Chapter 4

Grammars and Parsing

Grammar

- Grammars, or more precisely, **context-free grammars**, are the formalism for describing the structure of program in programming languages.
- A **grammar** consists of a set of production rules and a start symbol (left symbol of first rule).
- A **production rule** consists of two parts: a left-hand side and a right-hand side.
  - ex: expression → expression ‘+’ term

left-hand side

right-hand side

Grammar (Cont.)

- The **left-hand side** is the **name** of the syntactic construct.
- The **right-hand side** shows a **possible form** of the syntactic construct.
- There are two possible forms (rules) derived by the name "expression":

  expression → expression '+' term (rule 1)

  expression → expression '-' term (rule 2)

Grammar (Cont.)

- The right-hand side of a production rule can contain two kinds of symbols:

  terminal and non-terminal.
- A **terminal symbol** (or *terminal*) is an end point of the production process, also called **token**. Use lower-case letters such as a, b.
- A **non-terminal symbol** (or *non-terminal*) must occur as the left-hand side of one or more production rules.

  Use upper-case letters such as A, B, S.

- Non-terminal and terminal together are called **grammar symbols**.

production process

- A string of terminals can be produced from a grammar by applying **productions** to a **sentential form**. (see example next)

- The steps in the production process leading from the start symbol to a string of terminal are called:

  The **derivation** of that string of terminals.

An example of production process

- Grammar :
  - expression → ‘(‘ expression operator expression ‘)’
  - expression → ‘1’
  - operator → ‘+’
  - operator → ‘*’

An example of production process (Cont.)

- Derivation of the string (1*(1+1))
  - expression
  - ‘(‘ expression operator expression ‘)’
  - ‘(‘ ‘1’ operator expression ‘)’
  - ‘(‘ ‘1’ ‘*’ expression ‘)’
  - ‘(‘ ‘1’ ‘*’ ‘(‘ expression operator expression ‘)’ ‘)’
  - ‘(‘ ‘1’ ‘*’ ‘(‘ ‘1’ operator expression ‘)’ ‘)’
  - ‘(‘ ‘1’ ‘*’ ‘(‘ ‘1’ ‘+’ expression ‘)’ ‘)’
  - ‘(‘ ‘1’ ‘*’ ‘(‘ ‘1’ ‘+’ ‘1’ ‘)’ ‘)’

  - <u>Each of the above is a **sentential form**</u>

- It forms a **leftmost derivation**, in which it is always the leftmost non-terminal in the sentential form that is rewritten.

Think

- Why we need to discover some specific rules for derivation? Let's look at the original problem, I have too many possible ways to choose production rules and even you choose one, you don't know if this step can lead to derive the token streams you need.
- It is hard if we try to build such a SMART parser.
- Machine are stupid, we better clarify the problem and so that a parser can work economically.
- Sometimes this means we need to give up some freedom of the grammar we wrote.

The definition of a grammar

Context-free grammar **(CFG)** is defined by:

(1) A finite terminal vocabulary Vt; this is the token set produced by the scanner.

(2) A finite set of different, intermediate symbols, called the non-terminal vocabulary Vn.

(3) A start symbol S $\in$ Vn that starts all derivations. A start symbol is sometimes called a goal symbol.

(4) P, a finite set of productions (sometimes called rewriting rules) of the form $A \rightarrow X1 \ldots Xm$, where

$$A \in Vn,\ Xi \in Vn \cup Vt,\ 1 <= i <= m,\ m >= 0$$

The definition of a grammar (Cont.)

Given two sets of symbols V1, V2

A production rule is

(N, α) such that N $\in$ V1, α $\in$ V2*

Context free grammar G=(Vn, Vt, S, P)

Vn ∩ Vt = Φ

S $\in$ Vn

P $\subseteq$ { (N, α) | N $\in$ Vn, α $\in$ (Vn ∪ Vt)*}

BNF form of grammars

- Backus-Naur Form (BNF) is a formal grammar for expressing context-free grammars.
- The single grammar rule format:
  - Non-terminal $\rightarrow$ zero or more grammar symbols

- It is usual to combine all rules with the same left-hand side into one rule, such as:

  $N \rightarrow \alpha$

  $N \rightarrow \beta$

  $N \rightarrow \gamma$

  Greek letters α,β, or γ means a string of symbols.

  are combined into one rule:

  $N \rightarrow \alpha \mid \beta \mid \gamma$

  α, β and γ are called the ***alternatives*** of N.

