REPRESENTATIVE PIXELS COMPRESSION ALGORITHM USING GRAPH SIGNAL PROCESSING FOR COLORIZATION-BASED IMAGE CODING

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ABSTRACT

This paper deals with the colorization-based image coding algorithm. In this algorithm, a color image is compressed by encoding its luminance image by standard coding method such as JPEG coding and by storing several color pixels called as representative pixels (RPs). In decoding phase, a color image is restored from luminance image and color information of RPs using the image colorization technique. While previous studies have achieved a high coding performance, the compression method of RPs has not been considered because the positions of RPs are inhomogeneous. In order to improve the image coding performance, this paper proposes the RPs compression algorithm using the graph Fourier transform. Numerical results show that proposed algorithm achieves better performance than JPEG2000 coding.

***Index Terms*—** colorization, image compression, signal processing on graphs, graph Fourier transform

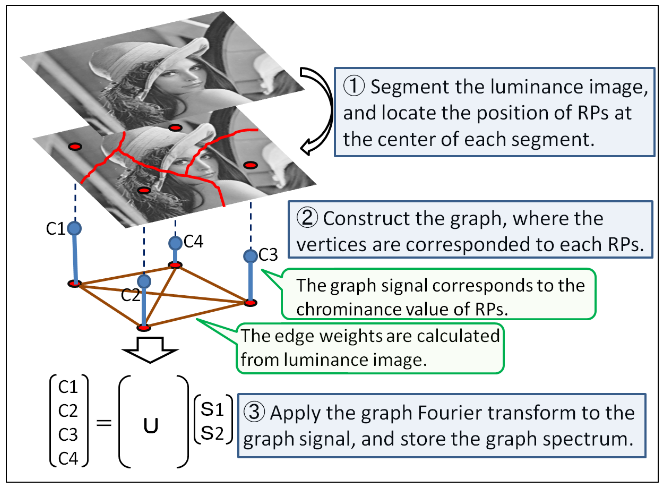

Fig. 1. Illustration of the proposed RPs coding algorithm based on the graph Fourier transform.

1. INTRODUCTION

The image colorization technique provides a color image from a luminance image and several color pixels called as representative pixels (RPs), which have the chrominance values and their positions [1–3]. Based on the image colorization, some image compression techniques have been proposed [4–8]. In encoding phase of these techniques, appropriate RPs are extracted from a color image, then the information of RPs are stored, and the luminance image is compressed by standard coding method such as JPEG coding. In decoding phase, the colorization algorithm recovers a color image from luminance image and RPs. Because the colorization algorithm can recover a full color image from a few number of RPs, this compression technique achieves a high coding performance. In [5], in order to minimize the recovery error, the chrominance values of RPs are optimized. In [6], Lee et al. have been proposed the colorization-based coding algorithm using sparse optimization.

The performance of the colorization-based image coding algorithm depends on its colorization accuracy and compression rate of the information about RPs, which consists of the information of their coordinates and chrominance values. Simple way to achieve high information compression rate of RPs is to choose pixels at grid points as RPs. If all RPs are on grid points, it is not required to store the positions of all RPs, and the chrominance values can be compressed by applying a standard coding method such as JPEG because the RPs can be represented as a small size image. However this idea leads to a low colorization accuracy because grid points pixels are not appropriate to be used as RPs. Although high colorization performance can be achieved by choosing RPs whose positions are not grid points, we cannot compress their chrominance values by any image coding method due to inhomogeneity of their positions.

In order to achieve high colorization accuracy with less information of RPs, this paper proposes a colorization-based image coding algorithm using a image segmentation technique and the graph Fourier transform. Empirical results show that the state-of-the-art superpixel method [12] can find good RPs by choosing a central pixel at each segment as RPs after image segmentation. Furthermore, it is not required to store the positions of RPs because we can obtain exact posi-

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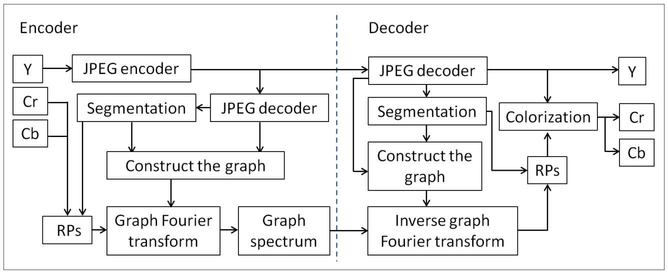

