Development of Photon Calibrator for Hardware Injection Test

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Abstract

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To my parents.

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Chapter 1

Introduction

When you got a new camera, you probably will take a lot of testing photos before you start to use it seriously.

Hardware injection test is a process to verify the performance of interferometer by sending real sample signal into interferometer. Ideally, we should prepare some real gravitational waves as test signal. But it is practically hard to generate large enough artificial gravitational waves that are detectable by current technology. Therefore, as an injection signal, people shake the mirror according to the waveform template, thereby changing arm length correspondingly.

1.1 Introduction to Gravitational Wave

1.1.1 What is gravitational wave

In the General Theory of Relativity proposed by Albert Einstein in 1915, phenomena caused by gravity can be interpreted as results of curved spacetime. This is one of his important work after his 'Happiest Thought', which recorded in his unpublished article "Fundamental Ideas and Methods of the Theory of Relativity, Presented in Their Development"[1]. Among different ways to curve our spacetime, which can be described by corresponding metric tensor fields, there exist wavelike solutions describing ripples of spacetime known as gravitational waves.

However, the physical reality of gravitational wave is not so clear to everyone in the early days, even to Even Einstein himself [2, 3]. The main problems is that there exist some gauge degree of freedom in the theory due to the arbitrariness of coordinate choices. We have to know whether the gravitational waves we found are just gauge waves (vibration of coordinate) or the wave can have some observable consequences.

One of the most important observational evidence implying the existence of real gravitational waves is Hulse-Taylor pulsar [4]. Taylor demonstrated that the change of pulsar rotation speed can be explained by emission of gravitational wave[5]. .... Finally, in 2015 September 14. GW150914

1.1.2 How to describe gravitational wave

In Einstein's General Relativity, gravitational effects are realized by geometry of spacetime. According to great mathematician Bernhard Riemann, we can describe the geometry of certain space by telling the "distance" between nearby points in the space. Practically, the information of distance between nearby spacetime points form a tensor called metric, which means the measure of distance. If such metric tensor exist everywhere in our universe, they become metric tensor field, which tell us the shape or geometry of our universe. By choosing a coordinate system, one can write down those corresponding components $g_{\mu\nu}$ of metric tensor g. Now, we can calculate spacetime distanceds between two nearby points by their coordinate separation:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1.1}$$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \tag{1.2}$$

mathematics.

TTgauge There is a very convenient gauge(coordinate) constrain, which is known as Traceless and Transverse gauge. for perturbation of wave like part metic over Schwarzschild background which represent Earths gravity.

Refer to [6]

How to generate gravitational wave The source of electromagnetic wave is time-dependent electrical charge distribution. Similarly, the source of gravitational wave is time-dependent mass(energy) distribution. Strictly speaking, the lowest order of mass multi-pole which can generate real gravitational waves is mass quadruple because we dont have negative mass, while the electromagnet wave can be generate though time-dependent electrical dipole moment. The gravitational wave strain generated by mass quadruple can be approximately described by famous quadrupole formula: (quadrupole formula) Practically, PN NR.... According to our current understand of universe, there are several kinds of astrophysical gravitational wave sources, whose h(t) amplitude is large enough to be detected by current ground based laser interferometer, like advanced-LIGO, advanced-Virgo or KAGRA. Compact Binary Coalescence BNS BBH

1.1.3 How to detect Gravitational wave

The interaction of detector and gravitational wave can have different interpretation due to different coordinate choice[7]. It is quiet similar to that the magnetic force in one observational frame may be electric force in the other frame. However, practically, I would like to use the .., which described in next section.

1.2 Detection of Gravitation wave

Interaction of GW wave when $\lambda_g w$ L of detector Limit of Michelson IFO IFO with dual-recycling and Fabry-Perot arms. Complex response WE NEED Calibration Calibration Calibration Calibration

1.3 Calibration and Reconstruction

Calibration is always the fist step before we measure something by some device. For example, to measure the weight of an apple, you should calibrate your scale by putting a standard kilogram on it. Then, you can either adjust the scale readout to be 1kg, or record the difference showed in scale readout, which may be used to reconstruct real weight of the apple. However, the spring constant of springs inside the scale could fluctuate due to temperature changes. To accurately measure the weight of the apple, we have to measure the calibration factor (scale readout when we put the standard mass on it.) when we measure the weight of apple, if possible, simultaneously.