Extended BNF form of grammars

- BNF is very suitable for expressing nesting and recursion, but less convenient for repetition and optionality.

- Three additional postfix operators +,?, and *, are thus introduced:
  - R+ indicates the occurrence of one or more Rs, to express repetition.
  - R? indicates the occurrence of zero or one Rs, to express optionality.
  - R* indicates the occurrence of zero or more Rs, to express repetition.
- The grammar that allows the above is called Extended BNF (EBNF).

Extended forms of grammars (Cont.)

An example is the grammar rule:

parameter_list →

('IN' | 'OUT')? identifier (',' identifier)*

which produces program fragments like:

a, b

IN year, month, day

OUT left, right

Extended forms of grammars (Cont.)

- Rewrite EBNF grammar to CFG
  - Given the EBNF grammar:

    expression → term (+ term)*

  Rewrite it to:

    expression → term term_tmp

    term_tmp → + term term_tmp

    | λ

Properties of grammars

- A non-terminal N is **left-recursive** if, starting with a sentential form N, we can produce another sentential form starting with N.
  - ex: expression → expression ‘+’ factor | factor

- right-recursion also exists, but is less important.
  - ex: expression → term ‘+’ expression

Properties of grammars (Cont.)

- A non-terminal N is **nullable**, if starting with a sentential form N, we can produce an empty sentential form.

  example:

  expression $\rightarrow$ λ

- A non-terminal N is **useless**, if it can never produce a string of terminal symbols.

  example:

  expression $\rightarrow$ + expression
  | - expression

Ambiguity

- A grammar can have more than one parse tree generating a given string of terminals. Such a grammar is said to be ***ambiguous***.

  Given the grammar:

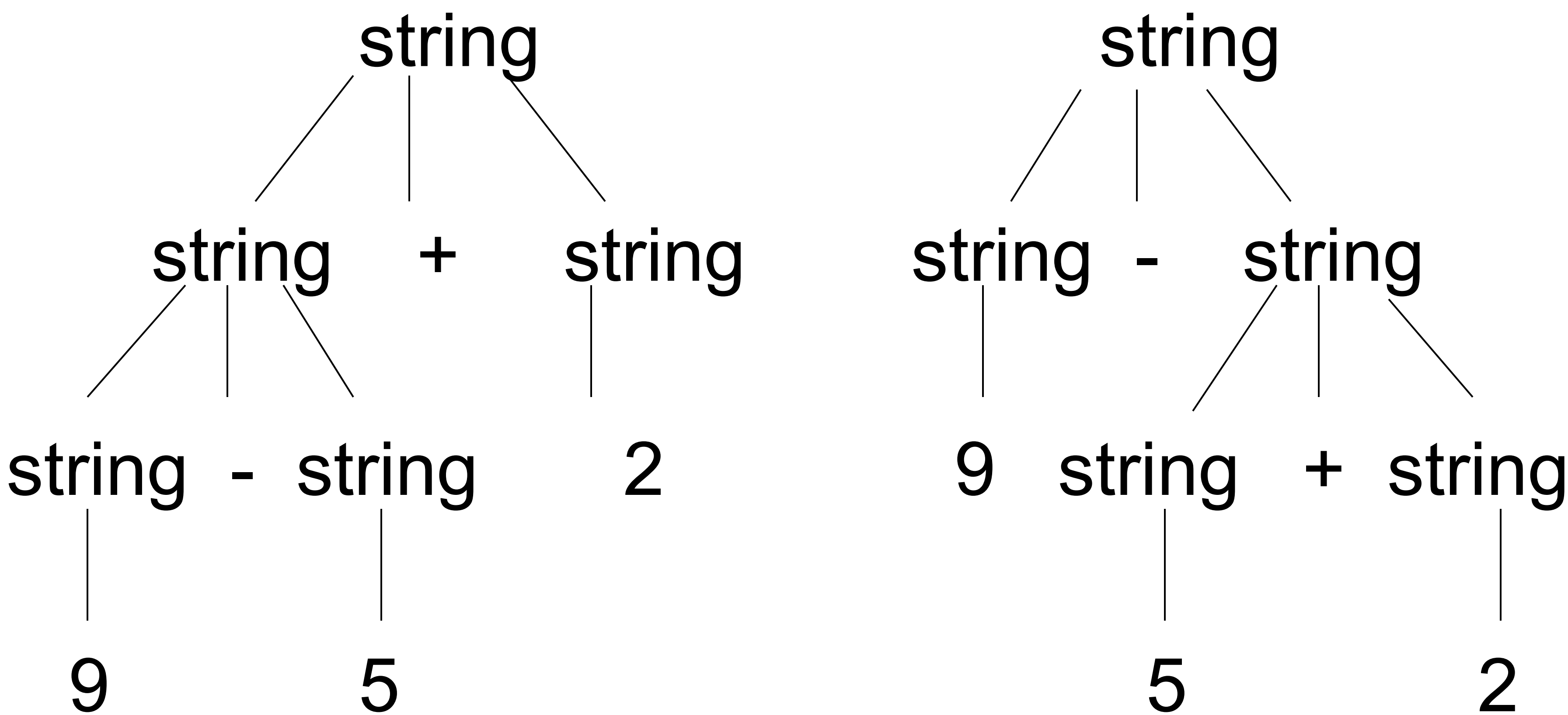
  string → string + string

  | string – string

  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

  Two parse trees for 9-5+2 can be constructed below. Thus, the grammar is ***ambiguous.***

Ambiguity


string
string
+
string
string
-
string
2
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5
string
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2

Associativity of operators

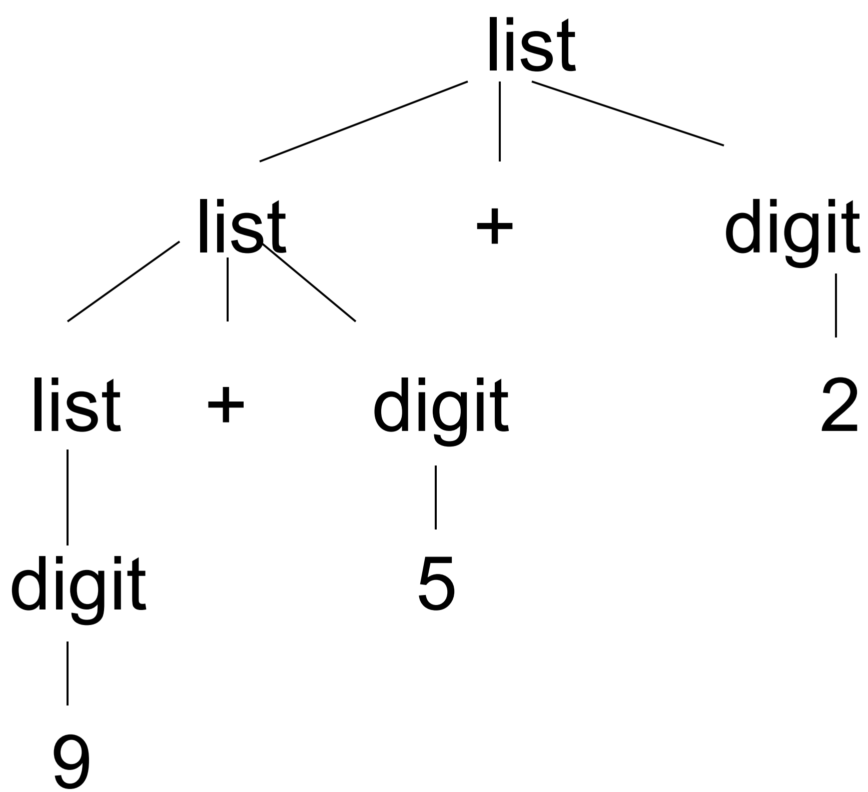
- Left-associativity:

  9+5+2 is equivalent to 9+5+2

- Given the grammar:
  - list → list + digit
  - | list – digit
  - | digit
  - digit → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Associativity of operators (Cont.)

