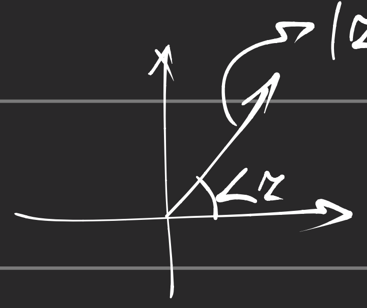
Chapter 6 时域频域特性

$X(j\omega)$ $X(e^{j\omega})$ → (抽象概念)

复数 → 极坐标表示：

$$Z = |Z| e^{j\angle Z}$$



$$X(j\omega) = |X(j\omega)| e^{j\angle X(j\omega)}$$

$$X(e^{j\omega}) = |X(e^{j\omega})| e^{j\angle X(e^{j\omega})}$$

$|X(e^{j\omega})|$ → 幅度谱　　$\angle X(e^{j\omega})$ → 相位谱/角

△ LTI 频率响应分析

$$y(t) = h(t) * x(t)$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega)$$

$$|Y(j\omega)| e^{j\angle Y(j\omega)} = |H(j\omega)| e^{j\angle H(j\omega)} \, |X(j\omega)| e^{j\angle X(j\omega)}$$

$$= |H(j\omega) X(j\omega)| \, e^{j(\angle H(j\omega) + \angle X(j\omega))}$$

群延迟　$\tau(\omega) = -\frac{d}{d\omega}\{\angle H(j\omega)\}$

Ex: $H(j\omega) = e^{-j\omega t_0}$

$|H(j\omega)| = 1$　　$\angle H(j\omega) = -\omega t_0$

$Y(j\omega)$:　　$\omega \to -\omega t_0 + \angle X(j\omega)$

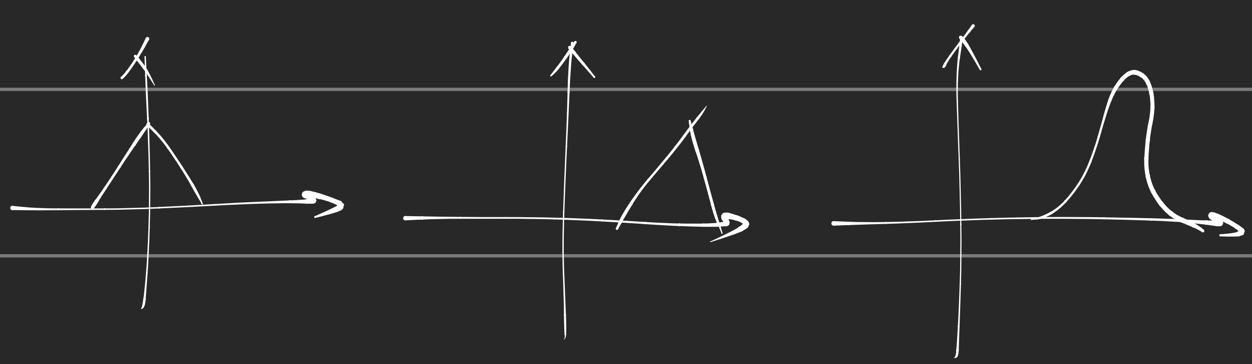
$$Y(j\omega) = e^{-j\omega t_0} X(j\omega)$$

$\Rightarrow y(t) = x(t - t_0)$ → 延迟 t_0

$x(t)$ → 拆成若干 ω 之分量叠加

每个分量（以 ω 区分）→ 延迟多少时间.

$\angle H(j\omega) = a + b\omega \Rightarrow \tau(\omega)$ 处处相等.



图示分析.