Due to the complexity of practical interferometer, the response of interferometer itself to external gravitational source is not only complex but also time-dependent. In reality, we inject several calibration lines, which means we shake the End-Test-Mirror by several known frequency and amplitude sine wave. Then, we try to see these standard signal in readout of interferometer. If we can solve .

1.3.1 Transfer function of Laser Interferometer with Fabry-Perot Cavity

1.3.2 Tracking Time-dependent Response by Calibration lines

1.4 Photon Calibrator (Pcal)

1.4.1 Principle of Photon Calibrator

Photon calibrator is an additional laser with high precision intensity modulator. It is installed in front of End-Test-Mass Mirror(ETM) and can push the ETM by radiation force due to its own Laser beam. To generate any artificial h(t) by Pcal, we have to translate desired h(t) into corresponding force F(t) exerting on ETM. This can be done by using equation of motion of the ETM suspend by its suspension system. Then, we control the Pcal Laser output intensity P(t) such that the radiation force exerted on ETM is F(t) we calculated before. If we analyze it frequency domain, the h(f) introduced by P(f) can be describe by eq:

Original Pcal is proposed by Glasgow group [8]. They use single laser beam hitting on the center of ETM. The problem is that it may introduce drumhead mode vibration of ETM surface (just like the vibration mode you see when you hit the center of a drum), which introduce unwanted h(t) effectively. This problem is solved by LIGO group[9], who separate the Pcal laser beam into two beams, hitting on the nodal point of drumhead mode on the ETM surface[10].

¿ KAGRA

In order to excite same amplitude h(t) in higher frequency regime, we have to give much larger F(t) since the relationship between x(t) and F(t) in an pendulum .

1.4.2 Why do we need Photon Calibrator

1.4.3 Tracking Time-dependent Response by Calibration lines

Chapter 2

Hardware Injection through Photon Calibrator

2.1 Principle

Validate IFO by Pcal (Hardware Injection Test) As I mentioned in last chapter, the practical response of IFO is very complex. To prevent some unexpected problem including no-linear response of IFO and The best way is to provide some test source of expected GW signal.

However, it is almost impossible to prepare, for example, a binary blackhole system in laboratory. Instead, we will generated some test signal by pushing the ETM with Pcal. This procedure is called Hardware Injection Test

Motivation To under whether we can successfully reconstruct the h(t) from our interferometer, the best way is to prepare an artificial signal, sending it to interferometer, reconstructing it, finally, comparing it with original one. However it is quite difficult to generate human made gravitational wave that can be detected by current gravitational wave detector.

Requirement

Low Frequency around 100Hz the nose should below the IFO sensitivity (absolute timing ¡ ?us ns)

High Frequency above 1kHz the transfer function should as flat as possible

Amplitude of Injection Signal

$$\frac{F(t)}{M} = \frac{1}{M}\frac{2P(t)\cos(\theta)}{c} = \ddot{x}(t) \tag{2.1}$$

For $x = x_0 \sin(\omega t)$,

$$\frac{1}{M}\frac{2P_0\cos(\theta)}{c}\sin(\omega t) = -\omega^2 x_0 \sin(\omega t) \tag{2.2}$$

Thus,

$$P_0 = -\omega^2 \frac{Mc}{2\cos(\theta)} x_0 = -\omega^2 \frac{Mc}{2\cos(\theta)} L h_0 \tag{2.3}$$

$$M = 23 \text{ kg}$$
$$L = 3 \text{ km}$$
$$\theta = 0.72 \text{ deg}$$
$$c = 2.998 \times 10^8 \text{ m/s}$$

$$P_0 \text{ (Watts)} \times \frac{Gain_{\text{ Power to OFSPD}}}{2} = \underbrace{V_{\text{OFSPD}}}_{\text{Same as V}_{\text{Injection}}} \quad \text{(Volts)}$$

Therefore, the overall gain should be set in injection channel, which is in Volt unit, is

$$\omega^2 \frac{Mc}{2\cos(\theta)} L \times \frac{Gain_{\text{ Power to OFSPD}}}{2} \tag{2.4}$$

Chapter 3

Signal Generating System

3.1 KAGRA Digital System as Signal Generator

The Digital Control System used in KAGRA is based on Advanced LIGO Digital System. In this system, analog control signals can be generated from a Digital-Analog Convertor(DAC) installed in any realtime computer known as Front-End machine in the tunnel. Between the DAC output and experimental device, a customized analog low-pass filter called Anti-Image filter has been installed for removing unwanted high frequency signal, the Image, due to digitized output from DAC.

