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Assignment C1 – Classroom Work

Problem 1 (Colour Spaces)

Let an RGB colour image (f_r, f_g, f_b) with $f_r = f_g = f_b$ be given.

(a) Describe the visual appearance of this image.

(b) How do the corresponding channels of the HSV-transformed image look like?

(c) How do the corresponding channels of the YCbCr-transformed image look like?

Problem 2 (Properties of the Continuous Fourier Transform)

Prove the following properties of the continuous Fourier transform for 1-D signals:

(a) Linearity: $\mathcal{F}[af + bg] \; = \; a\mathcal{F}[f] + b\mathcal{F}[g] \qquad \forall a, b \in \mathbb{R}$

(b) Shift Theorem: $\mathcal{F}[f(x - x_0)](u) \; = \; e^{-i2\pi u x_0}\mathcal{F}[f](u)$

(c) Convolution Theorem: $\mathcal{F}[f * g](u) = \mathcal{F}[f](u) \cdot \mathcal{F}[g](u)$

Problem 3 (Discrete Fourier Transform)

For complex vectors $\boldsymbol{f} = (f_i)_{i=0}^{M-1}$ and $\boldsymbol{g} = (g_i)_{i=0}^{M-1}$, one defines their Hermitian inner product as $\langle \boldsymbol{f}, \boldsymbol{g} \rangle := \sum_{m=0}^{M-1} f_m \bar{g}_m$ where $\bar{g}_m$ is the complex conjugate of g_m.

Show that with respect to this inner product, the M vectors

$$\boldsymbol{v}_p := \frac{1}{\sqrt{M}} \left(\exp\left(\frac{2\pi i \, p \, 0}{M}\right), \exp\left(\frac{2\pi i \, p \, 1}{M}\right), ..., \exp\left(\frac{2\pi i \, p \, (M-1)}{M}\right) \right)^\top$$

with $p = 0,...,M-1$ form an orthonormal basis of the M-dimensional complex vector space $\mathbb{C}^M$.

(This property allows to interpret the DFT as a change of basis.)

Assignment H1 – Homework

Problem 1 (Discrete Convolution) (6 points)

Consider the discrete signal $f = (f_i)_{i\in\mathbb{Z}}$ with

$$f_i = \begin{cases} \frac{1}{2} & (i = 0) \\ \frac{1}{2} & (i = 1) \\ 0 & (\text{else}) \end{cases}$$

and convolve it three times with itself (i.e. in each step the convolution kernel is f).

Problem 2 (Properties of the Convolution) (6 points)

Show that the discrete convolution in 1-D,

$$(f * w)_i := \sum_{k=-\infty}^{\infty} f_{i-k} \cdot w_k$$

of infinite discrete signals $f = (f_i)$, $g = (g_i)$ and $w = (w_i)$ possesses the following properties:

(a) Linearity: $(\alpha \cdot f + \beta \cdot g) * w = \alpha \cdot (f * w) + \beta \cdot (g * w) \quad$ for all $\alpha, \beta \in \mathbb{R}$.

(b) Commutativity: $f * w = w * f$.

(c) Identity: For which signal e does $f * e = f$ hold?

Remark:
You may assume that the signals fulfill all necessary conditions such that a reordering of an infinite series is allowed where this step is necessary.

Problem 3 (Continuous Fourier Transform of a Discrete Filter) (6 points)

Let f be a 2-D continuous signal and g be defined as

$$\begin{aligned} g(x,y) \quad := \quad & \frac{1}{8}\Big(-f\Big(x-1,y-1\Big) + f\Big(x+1,y-1\Big) - 2f\Big(x-1,y\Big) \\ & +2f\Big(x+1,y\Big) - f\Big(x-1,y+1\Big) + f\Big(x+1,y+1\Big)\Big) \end{aligned}$$

Compute the Fourier transform $\mathcal{F}[g](u,v)$ and express it in terms of $\mathcal{F}[f](u,v)$.

Problem 4 (Quantisation, Noise Models, Error Measures) (12 points)

Please download the required file `is20_ex01.tgz` from ILIAS into your own directory. To unpack the data use the command `tar xvzf is20_ex01.tgz`.

Usually, the grey values in an image are represented by $2^8 = 256$ different grey values. In this problem, we want to quantise the image `camera.pgm` such that it contains afterwards only 2^q different grey values, with $q < 8$.

