CS190C Lec1

Word Embedding & Language Modeling

Overview

- How to embed words
- What is language modeling
- Some naive and early language models
    - N-grams
    - RNN
    - LSTM
- Thinking

PART1：How to embed words?

Problem: Computer cannot understand natural language……

- We should try to convert them, such as words, into digital form.
- How to represent words formally?

A naive idea

- Like a dictionary, each word has its position in it
- Can we represent a word using a certain number?
  - That is: each word has its "one-to-one mapping value"
- Just like this:

Word	**I**	**love**	**Natural**	**Language**	**Processing**	**" "**	**.**
Value	0	1	2	3	4	5	6

`"I love Natural Language Processing."` $\Rightarrow$ `0,5,1,5,2,5,3,5,4,6`

Mapping has its defect

- Can we have a method to infer this word's part of speech or meaning, just based on the mapping value?
- For example: If you only receive a string of numbers: `0,5,1,5,2,5,3,5,4,6`, can you successfully infer the meaning of this sentence?
- Mathematically, one-to-one mapping can be understood as: **The semantics of different words constitute a one-dimensional vector space!**
  - Each word is a 1-dim vector in this 1-dim space
  - Using 1-dim vector space to represent natural language is obviously not enough!

Enlarge the dimension?

- If we use 2-dim space to represent some words?
  - x-aixs represent "area", y-axis represent "population"
  - Can we use 2-dim vectors to approximatly represent: `China` `India` `Canada` `Luxembourg`? (Plot a graph of it)
- Similarly, the higher the dimension, the richer the semantics it can represent.
- For example, GPT-3 uses 2048-dim to represent words.

What is word embedding?

So far we know that:

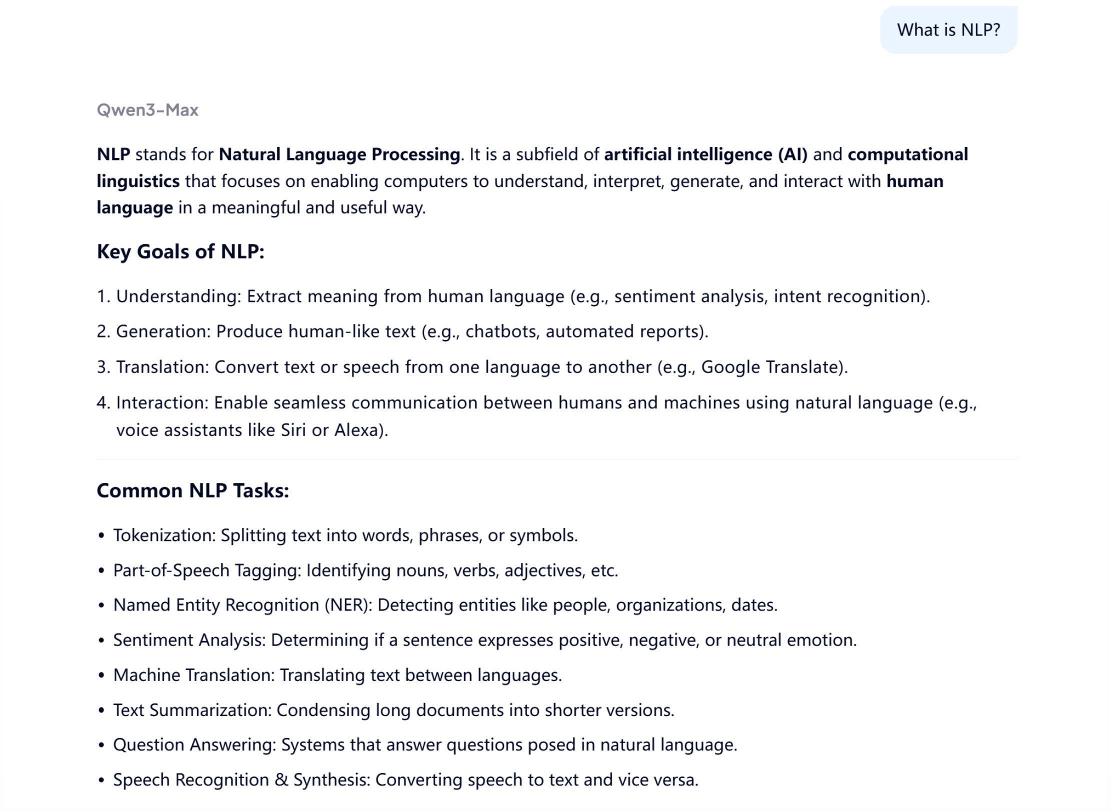
- We can use n-dim vector to represent the meaning of the word formally.
- Each dim can be understand as a kind of semantics, it just like one more dimension of space in linear algebra.

We call vectors of word `Word Embedding`.

- Encoding words with appropriate word embeddings is one of the key points in natural language processing technology
- In later lecs, we will introduce some important encoder models.

PART2: What is language modeling?

Look at a LLM



What is NLP?

Qwen3-Max

NLP stands for **Natural Language Processing**. It is a subfield of **artificial intelligence (AI)** and **computational linguistics** that focuses on enabling computers to understand, interpret, generate, and interact with **human language** in a meaningful and useful way.

Key Goals of NLP:

1. Understanding: Extract meaning from human language (e.g., sentiment analysis, intent recognition).
2. Generation: Produce human-like text (e.g., chatbots, automated reports).
3. Translation: Convert text or speech from one language to another (e.g., Google Translate).
4. Interaction: Enable seamless communication between humans and machines using natural language (e.g., voice assistants like Siri or Alexa).

Common NLP Tasks:

- Tokenization: Splitting text into words, phrases, or symbols.
- Part-of-Speech Tagging: Identifying nouns, verbs, adjectives, etc.
- Named Entity Recognition (NER): Detecting entities like people, organizations, dates.
- Sentiment Analysis: Determining if a sentence expresses positive, negative, or neutral emotion.
- Machine Translation: Translating text between languages.
- Text Summarization: Condensing long documents into shorter versions.
- Question Answering: Systems that answer questions posed in natural language.
- Speech Recognition & Synthesis: Converting speech to text and vice versa.