Inside the Front-End machines, the signal that will be sent to DAC are prepared by realtime softwares, which are generated from several building blocks, the realtime models, by customized parser and compiler. Each realtime model will be running at specifiable sample rate on a dedicated CPU core. However, currently, all DAC cards installed at KAGRA site are 16bit, 64kHz (65536Hz) one. Therefore, a mandatary model named Input/Output Processor(IOP) model will always running at 64kHz, communicating with DAC card and other "slave" models, in which people can put digital filters, signal generators, etc.

3.2 Noise Problem of Injection Signal

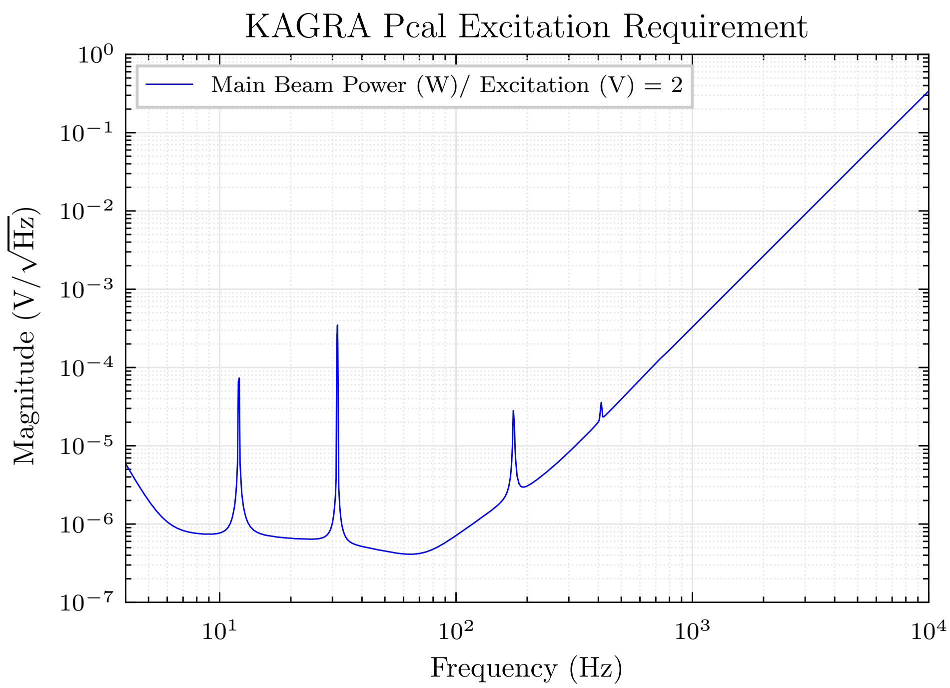

Figure 3.1: Injection Channel Noise Requirement

$$\Delta L(f) < \frac{1}{10} \times (\text{KAGRA length sensitivity}) \tag{3.1}$$

$$\Delta L(f) = \frac{2\Delta P(f)\cos(\theta)}{c}\frac{1}{M(2\pi f)^2} < \frac{1}{10}\Delta h(f)L \tag{3.2}$$

3.2.1 Noise Sources from Control Signal

Quantization Noise of DAC

The origin of quantization error is coming from the difference between desired analog output and quantized Digital to Analog Converter(DAC) output value. Roughly speaking, it shows like white noise spreading from DC to Nyquist frequency i.e. $Fs/2$. The Root Mean Square value of quantization noise has the order of voltage difference corresponding to last digit or Least Significant Bit(LSB). In time domain, we can calculate standard deviation.

$$\sigma_x = \sqrt{\frac{1}{12}}\delta x_{LSB} \tag{3.3}$$

For a 16-bit 64kHz DAC with output range between ±10Volts,

$$\begin{aligned}
\sigma_x &= \sqrt{\frac{1}{12}}\delta x_{LSB} && (3.4)\\
&= \sqrt{\frac{1}{12}}\frac{(+10) - (-10)\text{Volts}}{2^{16}} && (3.5)\\
&= 8.81 \times 10^{-5} \text{ Volts} && (3.6)
\end{aligned}$$

In frequency Domain, the quantization noise is distributed from DC to 32768Hz; therefore, we have ASD

$$\begin{aligned}
ASD &= \sqrt{PSD} && (3.7)\\
&= \sqrt{\frac{\sigma_x^2}{32768}} && (3.8)\\
&= 8.81 \times 10^{-5}\sqrt{\frac{1}{32768}} && (3.9)\\
&= 4.87 \times 10^{-7} \text{ Volts}/\sqrt{\text{Hz}} && (3.10)
\end{aligned}$$

