CS182 Introduction to Machine Learning

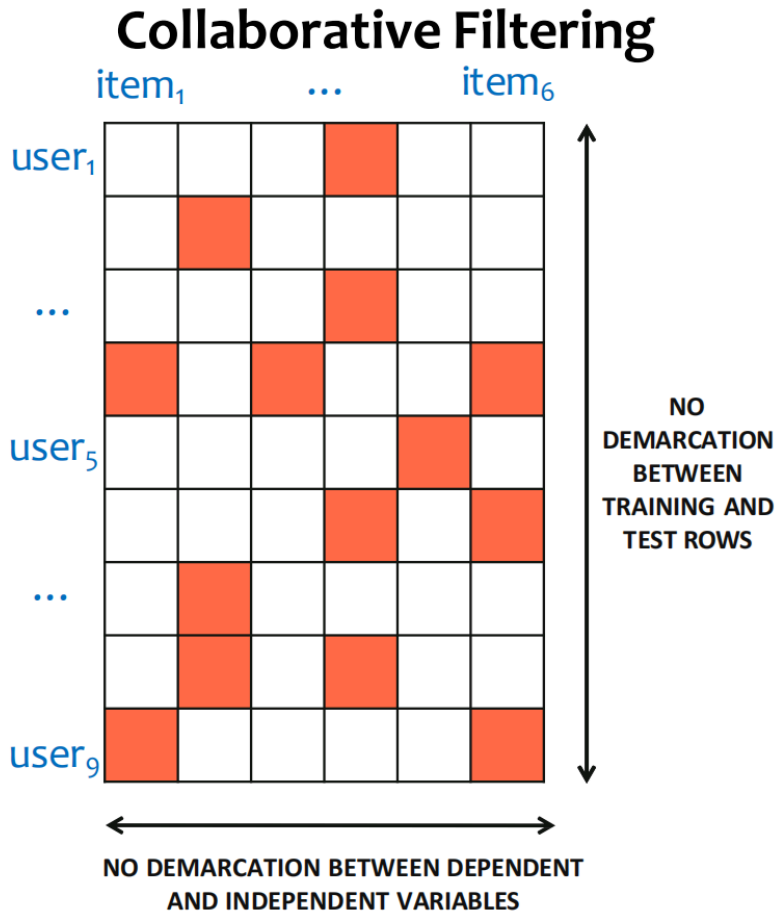
Recitation 10

2025.5.21

Outline

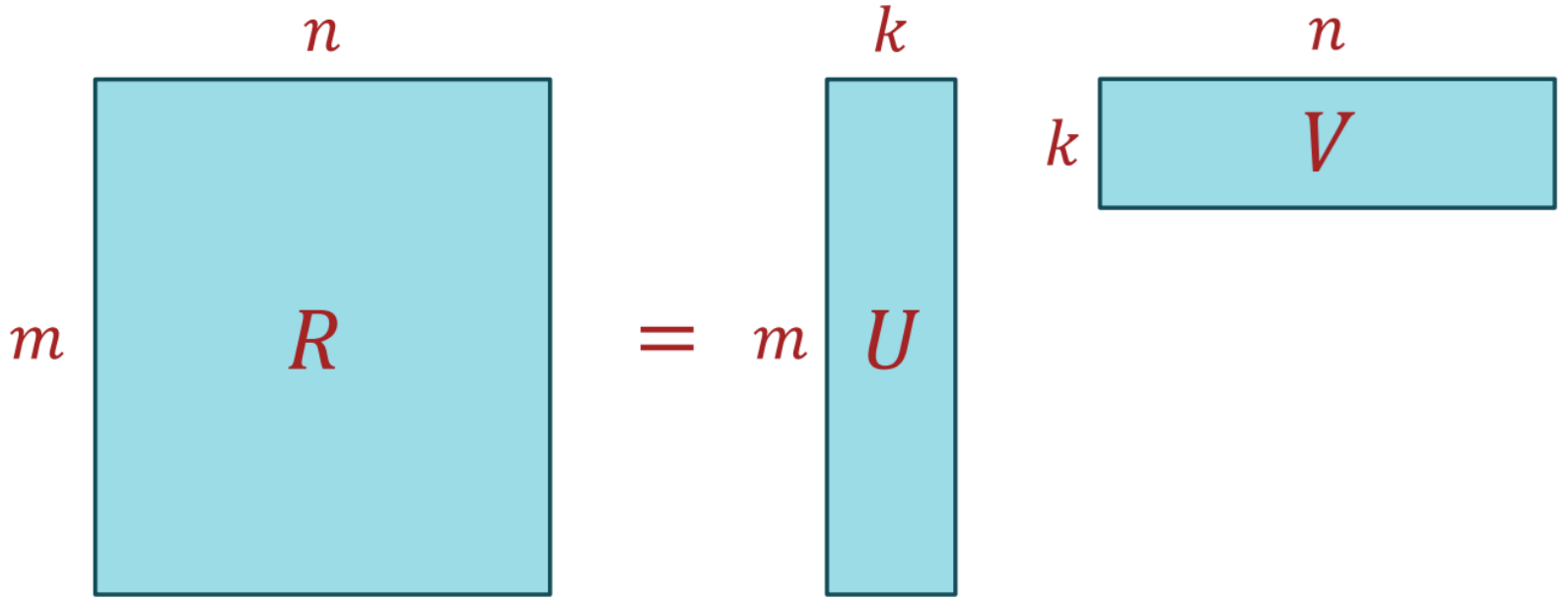
