Development of Photon Calibrator for Hardware Injection Test

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Abstract

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To my parents.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Introduction to Gravitational Wave

1.1.1 What is gravitational wave

In the General Theory of Relativity proposed by Albert Einstein in 1915, phenomena caused by gravity can be interpreted as results of curved spacetime. This is one of his important work after his 'Happiest Thought', which recorded in his unpublished article "Fundamental Ideas and Methods of the Theory of Relativity, Presented in Their Development"[1]. Among different ways to curve our spacetime, which can be described by corresponding metric tensor fields, there exist wavelike solutions describing ripples of spacetime known as gravitational waves.

However, the physical reality of gravitational wave is not so clear to everyone in the early days, even to Even Einstein himself [2, 3]. The main problems is that there exist some gauge degree of freedom in the theory due to the arbitrariness of coordinate choices. We have to know whether the gravitational waves we found are just gauge waves (vibration of coordinate) or the wave can have some observable consequences.

One of the most important observational evidence implying the existence of real gravitational waves is Hulse-Taylor pulsar [4]. Taylor demonstrated that the change of pulsar rotation speed can be explained by emission of gravitational wave[5]. .... Finally, in 2015 September 14. GW150914

1.1.2 How to describe gravitational wave

In Einstein's General Relativity, gravitational effects are realized by geometry of spacetime. According to great mathematician Bernhard Riemann, we can describe the geometry of certain space by telling the "distance" between nearby points in the space. Practically, the information of distance between nearby spacetime points form a tensor called metric, which means the measure of distance. If such metric tensor exist everywhere in our universe, they become metric tensor field, which tell us the shape or geometry of our universe. By choosing a coordinate system, one can write down those corresponding components $g_{\mu\nu}$ of metric tensor g. Now, we can calculate spacetime distanceds between two nearby points by their coordinate separation:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1.1}$$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \tag{1.2}$$

mathematics.

TTgauge There is a very convenient gauge(coordinate) constrain, which is known as Traceless and Transverse gauge. for perturbation of wave like part metic over Schwarzschild background which represent Earths gravity.

Refer to [6]

How to generate gravitational wave The source of electromagnetic wave is time-dependent electrical charge distribution. Similarly, the source of gravitational wave is time-dependent mass(energy) distribution. Strictly speaking, the lowest order of mass multi-pole which can generate real gravitational waves is mass quadruple because we dont have negative mass, while the electromagnet wave can be generate though time-dependent electrical dipole moment. The gravitational wave strain generated by mass quadruple can be approximately described by famous quadrupole formula: (quadrupole formula) Practically, PN NR.... According to our current understand of universe, there are several kinds of astrophysical gravitational wave sources, whose h(t) amplitude is large enough to be detected by current ground based laser interferometer, like advanced-LIGO, advanced-Virgo or KAGRA. Compact Binary Coalescence BNS BBH

1.1.3 How to detect Gravitational wave

The interaction of detector and gravitational wave can have different interpretation due to different coordinate choice[7]. It is quiet similar to that the magnetic force in one observational frame may be electric force in the other frame. However, practically, I would like to use the .., which described in next section.

1.2 Detection of Gravitation wave

Interaction of GW wave when $\lambda_g w$ L of detector Limit of Michelson IFO IFO with dual-recycling and Fabry-Perot arms. Complex response WE NEED Calibration Calibration Calibration

1.3 Calibration and Reconstruction

Calibration is always the fist step before we measure something by some device. For example, to measure the weight of an apple, you should calibrate your scale by putting a standard kilogram on it. Then, you can either adjust the scale readout to be 1kg, or record the difference showed in scale readout, which may be used to reconstruct real weight of the apple. However, the spring constant of springs inside the scale could fluctuate due to temperature changes. To accurately measure the weight of the apple, we have to measure the calibration factor (scale readout when we put the standard mass on it.) when we measure the weight of apple, if possible, simultaneously.