- Input a prompt (a string of words)
- Give an answer based on the prompt

How does it generate words, finally forms an answer?

- Like normal speaking of human, each word should generate after formal words, and based on it logically.
- That is: based on old words, and generate new words.
- This called Language Modeling!

Tips: Difference between `Language Model` and `Language Modeling`

- Language Model: Is a tool to generate certain answer based on prompts.
- Language Modeling: Is the methods to generate "new words" based on "old words".

So, what's the "methods"?

General methods

There are maybe several words suitable to be the generated new word……

- But different word may have different levels of suitability.
- We can try to model "levels of suitability" into probabilistic distribution.
- That is, find a way to calculate the probability of generating a certain word as the "new word", based on "old words".

Language Modeling Methods……

These are different models, using different certain methods to calculate the probabilistic distribution.

- N-grams
- RNN, LSTM (An optimized architecture based on RNN)
- Transformer
- GPT, BERT .etc (Based on Transformer)

PART3： N-grams

Another naive idea

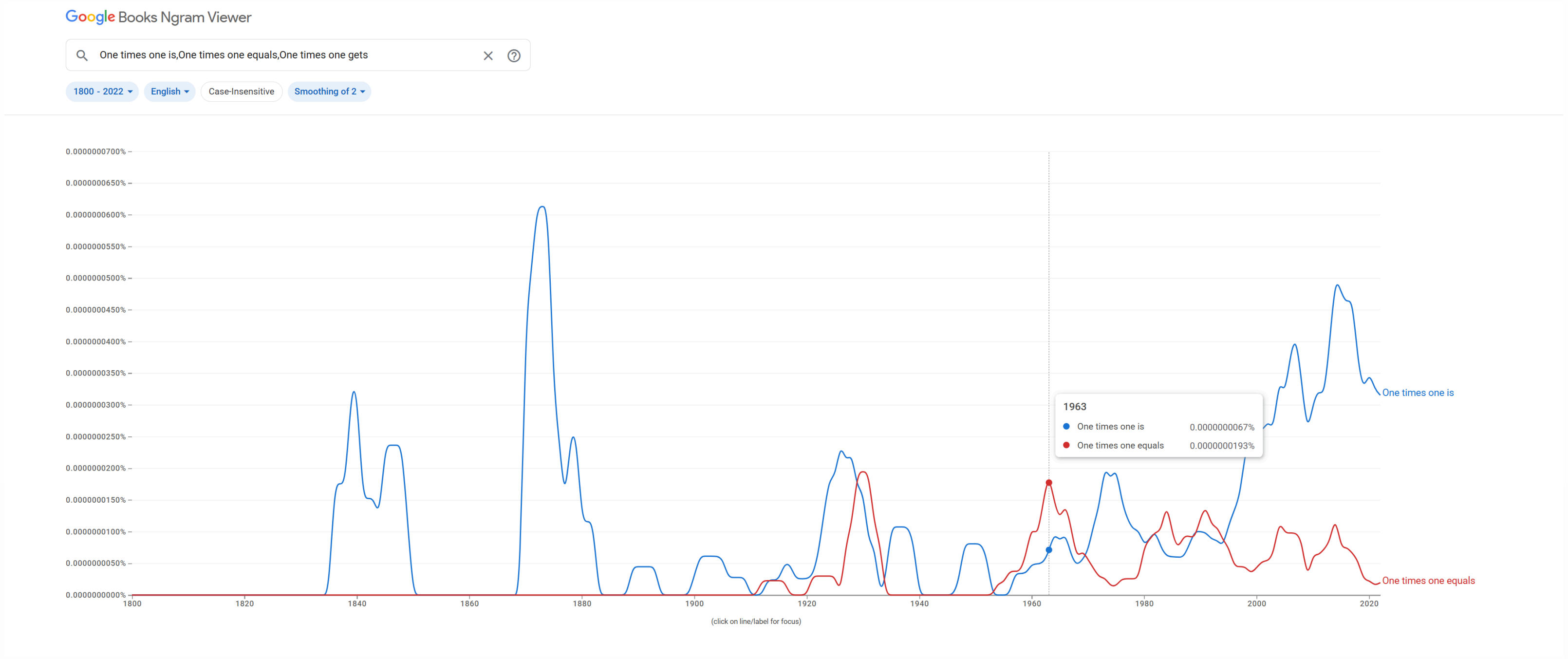
Simply consider: If we just focus on a fix window of old words to generate a new word, it can still work at most of time.

For example:

- We only focus on latest 3 old words to generate a new word
- We call it `4-grams`, which means a fixed window contain old words + a new word has 4 words.
- Prompt: "Let's calculate simple multiplication! One times one......"

A naive 4-grams

- old words focused: `One times one`
- Generate new word directly use statistical laws!
- We can use https://books.google.com/ngrams/