Fig. 2. The flowchart of the proposed colorization-based image coding algorithm using graph signal processing.

tions of RPs at decoding phase by applying the same image segmentation algorithm for encoding phase to a luminance image. While the positions of RPs extracted based on the image segmentation are inhomogeneous, we can compress their chrominance values by using the graph Fourier transform, which has been studied in the field of the graph signal processing [9–11]. Figure 1 illustrates the proposed RPs coding algorithm, and Fig. 2 shows the flowchart of the proposed algorithm. First the luminance image is segmented based on the image segmentation algorithm, and the RPs are extracted as the center pixels of each segment. Then the graph is constructed by utilizing the each RPs as vertex, and the weight of edges are calculated from luminance image. The chrominance values of RPs are represented as the graph signal, which is transformed to the graph spectrum based on the graph Fourier transform. Then the value of graph spectrum corresponding to the low frequency is stored. In decoding phase, we obtain the same graph constructed in encoding phase by applying the same image segmentation method to a luminance image. Then the stored graph spectrum is transformed to the chrominance values of RPs based on the inverse graph Fourier transform. Finally, the colorization algorithm recovers the color image using a luminance image and RPs. Numerical results show that the proposed algorithm achieves better performance than JPEG2000 coding.

2. GRAPH FOURIER TRANSFORM

This work uses the Fourier transform of a function on a graph [9, 10], and this section gives its brief description.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$ denote an undirected connected weighted graph consisting of a finite set of vertices $\mathcal{V}$ with $|\mathcal{V}| = P$, a set of edges $\mathcal{E}$, and a weighted adjacency matrix W whose (i, j)th element $w_{i,j}$ represents the weight of the edge $e = (i, j)$ connecting vertices i and j. The degree matrix D denotes a diagonal matrix whose ith diagonal element is equal to the sum of the weights of all the edges incident to vertex i, and the normalized graph Laplacian is defined as follows,

$$L = D^{-1/2}(D - W)D^{-1/2}. \tag{1}$$

Because L is a real symmetric matrix, it has a set of orthogonal eigenvectors. Let $\boldsymbol{u}_i, (i = 1, \ldots, P)$ and $\lambda_i, (i = 1, \ldots, P)$ denote eigenvectors and eigenvalues, respectively, which satisfy $L\boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i$ and $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_P$. Then we obtain the following equation,

$$L = U \Lambda U^T, \tag{2}$$

where $U = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_P]$ and $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_P)$. The Fourier transform of the graph signal $\boldsymbol{f} \in R^P$ is defined by

$$\boldsymbol{s} = U^T \boldsymbol{f}, \tag{3}$$

where $\boldsymbol{s} \in R^P$ is graph spectrum of graph signal, and the inverse graph Fourier transform is given as follows,

$$\boldsymbol{f} = U\boldsymbol{s}. \tag{4}$$

Some empirical characteristics of eigenvectors are reported in [9, 10]. The values of ith and jth element of each eigenvector tend to be similar when they are connected by edge with a large weight value $w_{i,j}$. The each element of $\boldsymbol{u}_1$ have almost the same values, and the element of each eigenvector tend to have various values according to increasing eigenvalues.

3. REPRESENTATIVE PIXELS COMPRESSION USING GRAPH FOURIER TRANSFORM

This paper focuses on the compression of RPs in order to achieve a high coding performance. As mentioned in Section 1, the performance of the colorization-based image coding algorithm depends on its colorization accuracy and compression rate of the information about RPs. To achieve high colorization accuracy with less information of RPs, this paper proposes a colorization-based image coding algorithm using a image segmentation technique and the graph Fourier transform.

First we consider the extraction method of RPs. Because colorization algorithm colorizes an image by expanding the color information from RPs, RPs are preferably to be in the center of objects in the image rather than in regions near the edges. Hence this paper introduces the image segmentation technique in order to choose desirable RPs. In encoding phase, the state-of-the-art superpixel method proposed in [12] is applied, and the center pixel in each segment is extracted as RPs. The advantage of this method is that the volume of information storing the positions of RPs is not required because we can obtaine the postions of RPs by applying the same segmentation method to the luminance image in decoding phase.