Due to the complexity of practical interferometer, the response of interferometer itself to external gravitational source is not only complex but also time-dependent. In reality, we inject several calibration lines, which means we shake the End-Test-Mirror by several known frequency and amplitude sine wave. Then, we try to see these standard signal in readout of interferometer. If we can solve .

1.3.1 Transfer function of Laser Interferometer with Fabry-Perot Cavity

1.3.2 Tracking Time-dependent Response by Calibration lines

1.4 Photon Calibrator (Pcal)

1.4.1 Principle of Photon Calibrator

Photon calibrator is an additional laser with high precision intensity modulator. It is installed in front of End-Test-Mass Mirror(ETM) and can push the ETM by radiation force due to its own Laser beam. To generate any artificial h(t) by Pcal, we have to translate desired h(t) into corresponding force F(t) exerting on ETM. This can be done by using equation of motion of the ETM suspend by its suspension system. Then, we control the Pcal Laser output intensity P(t) such that the radiation force exerted on ETM is F(t) we calculated before. If we analyze it frequency domain, the h(f) introduced by P(f) can be describe by eq:

Original Pcal is proposed by Glasgow group [8]. They use single laser beam hitting on the center of ETM. The problem is that it may introduce drumhead mode vibration of ETM surface (just like the vibration mode you see when you hit the center of a drum), which introduce unwanted h(t) effectively. This problem is solved by LIGO group[9], who separate the Pcal laser beam into two beams, hitting on the nodal point of drumhead mode on the ETM surface[10].

¿ KAGRA

In order to excite same amplitude h(t) in higher frequency regime, we have to give much larger F(t) since the relationship between x(t) and F(t) in an pendulum .

1.4.2 Why do we need Photon Calibrator

1.4.3 Tracking Time-dependent Response by Calibration lines

Chapter 2

Hardware Injection through Photon Calibrator

2.1 Principle

Validate IFO by Pcal (Hardware Injection Test) As I mentioned in last chapter, the practical response of IFO is very complex. To prevent some unexpected problem including no-linear response of IFO and The best way is to provide some test source of expected GW signal.

However, it is impossible to prepare an BBH system in laboratory. Instead, we will generated some test signal by pushing the ETM with Pcal. This procedure is called Hardware Injection Test

Motivation To under whether we can successfully reconstruct the h(t) from our interferometer, the best way is to prepare an artificial signal, sending it to interferometer, reconstructing it, finally, comparing it with original one. However it is quite difficult to generate human made gravitational wave that can be detected by current gravitational wave detector.

Requirement

Low Frequency around 100Hz the nose should below the IFO sensitivity (absolute timing ¡ ?us ns)

High Frequency above 1kHz the transfer function should as flat as possible

Amplitude of Injection Signal

$$\frac{F(t)}{M} = \frac{1}{M}\frac{2P(t)\cos(\theta)}{c} = \ddot{x}(t) \tag{2.1}$$

For $x = x_0 \sin(\omega t)$,

$$\frac{1}{M}\frac{2P_0\cos(\theta)}{c}\sin(\omega t) = -\omega^2 x_0 \sin(\omega t) \tag{2.2}$$

Thus,

$$P_0 = -\omega^2 \frac{Mc}{2\cos(\theta)} x_0 = -\omega^2 \frac{Mc}{2\cos(\theta)} L h_0 \tag{2.3}$$

$$M = 23 \text{ kg}$$
$$L = 3 \text{ km}$$
$$\theta = 0.72 \text{ deg}$$
$$c = 2.998 \times 10^8 \text{ m/s}$$

$$P_0 \text{ (Watts)} \times \frac{Gain_{\text{ Power to OFSPD}}}{2} = \underbrace{V_{\text{OFSPD}}}_{\text{Same as V}_{\text{Injection}}} \quad \text{(Volts)}$$

Therefore, the overall gain should be set in injection channel, which is in Volt unit, is

$$\omega^2 \frac{Mc}{2\cos(\theta)} L \times \frac{Gain_{\text{ Power to OFSPD}}}{2} \tag{2.4}$$