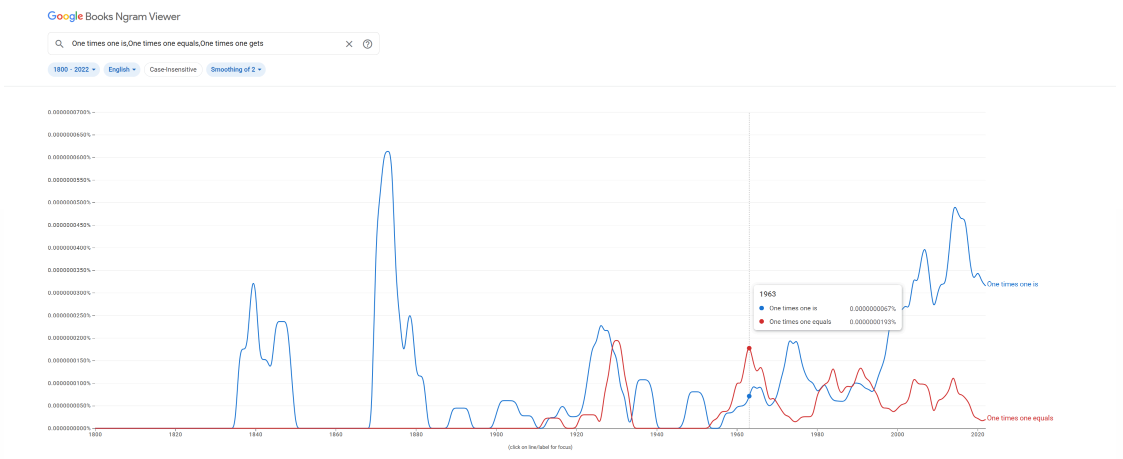
Google Books Ngram Viewer

One times one is,One times one equals,One times one gets

1800 - 2022 ▾ English ▾ Case-Insensitive Smoothing of 2 ▾

1963
● One times one is 0.0000000067%
● One times one equals 0.0000000193%

A naive 4-grams



One times one is,One times one equals,One times one gets

1800 - 2022 · English · Case-Insensitive · Smoothing of 2

1963
- ● One times one is 0.0000000067%
- ● One times one equals 0.0000000193%

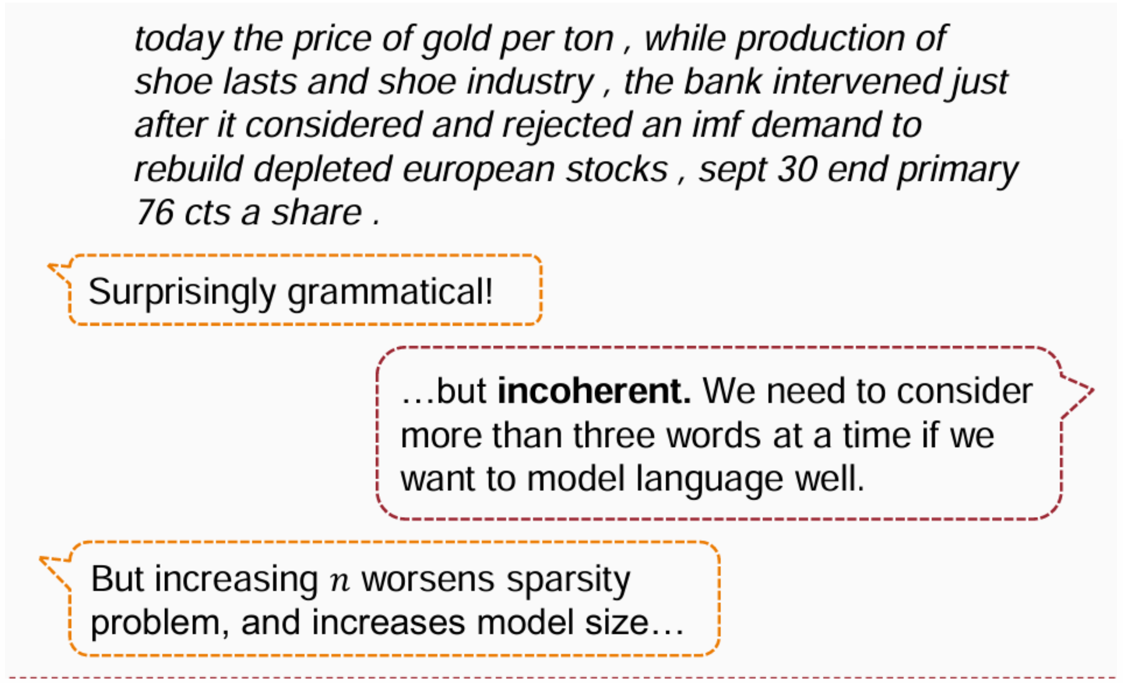
- We can try all words in vocabulary
- We can also get $P(new, old)$ (marginal probability of 4-grams)
- What we want to model is $P(new|old) \propto P(new, old)$

- Suppose we've modeled $P(w_i|old)$
- We choose $\mathrm{argmax}_{w_i} P(w_i|old)$ or sample words according to the distribution
- Suppose we decide `is` to be the new word
- Sentence now: `One times one is`
- Next turn: use `times one is` to generate a new word, and so on.

Pros and cons?

- Actually it is quite simple, and do not need much calculation (especially model training)
- If you really use N-grams to generate text, what will happen?

Generate text?



today the price of gold per ton , while production of shoe lasts and shoe industry , the bank intervened just after it considered and rejected an imf demand to rebuild depleted european stocks , sept 30 end primary 76 cts a share .

Surprisingly grammatical!

…but **incoherent.** We need to consider more than three words at a time if we want to model language well.

