SI 140A: Probability and Statistics for EECS
Spring 2025, Instructor: Xavier Lagorce, TA: Junye Wang, Chuyan Zhou

Homework 6

Due: 23:59 (CST), May 2nd, 2025
Full Mark: 50 points, 5 problems including 1 bonus question.

Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- You are required to write down all the major steps towards making your conclusions; otherwise you may obtain limited points of the problem.
- Write your homework in English; otherwise you will get no points of this homework.
- Any form of plagiarism will lead to 0 point of this homework.
- If there are any descriptions unclear, we recommend you to ask on Piazza.

Problem 1

(Laplace Distribution *5'+5'***)**

The *Laplace distribution* has PDF:

$$f(x) = \frac{1}{2}e^{-|x|}, \text{ for } x \in \mathbb{R}$$

The Laplace distribution is also called a *symmetrized Exponential distribution.* Explain this in the following two ways:

(a) Plot the PDFs and explain how they relate.

(b) Let $X \sim \text{Expo}(1)$ and S be a random sign (1 or -1) with equal probabilities, with S and X independent. Find the PDF of SX (by first finding the CDF), and compare the PDF of SX with the PDF of the Laplace distribution.

Problem 2

(Expectation *10'***)**

Let $Z \sim \mathcal{N}(0,1)$ and c be a nonnegative constant. Find $E(max(Z-c,0))$, in terms of the standard Normal CDF Φ and PDF ϕ.

Problem 3

(Conditional probabilities *5'+5'***)**

Let X and Y be two continuous random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} cx^2y & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of constant c.

(b) Find the conditional probability $P\left(Y \le \frac{X}{4} \,\middle|\, Y \le \frac{X}{2}\right)$.

Problem 4

(Play with Distributions *5'+5'***)**

Let X and Y be i.i.d. Expo(λ), and $T = X + Y$.

(a) Find the conditional CDF of T given $X = x$. Be sure to specify where it is zero.

(b) Find the conditional PDF $f_{T|X}(t \mid x)$, and verify that it is a valid PDF.

(c) Find the conditional PDF $f_{X|T}(x \mid t)$, and verify that it is a valid PDF.

(d) In class we have shown that the marginal PDF of T is $f_T(t) = \lambda^2 t e^{-\lambda t}$, for $t > 0$. Give a short alternative proof of this fact, based on the previous parts and Bayes' rule.

Problem 5

(Integer limitations *5'+5'+ optional 10'***)**

Suppose $X \sim \mathcal{N}(m, \sigma^2)$, where m is an integer and σ is a real number. Let $Y = \lfloor X \rfloor$ be the integer part of X.

(a) Find the PMF of Y.

(b) Find $E[Y]$.

(c) **(Optional)** Find $Var[Y]$.