

3.

1)

$$\theta_{11}^{(1)} = 0.5, \theta_{12}^{(1)} = 0.5, \theta_{21}^{(1)} = 0.1, \theta_{22}^{(1)} = 0.7, \theta_{11}^{(2)} = 1, \theta_{12}^{(2)} = 2$$

$$a_1^{(1)} = x_1 = 0.5, a_2^{(1)} = x_2 = 0.9$$

$$Z_1^{(2)} = \theta_{10} * 1 + \theta_{11} * x_1 + \theta_{12} * x_2 = 0.2 + 0.5 * 0.5 + 0.5 * 0.9 = 0.9$$

$$Z_2^{(2)} = \theta_{20} * 1 + \theta_{21} * x_1 + \theta_{22} * x_2 = 0.2 + 0.1 * 0.5 + 0.7 * 0.9 = 0.88$$

$$a_1^{(2)} = g(Z_1^{(2)}) = 1 / (1 + e^{-Z_1^{(2)}}) = 1 / (1 + e^{-0.9}) = 0.71$$

$$a_2^{(2)} = g(Z_2^{(2)}) = 1 / (1 + e^{-Z_2^{(2)}}) = 1 / (1 + e^{-0.88}) = 0.7$$

$$Z_1^{(3)} = \theta_{10}^{(2)} * 1 + \theta_{11}^{(2)} * a_1^2 + \theta_{12}^{(2)} * a_2^2 = 0.2 + 1 * 0.71 + 2 * 0.7 = 2.31$$

$$a_1^{(3)} = g(Z_1^{(3)}) = 1 / (1 + e^{-Z_1^{(3)}}) = 1 / (1 + e^{-2.31}) = 0.9$$

2)

Errors:

$$\delta_1^{(3)} = y - a_1^{(3)} = 1 - 0.9 = 0.1$$

$$\delta_2^{(2)} = \theta_{12}^{(2)} * \delta_1^{(3)} = 2 * 0.1 = 0.2$$

$$\delta_1^{(2)} = \theta_{11}^{(2)} * \delta_1^{(3)} = 1 * 0.1 = 0.1$$

$$\delta_2^{(1)} = \theta_{12}^{(1)} * \delta_1^{(2)} + \theta_{22}^{(1)} * \delta_2^{(2)} = 0.5 * 0.1 + 0.7 * 0.2 = 0.05 + 0.14 = 0.19$$

$$\delta_1^{(1)} = \theta_{11}^{(1)} * \delta_1^{(2)} + \theta_{21}^{(1)} * \delta_2^{(2)} = 0.5 * 0.1 + 0.1 * 0.2 = 0.05 + 0.02 = 0.07$$

Updates:

$$\Delta_{11}^{(1)} = \Delta_{11}^{(1)} + a_1^{(1)} * \delta_1^{(2)} = 0.5 + 0.5 * 0.1 = 0.55$$

$$\Delta_{12}^{(1)} = \Delta_{12}^{(1)} + a_2^{(1)} * \delta_1^{(2)} = 0.5 + 0.9 * 0.1 = 0.59$$

$$\Delta_{21}^{(1)} = \Delta_{21}^{(1)} + a_1^{(1)} * \delta_2^{(2)} = 0.1 + 0.5 * 0.2 = 0.2$$

$$\Delta_{22}^{(1)} = \Delta_{22}^{(1)} + a_2^{(1)} * \delta_2^{(2)} = 0.7 + 0.9 * 0.2 = 0.88$$

$$\Delta_{11}^{(2)} = \Delta_{11}^{(2)} + a_1^{(2)} * \delta_1^{(3)} = 1 + 0.71 * 0.1 = 1.071$$

$$\Delta_{12}^{(2)} = \Delta_{12}^{(2)} + a_2^{(2)} * \delta_1^{(3)} = 2 + 0.7 * 0.1 = 2.07$$

4.1

Finding the decision surface

$$x_2 = a + b * x_1 \quad (-1,0), (0,2)$$

From the two points on a line, we can infer that decision line:

$$2 + 2 * x_1 - x_2 = 0$$

According to the graph, the point $(-1,1)$ should be categorized as positive, so the decision line is

$$-2 - 2x_1 + x_2 = 0$$

So

$$w_0 = -2, w_1 = -2, w_2 = 1$$

4.2

a)

A AND (NOT B)

A, B and weights should be like below:

$$w_0 = -10, w_A = 20, w_B = -20$$

A	B	$h_{\theta}0$
0	1	0
1	0	1
0	0	0
1	1	0

b)

A XOR B

A, B and weights should be like below:

$$w_{10}^{(1)} = -30, w_{11}^{(1)} = 20, w_{12}^{(1)} = 20$$

$$w_{20}^{(1)} = 10, w_{21}^{(1)} = -20, w_{22}^{(1)} = -20$$

$$w_{10}^{(2)} = 10, w_{11}^{(2)} = -20, w_{12}^{(2)} = -20$$

$A(x_1)$	$B(x_2)$	a_1^2	a_2^2	h_θ
0	1	0	0	1
1	0	0	0	1
1	1	1	0	0
0	0	0	1	0