

Question 1

For this problem in both parts I used LINDO to solve them. In each of the part sections there is a figure that will have the data output from LINDO's solver, and the code that I ran in order to get that output. Also the the orientation of the figure is like so, on the left there is the LINDO code used to represent the problem at hand, and the on the right is the output data that shows the optimal solution.

- (a) In this problem we are trying to find the shortest path from G to C, below in figure 1, the objective function is the first line of code and the constraints that the problem is subjected to are all the lines of code in between ST and end, which includes lines 2.

Objective Function:

Stretched_out(maximize) distance between G and C; finds shortest distance from $G \rightarrow C$.

Constraints:

$$\begin{array}{lll}
 dg = 0 & db - da \leq 8 & dc - db \leq 4 \\
 dg - de \leq 7 & db - dh \leq 9 & db - df \leq 7 \\
 dh - dg \leq 3 & de - db \leq 10 & dd - dg \leq 2 \\
 da - dh \leq 4 & dd - de \leq 9 & df - dd \leq 18 \\
 da - df \leq 5 & de - dd \leq 25 & de - df \leq 2 \\
 df - da \leq 10 & dd - dc \leq 3 &
 \end{array}$$

The optimal solution is shown below in figure 1 of the LINDO solver program. The shortest path between G and C is 16.

```

max dc
ST
    dg = 0
    dg - de <= 7
    dh - dg <= 3
    da - dh <= 4
    da - df <= 5
    df - da <= 10
    db - da <= 8
    db - dh <= 9
    de - db <= 10
    dd - de <= 9
    de - dd <= 25
    dd - dc <= 3
    dc - db <= 4
    db - df <= 7
    dd - dg <= 2
    df - dd <= 18
    de - df <= 2
end

```

LP OPTIMUM FOUND AT STEP 6

OBJECTIVE FUNCTION VALUE

1) 16.000000

VARIABLE	VALUE	REDUCED COST
DC	16.000000	0.000000
DG	0.000000	0.000000
DE	0.000000	0.000000
DH	3.000000	0.000000
DA	4.000000	0.000000
DF	5.000000	0.000000
DB	12.000000	0.000000
DD	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	7.000000	0.000000
4)	0.000000	1.000000
5)	3.000000	0.000000
6)	6.000000	0.000000
7)	9.000000	0.000000
8)	0.000000	0.000000
9)	0.000000	1.000000
10)	22.000000	0.000000
11)	9.000000	0.000000
12)	25.000000	0.000000
13)	19.000000	0.000000
14)	0.000000	1.000000
15)	0.000000	0.000000
16)	2.000000	0.000000
17)	13.000000	0.000000
18)	7.000000	0.000000

NO. ITERATIONS= 6

Figure 1: shortest path from G to C, with LINDO code and data output.

Question 3

For parts a-c I will be using LINDO to running linear programming optimizations to find the optimal solutions that each part asks for. The figures that I include in each section follows this layout; on the left is the LINDO code and on the right is the data output from the code that is solved, the objective function's output and answers to the question are displayed.[\[1\]](#)

(a) Minimize Shipping Costs of Refrigerators:

Objective Function:

minimize the shipping cost of the refrigerators ($10s1_1 + 15s1_2 + 11s2_1 + 8s2_2 + 13s3_1 + 8s3_2 + 9s3_3 + 14s4_2 + 8s4_3 + 5d1_1 + 6d1_2 + 7d1_3 + 10d1_4 + 12d2_3 + 8d2_4 + 10d2_5 + 14d2_6 + 12d3_4 + 12d3_5 + 12d3_6 + 6d3_7$)

Constraints:

$$s1_1 + s1_2 \leq 150$$

$$s2_1 + s2_2 \leq 450$$

$$s3_1 + s3_2 + s3_3 \leq 250$$

$$s4_2 + s4_3 \leq 150$$

$$d1_1 \geq 100$$

$$d1_2 \geq 150$$

$$d1_3 + d2_3 \geq 100$$

$$d1_4 + d2_4 + d3_4 \geq 200$$

$$d2_5 + d3_5 \geq 200$$

$$d2_6 + d3_6 \geq 150$$

$$d3_7 \geq 100$$

$$s1_1 + s2_1 + s3_1 - d1_1 - d1_2 - d1_3 - d1_4 = 0$$

$$s1_2 + s2_2 + s3_2 + s4_2 - d2_3 - d2_4 - d2_5 - d2_6 = 0$$

$$s3_3 + s4_3 - d3_4 - d3_5 - d3_6 - d3_7 = 0$$

$$s1_1 \geq 0$$

$$s1_2 \geq 0$$

$$s2_1 \geq 0$$

$$s2_2 \geq 0$$

$$s3_1 \geq 0$$

$$s3_2 \geq 0$$

$$s3_3 \geq 0$$

$$s4_2 \geq 0$$

$$s4_3 \geq 0$$

$$d1_1 \geq 0$$

$$d1_2 \geq 0$$

$$d1_3 \geq 0$$

$$d1_4 \geq 0$$

$$d2_3 \geq 0$$

$$d2_4 \geq 0$$

$$d2_5 \geq 0$$

$$d2_6 \geq 0$$

$$d3_4 \geq 0$$

$$d3_5 \geq 0$$

$$d3_6 \geq 0$$

$$d3_7 \geq 0$$

From figure 4, we get the optimal solution (minimized shipping cost is \$17100.00 dollars, and the optimal shipping routes with the number of refrigerators that should be shipped on each route are as follows:

$$P1 \rightarrow W1 = 150$$

$$P1 \rightarrow W2 = 0$$

$$P2 \rightarrow W1 = 200$$

$$P2 \rightarrow W2 = 250$$

$$P3 \rightarrow W1 = 0$$

$$P3 \rightarrow W2 = 150$$

$$P3 \rightarrow W3 = 100$$

$$P4 \rightarrow W2 = 0$$

$$P4 \rightarrow W3 = 150$$

$$W1 \rightarrow R1 = 100$$

$$W1 \rightarrow R2 = 150$$

$$W1 \rightarrow R3 = 100$$

$$W1 \rightarrow R4 = 0$$

$$W2 \rightarrow R3 = 0$$

$$W2 \rightarrow R4 = 200$$

$$W2 \rightarrow R5 = 200$$

$$W2 \rightarrow R6 = 0$$

$$W3 \rightarrow R4 = 0$$

$$W3 \rightarrow R5 = 0$$

$$W3 \rightarrow R6 = 150$$

$$W3 \rightarrow R7 = 100$$

```

min      10s1_1 + 15s1_2 + 11s2_1 + 8s2_2 + 12s3_1 + 8s3_2 +
          9s3_3 + 14s4_2 + 8s4_3 + 5d1_1 + 6d1_2 + 7d1_3 +
          10d1_4 + 12d2_3 + 8d2_4 + 10d2_5 + 14d2_6 + 12d3_4 +
          12d3_5 + 12d3_6 + 6d3_7
ST
s1_1 + s1_2 <= 150
s2_1 + s2_2 <= 450
s3_1 + s3_2 + s3_3 <= 250
s4_2 + s4_3 <= 150

d1_1 >= 100
d1_2 >= 150
d1_3 + d2_3 >= 100
d1_4 + d2_4 + d3_4 >= 200
d2_5 + d3_5 >= 200
d2_6 + d3_6 >= 150
d3_7 >= 100

s1_1 + s2_1 + s3_1 - d1_1 - d1_2 - d1_3 - d1_4 = 0
s1_2 + s2_2 + s3_2 + s4_2 - d2_3 - d2_4 - d2_5 - d2_6 = 0
s3_3 + s4_3 - d3_4 - d3_5 - d3_6 - d3_7 = 0

s1_1 >= 0
s1_2 >= 0
s2_1 >= 0
s2_2 >= 0
s3_1 >= 0
s3_2 >= 0
s3_3 >= 0
s4_2 >= 0
s4_3 >= 0

d1_1 >= 0
d1_2 >= 0
d1_3 >= 0
d1_4 >= 0
d2_3 >= 0
d2_4 >= 0
d2_5 >= 0
d2_6 >= 0
d3_4 >= 0
d3_5 >= 0
d3_6 >= 0
d3_7 >= 0
end

```

LP OPTIMUM FOUND AT STEP 13

OBJECTIVE FUNCTION VALUE

1) 17100.00

VARIABLE	VALUE	REDUCED COST
S1_1	150.000000	0.000000
S1_2	0.000000	8.000000
S2_1	200.000000	0.000000
S2_2	250.000000	0.000000
S3_1	0.000000	1.000000
S3_2	150.000000	0.000000
S3_3	100.000000	0.000000
S4_2	0.000000	7.000000
S4_3	150.000000	0.000000
D1_1	100.000000	0.000000
D1_2	150.000000	0.000000
D1_3	100.000000	0.000000
D1_4	0.000000	5.000000
D2_3	0.000000	2.000000
D2_4	200.000000	0.000000
D2_5	200.000000	0.000000
D2_6	0.000000	1.000000
D3_4	0.000000	5.000000
D3_5	0.000000	3.000000
D3_6	150.000000	0.000000
D3_7	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	1.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-16.000000
10)	0.000000	-18.000000
11)	0.000000	-21.000000
12)	0.000000	-15.000000
13)	0.000000	-11.000000
14)	0.000000	-8.000000
15)	0.000000	-9.000000
16)	150.000000	0.000000
17)	0.000000	0.000000
18)	200.000000	0.000000
19)	250.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	200.000000	0.000000
31)	200.000000	0.000000
32)	0.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000
36)	100.000000	0.000000