But increasing n worsens sparsity problem, and increases model size…

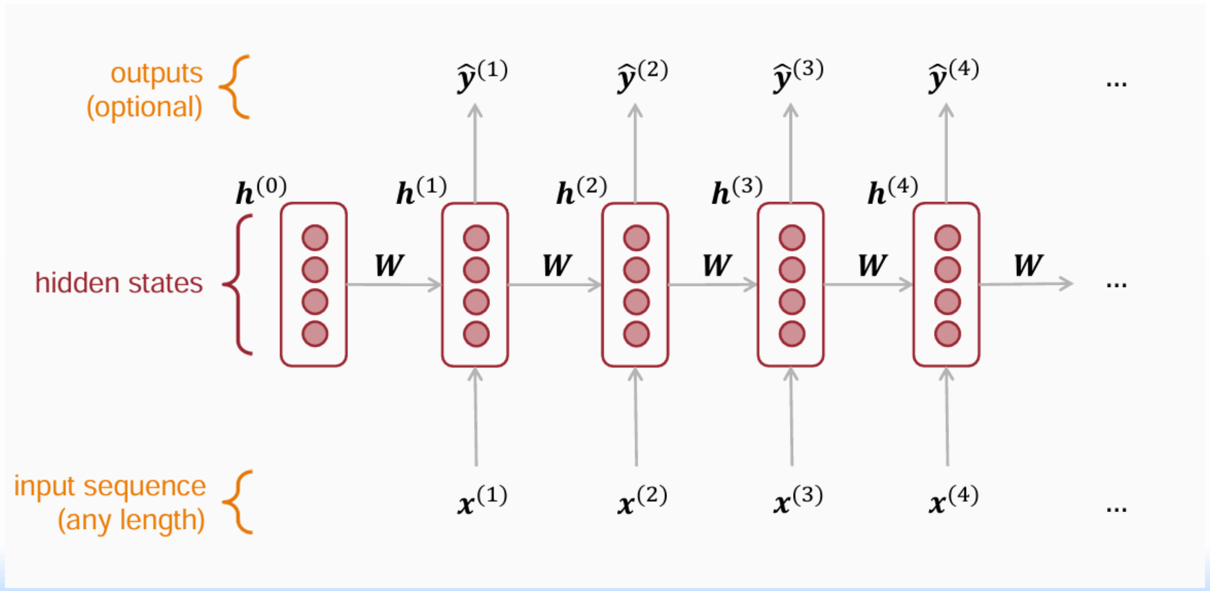
- For grammar……
  - Subject-predicate/verb-object…… part-of-speech collocation follows statistical rules
- But for semantics……
  - Missing of global context
  - One mistake will cause accumulation of subsequent errors

PART4: Recurrent Nerual Network (RNN)

How to make use of global context?

Encodding……

- We encode words as vectors……
- Can we encode text as vectors too?
- If we get the encoded vector of text, we can generate new words with "understanding" of text!



How to encode words and text?

- For words……
  - At first, each word still encoded with single number.
  - We hope to train a matrix $W_e \in R^{d*|V|}$, each column is the embedding of i-th word.
  - So for i-th word, we can get its embedding through $W_e e$, e is the one-hot vector of this word (only get value 1 in i-th element) $\Rightarrow W_e e$ is the i-th column of W_e

How to encode words and text?

- For text……
    - Like reading word by word, with each word read, the understanding of the text will be more substantial.
    - When a new word read: the "understanding" of the text will mixed with: Former understanding of the text, and the information of the new word.
    - That is: the embedding of the text in this time step should mix former embedding of text and embedding of the new word together.

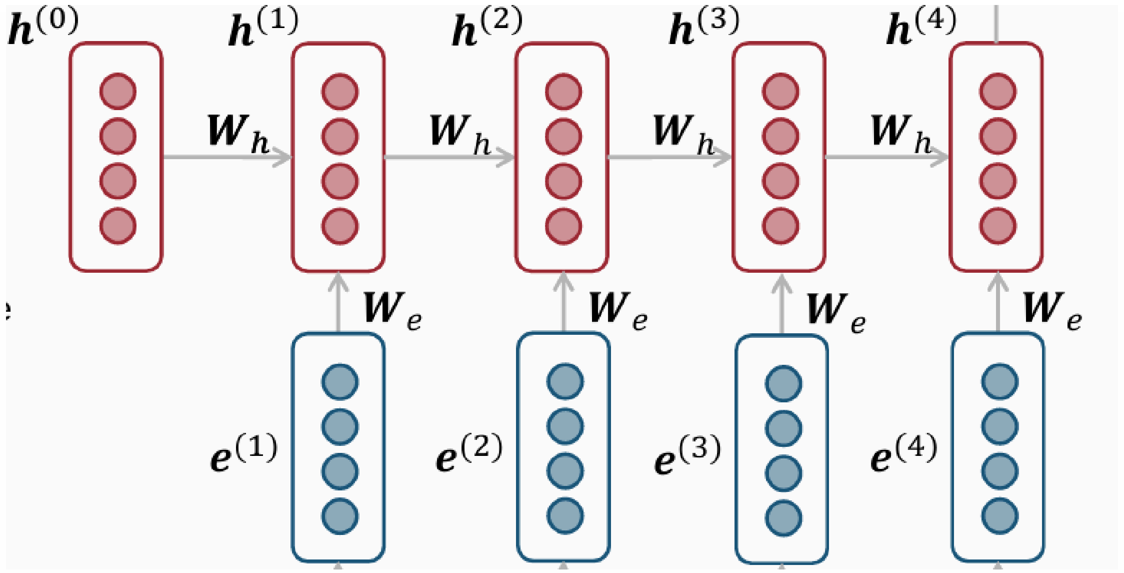
How to encode words and text?

Let embedding of text in step-t be h^t

- Inherit former embedding: $W_h h^{t-1}$. We hope to train W_h, which stands for how to proper inherit former information
- New word's information: $W_e e^t$
- Combine:$W_h h^{t-1} + W_e e^t + b_1$. ($b_1$ is the optional bias term)
- Add a nonlinear activation (usually use sigmoid)

How to encode words and text?

