STRUCTURE-ADAPTIVE VECTOR MEDIAN FILTER FOR IMPULSE NOISE REMOVAL IN COLOR IMAGES

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ABSTRACT

A structure-adaptive vector median filter (SAVMF) for removal of impulse noise from color images is presented in this paper. A color image is represented in quaternion form, and then quaternion Fourier transform is employed to detect the dominant orientation of the pattern in a local neighborhood. Based on the local orientation and its strength, the size, shape and orientation of the support window of vector median filter (VMF) can be adaptively computed, and thus structure-adaptive VMF is implemented. Experimental results exhibit the validity of the proposed method by showing clearly performance improvements both in noise removal and in detail preservation, compared to other VMF-based vector filters.

Index Terms— Vector median filter, orientation detection, impulse noise, color image.

1. INTRODUCTION

Noise removal is one of the most important and fundamental tasks in image processing. As a common and typical contamination, impulse corruption degrades image quality severely, since the corruption significantly destroys image structures and contents. Therefore, numerous filtering techniques for impulse noise suppression were developed in the past years. For color images, impulse noise removal techniques [1] mainly include component-wise methods and vector filtering methods. Vector filtering techniques, which treat color images as vector fields, are often more effective and preferred than component-wise methods, due to the strong correlation among color channels.

The most classical vector filters are vector median filter (VMF) [2], basic vector directional filter (BVDF) [3], and directional-distance filter (DDF) [4]. These methods are based on the reduced ordering principle and output the median vector samples according to different vector distance criteria. However, these classical filters perform maximum amount of smoothing on each pixel, regardless whether it is contaminated or not. Therefore, some natural improvements have been developed introducing more efficient switching vector filters [5-7] and weighted vector filters [8,9]. Weighted vector filters perform different amount of smoothing according to different weighting coefficient sets. Switching vector filters restore only the noisy pixels detected, leaving the uncorrupted ones unaltered. Other techniques such as partial differential equation (PDE), wavelets, fuzzy sets, quaternions, sparse representation, and low-rank matrix approximation, also have been developed to suppress impulse noise in color images.

Some structure- or window- adaptive filters have been proposed for image smoothing or denoising. Yang et al. [10] introduced an anisotropic smoothing method based on local orientation estimation. Later Greenberg and Kogan [11] improved Yang et al.'s method for image denoising. Goshtasby and Satter [12] implemented anisotropic image smoothing using adaptive windows based on image gradients. However, all the above methods are developed for grayscale image and mainly used for image smoothing.

In this paper, we propose a structure-adaptive vector median filter (SAVMF) for suppressing impulse noise in color images. By computing the orientation of any local pattern and the strength of orientation, the shape, size and orientation of filter window can be determined, resulting in the proposed SAVMF.

2. PROPOSED METHOD

By adaptively adjusting the shape, size and orientation of the window of VMF on each pixel, the proposed SAVMF is implemented. To compute the orientation of a local pattern in color images, we need to analyze the power spectrum of quaternion Fourier transform of the color pattern.

2.1. Quaternion

The quaternion algebra [13] is a non-commutative four-dimensional algebra. A quaternion includes one real part and three orthogonal imaginary components, and is usually represented in the following form:

$$q = a + b\,i + c\,j + d\,k\,, \tag{1}$$

where a, b, c and d are real coefficients, and i, j and k are complex operators that satisfy the following rules:

$$\begin{cases} i^2 = j^2 = k^2 = ijk = -1 \\ ij = -ji = k, \ \ jk = -kj = i, \ \ ki = -ik = j \end{cases} . \tag{2}$$

It can be observed that the multiplication of quaternions is not commutable. The modulus and conjugate of quaternion q are respectively defined as follows:

$$|q| = \sqrt{a^2 + b^2 + c^2 + d^2} , \tag{3}$$

$$q^* = a - bi - cj - dk . \tag{4}$$

A quaternion is a pure quaternion if its real component is equal to zero. A quaternion with unit modulus is called a unit quaternion.

A color image is usually represented in pure quaternion form, which was initially proposed by Sangwine et al. [14, 15].

