University of Stuttgart
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Complex Network Systems

Preferential attachment

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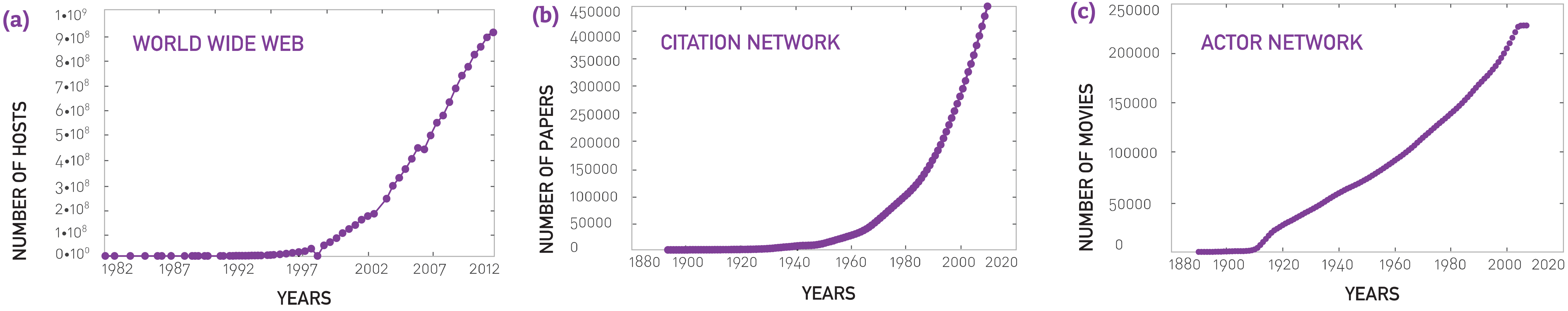
Why are hubs and power laws absent in random networks?

Why do so different systems as the WWW or the cell converge to a similar scale-free architecture?

Networks expand through the addition of new nodes



Random network model assumes that there is a fixed number of nodes

In real networks, the number of nodes continually grows through the addition of new nodes

Nodes prefer to link to the more connected nodes

- **WWW**
  - Nodes we know are not entirely random: Google, Facebook, Twitter
  - Rarely we encounter the billions of less-prominent nodes
- **Citation network**
  - The more cited a paper is, the more likely we heard about it
- **Actor network**
  - The more movies an actor has played in, the more familiar the casting director is with their skill

Random network model assumes that the interaction partners of a node are randomly chosen