Analog circuits

AC Power Line

Chapter 4

Noise Reduction through De-Whitening Filter

4.1 Concept of De-Whitening Filter

Consider a situation in which the Photon Calibrator received white, frequency independent, excitation signal noise, it will generate $1/f^2$ displacement noise on End-Test-Mirror because the force to displacement transfer function contains $1/f^2$ feature. If we put an analog filter that has frequency response proportional to f^2 between excitation signal and PCal excitation input port, we may create colored, f^2, laser intensity noise from white electrical noise of excitation signal. Then, such colored noise will be whitened by $1/f^2$ force to displacement transfer function. We call the f^2 analog filter *De-Whitening filter*

Practically, we use a one pole one zero analog filter with transfer function described in (ref) as our De-Whitening filter.

4.2 Circuit Design

The main circuit design is described in Fig. 4.1. The pole and zero frequency is determined by resistors and capacitors between A and B. We use 0.01μF Mica capacitor(CD30FD103FO3F made by Cornell Dubilier Electronics), whose capacitance tolerance is within 1% and capacitance drift is within $\pm(0.05\% + 0.1pF)$ to reduce filter shape uncertainty caused by pole-zero frequency drifting.

The Transfer function of this circuit is

$$\text{DewTF} = \underbrace{\frac{Z_A}{Z_A + Z_B}}_{\text{pole-zero stage}} \times \underbrace{2}_{\text{Single to Differential}} \tag{4.1}$$

where

$$A//B \equiv \frac{1}{\frac{1}{A} + \frac{1}{B}} \tag{4.2}$$

$$Z_B = (R_3 + \frac{1}{i\omega C_2}) \ // \ R_b \tag{4.3}$$

$$Z_A = (R_4 + \frac{1}{i\omega C_1}) \ // \ R_a \tag{4.4}$$

When $R_3 = R_4 = a$, $C1 = C2 = C$, Eq. (4.1) will reduce to

$$\text{DewTF} = \frac{1 + i\omega C(a + R_b)}{1 + \frac{R_b}{R_a} + i\omega C(2R_b + a(1 + \frac{R_b}{R_a}))} \tag{4.5}$$

Practically, we will choose $a = 100\Omega << R_a = 8.37\text{k}\Omega < R_b = 159\text{k}\Omega$ For DC, the gain is