2.2. Local orientation detection

Let a color image $\boldsymbol{f}(x,y)$ be represented in pure quaternion form:

$$\boldsymbol{f}(x,y) = R(x,y)\,i + G(x,y)\,j + B(x,y)\,k \tag{5}$$

where $R(x,y)$ 、 $G(x,y)$ 、 $B(x,y)$ are real functions and denote the red, green, and blue channel values at position (x,y). Then, the quaternion Fourier transform (QFT) of $\boldsymbol{f}(x,y)$ is expressed as follows [14,15]:

$$F(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,e^{-\mu(ux+vy)}dxdy \tag{6}$$

where μ is a unit pure quaternion and typically set to $(i+j+k)/\sqrt{3}$. Note that QFT has three forms (left-, right- and two- side QFT) due to the non-commutativity of quaternion multiplication. Without loss of generality, we use the right-side QFT in this paper.

According to the slice theorem of Fourier transform, the orientation of a spatial pattern can be detected in frequency domain: the power spectrum of such a pattern lies along a line through the origin in the Fourier domain, and the direction of the line is perpendicular to the dominant spatial orientation of the pattern [10]. Fig. 1 demonstrates two oriented color patterns and the corresponding power spectrum images in quaternion frequency domain.

So, the detection of spatial color orientation becomes a minimization problem in QFT: finding a directional line $\mathbf{n}$ through the origin in the quaternion power spectrum image which minimizes the following distance sum $D(\mathbf{n})$:

$$D(\boldsymbol{n}) = \iint \left(d_{\boldsymbol{n}}(u,v)\ |F(u,v)|\right)^2 dudv \tag{7}$$

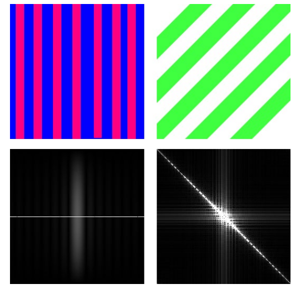

Fig. 1. Quaternion power spectrum images: the first and second rows are the oriented color patterns and the power spectrums of QFTs.

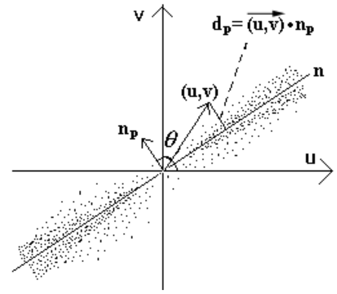

Fig. 2. The distance of point (u,v) to directional line $\mathbf{n}$, where $\mathbf{n}_{\mathrm{p}} = (\cos\theta \ \sin\theta)$ is the normal vector of $\mathbf{n}$.

where $d_{\boldsymbol{n}}(u,v)$ is the shortest Euclidean distance between point (u,v) and the directional line $\mathbf{n}$, which is equal to the projection of vector $\overrightarrow{(u,v)}$ to the unit normal vector $\mathbf{n}_{\mathrm{p}} = (\cos\theta \ \sin\theta)$ (i.e. the dominant spatial orientation), as shown in Fig. 2. Thus,

$$\begin{aligned} D(\mathbf{n}) &= \iint \left(d_{\mathbf{n}}(u,v)\ |F(u,v)|\right)^2 dudv \\ &= \iint \left(\overrightarrow{(u,v)} \cdot \mathbf{n_p}\right)^2 \ |F(u,v)|^2 dudv \\ &= \iint \left((u\ v)\,\mathbf{n}_{\mathrm{p}}^{\mathrm{T}}\right)\left(\mathbf{n_p}\begin{pmatrix} u \\ v \end{pmatrix}\right) |F(u,v)|^2 dudv \\ &= \iint (u\ v)\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} |F(u,v)|^2 dudv \end{aligned} .$$

For simplicity, we denote the above equation as

$$D(\mathbf{n}) = E\ \cos^2\theta \ + \ 2F\ \cos\theta\sin\theta \ + \ G\ \sin^2\theta \tag{8}$$

where

$$\begin{cases} E = \iint u^2 |F(u,v)|^2 \ dudv \\ F = \iint uv |F(u,v)|^2 \ dudv \\ G = \iint v^2 |F(u,v)|^2 \ dudv \end{cases} . \tag{9}$$