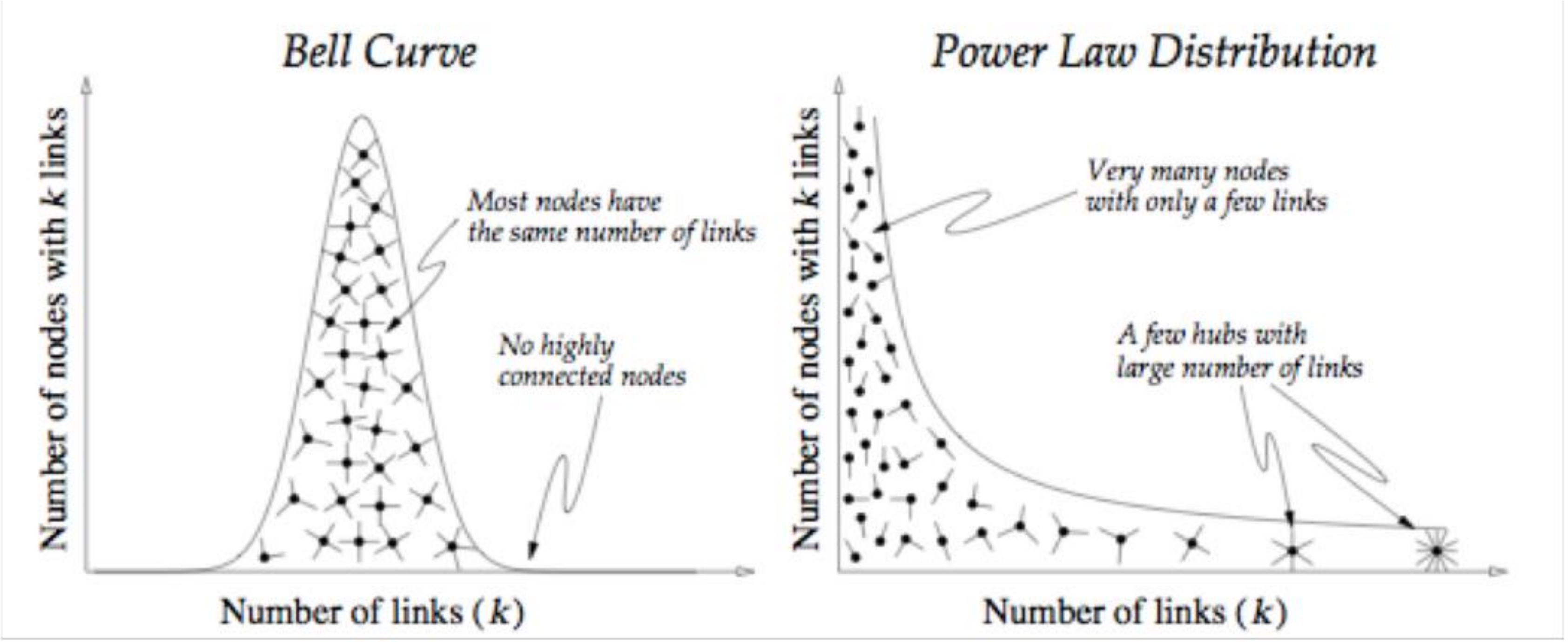
Most real networks prefer to link to the more connected nodes

Preferential attachment

Preferential attachment

- Process also known as
  - Rich get richer
  - Matthew effect
    - “For whoever has will be given more, and they will have an abundance. Whoever does not have, even what they have will be taken from them.”

Models



Bell Curve
Number of nodes with k links
Most nodes have
the same number of links
No highly
connected nodes
Number of links (k)
Power Law Distribution
Number of nodes with k links
Very many nodes
with only a few links
A few hubs with
large number of links
Number of links (k)

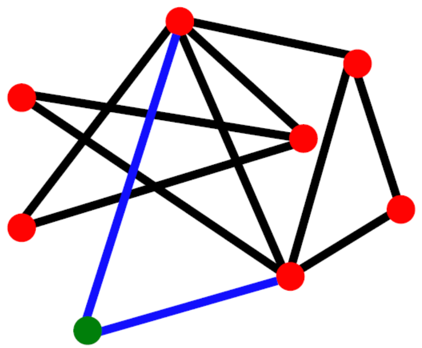
$G(n, p)$

?

Barabási-Albert preferential attachment model

- Nodes are in order $1,2,\ldots,n$
- At step j, let k_i be the indegree of node $i < j$
- A new node j arrives and creates m out-links
- Probability of j linking to a previous node i is proportional to degree k_i of node i