$$\text{DewTF} \mid_{\omega=0} = \frac{1}{1 + \frac{R_b}{R_a}} \tag{4.6}$$

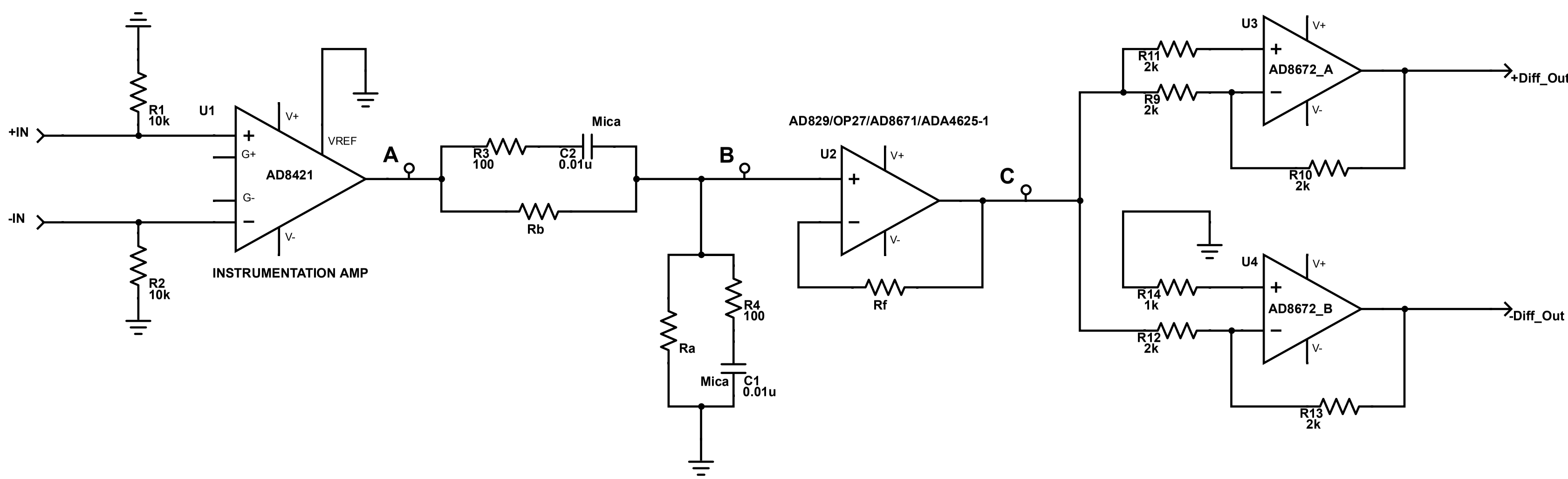

Figure 4.1: De-Whitening filter circuit. There are three points labeled as A, B,and C in the diagram. Before A, the differential input signal provided by Digital System will be converted into single single by a instrumentation amplifier AD8421. After that, a passive pole zero stage between A and B defines the dominate transfer function of De-Whitening filter. Then we put a voltage follower between B and C as a buffer to keep passive filter response. Finally, we convert signal back to differential signal to match downstream device input.

4.3 Fidelity of Injection Signal

The fidelity of injected signal is the fundamental requirement of hardware injection test. One can estimate the distortion of injected waveforms by measuring transfer function between excitation port in the software and PCal laser intensity. With De-Whitening filter, although we