NO. ITERATIONS= 13

Figure 4: Transshipment Optimal Solution, with LINDO code and data output.

- (b) If we take out warehouse 2 from the transshipment graph, the following LINDO code will represent the scenario where there will only be warehouses 1 and 3 that are used to process shipments. This situation is non feasible since some of the constraints that are bounding the problem can not be met; for example if retailers 5, 6 and 7 together needed 450 units of refrigerators, which would mean that only warehouse 3 can supply them. We also know that the only plants 3 and 4 can supply warehouse 3. So if we pull all of the units supplied by plants 3 and 4 we get only 400 units which would not be enough to meet the combined demand from retailers 5, 6, and 7's 450 units of refrigerators, hence a non feasible solution will result from LINDO solver (error message). Below in figure 5 we can see that the error message on the left confirms my conclusions about this situation. The code that is used to represent the situation in on the right of the error message.

$$\begin{array}{lll} d2_6 \geq 0 & d3_5 \geq 0 & d3_7 \geq 0 \\ d3_4 \geq 0 & d3_6 \geq 0 & \end{array}$$

From figure 6, we get that the optimal solution (minimized shipping costs) is \$18300.00 dollars, with the optimal shipping routes and the optimal number of refrigerators per route is as follows:

P1 → W1 = 150	P4 → W2 = 0	W2 → R4 = 50
P1 → W2 = 0	P4 → W3 = 150	W2 → R5 = 50
P2 → W1 = 350	W1 → R1 = 100	W2 → R6 = 0
P2 → W2 = 100	W1 → R2 = 150	W3 → R4 = 0
P3 → W1 = 0	W1 → R3 = 100	W3 → R5 = 150
P3 → W2 = 0	W1 → R4 = 150	W3 → R6 = 150
P3 → W3 = 250	W2 → R3 = 0	W3 → R7 = 100

```

min      10s1_1 + 15s1_2 + 11s2_1 + 8s2_2 + 13s3_1 + 8s3_2 +
          9s3_3 + 14s4_2 + 8s4_3 + 5d1_1 + 6d1_2 + 7d1_3 +
          10d1_4 + 12d2_3 + 8d2_4 + 10d2_5 + 14d2_6 + 12d3_4 +
          12d3_5 + 12d3_6 + 6d3_7
ST
s1_1 + s1_2 <= 150
s2_1 + s2_2 <= 450
s3_1 + s3_2 + s3_3 <= 250
s4_2 + s4_3 <= 150
s1_2 + s2_2 + s3_2 + s4_2 <= 100

d1_1 >= 100
d1_2 >= 150
d1_3 + d2_3 >= 100
d1_4 + d2_4 + d3_4 >= 200
d2_5 + d3_5 >= 200
d2_6 + d3_6 >= 150
d3_7 >= 100

s1_1 + s2_1 + s3_1 - d1_1 - d1_2 - d1_3 - d1_4 = 0
s1_2 + s2_2 + s3_2 + s4_2 - d2_3 - d2_4 - d2_5 - d2_6 = 0
s3_3 + s4_3 - d3_4 - d3_5 - d3_6 - d3_7 = 0

s1_1 >= 0
s1_2 >= 0
s2_1 >= 0
s2_2 >= 0
s3_1 >= 0
s3_2 >= 0
s3_3 >= 0
s4_2 >= 0
s4_3 >= 0

d1_1 >= 0
d1_2 >= 0
d1_3 >= 0
d1_4 >= 0
d2_3 >= 0
d2_4 >= 0
d2_5 >= 0
d2_6 >= 0
d3_4 >= 0
d3_5 >= 0
d3_6 >= 0
d3_7 >= 0
end