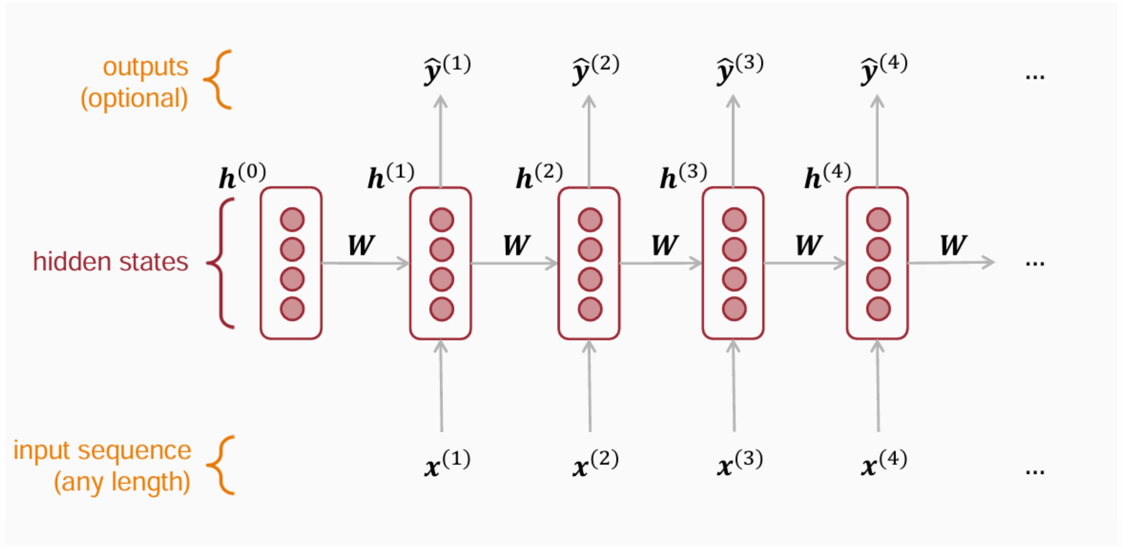
- $h^t = \sigma(W_h h^{t-1} + W_e e^t + b_1)$
- Parameters to train:
  - W_e
  - W_h
  - b_1



How to language modeling?

Just use a linear activation to h^t, generate the probability distribution of the new words!

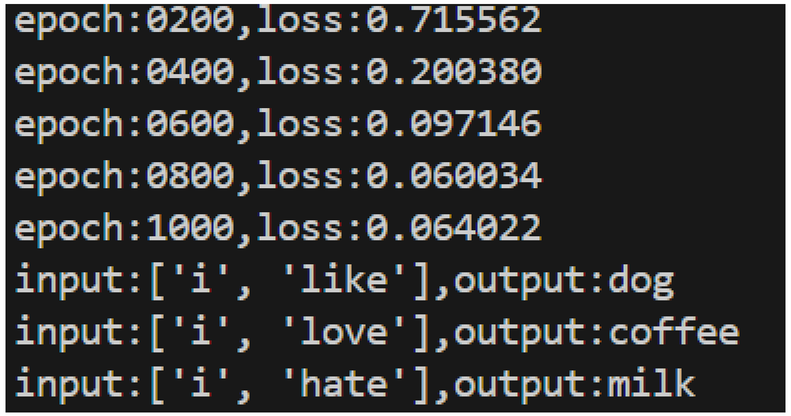
- $\hat{y}^t = \mathrm{Softmax}(Uh^t + b_2)$
- Parameters to train:
    - U: the linear activation matrix
    - b_2: the optional bias term



Implement a simple RNN

https://github.com/kuangpenghao/NLP_models_by_hand/blob/main/toy_RNN.py

- 3 training sentences
- Train RNN
- Input sentence except the last word
- Hope to output the correct word



```
epoch:0200,loss:0.715562
epoch:0400,loss:0.200380
epoch:0600,loss:0.097146
epoch:0800,loss:0.060034
epoch:1000,loss:0.064022
input:['i', 'like'],output:dog
input:['i', 'love'],output:coffee
input:['i', 'hate'],output:milk
```

Implement a simple RNN

- main function

```python
if __name__=="__main__":

    config=TextRNNConfig()
    model=TextRNN(config)

    trainer=TextRNNTrainer(config,model)
    predictor=TextRNNPredictor(config,model)

    trainer.train()

    test_sentences=["i like dog", "i love coffee", "i hate milk"]
    for sentence in test_sentences:
        sentence=sentence.split()
        input_sentence=sentence[:-1]
        predicted_word=predictor.predict(input_sentence)
        print(f"input:{input_sentence},output:{predicted_word}")
```

Implement a simple RNN

```
class TextRNNConfig:
    def __init__(self):
        self.n_hidden=5
        self.sentences=["i like dog", "i love coffee", "i hate milk"]

        word_list=' '.join(self.sentences).split()
        word_list=list(set(word_list))
        self.word_dict={w:i for i,w in enumerate(word_list)}
        self.number_dict={i:w for i,w in enumerate(word_list)}

        self.n_class=len(word_list)

        self.batch_size=2
        self.learning_rate=0.001
        self.epochs=1000
        self.interval=200
```

Implement a simple RNN

```
class TextRNN(nn.Module):
    def __init__(self,config):
        super(TextRNN,self).__init__()
        self.config=config
        self.rnn=nn.RNN(self.config.n_class,self.config.n_hidden)
        self.W=nn.Linear(self.config.n_hidden,self.config.n_class,bias=True)

    def forward(self,X):
        X=X.transpose(0,1)
        ori_hidden=torch.zeros(1,X.shape[1],self.config.n_hidden)
        outputs,last_hidden=self.rnn(X,ori_hidden)
        output=outputs[-1]
        output=self.W(output)
        return output
```

Implement a simple RNN

```python
class TextRNNDataset(Dataset):
    def __init__(self,config):
        super(TextRNNDataset,self).__init__()
        self.config=config

    def __len__(self):
        return len(self.config.sentences)

    def __getitem__(self,idx):
        sentence=self.config.sentences[idx]
        words=sentence.split()
        words=[self.config.word_dict[i] for i in words]

        input_idx=words[:-1]
        one_hot=np.eye(self.config.n_class)[input_idx]
        input_one_hot=torch.tensor(one_hot,dtype=torch.float32)

        target_idx=words[-1]
        target_idx=torch.tensor(target_idx,dtype=torch.int64)

        return input_one_hot,target_idx
```