- Parse tree for 9+5+2
  using a left-associative grammar



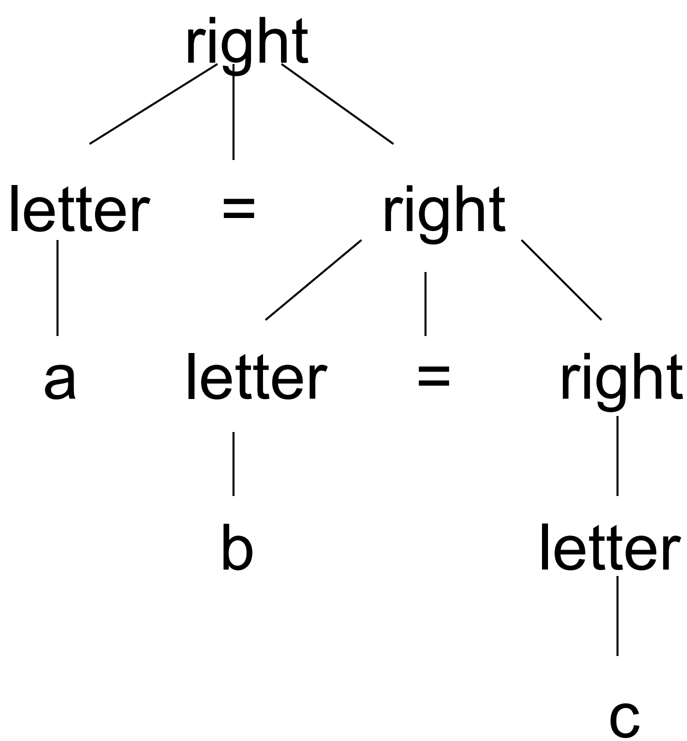
Associativity of operators (Cont.)

- Right-associativity: expression a=b=c is treated in the same way as the expression a=<u>b=c</u>

- Given the grammar:
  - right $\rightarrow$ letter = right
  - | letter
  - letter $\rightarrow$ a | b | … | z

Associativity of operators (Cont.)

- Parse tree for a=b=c using a right-associative grammar.



From tokens to parse tree

The process of finding the structure (parse tree) in the flat stream of tokens is called **parsing**,

and the module that performs this task is called **parser**.

Parsing methods

The way to construct the parse tree:

- Leaf nodes are labeled with terminals and inner nodes are labeled with non-terminals.
- The top node is labeled with the start symbol.
- The children of an inner node labeled N correspond to the members of an alternative of N, in the same order as they occur in that alternative.
- The terminals labeling the leaf nodes correspond to the sequence of tokens, in the same order as they occur in the input.

Parsing methods

There are two well-known ways to parse:

1) top-down

Left-scan, **L**eftmost derivation (**LL**).

2) bottom-up

Left-scan, **R**ightmost derivation in reverse (**LR**).

- LL constructs the parse tree in pre-order;
- LR in post-order.