Chapter 3

Signal Generating System

3.1 Noise Problem of Injection Signal

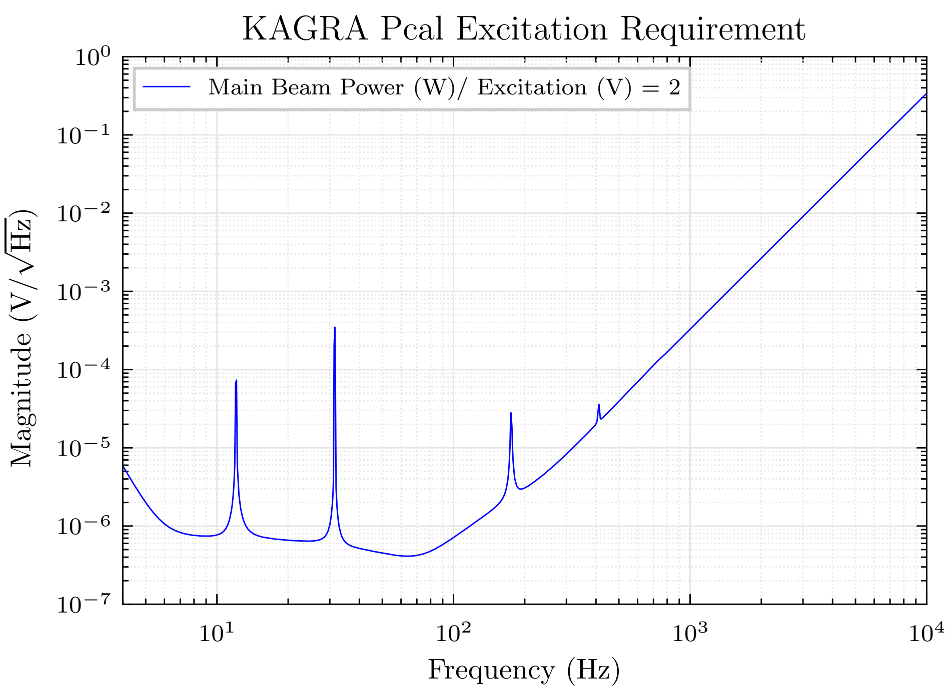

Figure 3.1: Injection Channel Noise Requirement

$$\Delta L(f) < \frac{1}{10} \times (\text{KAGRA length sensitivity}) \tag{3.1}$$

$$\Delta L(f) = \frac{2\Delta P(f)\cos(\theta)}{c}\frac{1}{M(2\pi f)^2} < \frac{1}{10}\Delta h(f)L \tag{3.2}$$

3.1.1 Noise Sources from Control Signal

Quantization Noise of DAC

The origin of quantization error is coming from the difference between desired analog output and quantized Digital to Analog Converter(DAC) output value. Roughly speaking, it shows like white noise spreading from DC to Nyquist frequency i.e. $Fs/2$. The Root Mean Square value of quantization noise has the order of voltage difference corresponding to last digit or Least Significant Bit(LSB). In time domain, we can calculate standard deviation.

$$\sigma_x = \sqrt{\frac{1}{12}}\delta x_{LSB} \tag{3.3}$$

For a 16-bit 64kHz DAC with output range between ±10Volts,

$$\sigma_x = \sqrt{\frac{1}{12}}\delta x_{LSB} \tag{3.4}$$

$$= \sqrt{\frac{1}{12}}\frac{(+10) - (-10)\text{Volts}}{2^{16}} \tag{3.5}$$

$$= 8.81 \times 10^{-5} \text{ Volts} \tag{3.6}$$

In frequency Domain, the quantization noise is distributed from DC to 32768Hz; therefore, we have ASD

$$\begin{aligned} ASD &= \sqrt{PSD} && (3.7) \\ &= \sqrt{\frac{\sigma_x^2}{32768}} && (3.8) \\ &= 8.81 \times 10^{-5}\sqrt{\frac{1}{32768}} && (3.9) \\ &= 4.87 \times 10^{-7} \text{ Volts}/\sqrt{\text{Hz}} && (3.10) \end{aligned}$$