4.4 Noise Reduction Performance

In principle, a

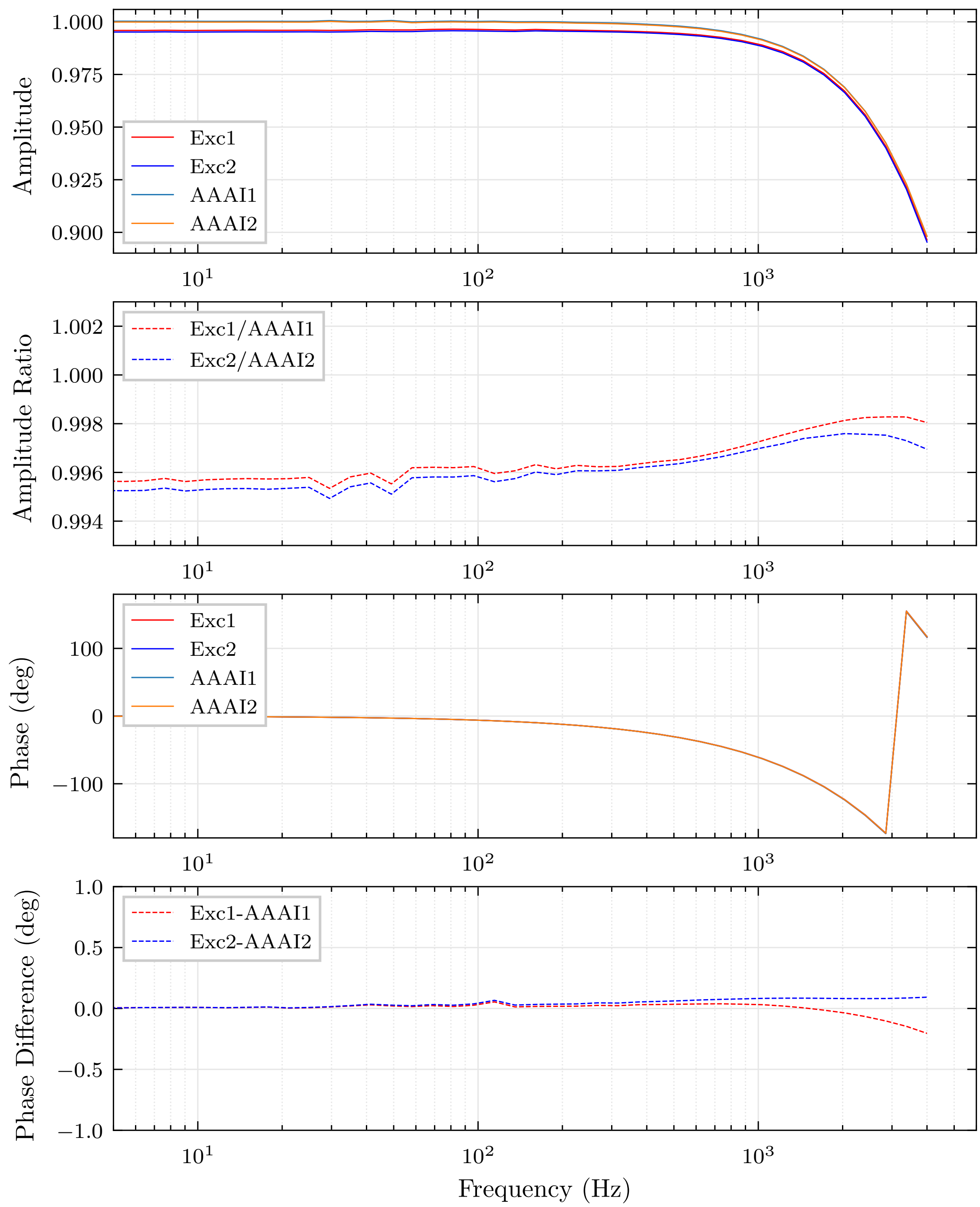

Figure 4.2: Transfer function of De-Whitening Filter with Digital Inverse Filter. The transfer function is measured in KEK cryogenic center with KAGRA standalone digital system and 64kHz salve model.