- Recommendation system
- PCA
- Kmeans

Recommendation system



Sparse -> Low rank

Matrix Factorization



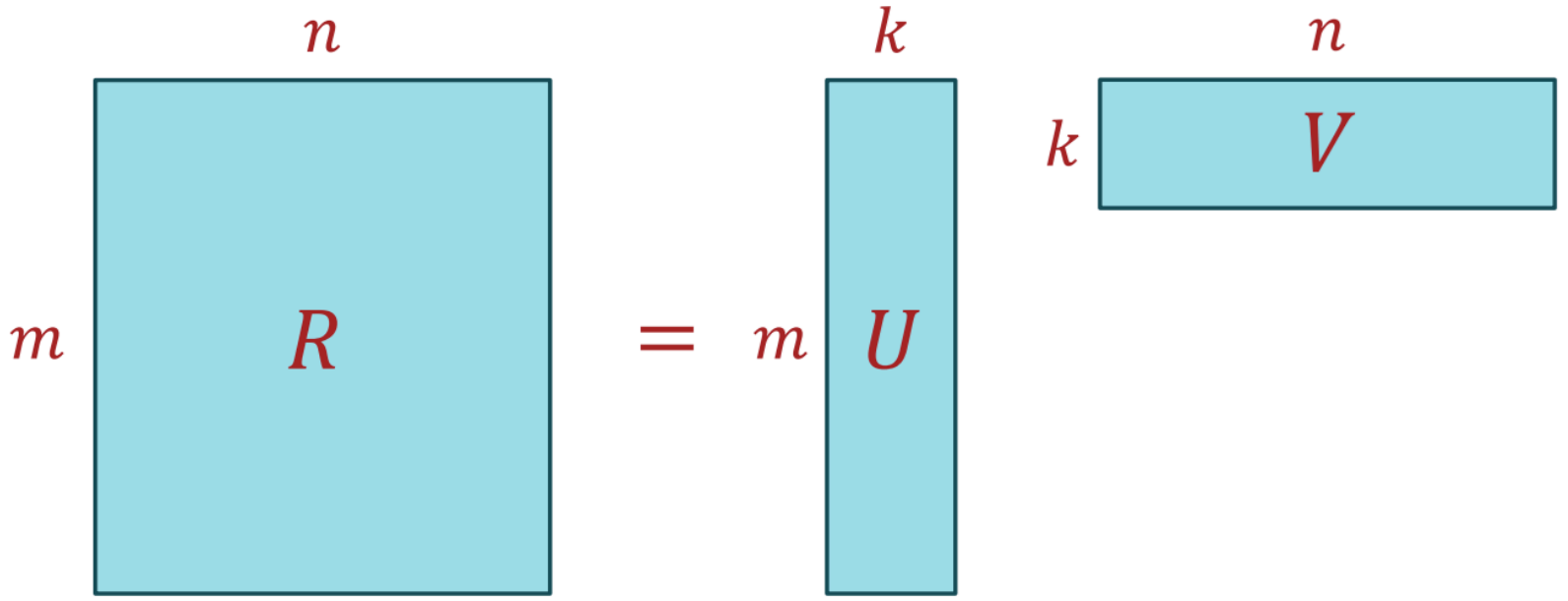
