Class Notes
CIS 502 Analysis of Algorihtm
Week 4 - Computational Geometry

Da Kuang

University of Pennsylvania

1 Polynomial Multiplication

A polynomial in the variable x over an algebraic field F represents a function $A(x)$ as a formal sum:

$$A(x) = \sum_{j=0}^{n-1} a_j x^j$$

We call the values $a_0, a_1, \cdots, a_{n-1}$ the **coefficients** of the polynomial. The coefficients are drawn from a field F , typically the set $\mathbb{C}$ of complex numbers.

A polynomial $A(x)$ has **degree** k if its highest nonzero coefficient is a_k. we write that degree$(A) = k$. Any integer strictly greater than the degree of a polynomial is a **degree-bound** of that polynomial. Therefore, the degree of a polynomial of degree-bound n may be any integer between 0 and $n-1$, inclusive.

Polynomial Multiplication: if $A(x)$ and $B(x)$ are polynomials of degree-bound n, their product $C(x)$ is a polynomial of degree-bound $2n-1$ such that $C(x) = A(x)B(x)$ for all x in the underlying field. For example,

$$\begin{array}{rrrrrrr}
 & & & 6x^3 & +\ 7x^2 & -\ 10x & +\ 9 \\
 & & & -\ 2x^3 & & +\ 4x & -\ 5 \\
\hline
 & & & -\ 30x^3 & -\ 35x^2 & +\ 50x & -\ 45 \\
 & & 24x^4 & +\ 28x^3 & -\ 40x^2 & +\ 36x & \\
-\ 12x^6 & -\ 14x^5 & +\ 20x^4 & -\ 18x^3 & & & \\
\hline
-\ 12x^6 & -\ 14x^5 & +\ 44x^4 & -\ 20x^3 & -\ 75x^2 & +\ 86x & -\ 45
\end{array}$$

2 Convex Hull on the Plane