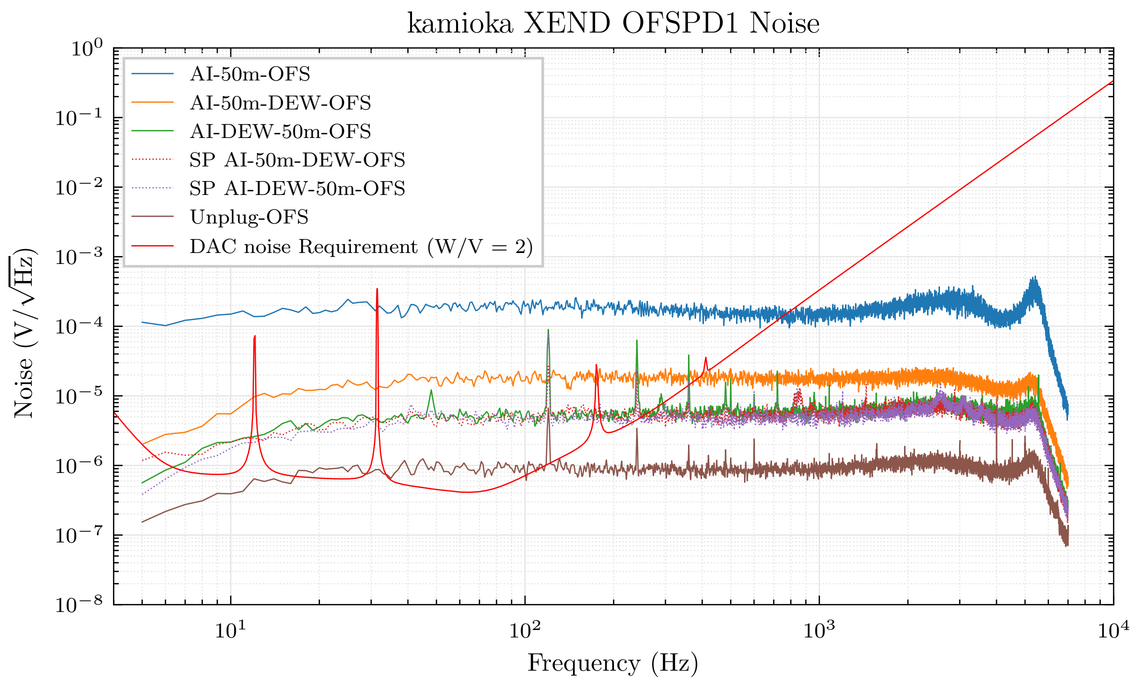

Figure 4.3: Noise

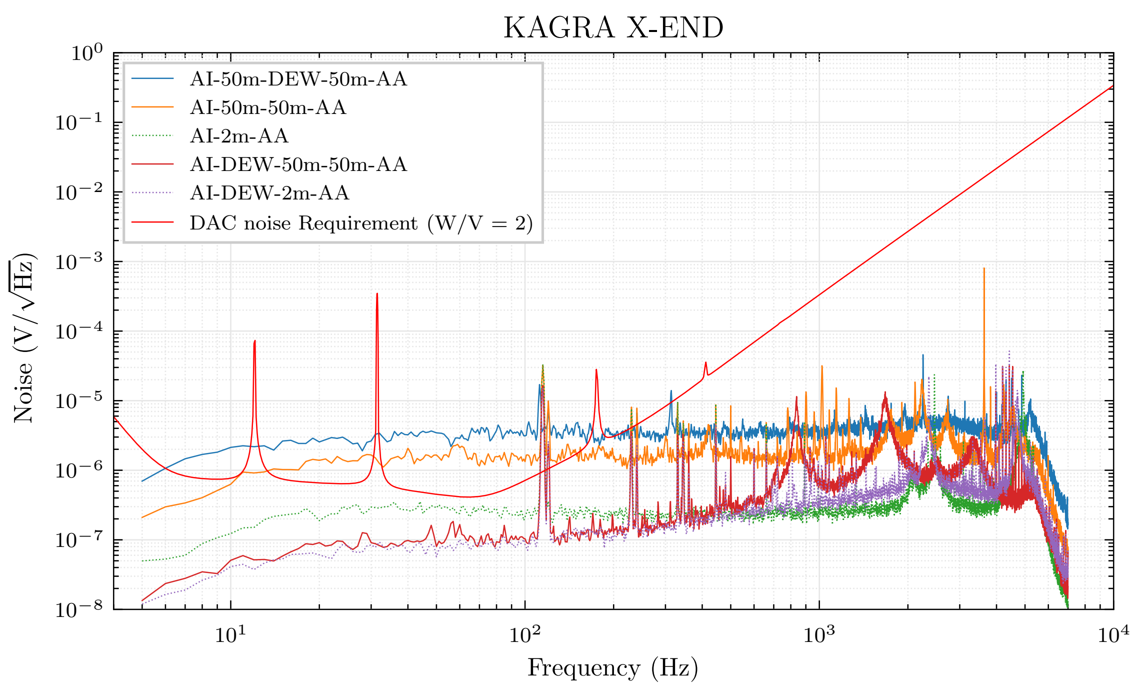

Figure 4.4: Noise

Chapter 5

Validation of Injection Channel

Without loss of generality, We injected binary blackhole signals to test performance of our De-Whitening Filter.....

Besides, We have tried to inject Sine-Gaussian signals.

Noise measurement around 100Hz the nose should below the IFO sensitivity Transfer Function measurement above 1kHz performance time delay of excitation channel (absolute timing measurement?) Distortion of Scientific Signal BBH BNS post merger