Next we focus on the method to store the chrominance values of RPs. Let $\Omega = \{(m_i, n_i) | i = 1, \ldots, P\}$ and C_{m_i,n_i} denote the set of coordinates of RPs and a chrominace value of the (m_i, n_i)th pixel, where P is the number of RPs. This

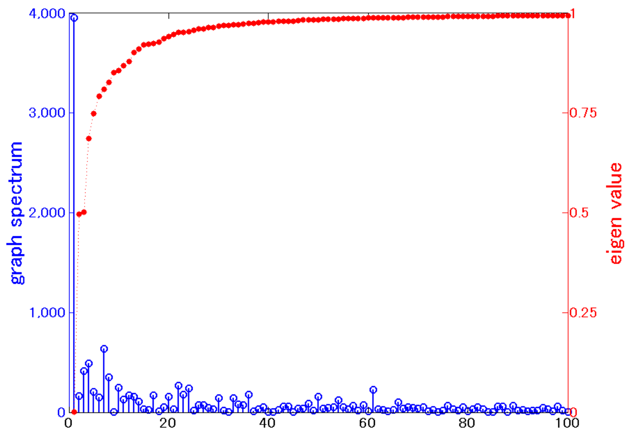

Fig. 3. The graph spectrum of the Cb values of RPs (Lenna image).

paper constructs the graph such that the pixels in Ω are used as the vertices and proposes to compress the chrominance values of RPs as the graph signal. Let us define the graph signal $\boldsymbol{f} \in R^P$ as follows,

$$\boldsymbol{f} = [C_{m_1,n_1}, C_{m_2,n_2}, \ldots, C_{m_P,n_P}]^T. \tag{5}$$

In order to compress the graph signal effectively, we consider the weighted adjacency matrix W such that the graph signal is represented by a few eigenvetors of the graph Laplacian. This paper makes an assumptions that the pixels have similar chrominance values when the their distance is small, or they have similar luminance values. Based on this assumption, this paper defines a graph of RPs and gives its (i, j)th element of the weighted adjacency matrix as follows,

$$w_{i,j} = \exp\left(-\alpha \Delta d_{(i,j)}\right) \exp\left(-\beta \Delta y_{(i,j)}\right), \tag{6}$$

where α and β are given constants. In this equation, $\Delta d_{(i,j)}$ and $\Delta y_{(i,j)}$ denote the distance between vertices and the difference of the luminance values between vertices defined by,

$$\Delta d_{(i,j)} = \sqrt{\left(\frac{m_i}{M} - \frac{m_j}{M}\right)^2 + \left(\frac{n_i}{N} - \frac{n_j}{N}\right)^2}, \tag{7}$$

and

$$\Delta y_{(i,j)} = |Y_{m_i,n_i} - Y_{m_j,n_j}|, \tag{8}$$

where M, N denote the image size, and $Y_{m_i,n_i} \in [0, 1]$ denotes the luminance value of the (m_i, n_i)th pixel. Based on the equations from (1) to (3), the graph signal $\boldsymbol{f}$ is transformed to the graph spectrum $\boldsymbol{s}$.

Figure 3 shows an example of the 100 smallest eigenvalues of graph Laplacian and the absolute value of its corresponding graph spectrum transformed from the graph with Cb values of 8000 RPs. As can be seen, the values of spectrum are decreased according to increasing eigenvalues, and therefore this paper proposes an RPs compression method to store the information of the graph spectrum corresponding to the c smallest eigenvalues. Then the proposed algorithm requires the following volume of information Q to store a color image,

$$Q = 2\{(c-2)q_1 + 2q_2\} + Q_y, \tag{9}$$

where q_1 and q_2 denote the quantization bit size, and Q_y denotes the volume of information to store a luminance image. $2q_2$ represents the maximum value and minimum value of graph spectrum.

In decodering phase, we can construct the same graph in encoding phase from the luminance image, and the graph Laplacian eigenvectors corresponding to the c small eigenvalues are calculated. Then the stored c graph spectrums are transformed to the chrominance values of RPs based on the inverse graph Fourier transform (4). Finally, the chrominance values of the whole image are recovered using Levin's colorization algorithm [1].

(a) (b) (c) (d)

Fig. 4. Test images: (a) Lenna, (b) Pepper, (c) Airplane and (d) Sailboat.

