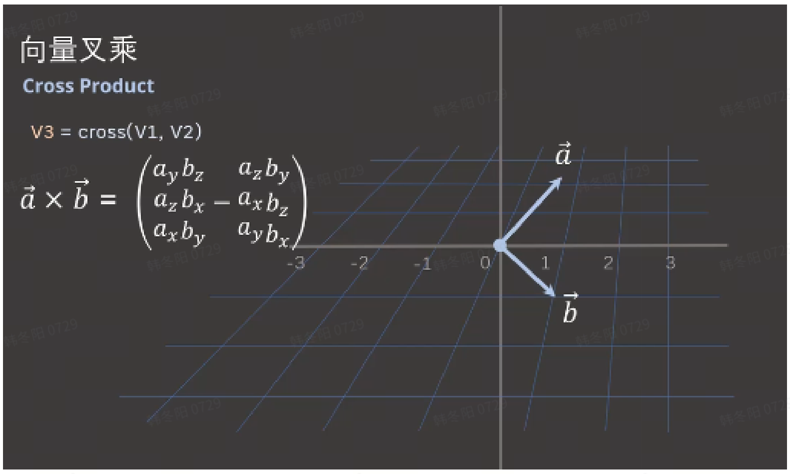
dual matrix of vector a

可矩阵表示叉乘

叉乘用途

- Determine left / right
- Determine **inside / outside**



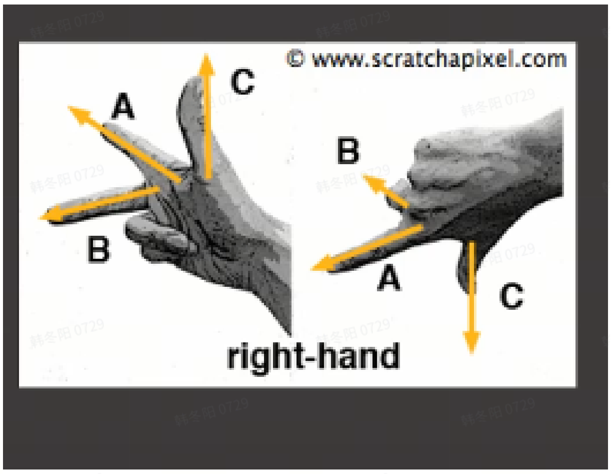


向量叉乘

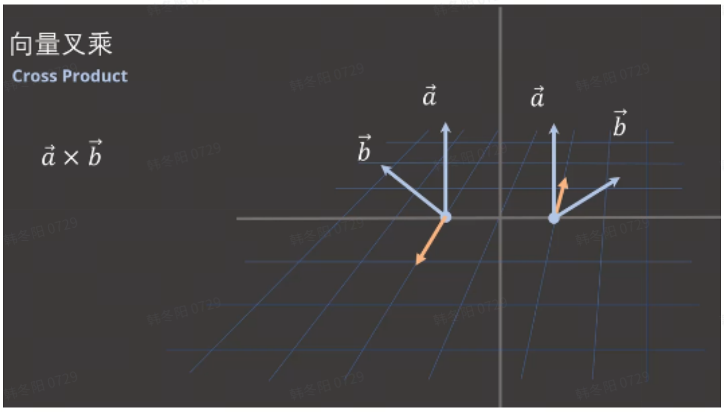
Cross Product

V3 = cross(V1, V2)

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_y b_z & a_z b_y \\ a_z b_x & - a_x b_z \\ a_x b_y & a_y b_x \end{pmatrix}$$