Analog circuits

AC Power Line

Chapter 4

Noise Reduction through De-Whitening Filter

4.1 Principle of De-Whitening Filter

De-Whitening Filter is an equalizer type analog filter. It has a specific frequency response such that

4.2 Circuit Design

Problem of 16kHz excitation channel Implementation of 64kHz Excitation channel in KAGRA digital system

Principle of Analog filter Design of De-Whitening filter Performance test Transfer function measurement Noise requirement Create Inverse De-Whitening filter

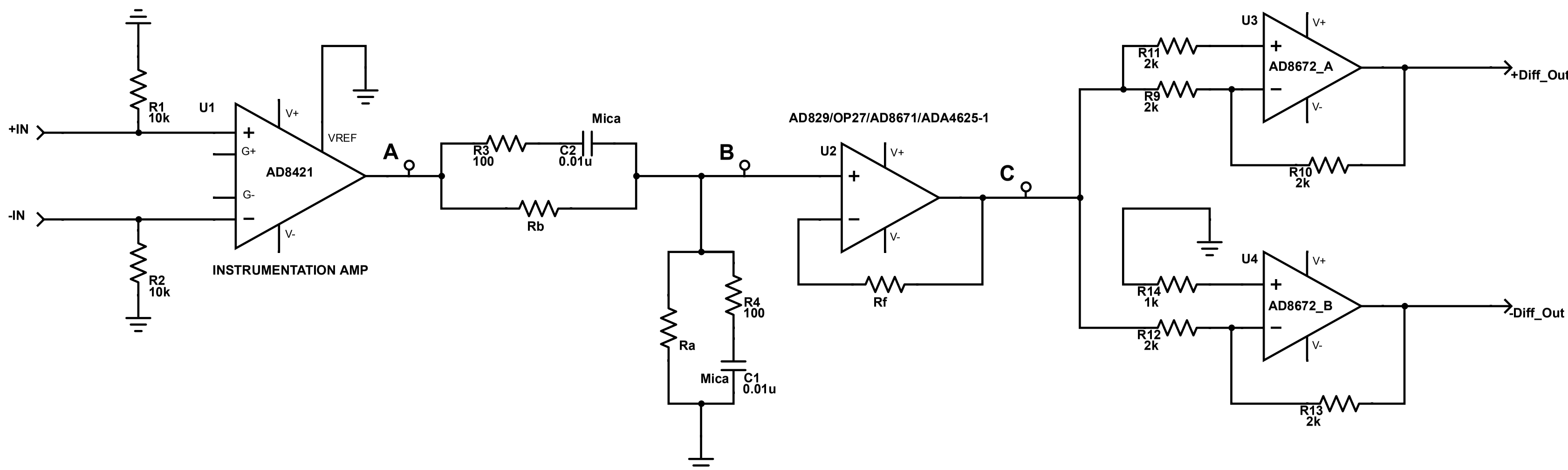

Figure 4.1: De-Whitening filter circuit.

Chapter 5

Validation of Injection Channel

Without loss of generality, We injected binary blackhole signals to test performance of our De-Whitening Filter.....

Besides, We have tried to inject Sine-Gaussian signals.