$r_{i,j}$是用户对物品的评分, 打分较少, 所以R是稀疏低秩矩阵

$Rank(R) = k \ll \min(m, n)$, k是超参数

$R = UV^\top$

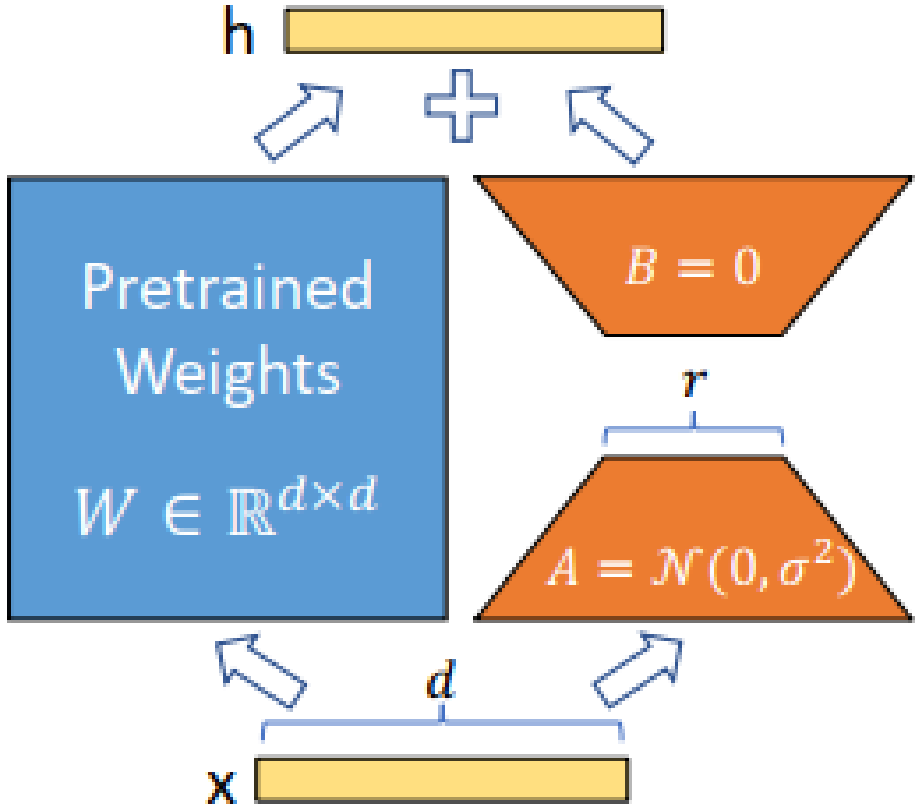
预测用户i对物品j的评分: $\hat{r}_{ij} = U_i V_j^\top$