4. NUMERICAL EXAMPLES

This section provides the numerical results. In order to evaluate the effectiveness of the proposed algorithm, this paper compares the proposed algorithm with the JPEG2000 and the colorization-based coding algorithm proposed in [6]. Figure 4 shows test images, and the size of these images is 256×256. In the proposed algorithm and the algorithm [6], the luminance image is compressed by JPEG2000. The proposed algorithm uses the image segmentation algorithm proposed in [12], and each image is divided into 8000 segments. The parameters of the proposed algorithm are set as $\alpha = 3.5$, $\beta = 2.5$, $q_1 = 8$ and $q_2 = 64$, and the value of c is determined such that the volume of information of the proposed algorithm is equal to that of JPEG2000. In coding phase of the proposed algorithm, the chrominance values of RPs are optimized to minimize the recovery error based on the method proposed in [5].

Table 1, Fig. 5 and 6 show the results. Table 1 shows the size of the volume of information storing the color image and the peak signal to noise ratio (PSNR). In order to calculate the PSNR, the recovered chrominance images and the luminance image are transformed to the RGB color image, and they are

Table 1. Numerical results.

Image	algorithm	byte	PSNR[dB]
Lenna	JPEG2000	3641	29.19
	algorithm proposed in [6]	3641	29.24
	Proposed algorithm	3641	**29.63**
Pepper	JPEG2000	3634	**27.09**
	algorithm proposed in [6]	3634	25.70
	Proposed algorithm	3634	26.82
Airplane	JPEG2000	3627	28.07
	algorithm proposed in [6]	3627	27.82
	Proposed algorithm	3627	**28.43**
Sailboat	JPEG2000	3652	26.98
	algorithm proposed in [6]	3654	27.33
	Proposed algorithm	3652	**27.77**
Average	JPEG2000	–	27.83
	algorithm proposed in [6]	–	27.52
	Proposed algorithm	–	**28.16**

compared. As can be seen, the proposed algorithm achieves the high coding performance. Figure 5 and 6 show the recovered lenna images and those zoomed images, respectively. The proposed algorithm provides less false-color pixels than JPEG2000 and the method proposed in [6].

5. CONCLUSIONS

This paper proposes the colorization-based image coding algorithm using the graph Fourier transform. In encoding phase, the graph Fourier transform is applied to the compression of the chrominace values of RPs, and the inverse Fourier transform recovers the chrominance values of RPs in decoding phase. In the algorithm, the coordinates of RPs is not stored because the same segmentation method is used to choose RPs, and the information of chrominance values in RPs are compressed by the graph Fourier transform. Numerical results show that the effectiveness of the proposed algorithm.

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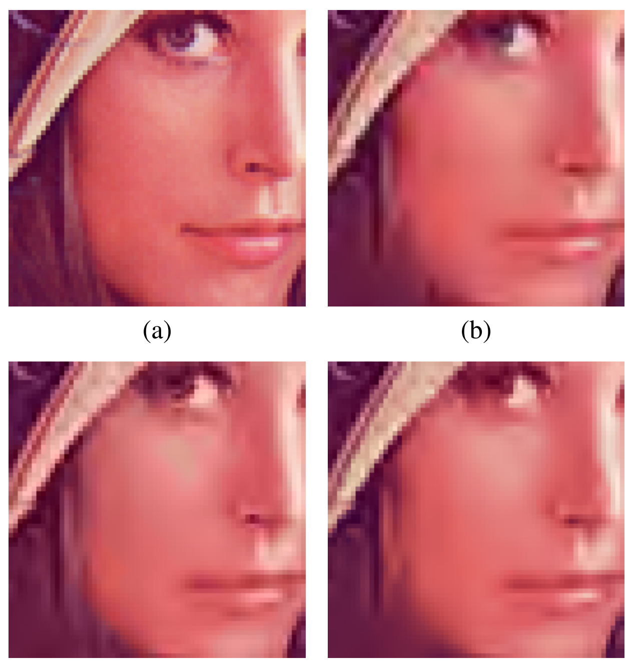
(a) (b) (c) (d)

Fig. 5. Visual comparison of lenna image: (a) original image, (b) JPEG2000, (c) algorithm proposed in [6] and (d) proposed algorithm.



(a) (b) (c) (d)

Fig. 6. Visual comparison of lenna image (zoom in): (a) original image, (b) JPEG2000, (c) algorithm proposed in [6] and (d) proposed algorithm.

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