$$P(j \to i) = \frac{k_i}{\sum_t k_t}$$



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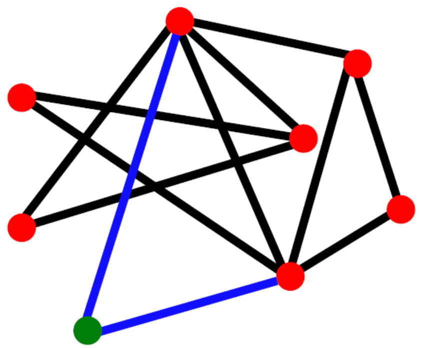
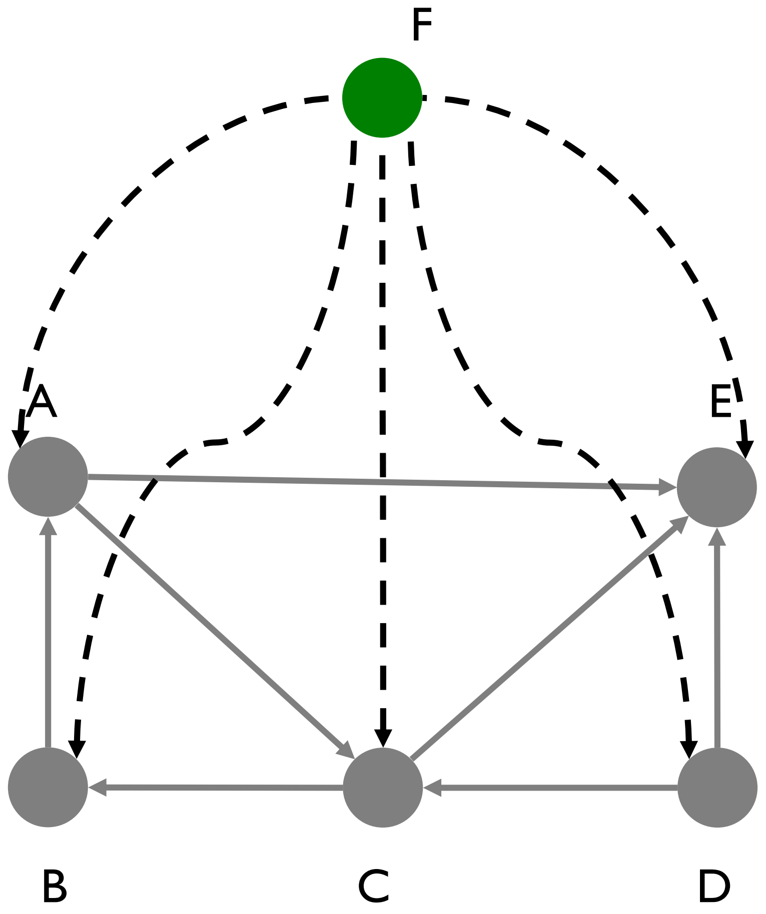


Example



Node F arrives

$$P(F \to A) = \frac{1}{7} \qquad P(F \to D) = 0$$

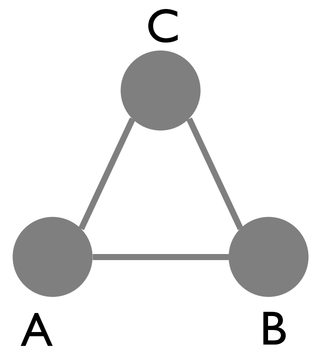
$$P(F \to B) = \frac{1}{7} \qquad P(F \to E) = \frac{3}{7}$$