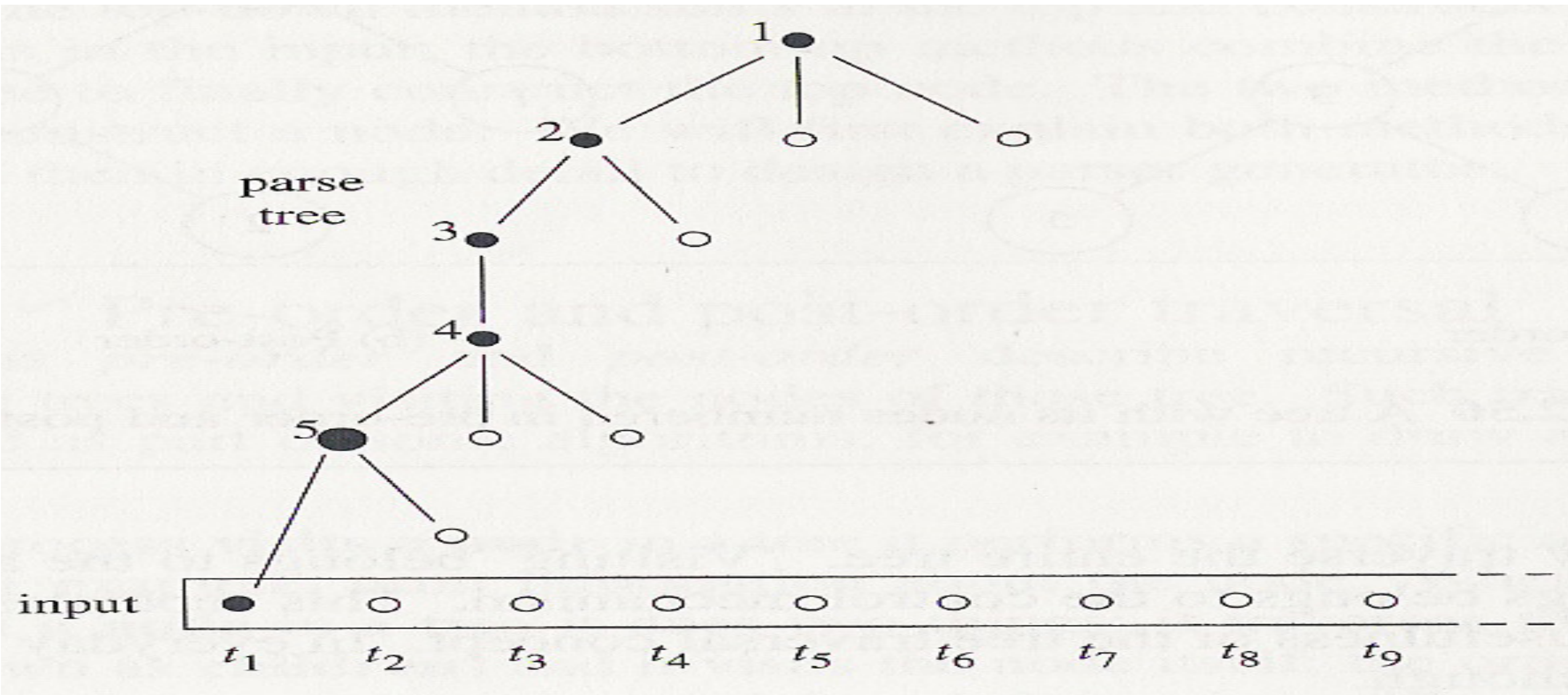
Pre-order vs. post-order traversal

- When traversing a node N in pre-order, the process first visits the node N and then traverses N’s subtrees in left-to-right order.

- When traversing a node N in post-order, the process first traverses N’s subtrees in left-to-right order and then visits the node N.