Implement a simple RNN

```python
class TextRNNTrainer:
    def __init__(self,config,model):
        self.config=config
        self.model=model
        self.loss_function=nn.CrossEntropyLoss()
        self.optimizer=torch.optim.SGD(model.parameters(),lr=self.config.learning_rate,momentum=0.9)

        self.datagetter=TextRNNDataset(config)
        self.dataloadder=DataLoader(self.datagetter,batch_size=self.config.batch_size,shuffle=True)

    def train(self):
        for epoch in range(self.config.epochs):
            for input_batch,target_batch in self.dataloadder:
                self.optimizer.zero_grad()
                output=self.model(input_batch)
                loss=self.loss_function(output,target_batch)
                loss.backward()
                self.optimizer.step()
            if (epoch+1)%self.config.interval==0:
                print(f"epoch:{epoch+1:04d},loss:{loss.item():.6f}")
```

Implement a simple RNN

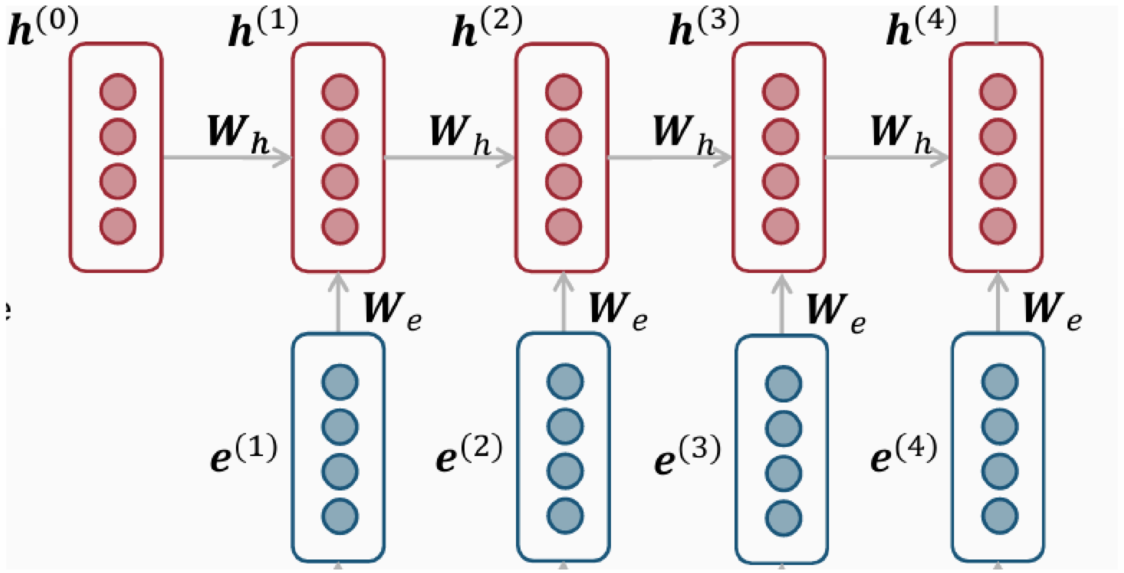
```
class TextRNNPredictor:
    def __init__(self,config,model):
        self.config=config
        self.model=model

    def predict(self,input_sentence):
        with torch.no_grad():
            word=[self.config.word_dict[i] for i in input_sentence]
            word=np.eye(self.config.n_class)[word]
            word=torch.tensor(word,dtype=torch.float32).unsqueeze(0)

            output=self.model(word)
            outcome=output.max(1,keepdim=False)[1]
            predicted_word=self.config.number_dict[outcome.item()]

            return predicted_word
```

Pros and cons?

- Actually, it can make use of the information of the global text……
- When the text is extremely long, step t is very large?

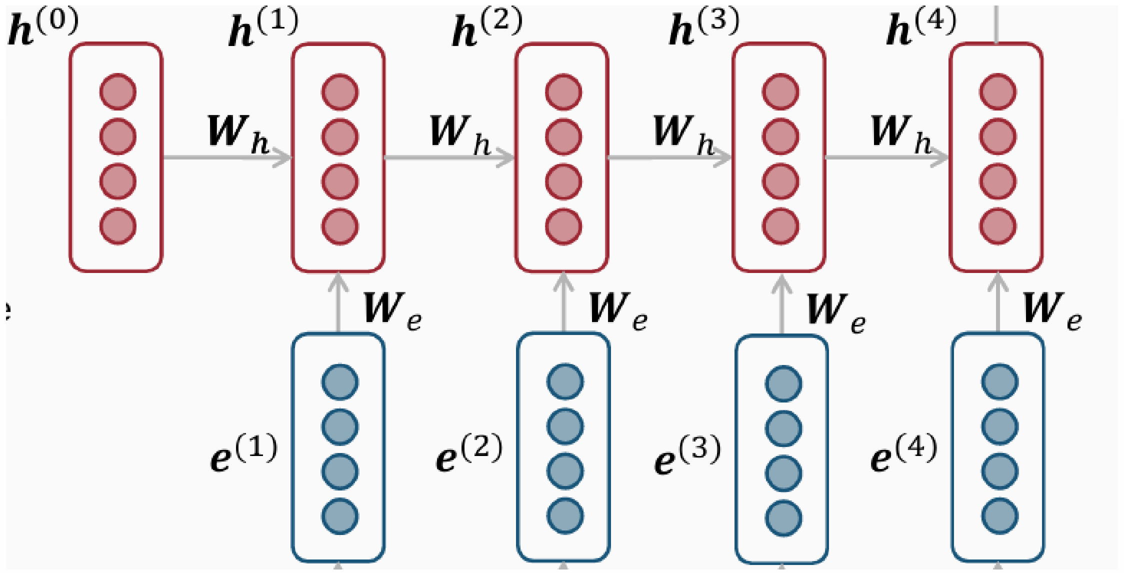
Update parameters (W_h)