```

OBJECTIVE FUNCTION VALUE		
1) 18300.00		
VARIABLE	VALUE	REDUCED COST
S1_1	150.000000	0.000000
S1_2	0.000000	8.000000
S2_1	350.000000	0.000000
S2_2	100.000000	0.000000
S3_1	0.000000	4.000000
S3_2	0.000000	2.000000
S3_3	250.000000	0.000000
S4_2	0.000000	9.000000
S4_3	150.000000	0.000000
D1_1	100.000000	0.000000
D1_2	150.000000	0.000000
D1_3	100.000000	0.000000
D1_4	150.000000	0.000000
D2_3	0.000000	7.000000
D2_4	50.000000	0.000000
D2_5	50.000000	0.000000
D2_6	0.000000	4.000000
D3_4	0.000000	2.000000
D3_5	150.000000	0.000000
D3_6	150.000000	0.000000
D3_7	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	2.000000
5)	0.000000	3.000000
6)	0.000000	5.000000
7)	0.000000	-16.000000
8)	0.000000	-17.000000
9)	0.000000	-18.000000
10)	0.000000	-21.000000
11)	0.000000	-23.000000
12)	0.000000	-23.000000
13)	0.000000	-17.000000
14)	0.000000	-11.000000
15)	0.000000	-13.000000
16)	0.000000	-11.000000
17)	150.000000	0.000000
18)	0.000000	0.000000
19)	350.000000	0.000000
20)	100.000000	0.000000
21)	0.000000	0.000000
22)	0.000000	0.000000
23)	250.000000	0.000000
24)	0.000000	0.000000
25)	150.000000	0.000000
26)	100.000000	0.000000
27)	150.000000	0.000000
28)	100.000000	0.000000
29)	150.000000	0.000000
30)	0.000000	0.000000
31)	50.000000	0.000000
32)	50.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000
36)	150.000000	0.000000
37)	100.000000	0.000000

NO. ITERATIONS= 15

Figure 6: Transshipment optimal solution with limited warehouse 2, with LINDO code and data output.

Question 4

For the following coining problem sets a) and b), I will be using Excel Solver to come up with the minimum amount of coin used to make the exact change. For each of the problem sets I will include the the decision variables, objective function, the constraints to that problem, and the optimal solution. I will also include a figure that will show the equations used for the objection function and constraints used to the solve the problem in each coin set.[2]

(a) $V = [1, 5, 10, 25]$ where $A = 202$.

A will also be known as "Desired Amount".

Objective Function and Decision Variables:

Minimize: $w + x + y + z$ coins used to make A , where decision variables are:

w = amount of 1's coins used.

x = amount of 5's coins used.

y = amount of 10's coins used.

z = amount of 25's coins used.

Constraints:

$$w \geq 0$$

$$x \geq 0$$

$$y \geq 0$$

$$z \geq 0$$

$$w = \text{Integers}$$

$$x = \text{Integers}$$

$$y = \text{Integers}$$

$$z = \text{Integers}$$

$$w + 5x + 10y + 25z = A \text{ (Desired Amount)}$$

From figure 7 below, the optimal solution (minimum coins used to make exact desired change) is 10 coins. The solution uses 8 25's coins, and 2 1's coins.