(a) In order to reduce the number of grey values we subdivide the co-domain into uniform intervals of size $d = 2^{8-q}$. All values that lie in an interval are mapped to the mean value of the interval:

$$u_{i,j} := \left(\left\lfloor \frac{u_{i,j}}{d} \right\rfloor + \frac{1}{2} \right) \cdot d$$

where $\lfloor x \rfloor := \max\{m \in \mathbb{Z} \mid m \leq x\}$ denotes the floor function.

Consider the file `quantisation.c` and supplement the missing code such that it performs the quantisation described above. Compile the programme using the command

```
gcc -O2 -o quantisation quantisation.c -lm
```

and apply it to the image `camera.pgm` using different values for q.

Remark: You can display an image `img.pgm` by using the command `display img.pgm`. As you have not yet implemented any algorithm for creating noise at this stage, you may use 0 for all program parameters regarding noise.

(b) Integrate a routine into the quantisation program that computes the mean squared error (MSE) and the peek-signal-to-noise ratio (PSNR) between two given images.

(c) Implement routines that generate uniformly distributed noise $n^{\mathrm{U}}_{i,j}$ in the interval $[-a, a]$ and Gaussian noise $n^{\mathrm{G}}_{i,j}$ by using the Box-Muller algorithm with standard deviation $\sigma = a$ and mean $\mu = 0$. Integrate this computed noise additively into the quantisation process:

$$u_{i,j} := \left(\left\lfloor \frac{u_{i,j}}{d} + n_{i,j} \right\rfloor + \frac{1}{2} \right) \cdot d \; ,$$

Use $a = 0.125$, $a = 0.25$, $a = 0.5$, $a = 1$ and $a = 5$. How do the results change for both noise models? Try out different values for q. Consider also the values for the MSE and the PSNR.

Problem 5 (Interpretation of the Fourier Spectrum) (6 points)

Please download the archive `is20_ex01.tgz` from ILIAS into your own directory. You can unpack them with the command `tar xvzf is20_ex01.tgz`.

In this archive you can find the programme `fourierspectrum`. It computes the logarithmically transformed Fourier spectrum $c \ln(1 + \hat{f}(u, v))$ of an image $f(x, y)$ by means of the FFT. The lowest frequencies have been shifted towards the centre of the image.

Apply this programme to the images `pattern.pgm`, `gauss1.pgm`, `gauss2.pgm`, `gauss3.pgm` and `tile.pgm` by typing `./fourierspectrum`, and visualise them by using `display image_name.pgm &`.

(a) Why do you observe a three-point spectrum for `pattern.pgm` and why it is located this way?

(b) Why is the DFT of `gauss3.pgm` not rotationally symmetric?

(c) Can you find aliasing artifacts in the spectrum of `tile.pgm`? If you can, describe them. (This image has been downsampled with `display` to half its size.)

Submission

Please remember that up to three people from the same tutorial group can work and submit their results together. Note that in order to submit results as a group you have to create a submission group in ILIAS. A submission result will only be accepted for the submitting group as created in ILIAS!

The solutions have to be submitted in two parts:

1. The solutions to the theoretical (H1) problems 1, 2, and 3 have to be submitted in a single `pdf` file, which can be either digitally created or contain a scanned document.
2. The solutions to the practical problems 4 and 5 have to be submitted in a single archive file:
   - Rename the main directory `Ex01` to `Ex01_<your_name>` and use the command
     ```
     tar czvf Ex01_<your_name>.tgz Ex01_<your_name>
     ```
     to pack the data. The directory that you pack and submit should contain the following files:
     - the supplemented source code used in 4(a)-4(c).
     - one image using $q = 3$ for subproblem 4(a) and
     - one image for each noise model for $q = 3$ and $a = 0.25$ for subproblem 4(c).
     - a text file README that contains the answers to the questions of 4(c) and of 5 as well as information on all people working together for this assignment.
   - The file format can be a gzipped tar archive (`.tgz`) or a zip archive (`.zip`). No other file formats are accepted.
   - Please make sure that only your final version of the programmes and images are included.

Submit the two files via ILIAS.
(Remark: Please do **not** use the button "Upload Multiple Files as Zip-Archive").

Deadline for submission: Thursday, May 21st, 11:59 pm