For Eq. (8), the maximum and minimum of $D(\mathbf{n})$, and the corresponding θ values are computed as follows [16]:

$$\lambda_{\max} = \frac{1}{2}\left((E+G) + \sqrt{(E-G)^2 + (2F)^2}\right) , \tag{10}$$

$$\theta_{\max} = \begin{cases} \mathrm{sgn}(F)\ \sin^{-1}\left(\dfrac{\lambda_{\max} - E}{2\lambda_{\max} - E - G}\right)^{1/2} + k\pi \\ \qquad\qquad\qquad\qquad (E-G)^2 + F^2 \neq 0 \\ Undefined\ , \qquad\qquad (E-G)^2 + F^2 = 0 \end{cases} , \tag{11}$$

$$\lambda_{\min} = \frac{1}{2}\left((E+G) - \sqrt{(E-G)^2 + (2F)^2} \right), \tag{12}$$

$$\theta_{\min} = \theta_{\max} + \pi/2 , \tag{13}$$

where $\mathrm{sgn}(\bullet)$ is a sign function:

$$\mathrm{sgn}(F) = \begin{cases} 1 , & F \geq 0 \\ -1 , & F < 0 \end{cases} . \tag{14}$$

Therefore, $\theta_{\min}$ denotes the dominant spatial orientation (notice that θ in Eq. (8) denotes the direction of $\mathbf{n}_p$ rather than $\mathbf{n}$). E, F and G can be computed by the fast QFT algorithms described in [14,15].

For a color pattern that is strongly orientated, its power spectrum in quaternion frequency domain will be very close to the line $\mathbf{n}$, resulting in that $\lambda_{\max}$ is far greater than $\lambda_{\min}$, whereas for an isotropic pattern $\lambda_{\max}$ will be very close to $\lambda_{\min}$. The stronger the orientation of a pattern, the larger difference between $\lambda_{\max}$ and $\lambda_{\min}$. So, we can define the orientation strength g as

$$g = \left(\frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}} \right)^2 . \tag{15}$$

2.3. Proposed SAVMF

By adaptively adjust the orientation, shape, and size of the VMF window, the proposed structure adaptive vector median filter (SAVMF) is obtained.

Denote the orientation and orientation strength of the pattern in neighborhood centered at any position $\boldsymbol{x}_0$ as $\boldsymbol{l}_{\boldsymbol{x}_0}$ and $g(\boldsymbol{x}_0)$, then the support window W is defined as the following elliptic shape:

$$W = \left\{ \boldsymbol{x} \mid \rho(\boldsymbol{x}, \boldsymbol{x}_0) \leq 1 \right\} \tag{16}$$

with

$$\rho(\boldsymbol{x}, \boldsymbol{x}_0) = \frac{\left((\boldsymbol{x} - \boldsymbol{x}_0) \cdot \boldsymbol{l}_{\boldsymbol{x}_0} \right)^2}{r^2} + \frac{\left((\boldsymbol{x} - \boldsymbol{x}_0) \cdot \boldsymbol{l}_{\boldsymbol{x}_0}^{\perp} \right)^2}{\left((1 - g(\boldsymbol{x}_0)) r \right)^2} \tag{17}$$

where $\boldsymbol{l}_{\boldsymbol{x}_0}^{\perp}$ represents the normal unit vector of $\boldsymbol{l}_{\boldsymbol{x}_0}$, and r is a predefined radius of the long axis of ellipse. It is seen that the orientation and shape of elliptic window are determined by pattern orientation $\boldsymbol{l}_{\boldsymbol{x}_0}$ and orientation strength $g(\boldsymbol{x}_0)$, respectively. The larger the orientation strength $g(\boldsymbol{x}_0)$, the flatter the elliptic window, leading to stronger structure preservation.