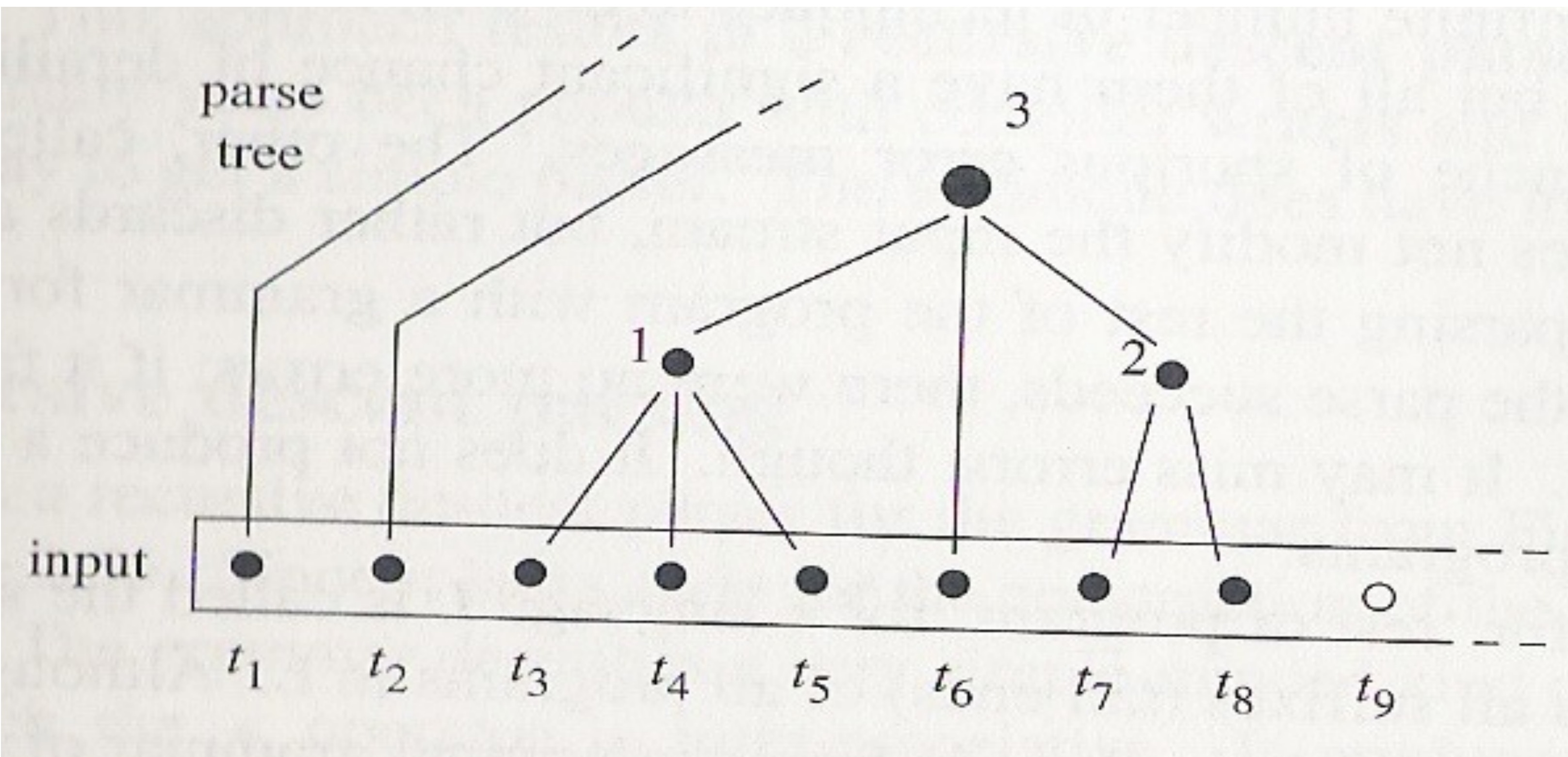
Principle of top-down parsing

- A top-down parser begins by constructing the top node of the parse tree, which is the start symbol.



Principles of bottom-up parsing

- The bottom-up parsing method constructs the nodes in the parse tree in post-order.



First and Follow

- The construction of both top-down and bottom-up parsers is aided by two functions: FIRST and FOLLOW.

- Define FIRST(α),where α is any string of grammar symbols,to be:

  the set of terminals

  that begin strings derived from α.

Why we want to compute First and FOLLOW set?

- As stated in previous section, when you have two production rules to choose from, a deterministic choices allow efficient/cheap parser to be built.
- A-> aB

First and Follow (Cont.)

Given the grammar:

input → expression

expression → term rest_expression

term → ID | parenthesized_expression

parenthesized_expression → '(' expression ')'

rest_expression → '+' expression | λ

FIRST (input) = FIRST(expression) =FIRST (term)

={ ID, '(' }

FIRST (parenthesized_expression) = { '(' }

FIRST (rest_expression) = { '+' λ}

First and Follow (Cont.)

Given the grammar (E for expression, T for term, F for factor) :

- $E \rightarrow TE'$
- $E' \rightarrow +TE' \mid \lambda$
- $T \rightarrow FT'$
- $T' \rightarrow *FT' \mid \lambda$
- $F \rightarrow (E) \mid id$

Find the first set of each symbol.

First and Follow (Cont.)

Answer:

FIRST(F) = FIRST(T) = FIRST(E) = {(, id }

FIRST(E') = {+, λ}

FIRST(T') = {*, λ}

First and Follow (Cont.)