左边向量a×b 右边向量b×a

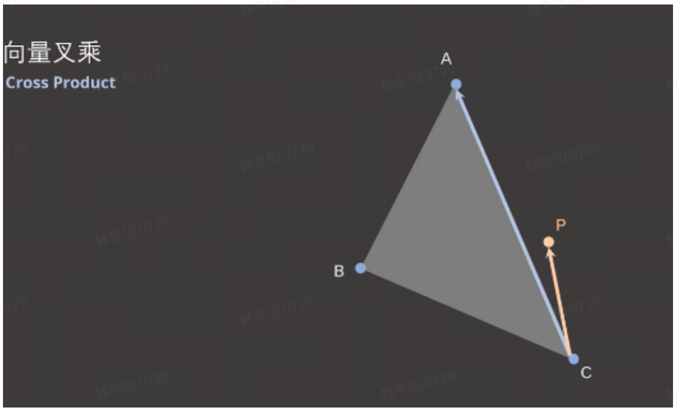


向量叉乘

Cross Product

$\vec{a} \times \vec{b}$

叉乘应用 判断向量左右



向量叉乘

Cross Product

叉乘应用 判断点是否在三角形内

1. 点乘:

   $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

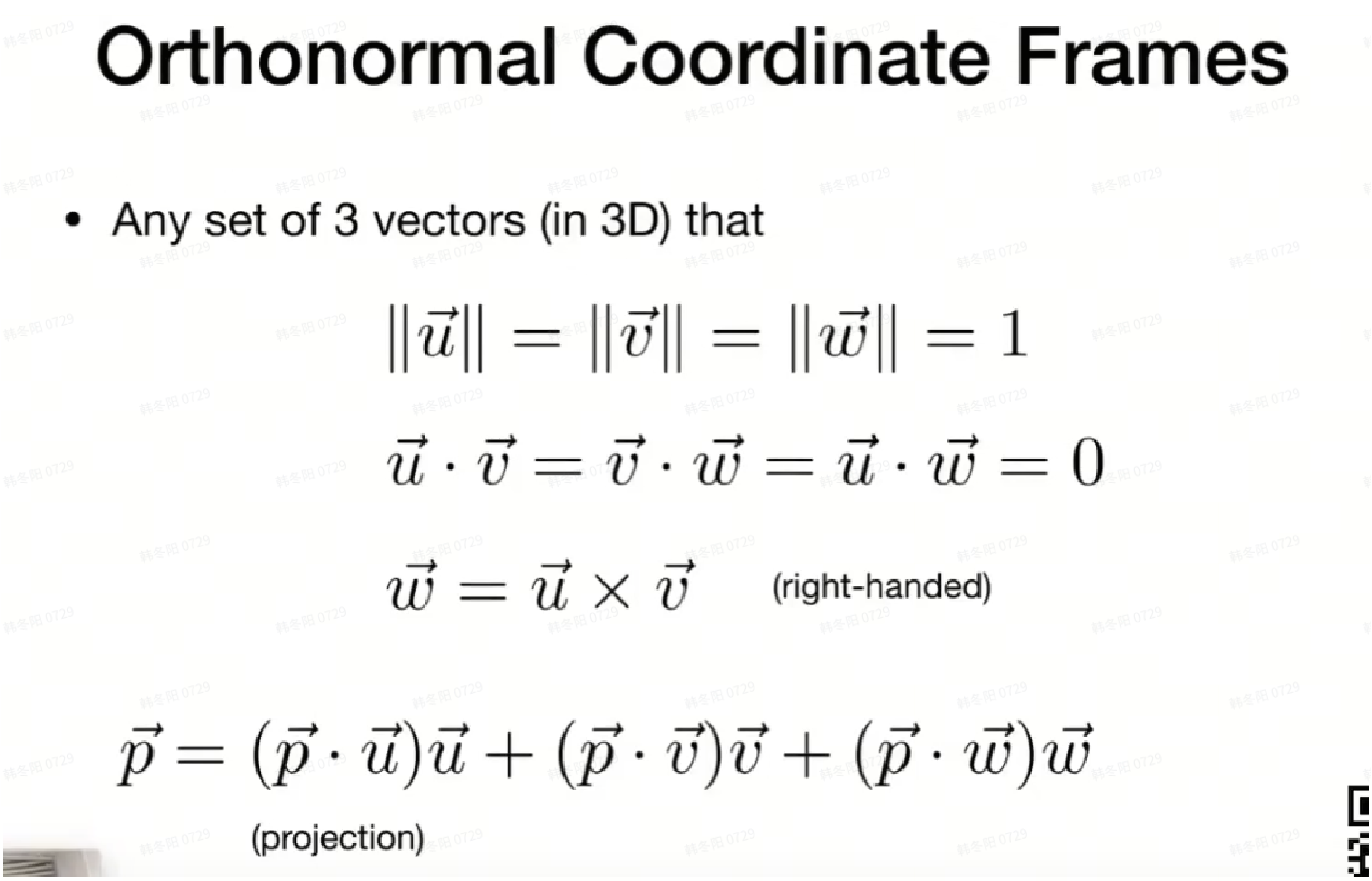
   点乘是一个标量,表示两个向量的"长度"乘以它们夹角的余弦。

2. 叉乘:

   $\vec{c} = \vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$

   叉乘是一个向量,垂直于$\vec{a}$和$\vec{b}$所在平面,方向由右手定则确定。

点乘算长 叉乘换维



Orthonormal Coordinate Frames

- Any set of 3 vectors (in 3D) that

$$\|\vec{u}\| = \|\vec{v}\| = \|\vec{w}\| = 1$$

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{w} = \vec{u} \cdot \vec{w} = 0$$

$$\vec{w} = \vec{u} \times \vec{v} \qquad \text{(right-handed)}$$

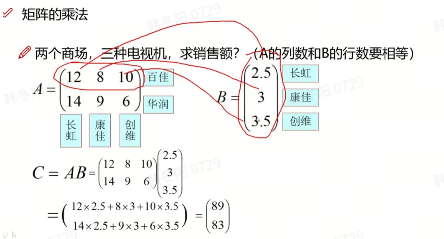
$$\vec{p} = (\vec{p} \cdot \vec{u})\vec{u} + (\vec{p} \cdot \vec{v})\vec{v} + (\vec{p} \cdot \vec{w})\vec{w}$$

(projection)

坐标系设计原理

矩阵相关

基本性质