Denote that $\boldsymbol{l} = (\cos\theta \ \sin\theta)$, $\boldsymbol{x}_0 = (x_0, y_0)$ and $\boldsymbol{x} = (x, y)$, then Eq. (17) can be written as

$$\rho(\boldsymbol{x}, \boldsymbol{x}_0) = \frac{\left((x - x_0)\cos\theta + (y - y_0)\sin\theta \right)^2}{r^2} + \frac{\left(-(x - x_0)\sin\theta + (y - y_0)\cos\theta \right)^2}{\left((1 - g(x_0, y_0)) r \right)^2} . \tag{18}$$

Thus, the output of proposed SAVMF on position $\boldsymbol{x}_0$ of color image $\mathbf{I}$ can be expressed as follows:

$$\mathbf{y}^{\mathrm{SAVMF}}(\boldsymbol{x}_0) = \min_{\mathbf{I}(\boldsymbol{x})} \sum_{\boldsymbol{x} \in W} \left\| \mathbf{I}(\boldsymbol{x}) - \mathbf{I}(\boldsymbol{x}_0) \right\|. \tag{19}$$

3. EXPERIMENTS

In the experiments, the long axis radius r is set to 3 (for slightly corrupted images r takes 2) and the minimum radius of short axis is forced to 1. The denoised performance is evaluated by three widely-used criteria [1], peak signal-to-noise ratio (PSNR), mean absolute error (MAE), and normalized color difference (NCD). Some representative methods, the classical VMF [2], a widely-used switching VMF (fast averaging peer group filter, FAPGF) [5], and a more recent and highly effective quaternion switching VMF (QSVMF) [7], are chosen for comparison. In the switching VMFs (FAPGF and QSVMF), the original VMFs that are used to restore the noisy pixels detected are replaced with the proposed SAVMF and the noise detection mechanisms are kept unchanged. For convenience, we name the modified versions as FAPGF_SAVMF and QSVMF_SAVMF.

Two noisy images are shown in Fig. 3 and the denoised results by different methods are presented in Figs. 4 and 5. It is seen that the proposed SAVMF, FAPGF_SAVMF and QSVMF_SAVMF excellently suppress noise and at the same preserve edges and structures well, and obviously outperform their respective opponents VMF, FAPGF, and QSVMF. Table 1 lists the objective evaluation results PSNR, MAE, and NCD values. The data presented in the table clearly show that the proposed SAVMFs outperform their respective counterparts in terms of all the three objective criteria.

(a) Lena (256x256) (b) Parrots (256x256)

Fig. 3. Noisy images (256x256): (a) Lena and (b) Parrots are corrupted by 25% and 50% impulse noise, respectively.

Fig. 4. Denoised results on noisy *Lena* image shown in Fig. 3(a). (d) and (f) are the results by modified FAPGF and QSVMF, in which the standard VMF in the original versions are replaced with the proposed SAVMF.

Fig. 5. Denoised results on noisy *Parrots* image shown in Fig. 3(b). (d) and (f) are the results by modified FAPGF and QSVMF, in which the standard VMF in the original versions are replaced with the proposed SAVMF.

Table 1. PSNR, MAE and NCD values of the images shown in Figs. 3, 4, and 5.

Indexes / Methods	Fig. 4			Fig. 5		
	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy image	16.40	12.90	0.1437	13.28	26.08	0.2868
VMF	29.63	3.85	0.0246	23.90	6.55	0.0473
SAVMF	30.44	3.65	0.0233	25.74	5.76	0.0363
FAPGF	30.74	2.08	0.0156	23.95	5.68	0.0454
FAPGF_SAVMF	31.83	1.85	0.0141	25.90	4.71	0.0336
QSVMF	33.64	1.42	0.0092	27.53	3.55	0.0220
QSVMF_SAVMF	34.61	1.26	0.0084	27.89	3.40	0.0210

4. CONCLUSION

A structure-preserving vector median filter for suppressing color impulse noise is proposed. The key technique in the method is the estimation of orientation of local pattern. This is done by computing the direction of power spectrum distribution of quaternion Fourier transform of the local pattern. Based on the local spatial orientation and its strength, the orientation, size, and shape of support window are determined. Thus, the proposed structure-adaptive VMF is obtained. Experimental results show the superiority of the proposed method both in noise removal and in structure preservation, compared to other VMF-based vector filters.

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