$$P(F \to C) = \frac{2}{7}$$

Each node has an equal number of edges

- Probability of choosing any node is 1/3

A A B B C C



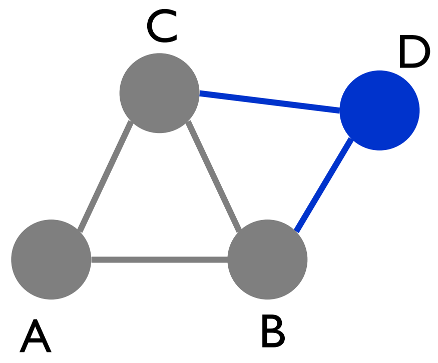
Add a new node, it will have m edges

- Take $m = 2$

Draw two random elements from the array

- Suppose they are B and C

A A B B B C C C D D

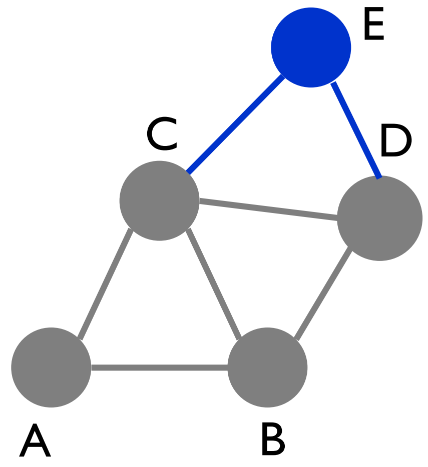


Probabilities of choosing A, B, C, D are 1/5, 3/10, 3/10, and 1/5

Add a new node

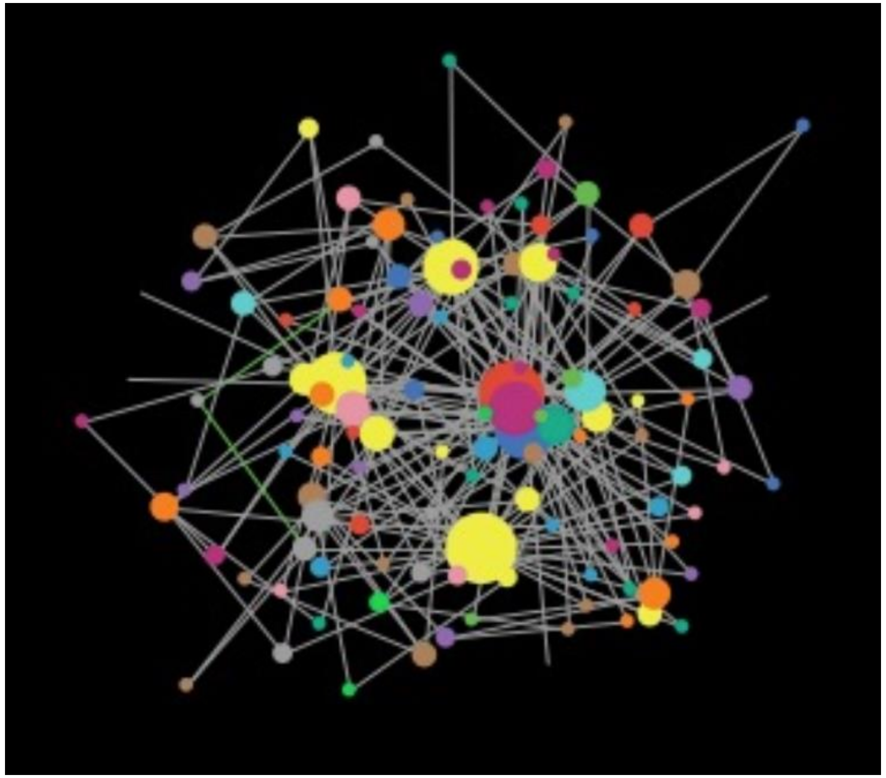
Draw nodes from the array

A A B B B C C C C D D D E E

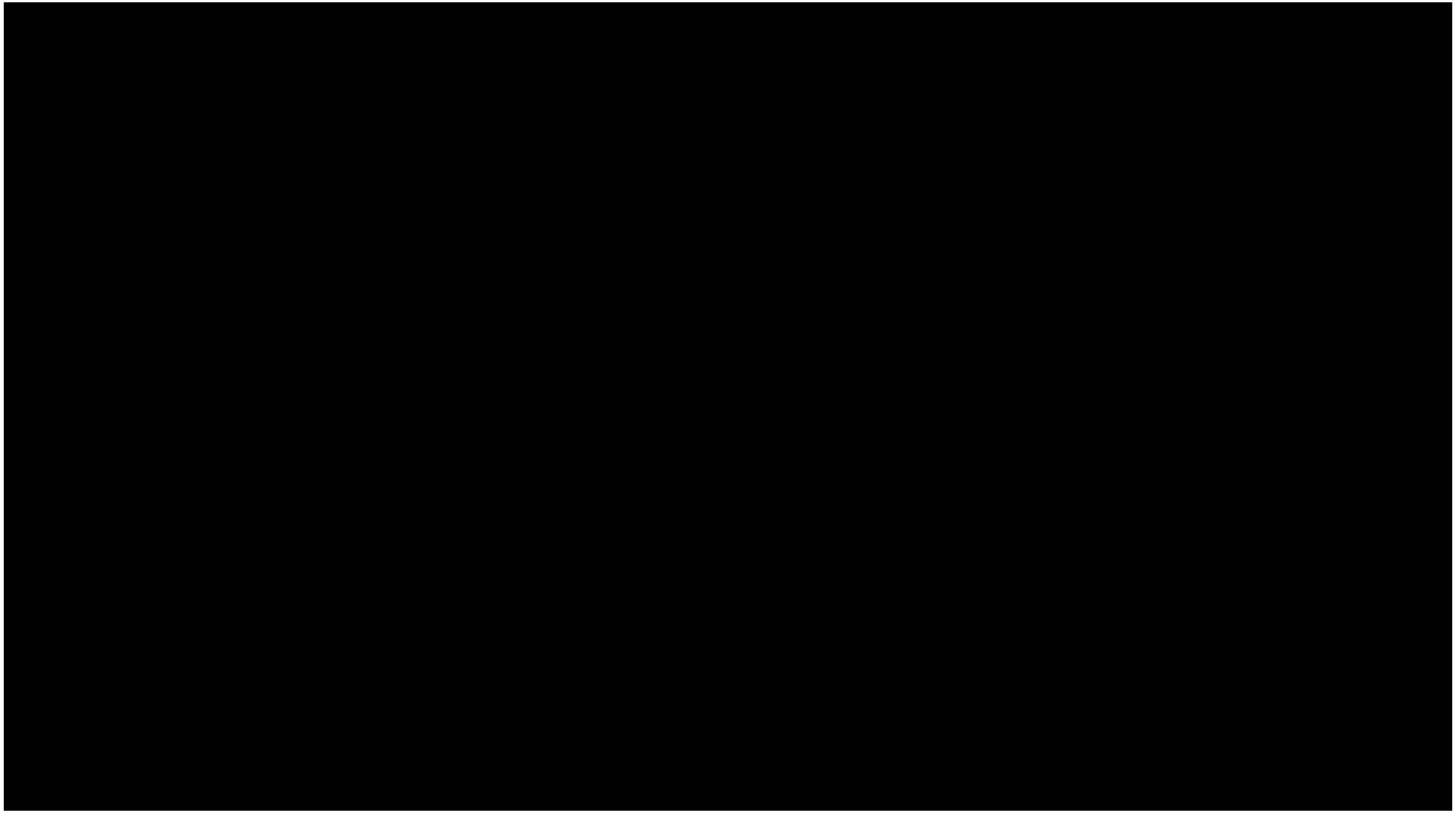


After a while

Preferential attachment ($m = 2$)