- To compute FIRST(X) for grammar symbol X, apply the following rules until no more terminals or λ can be added to it.
  - 1. If X is a terminal , then FIRST(X)={X}
  - 2. If X is a non-terminal and $X \rightarrow Y_1Y_2\ldots Y_k$ is a production for some k>=1, then place "a" in FIRST(X) if for some i, "a" is in FIRST(Y_i), and λ is in all of FIRST(Y_1),…,FIRST(Y_{i-1}). If λ is in FIRST(Y_j) for all j=1,2,…,k, then add λ to FIRST(X).
  - 3. If $X \rightarrow \lambda$ is a production, then add λ to FIRST(X).

First and Follow (Cont.)

- To compute FOLLOW(B) for non-terminal B:
  - 1. Place $\$$ in FOLLOW(S), where S is the start symbol, and $\$$ is the input right end-marker.
  - 2. if there is a production $A \rightarrow \alpha\ B\ \beta$, then everything in FIRST(β) except λ is in FOLLOW(B).
  - 3. (a) if there is a production $A \rightarrow \alpha\ B$,

    (b) or $A \rightarrow \alpha\ B\ \beta$, where FIRST($\beta$) contains λ,

    then everything in FOLLOW(A) is in FOLLOW(B).

First and Follow (Cont.)

input → expression
expression → term rest_expression
term → ID | parenthesized_expression
parenthesized_expression → ‘(‘ expression ‘)’
rest_expression → ‘+’ expression | λ

FOLLOW (input) = { $ } rule 1

FOLLOW (expression) = { $ ‘)’} rule 3(a) got $; rule 2 got)

FOLLOW (term) = FOLLOW (parenthesized_expression) rule3(a)
= {‘+’ $ ‘)’ } rule 2 got +; rule 3(b) got $)

FOLLOW (rest_expression) = { $ ‘)’} rule 3(a)

First and Follow (Cont.)

- For example, given the grammar :
  - E → TE’
  - E’ → +TE’ | λ
  - T → FT’
  - T’ → *FT’ | λ
  - F → (E) | id

  Find the follow set of each symbol.

First and Follow (Cont.)

Answers:

FOLLOW(E) = FOLLOW(E’) = {) , $ }

FOLLOW(T) = FOLLOW(T’) = { +,) , $ }

FOLLOW(F) = { $*$, + ,) , $ }

Homework

8. A grammar for infix expressions follows:

1	Start	$\rightarrow$	E $
2	E	$\rightarrow$	T plus E
3		|	T
4	T	$\rightarrow$	T times F
5		|	F
6	F	$\rightarrow$	(E)
7		|	num

Homework (Cont.)

(a) Show the leftmost derivation of the following string.

num plus num times num plus num $

(b) Show the rightmost derivation of the following string.

num times num plus num times num $

(c) Describe how this grammar structures expressions, in terms of the precedence and left- or right- associativity of operators.

Homework Solution 8

(a) Leftmost derivation

- Start
- E $
- T plus E $
- F plus E $
- num plus E $
- num plus T plus E $
- num plus T times F plus E $
- num plus F times F plus E $
- num plus num times F plus E $
- num plus num times num plus E $
- num plus num times num plus T $
- num plus num times num plus F $
- num plus num times num plus num $

Homework Solution 8 (Cont.)

(b) Rightmost derivation

- -Start
- -E $
- -T plus E $
- -T plus T $
- -T plus T times F $
- -T plus T times num $
- -T plus F times num $
- -T plus num times num $
- -T times F plus num times num $
- -T times num plus num times num $
- -F times num plus num times num $
- -num times num plus num times num $

Homework Solution 8 (Cont.)

(C) This grammar ensures that “times” precedes “plus”.

for 1+2+3 first 2+3 then 1+5 so operand 2 is associated with its right operator. that is, right-associativity for “plus” operator.

what if 1-2+3? This will get 1-5 or -4 wrong!

for 3*4*5 first 3*4 then 12*5 so operand 4 is associated with its left operator

that is, left-associativity for “times”

Homework (Cont.)

11 Compute First and Follow sets for the non-terminals of the following grammar

1	$S \rightarrow$	a S e
2	$\mid$	B
3	$B \rightarrow$	b B e
4	$\mid$	C
5	$C \rightarrow$	c C e
6	$\mid$	d

Homework Solution 11

First (S)={a, b, c, d}

First (B)={b, c, d}

First (C)={c, d}

Follow (S) = Follow (B) = Follow (C) = {e}