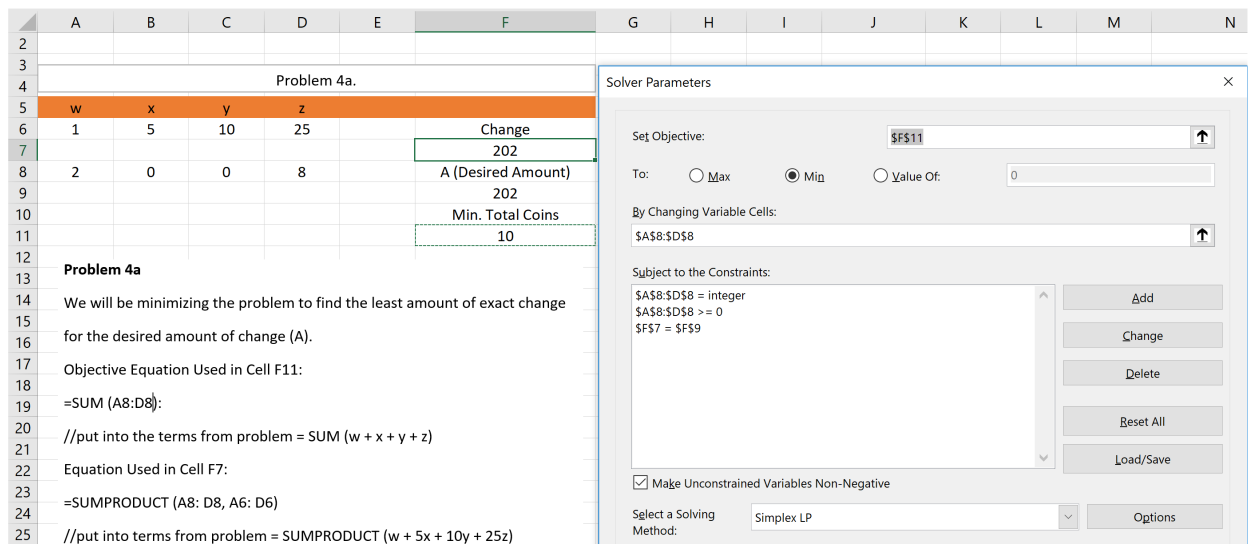


Figure 7: Coin Set 1 Optimal Output with Constraints and Objective Function using Microsoft Excel.

(b) $V = [1, 3, 7, 12, 27]$ where $A = 293$

A can also be known as "Desired Amount".

Objective Function and Decision Variables:

Minimize: $v + w + x + y + z$ coins used to make A , where decision variables are:

w = amount of 1's coins used.

w = amount of 3's coins used.

x = amount of 7's coins used.

y = amount of 12's coins used.

z = amount of 27's coins used.

Constraints:

$$w \geq 0$$

$$x \geq 0$$

$$y \geq 0$$

$$z \geq 0$$

$$w = \text{Integers}$$

$$x = \text{Integers}$$

$$y = \text{Integers}$$

$$z = \text{Integers}$$

$$v + 3w + 7x + 12y + 27z = A \text{ (Desired Amount)}$$

From figure 8 below, the optimal solution (minimum coins used to make exact desired change) is 14 coins. The solution uses 10 27's coins, 1 12's coin, 1 7's coin, 1 3's coin, and a 1's coin.

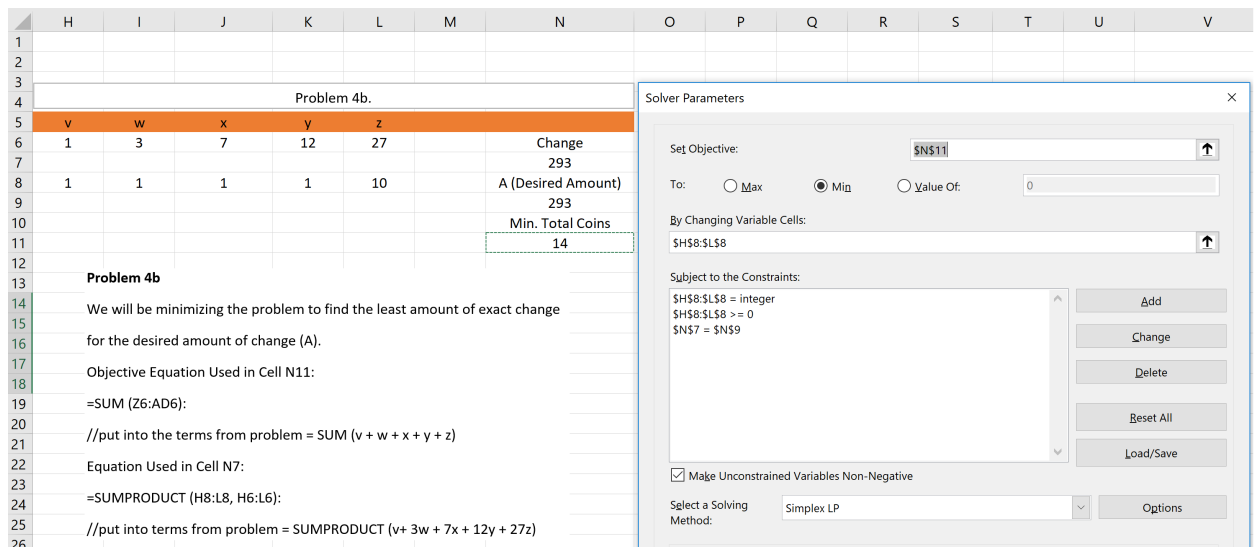


Figure 8: Coin Set 2 Optimal Output with Constraints and Objective Function using Microsoft Excel.

References

- [1] B. Greve, “Transshipment.” <https://www.youtube.com/watch?v=UGRIVTB9po8>, 2012.
- [2] M. C. Patterson, “The coin changing problem as a mathematical model.” <https://oregonstate.instructure.com/courses/1683584/files/71545501/download?wrap=1>, 2010.