✓ 矩阵的乘法

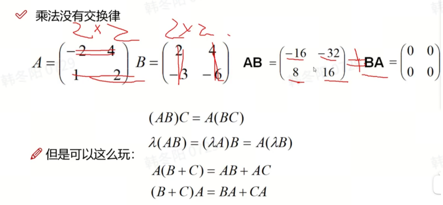
两个商场，三种电视机，求销售额？（A的列数和B的行数要相等）

$$A = \begin{array}{cccc} \begin{pmatrix} 12 & 8 & 10 \\ 14 & 9 & 6 \end{pmatrix} & \begin{matrix} \text{百佳} \\ \text{华润} \end{matrix} \\ \begin{matrix} \text{长虹} & \text{康佳} & \text{创维} \end{matrix} & \end{array} \qquad B = \begin{pmatrix} 2.5 \\ 3 \\ 3.5 \end{pmatrix} \begin{matrix} \text{长虹} \\ \text{康佳} \\ \text{创维} \end{matrix}$$

$$C = AB = \begin{pmatrix} 12 & 8 & 10 \\ 14 & 9 & 6 \end{pmatrix} \begin{pmatrix} 2.5 \\ 3 \\ 3.5 \end{pmatrix}$$

$$= \begin{pmatrix} 12\times2.5+8\times3+10\times3.5 \\ 14\times2.5+9\times3+6\times3.5 \end{pmatrix} = \begin{pmatrix} 89 \\ 83 \end{pmatrix}$$

两层宇宙相乘



✓ 乘法没有交换律

2×2　2×2.

$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix} \quad \mathbf{AB} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix} \neq \mathbf{BA} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