- $\frac{\partial L_T}{\partial W_h} = \sum_{t=1}^{T} \frac{\partial L_T}{\partial h^t} \frac{\partial h^t}{\partial W_h}$
- $\frac{\partial L_T}{\partial h^t} = \frac{\partial L_T}{\partial h^T} \frac{\partial h^T}{\partial h^{T-1}} \cdots\cdots \frac{\partial h^{t+1}}{\partial h^t}$
- $\frac{\partial h^{t+1}}{\partial h^t} = \frac{\partial \sigma(W_h h^t + W_e e^{t+1} + b_1)}{\partial h^t}$
  $= \sigma'(W_h h^t + W_e e^{t+1} + b_1) W_h$
- We usually use sigmoid function as σ.
  Then $\sigma'(z) = \sigma(z)(1 - \sigma(z))$
  $\in (0, 0.25]$
- $\frac{\partial L_T}{\partial h^t} = \frac{\partial L_T}{\partial h^T} \Pi_{k=t+1}^{T} [\sigma'(z_k) W_h]$



Update parameters (W_h)

- $\frac{\partial L_T}{\partial h^t} = \frac{\partial L_T}{\partial h^T} \Pi_{k=t+1}^{T} [\sigma'(z_k) W_h]$
- If $||W_h|| < 1$, each term must < 1 $\Rightarrow \frac{\partial L_T}{\partial h^t} \to 0$, which may leads to $\frac{\partial L_T}{\partial W_h} \to 0$
- Likewise, if $||W_h||$ is large enough, $\frac{\partial L_T}{\partial W_h} \to \infty$
- Called vanishing gradient & exploding gradient



PART5: Long Short Term Memory (LSTM)

- We have a sequence of inputs $\boldsymbol{x}^{(t)}$, and we will compute a sequence of hidden states $\boldsymbol{h}^{(t)}$ and cell states $\boldsymbol{c}^{(t)}$.
- On timestep t:

Sigmoid function: all gate values are between 0 and 1

Forget gate: controls what is kept vs forgotten, from previous cell state

$$\boldsymbol{f}^{(t)} = \sigma\left(\boldsymbol{W}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_f \boldsymbol{x}^{(t)} + \boldsymbol{b}_f\right)$$

Input gate: controls what parts of the new cell content are written to cell

$$\boldsymbol{i}^{(t)} = \sigma\left(\boldsymbol{W}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_i \boldsymbol{x}^{(t)} + \boldsymbol{b}_i\right)$$

Output gate: controls what parts of cell are output to hidden state

$$\boldsymbol{o}^{(t)} = \sigma\left(\boldsymbol{W}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_o \boldsymbol{x}^{(t)} + \boldsymbol{b}_o\right)$$

New cell content: this is the new content to be written to the cell

$$\tilde{\boldsymbol{c}}^{(t)} = \tanh\left(\boldsymbol{W}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_c \boldsymbol{x}^{(t)} + \boldsymbol{b}_c\right)$$

Cell state: erase ("forget") some content from last cell state, and write ("input") some new cell content

$$\boldsymbol{c}^{(t)} = \boldsymbol{f}^{(t)} \circ \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \circ \tilde{\boldsymbol{c}}^{(t)}$$

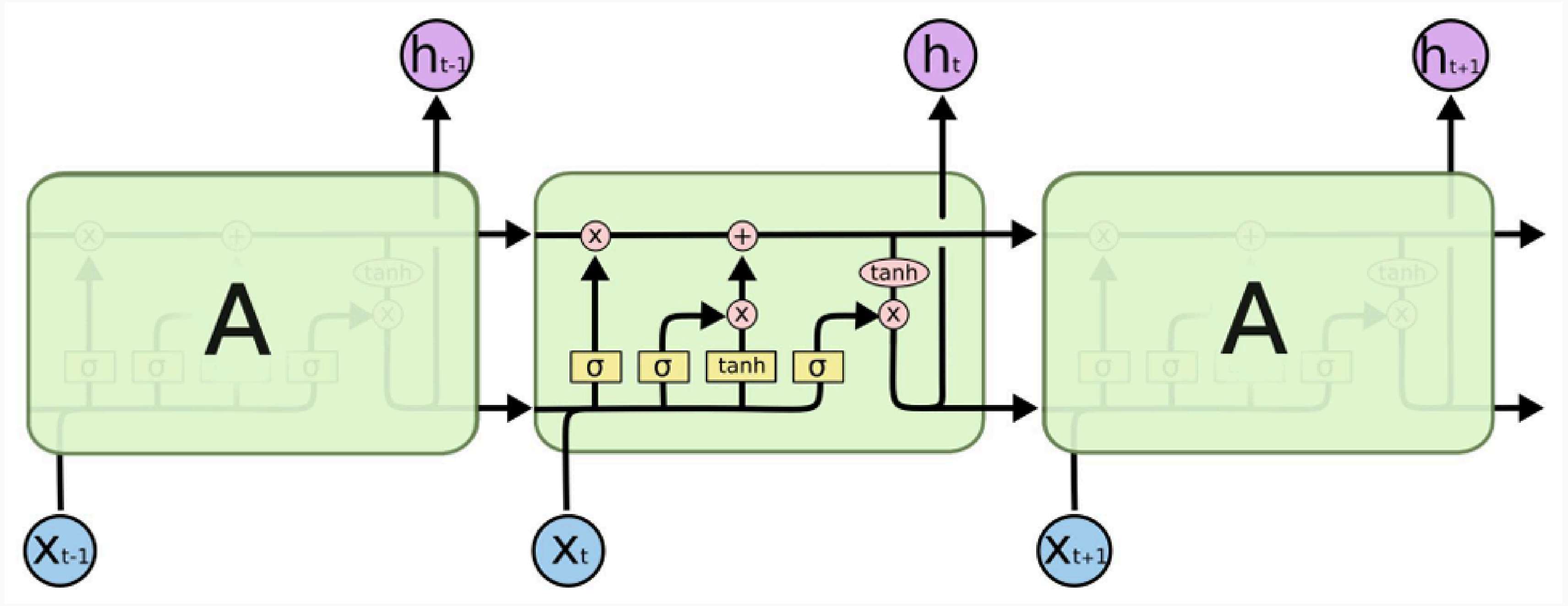
Hidden state: read ("output") some content from the cell