Matrix Factorization



$$J(U,V) = \left\| R - UV^\top \right\|_F^2 + \lambda(\|U\|_F^2 + \|V\|_F^2)$$

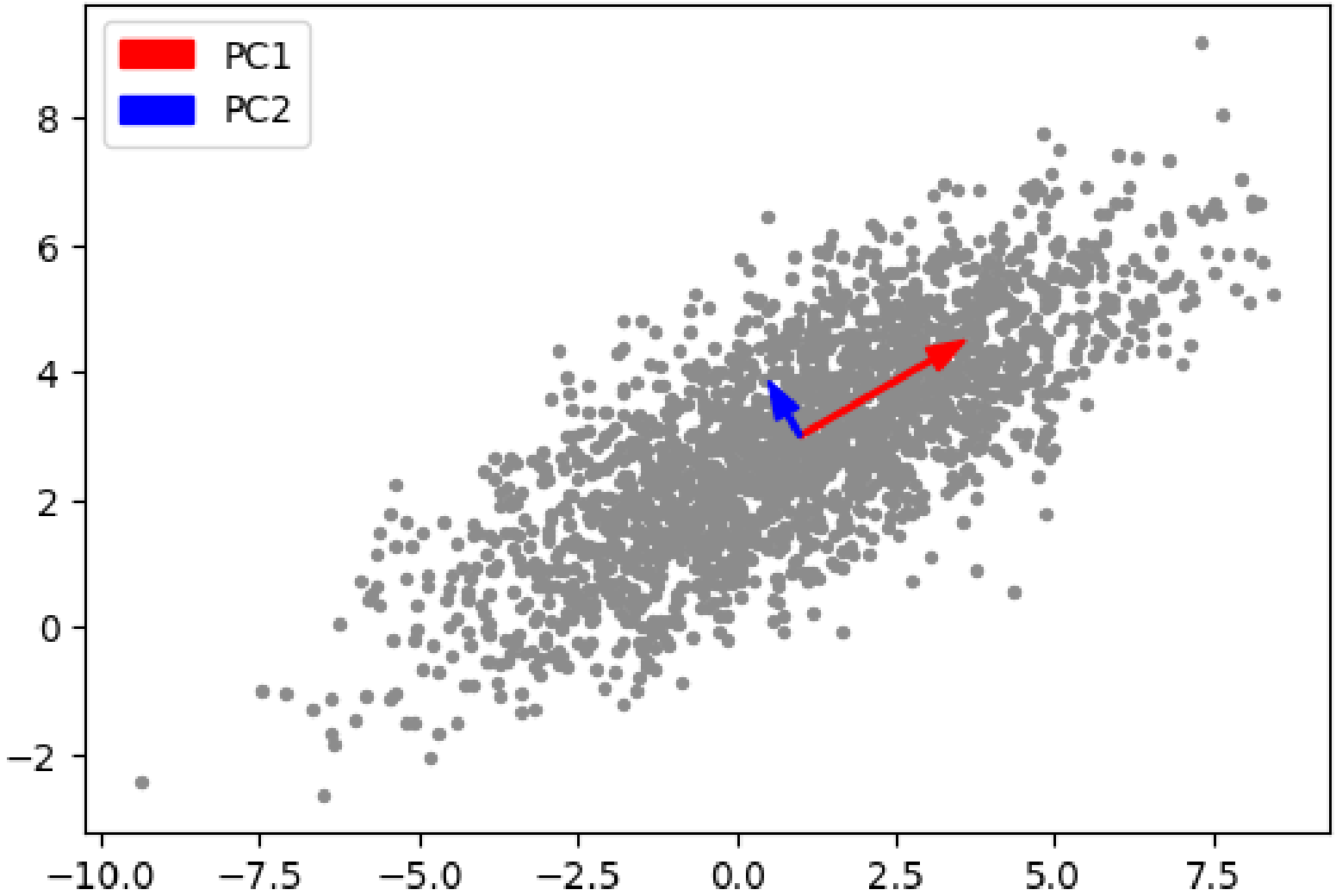
LoRA (Low Rank Adaptation)



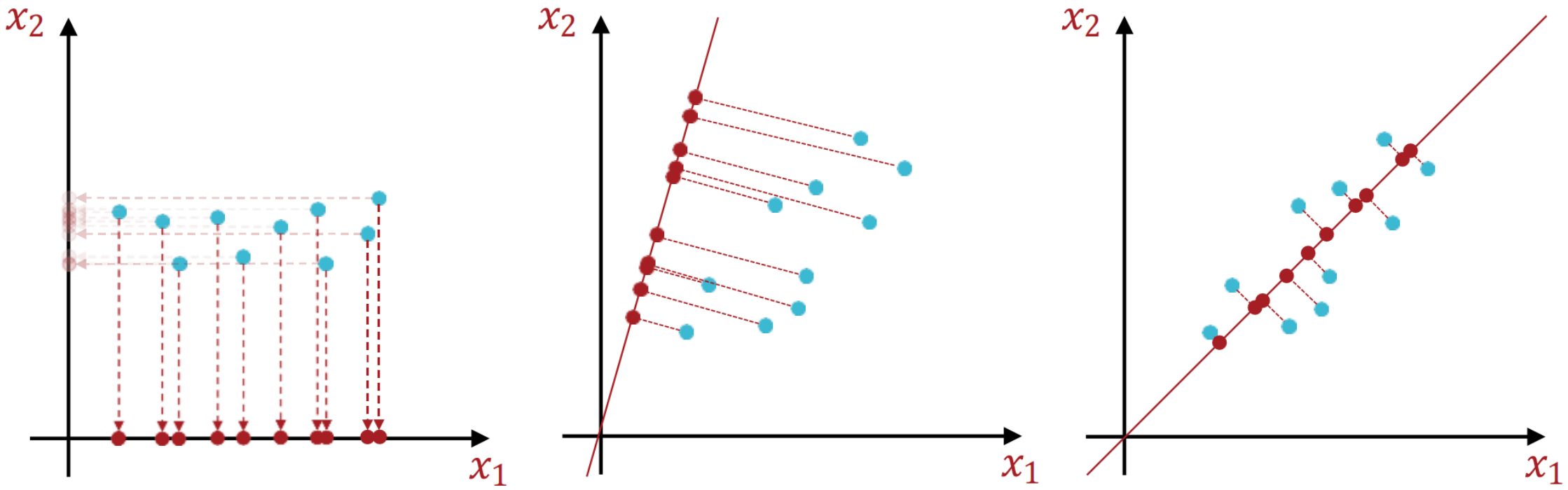
$$W_0 + \Delta W = W_0 + BA \qquad B \in \mathbb{R}^{d \times r}, A \in \mathbb{R}^{r \times k} \qquad r \ll \min(d, k)$$

