**Responses to Reviewers’ Comments**

R: This manuscript presents an extension of the 2D AxiSem method to 3D. The third dimension of the computations is introduced through a Fourier expansion along the azimuths. The results obtained with this technique are compared to those obtained with a 3D spectral element method and it is shown that this method is faster than specfem and that the speeding factor scales with the frequency.

The main motivation to develop this new method needs to be clarified and emphasized: Is it to compute synthetic seismograms in smooth 3D Earth models or to perform global tomography?

A: As stated in the introductory section, we aim at both forward modeling and tomography, and we don't see a reason to make such a distinction. Waveform tomography relies on accurate forward-modeled wavefields for a given Earth model, and that is exactly what our method provides.

R: Since the method makes different simplifying assumptions and thus obtains approximate synthetic seismograms this question is crucial. My feeling is that if one is willing to pay the cost of 3D full waveform inversion then computational efficiency becomes a secondary issue compared to the accuracy of the solution of the wave equation.

A: First of all, **our method does not simplify the Earth model**. The global tomographic models we used to compute synthetic seismograms in this paper are exactly the same as those in SPECFEM, i.e. state-of-the-art global models. The only difference to SPECFEM is the spatial parameterization of wavefields. In Section 5.1, we show the convergence of our solutions to the 3D solutions as the azimuthal expansion order increases, and the conclusions on performance are all drawn based on the same models and very strict accuracy criteria. In other words, **our method does not compromise on accuracy whatsoever; on the opposite it represents a fully convergent numerical method on which one can easily tune the desired accuracy and cost.** We now emphasize these features in the introduction. Please see **Update B**.

R: This method may still provide acceptable 3D kernels for global tomography applications, but this should be demonstrated.

A: We feel that this is a separate topic. The first concern when presenting an entirely new wave propagation method is to deliver precise and computationally efficient solutions to the wave equations – what this paper is all about. Of course, having full and accurate wavefields at hand one can compute accurate kernels for tomography, but this would drastically exceed the scope of this paper and is not commonplace in the literature to present all-in-one either. To clarify that our method is of course very suitable for computing kernels, we first noted it in Section 1.2, “Thanks to the self-adjointness of elastodynamics, the underlying wave solvers are identical to those used for forward simulations, i.e., the kernels are constructed by convolving a wavefield emanating from a seismic source with a receiver-originating adjoint wavefield that bears the misfit functional.”

In the benchmark section 5.2, we employed a set of stations covering the whole surface and demonstrated the excellent fit of waveforms of many different phases (Fig. 13), which, we believe, are sufficient to show the accuracy of the entire wavefield. Now we add a sentence about the kernels at the end of Section 5.2 to re-stress the statement in Section 1.2. Please see **Update F**.

R: The authors suggest that this method presents the greatest interest at short period, presumably to handle body waves. However, it appears that the method suffers from very serious limitations for doing so since it does not permit to include the effects of ellipticity, free surface topography, or variations of Moho depth.

A: This is a valid point to rise, but we do have solutions and ideas on this topic to be detailed below. Firstly, we would like to be clear that our method is not only geared towards short periods, but is computationally more efficient than 3D methods across the entire frequency band of global seismology (same for normal modes).

Secondly, as it is a new method in global seismology, the main purpose of this first paper is to establish the fundamental concepts and theories while showing the potential of the method, but not to announce it as a completed tool nor as a substitution of 3-D SEMs. We have reached a speedup by more than one order of magnitude concerning all the global mantle models for which we can find a reference solution. And we think this should be able to show its great potential. In fact, the method may be already useful in a number of applications as it stands, e.g., in the study of plumes.

And lastly, in Section 7.2, we proposed that we might handle these aspherical geometries (ellipticity, topography, and Moho undulation) by means of a “particle relabeling transformation” (Al- Attar & Crawford 2016). In fact, during the reviewing period of this paper, we have started to implement particle relabeling and have successfully benchmarked ellipticity, whereas 3D crustal models are under development. These will be the main focus of our next paper.

R: In the introduction, the review of the literature is incomplete. For example, for the forward modeling problem, recent studies have shown that hybrid methods that combine 1D or 2D methods at the global scale and a 3D method at the local or regional scale are also a good alternative to compute 3D synthetic seismograms at reasonable cost. These methods also open important perspectives for seismic tomography and in particular full waveform inversion.

A: We now add a new paragraph in the introduction. Please see **Update A**.

R: Some more specific comments: 1) section 2.1: How do you perform the FFT on the irregular GLL grid? Are your azimuthal basis functions in the Fourier domain orthogonal?

A: Sorry, we do not fully understand the first question, so we will try to explain the procedure in some detail. In the azimuthal direction, we use evenly spaced grids when performing FFT, such that we can make use of fast packages such as FFTW. But the number of azimuthal gird points (or equivalently the Fourier expansion order) at different GLL points in the 2D mesh varies, so we are capable of adapting the computational cost to the complexity of wavefields. FFT is a point-wise operation unlike computing displacement gradients, so it does not matter whether the GLL points are regular or not in the 2D mesh. The azimuthal basis functions are orthogonal.