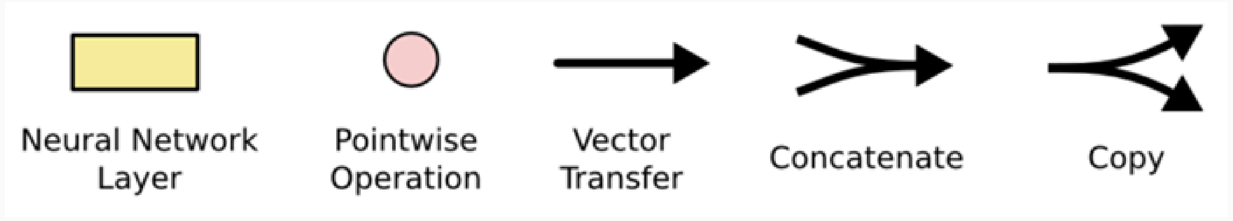
$$\boldsymbol{h}^{(t)} = \boldsymbol{o}^{(t)} \circ \tanh \boldsymbol{c}^{(t)}$$

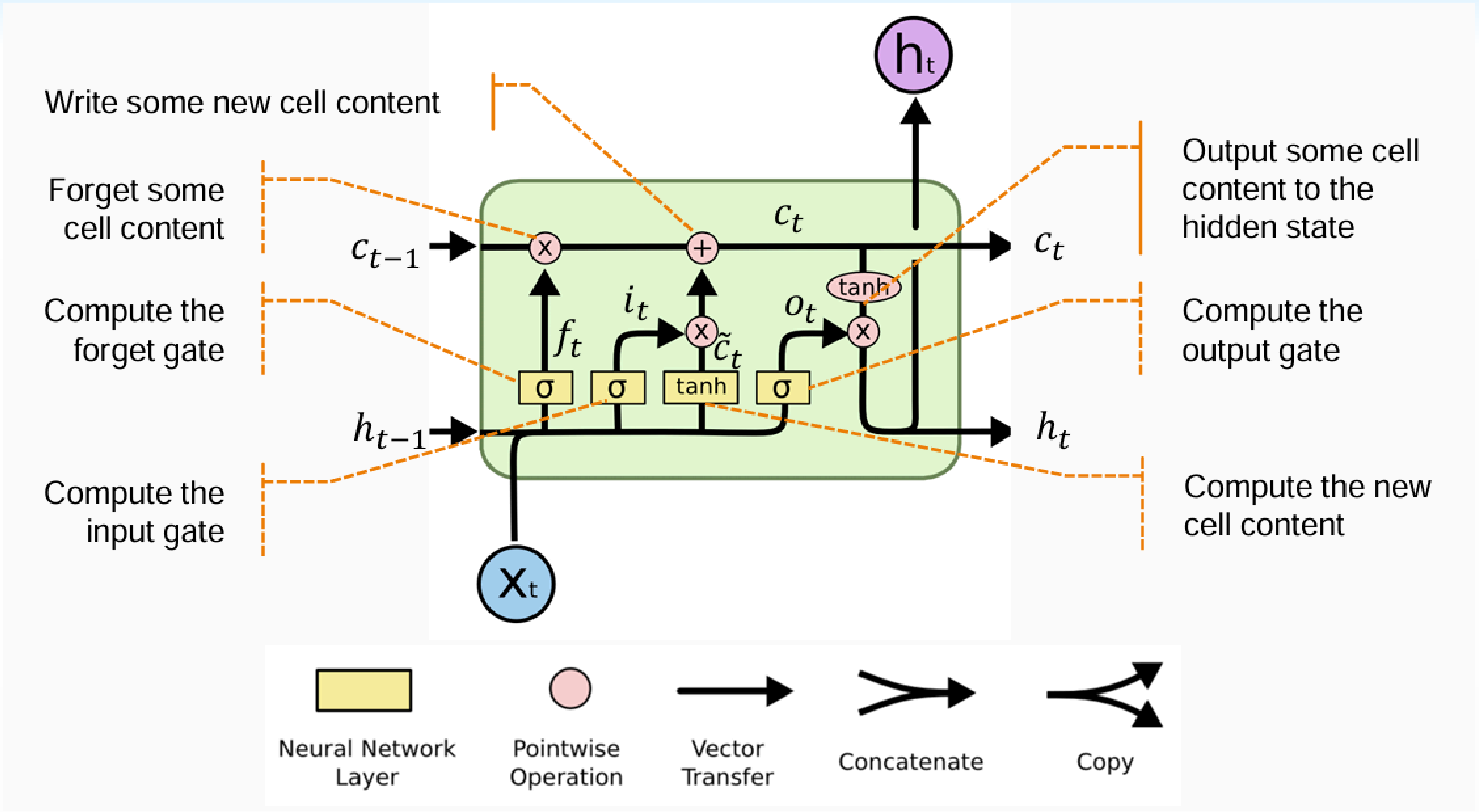
All these are vectors of same length n

Gates are applied using element-wise product

- You can think of the LSTM equations visually like this:






Write some new cell content
Forget some cell content
Compute the forget gate
Compute the input gate
Output some cell content to the hidden state
Compute the output gate
Compute the new cell content
h_t
c_{t-1}
c_t
c_t
f_t
i_t
$\tilde{c}_t$
o_t
tanh
σ
σ
tanh
σ
h_{t-1}
h_t
x_t
Neural Network Layer
Pointwise Operation
Vector Transfer
Concatenate
Copy

Thinkings

- *In N-grams, the model will crashed because of missing information of global context. Can we make N very large to solve this problem?*