$$h = W_0 x + \Delta W x = W_0 x + BAx$$

PCA (Principal Component Analysis)



PCA

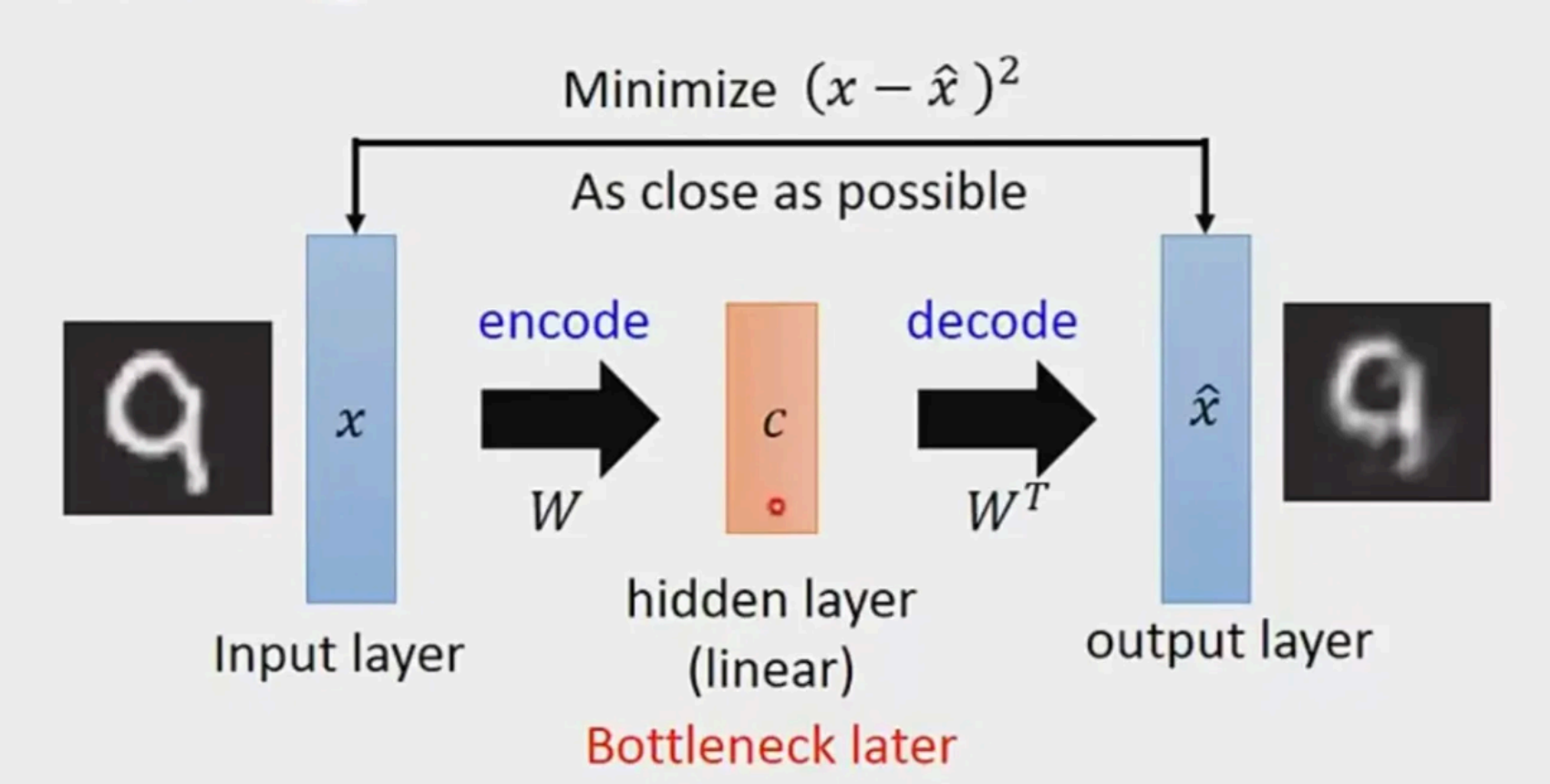


- 最大化投影后的方差 / 最小化重建误差

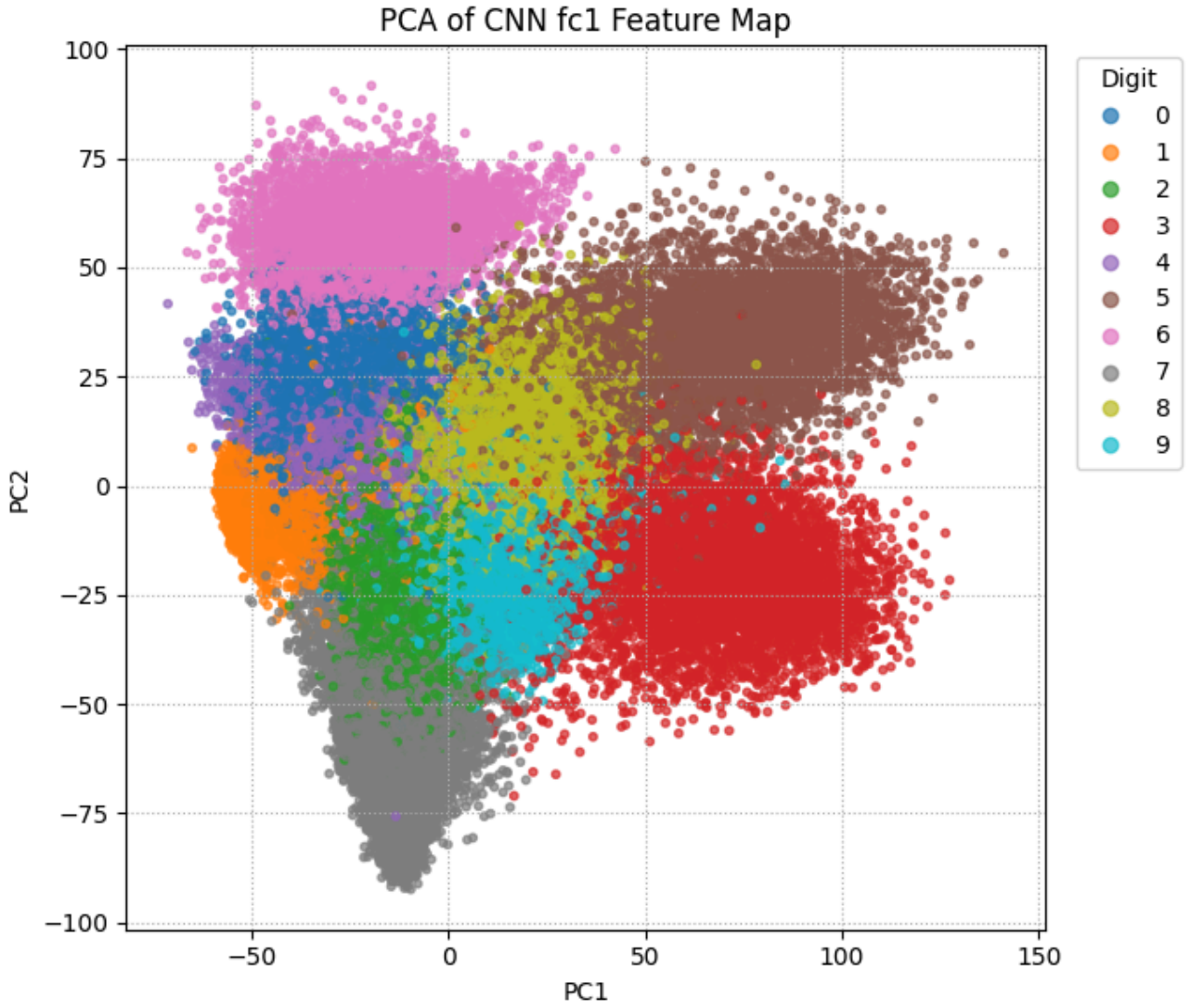
PCA

1. Centerization
   $X = X - \mu$
2. Eigenvalue Decomposition / SVD
   $X = U\Sigma V^\top$
   (U是$XX^\top$的特征向量, V是$X^\top X$的特征向量)