R: 2) Eqs (3), (8), etc... : please define beta and gamma

A: We clarified the usage of indices in a footnote in Section 3.3. Now we merge it with another footnote in Section 2.1 where *αβγ* first appears. Please see **Update C**.

R: 3) How do you derive the empirical relation (69)?

A: This relation is not derived but based on plenty of trial simulations, hence the notion “empirical”, as elaborated in both Section 4.2 and 7.4. From the users perspective, this field of expansion orders, nu(s, z), is the only additional input parameter compared with a full 3D SEM such as SPECFEM. It is technically difficult to find the optimal field that fits the wavefield best, so we temporarily use the empirical equation (69), which works well for global tomographic models. One can, of course, always use a constant order across the entire domain, but this will cost more than using our empirical relation. We are in the process of making the method fully self-adaptive, i.e., capable of determining nu(s, z) automatically, as stated in Section 7.4. This will be another issue addressed in our next paper.

R: 4) section 5 validation: Please define precisely your definitions of phase and amplitude misfits. Otherwise it is impossible to see if the misfits observed with respect to specfem are significant or not.

A: The phase and envelope misfits are defined exactly the same as in the original paper of Kristekova et al. 2009, and we use an Obspy routine to compute them.

We discussed with the original authors in a workshop about how to determine specific critical values of TF-misfits when judging the accuracy of synthetic seismograms. Their answer can be summarized as: one cannot have uniform criteria (critical values of EM and PM) that suit all problems, and the values themselves are not important in an absolute sense. The best practice is to determine the critical values by comparing typical waveforms intuitively and then to use these values to quantify the goodness-of-fit of similar comparisons. This is what we did. In fact, we employed very strict accuracy criteria when performing benchmark and performance comparison. As one can see from Fig 11 (the last two traces in each diagram), 12 and 13, the difference is negligible. Please note that the blue trace is almost invisible in Fig 12 and 13 due to its excellent fit. This is how we guarantee the benchmark quality. Based on the critical EM and PM corresponding to such fit, we measure and compare the performance.

R: 5) Figure 1: what is plotted exactly in this figure? scale?

A: As stated in the figure description, we plotted the displacement norm. A color bar is added as suggested.

R: The simplified approach proposed by the authors of this very interesting paper produces accurate results for global seismic wave propagation at significantly reduced computational cost. The paper is very well written and of great interest for the scientific community. The method is well described and has been properly validated on relevant benchmark problems. The computational results are of high quality and the reported speed ups compared to a standard fully 3D SPECFEM simulation are impressive.

A: Thank you, nothing can make us feel more honored than receiving such a review.

**Update Log (from beginning to end):**

1. **New paragraph in Section 1.1**

**The dimension-reduced fast methods can be used in conjunction with full 3-D methods to solve some multiscale problems, such as teleseismic wavefields scattered by strong 3-D local structures (e.g., Masson et al. 2013; Monteiller et al. 2015). Such a methodology proves to be a good alternative to 3-D global simulations for the purpose of shallow structure studies. The method we propose here could seamlessly be used either to provide the ambient teleseismic wavefield in 3-D models, or in a future local-scale version the box containing the local structures.**

1. **Last paragraph of Section 1.1**

… 3-D methods may lavish a large amount of computing power on **~~model~~** **wavefield** oversampling... In this paper, we shall be concerned with bridging the gap between fast methods for spherically symmetric media and slow methods for complex 3-D media by means of developing a method fully adapted to model complexity, **without simplifying the models nor compromising on the accuracy of the solution**. **~~The method is expected to be most efficient for Earth models that are laterally smooth relative to seismic wavelengths, but is to be seen as a fully convergent, general 3-D numerical wave solver.~~**  **This method should be a fully convergent numerical wave solver for general 3-D Earth models, and is expected to be particularly more efficient for state-of-the-art tomographic models, all of which naturally exhibit a structural smoothness relative to seismic wavelengths.**

1. **Two footnotes about notations are merged into one in Section 2.1**

**In this paper, we use *ijkl* as spatial indices and *αβγ* as Fourier series indices. Einstein summation convention only applies to spatial indices. Superscripts acting *directly* on field variables denote Fourier series indices; e.g., *ρ*2 represents the second order Fourier coefficient of *ρ*, whereas the square of *ρ*, if needed, will be written as (*ρ*) 2.**

1. **Add colour bar in Figure 1.**

As well as its unit in text.

1. **Fix errors in Equation 50 and 53**

One term was missing in the dipole term.

1. **Last paragraph of Section 5.2**

Note that though we only compare waveforms on the surface, the good fit of body wave phases, both direct and multiple, should imply a good fit of the interior wavefield**, and, based on the self-adjointness of the elastic wave operator, the capability of our method to produce accurate 3-D sensitivity kernels**.

1. **Fix a few errors in grammar**