但是可以这么玩：

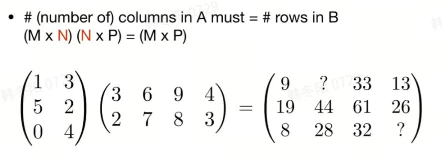
$$(AB)C = A(BC)$$

$$\lambda(AB) = (\lambda A)B = A(\lambda B)$$

$$A(B+C) = AB + AC$$

$$(B+C)A = BA + CA$$

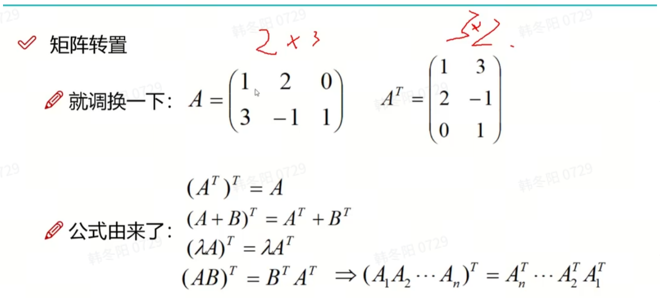
父级宇宙要在前边



- # (number of) columns in A must = # rows in B
(M x N) (N x P) = (M x P)

$$\begin{pmatrix} 1 & 3 \\ 5 & 2 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 3 & 6 & 9 & 4 \\ 2 & 7 & 8 & 3 \end{pmatrix} = \begin{pmatrix} 9 & ? & 33 & 13 \\ 19 & 44 & 61 & 26 \\ 8 & 28 & 32 & ? \end{pmatrix}$$

矩阵相乘 快速理解：第二行*第四列相加=26



✓ 矩阵转置

2×3　3×2.

就调换一下：$A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 3 \\ 2 & -1 \\ 0 & 1 \end{pmatrix}$

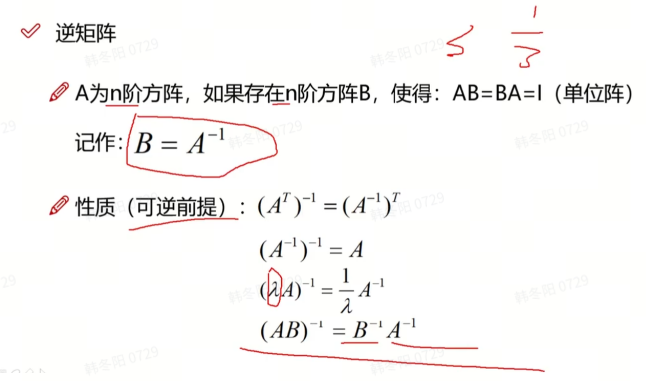
公式由来了：

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$(\lambda A)^T = \lambda A^T$$

$$(AB)^T = B^T A^T \Rightarrow (A_1 A_2 \cdots A_n)^T = A_n^T \cdots A_2^T A_1^T$$



✓ 逆矩阵

5　$\frac{1}{5}$

A为n阶方阵，如果存在n阶方阵B，使得：AB=BA=I（单位阵）

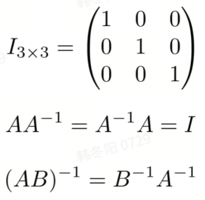
记作：$B = A^{-1}$

性质（可逆前提）：$(A^T)^{-1} = (A^{-1})^T$

$$(A^{-1})^{-1} = A$$

$$(\lambda A)^{-1} = \frac{1}{\lambda} A^{-1}$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