Noise measurement around 100Hz the nose should below the IFO sensitivity Transfer Function measurement above 1kHz performance time delay of excitation channel (absolute timing measurement?) Distortion of Scientific Signal BBH BNS post merger

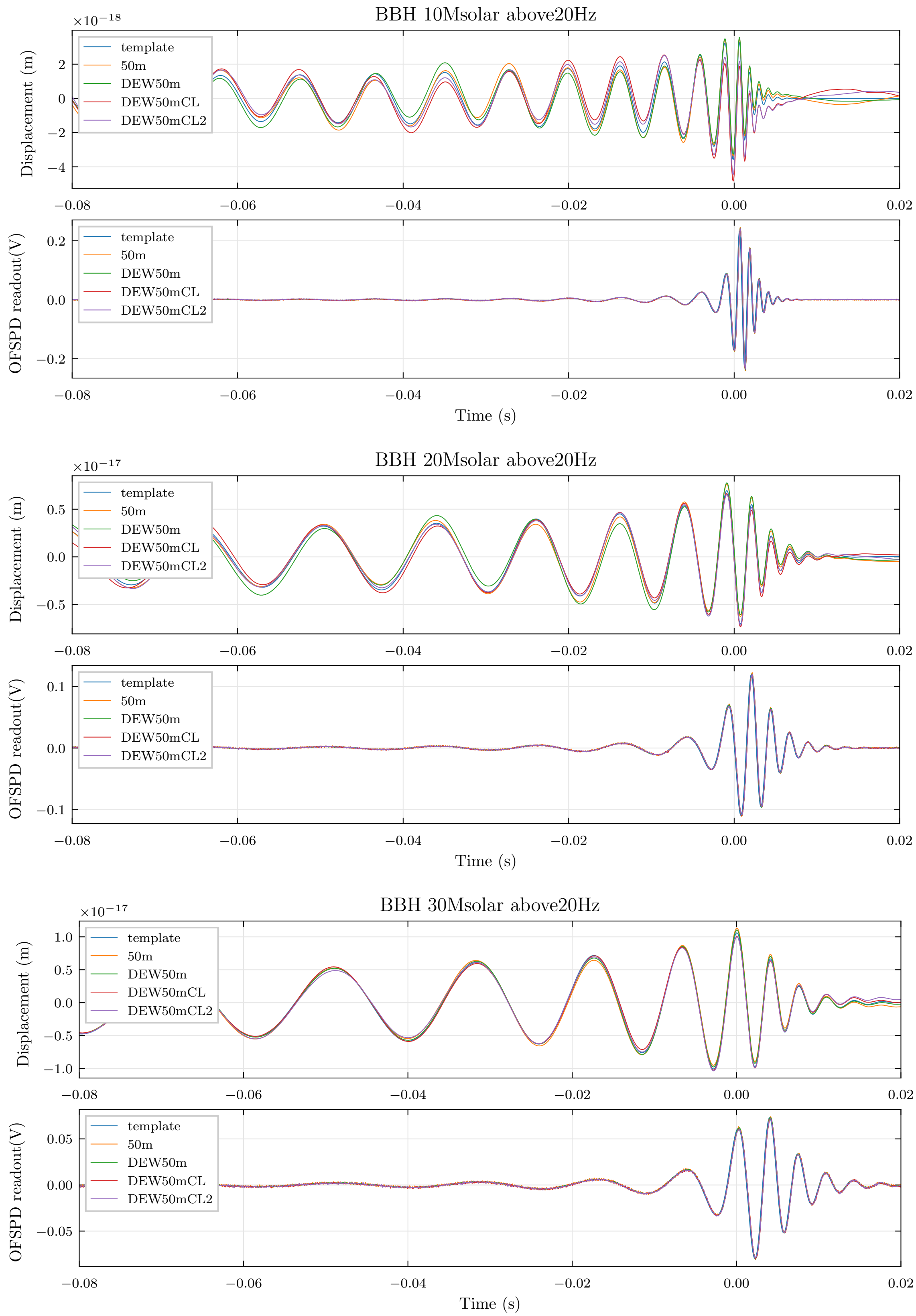

Figure 5.1: Injected Binary Blackhole Merger Signal

Chapter 6

Discussion and Future Works

Without loss of generality, Inject Binary Blackhole merger signal and sine-gaussian signals to test performance of our De-Whitening Filter.....

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