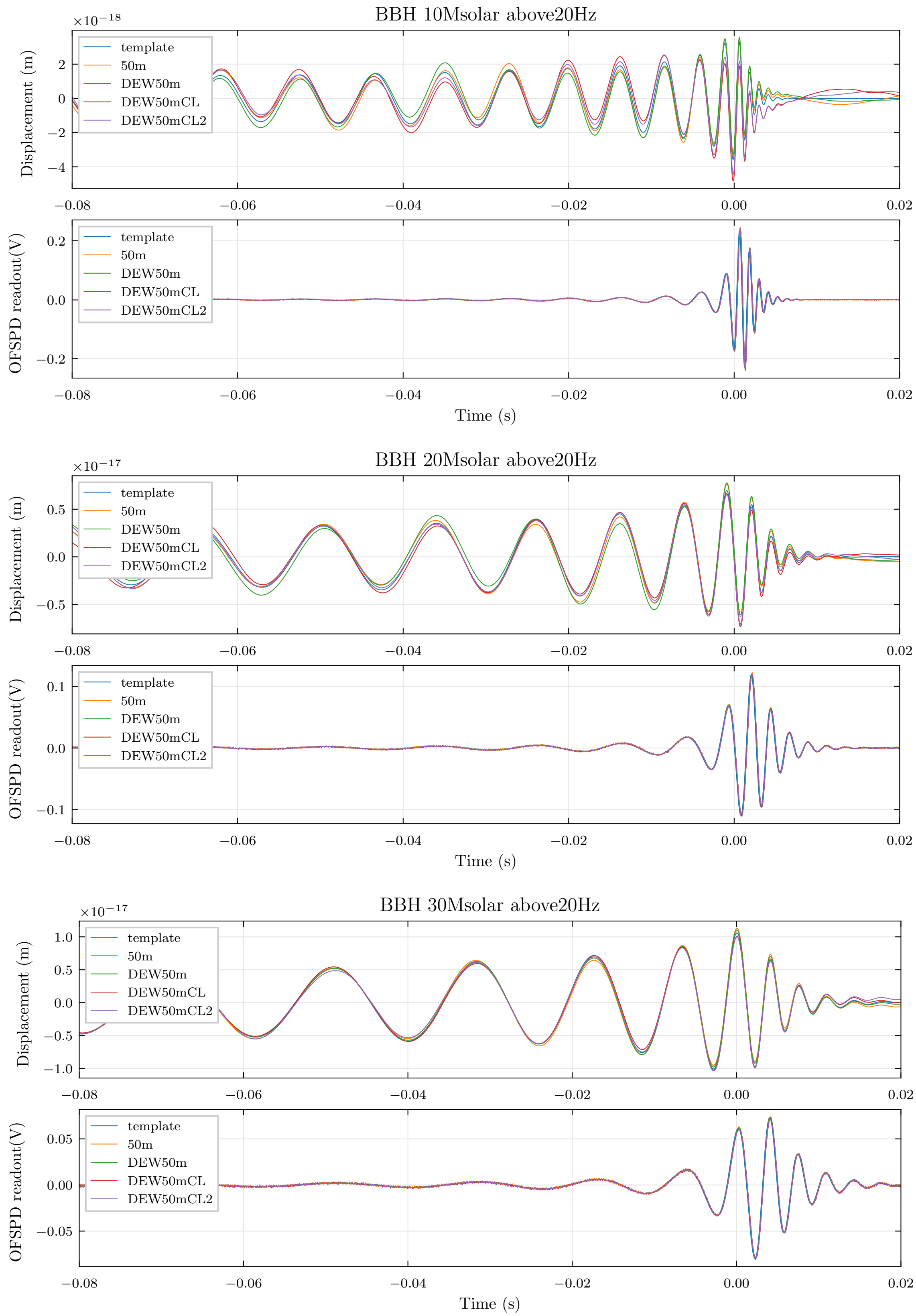

Figure 5.1: Injected Binary Blackhole Merger Signal

Chapter 6

Discussion and Future Works

Ideally, although the De-Whitening Filter can effectively increase low frequency signal resolution, it limits the maximum excitation signal can be sent to Photon Calibrator. In Fig. 6.1 we estimated how much dynamical range will be sacrificed.

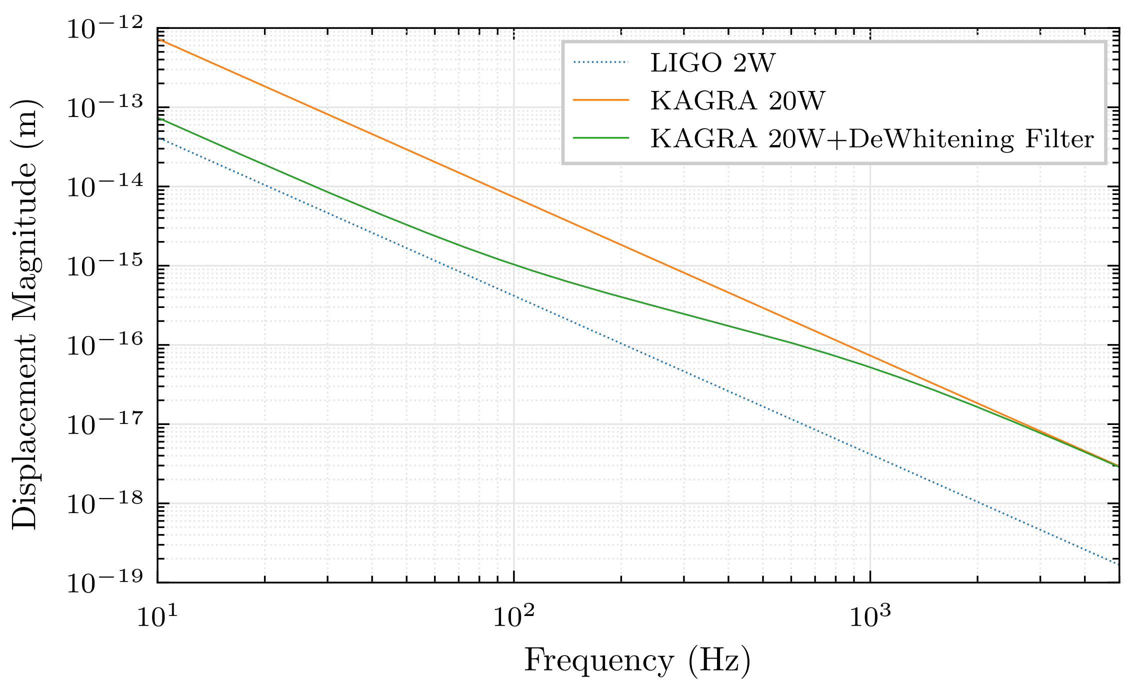

Figure 6.1: Maximum Injection Capability

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