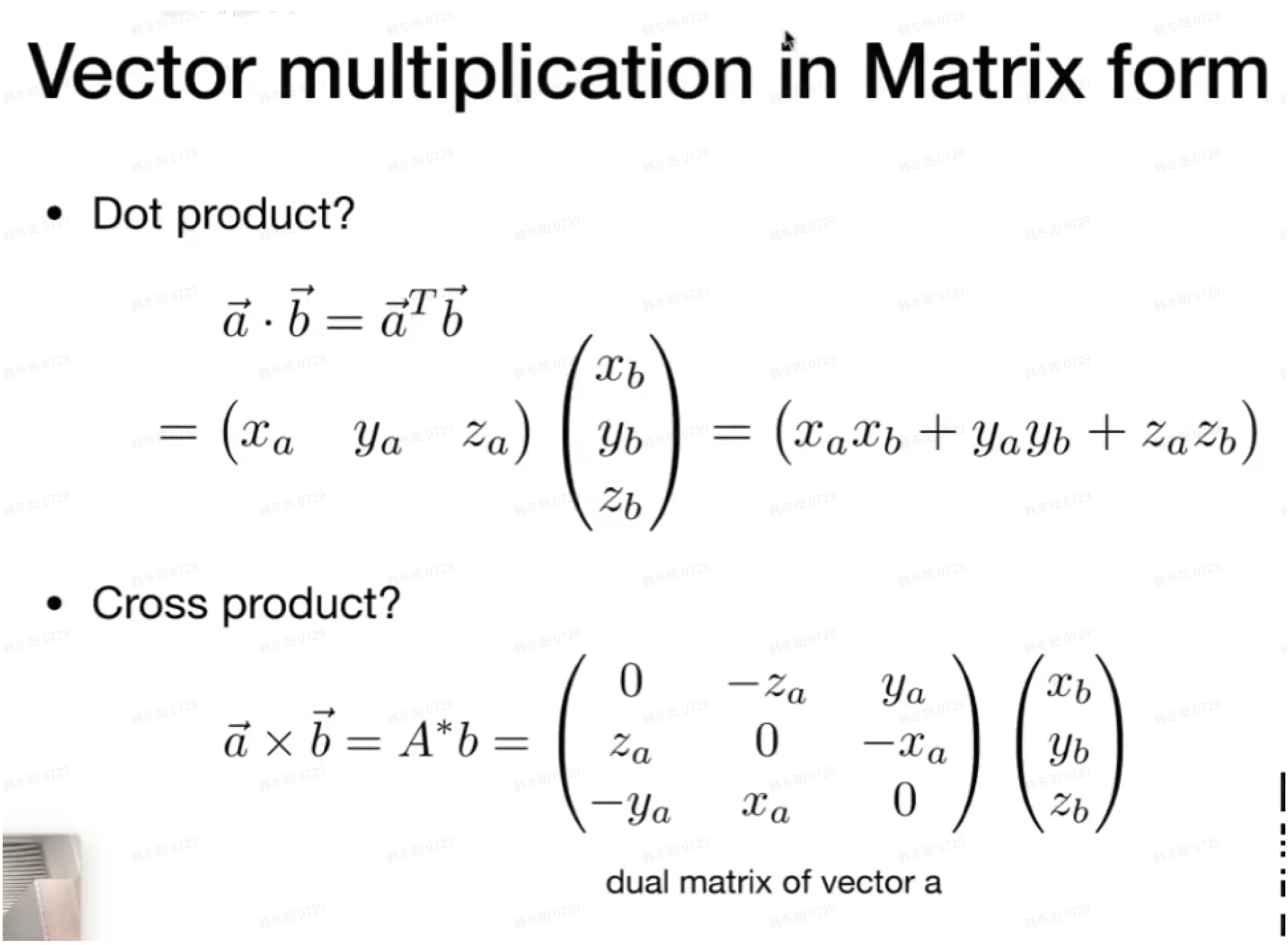


$$I_{3\times3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$AA^{-1} = A^{-1}A = I$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

I为单位矩阵 A-1为矩阵A的逆矩阵



Vector multiplication in Matrix form

- Dot product?

$$\vec{a}\cdot\vec{b} = \vec{a}^T\vec{b}$$

$$= \begin{pmatrix}x_a & y_a & z_a\end{pmatrix}\begin{pmatrix}x_b\\ y_b\\ z_b\end{pmatrix} = \left(x_ax_b + y_ay_b + z_az_b\right)$$

- Cross product?

$$\vec{a}\times\vec{b} = A^*b = \begin{pmatrix}0 & -z_a & y_a\\ z_a & 0 & -x_a\\ -y_a & x_a & 0\end{pmatrix}\begin{pmatrix}x_b\\ y_b\\ z_b\end{pmatrix}$$

dual matrix of vector a

以矩阵方式理解点乘叉乘 （*不是乘）

四元数描述的是一种旋转,其中标量部分表示旋转角度,矢量部分表示旋转轴

切线空间法线相对物体空间法线和世界空间法线是特殊的，它是根据每个顶点定义的，而其他空间法线都是一个具体的法线