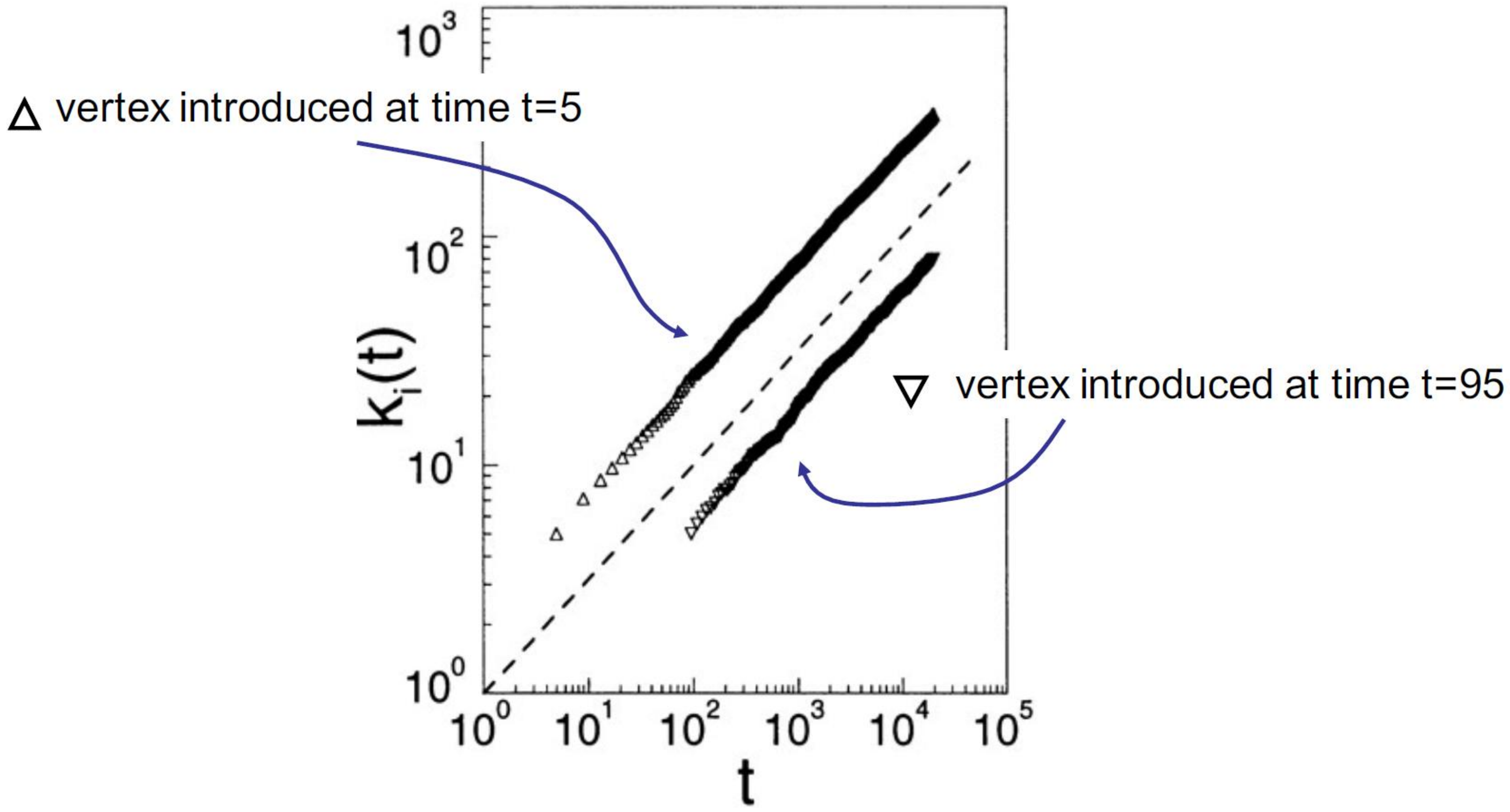
Random



https://vimeo.com/53071346

Barabási, A. and Albert, R., Emergence of Scaling in Random Networks, Science, 286(5439): 509-512, 1999.

Degree dynamics



Degree distribution

$$P(k) = 2m^2k^{-3}$$

- Power-law degree distribution with $\alpha = 3$

- Power-law indegree distribution

$$P(k) \cong k^{-(1+\frac{1}{1-p})}$$

α

Summary

- Is the combination of growth and preferential attachment why networks are scale free?
  - Growth and preferential attachment are jointly needed to generate scale-free networks
  - If both present, they lead to scale-free networks
- All known models and real systems that are scale-free have preferential attachment

Sources

- Leskovec, J. Analysis of Networks, CS224W, Stanford University (2018), http://web.stanford.edu/class/cs224w/
- Mateos, G. Degrees, Power Laws and Popularity, University of Rochester, 2018.
- Zafarani, R., Abbasi, M. A. and Liu, H. *Social Media Mining: An Introduction*, Cambridge University Press, 2014.
- Barabási, A. Network Science, http://networksciencebook.com