3. 取出$V_1, V_2, \ldots, V_k$(假设数据矩阵式**行向量**拼起来)
4. Projection
   $X_i' = X_i[V_1, V_2, \ldots, V_k]$

PCA是特殊的Encoder-Decoder结构(线性)



Visualize Latent Space



其他降维方式

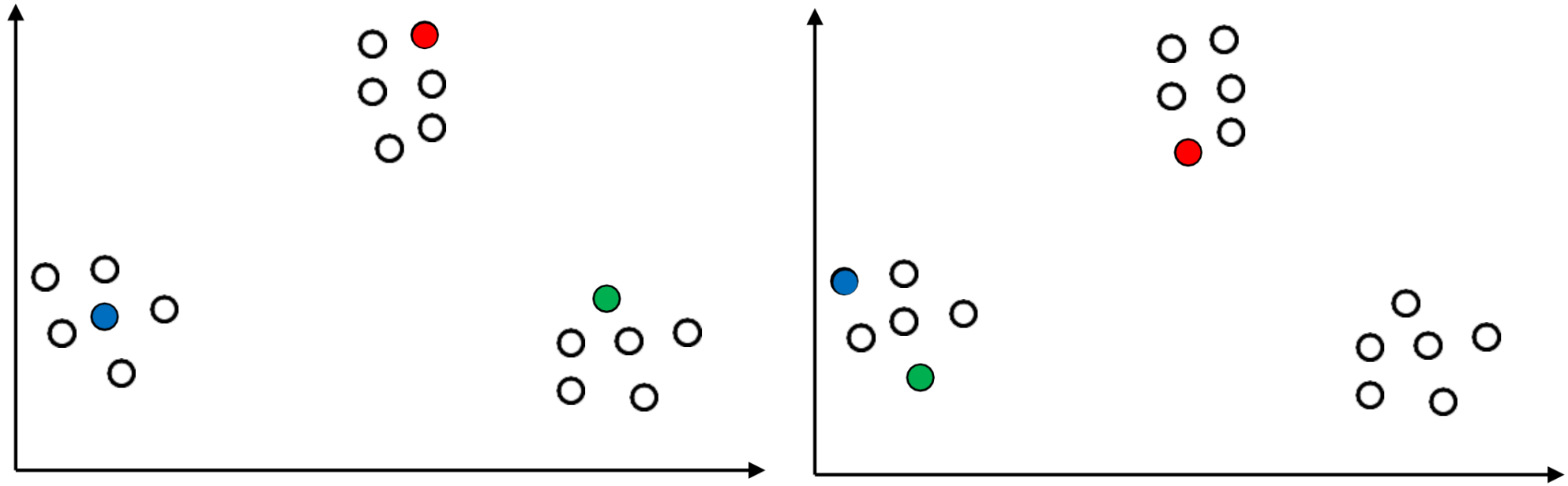
- PCA
- LDA
- t-SNE
- UMAP
- ...

Kmeans

E-step: $z_i = \arg\min_k \|x_i - \mu_k\|_2^2$

M-step: $\mu_k = \frac{1}{n_k} \sum_{i:z_i=k} x_i$

Initialization



Kmeans++

1. 从所有点中均匀随机选择一个, 作为第一个簇的中心c_1. 所有簇的中心的集合为 $C = \{c_1\}$
2. 对于每个非中心点x_i, 计算x_i到C中每个簇中心的距离
   $$D^2(x) = \min_{c \in C} \|x - c\|^2, \quad x \notin C$$
3. 选择下一个中心:
   $$\Pr(x_i\text{被选作下一个中心}) = \frac{D^2(x_i)}{\sum_{x \notin C} D^2(x)}$$
4. 重复步骤2和步骤3, 直到$|C| = k$