$$Y(j\omega) = X_1(j\omega) X_2(j\omega)$$

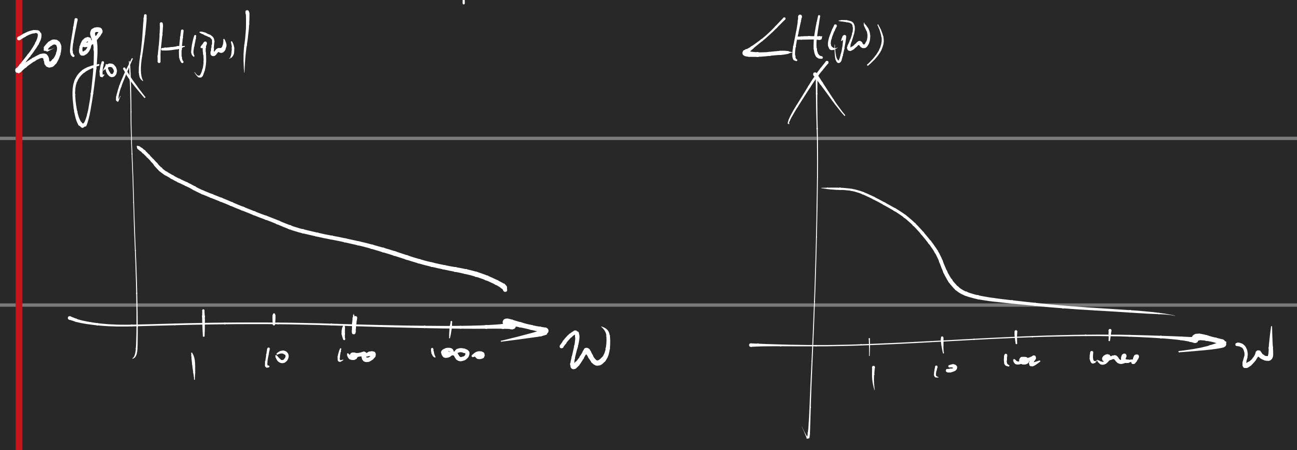
$$\lg|Y(j\omega)| = \lg|X_1(j\omega)| + \lg|X_2(j\omega)|$$

→ 相加运算比相乘更直观

Bode 图：

幅度谱：$\omega - 20\log_{10}|H(j\omega)|$　　相位谱：$\omega - \angle H(j\omega)$

横轴 ω 对数变化



一阶连续系统.

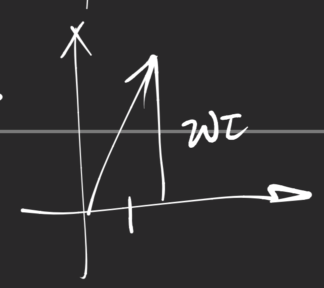
$$\tau \frac{dy(t)}{dt} + y(t) = x(t)$$

$\tau j\omega Y(j\omega) + Y(j\omega) = X(j\omega)$　　$Y(j\omega) = H(j\omega) X(j\omega)$

$\Rightarrow H(j\omega) = \frac{1}{1 + j\omega\tau}$　（频率响应）

$20\log_{10}|H(j\omega)|$　　$\angle H(j\omega)$

$1 + j\omega\tau$ →

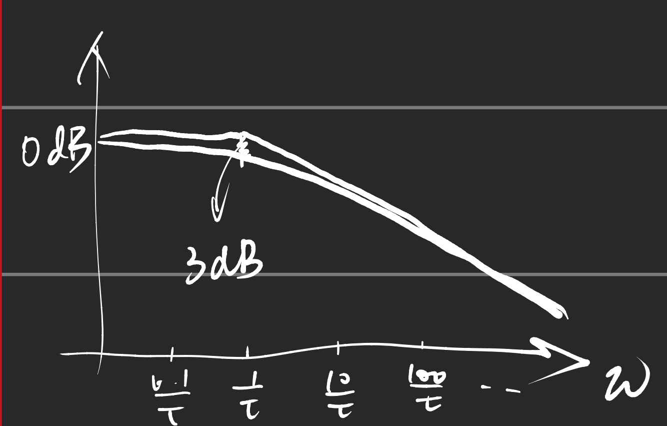


→ $\sqrt{1 + (\omega\tau)^2}$

$\arctan(\omega\tau)$

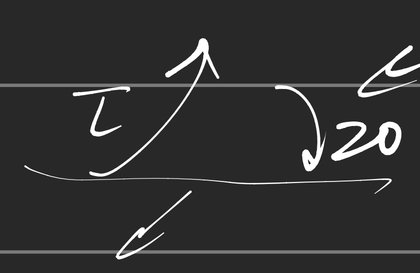
$$20\log_{10}|H(j\omega)| = -10\log_{10}((\omega\tau)^2 + 1)$$

$$\angle H(j\omega) = -\arctan(\omega\tau)$$



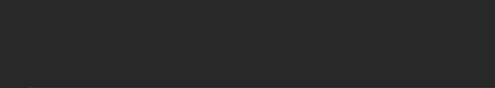
$\omega \ll \frac{1}{\tau}$　0

$\omega \gg \frac{1}{\tau}$　$-20\log_{10}(\omega) - 20\log_{10}(\tau)$ ↓ 常量



$$\angle H(j\omega) = -\arctan(\omega T)$$

$$\simeq \begin{cases} 0 & 0.1/T \\ -\frac{\pi}{4}\left(\lg_{10}(\omega T)+1\right) & \\ -\frac{\pi}{2} & 10/T \end{cases}$$



10倍 → 相位下降 $\frac{\pi}{4}$

取等点：$\frac{1}{T} \rightarrow -\frac{\pi}{4}$