

WEIGHTED MEDIAN-SHIFT ON GRAPHS FOR GEOMETRIC MODEL FITTING

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ABSTRACT

In this paper, we deal with geometric model fitting problems on graphs, where each vertex represents a model hypothesis, and each edge represents the similarity between two model hypotheses. Conventional median-shift methods are very efficient and they can automatically estimate the number of clusters. However, they assign the same weighting scores to all vertices of a graph, which can not show the discriminability on different vertices. Therefore, we propose a novel weighted median-shift on graphs method (WMSG) to fit and segment multiple-structure data. Specifically, we assign a weighting score to each vertex according to the distribution of the corresponding inliers. After that, we shift vertices towards the weighted median vertices iteratively to detect modes. The proposed method can adaptively estimate the number of model instances and deal with data contaminated with a large number of outliers. Experimental results on both synthetic data and real images show the advantages of the proposed method over several state-of-the-art model fitting methods.

Index Terms— weighted median-shift, geometric model fitting, graphs, modes

1. INTRODUCTION

Geometry model fitting is a key problem in various computer vision applications, such as optical flow [1], rang image segmentation [2], fundamental matrix estimation [3], and motion segmentation [4]. The task of model fitting is to estimate the models which can best explain the structured data. Generally, conventional fitting methods (such as least squares regression [5]) can achieve good results when there do not exist outliers. However, such conditions usually can not be satisfied in practical environments. Moreover, many statistical methods (e.g., RANSAC [6]) do not work very well when multiple structures exist in data.

Some clustering-based fitting methods [7–9] are proposed to solve the above problems. These methods cluster data points based on the principle that data points belonging to the same structures should be divided into the same cluster. Specifically, a distance measure is employed, and then, the

fitting results are obtained by performing clustering. For example, some works propose to use the Jaccard distance [7] and the Tanimoto distance [9] (T-linkage) to measure the similarities of data points, and employ an agglomerative clustering method to find the hidden structures. The above methods are generally very robust to gross outliers. However, they are usually ineffective in dealing with the intersection of inliers.

Another kind of clustering-based fitting methods is also proposed in the literature [10–16]. They fit models by clustering model hypotheses. However, most of them work in the feature domain [10, 11], where each hypothesis can be represented by a feature vector. Only a few methods focus on clustering in the graph domain [12–16]. These methods can be roughly divided into two categories. The first type of the methods is to map vertices from the graph domain to the feature domain [12], by which a vertex of the constructed graph is mapped to a feature vector using a kernel function. And then, they deal with the feature vectors by conventional clustering methods. However, mapping from the graph domain to the feature domain may cause information-loss, and the accuracy of these methods is thus affected. The second type of the methods is to directly cluster vertices in the graph domain (e.g., AKSWH [13], GMSF [15], AAS [14] and MSH [16]), which is called mode-seeking on graphs.

In this paper, we propose a novel method (WMSG) to deal with the model fitting problem in the parameter space, i.e., model hypotheses rather than data points are clustered. Here, model hypotheses are considered as the vertices on a graph, and the similarities of model hypotheses are modeled as the edges of the graph. Compared with the conventional median-shift methods [17, 18], where each vertex shifts towards the median vertex of a graph, each vertex constructed in this paper shifts towards the weighted median vertex of a graph. Therefore, vertices with high scores are easier to be chosen as the representative modes (i.e., candidate model hypotheses). Compared with several other model fitting methods, WMSG is very robust.

2. THE PROPOSED METHOD

In this paper, the geometry model fitting problem is formulated as a mode-seeking problem on a graph, where each vertex represents a model hypothesis, and each edge represents the

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relationship of two model hypotheses. A graph $G = (\mathcal{V}, \mathcal{E})$ consists of vertices \mathcal{V} and edges \mathcal{E} . Each vertex $v_i \in \mathcal{V}$ is assigned a weighting score w_{v_i} . Each edge $e_{ij} \in \mathcal{E}$ has a value S_{ij} , which represents the similarity of two vertices v_i and v_j .

Now, we describe how to construct such a graph G . Let the original data points be $\mathbf{X} = \{x_1, x_2, \dots, x_n\}$, we sample a number of Minimum Sample Sets. Then, we generate M model hypotheses $\Theta = \{\theta_1, \theta_2, \dots, \theta_M\}$ based on the Minimum Sample Sets. The model hypotheses are represented as the vertices of the graph G . And the weighting scores of the vertices can be estimated by using the non-parameter kernel density estimate technique [13]. Finally, we compute the similarities of the vertices based on the residual information of the corresponding hypotheses.

2.1. Similarity Measure and Weighting Score

In this subsection, we measure the similarities of vertices by comparing the preferences of model hypotheses to data points. For each model hypothesis θ_i , we compute the absolute residuals of n input data points to form a vector:

$$\mathbf{r}^i = [r_1^i, r_2^i, \dots, r_n^i]. \quad (1)$$

To obtain the preference information of model hypotheses to data points, we sort the elements in \mathbf{r}^i in non-descending order to form the permutation:

$$\boldsymbol{\pi}^{(i)} = [\pi_1^{(i)}, \pi_2^{(i)}, \dots, \pi_n^{(i)}], \quad (2)$$

which satisfies that $p < q \Rightarrow r_{\pi_p^{(i)}}^{(i)} < r_{\pi_q^{(i)}}^{(i)}$.

Then, we compute the “intersection” between the leading elements of two permutations corresponding to hypotheses θ_i and θ_j as [19]:

$$f(\theta_i, \theta_j) = |\boldsymbol{\pi}_{1:b_i}^{(i)} \cap \boldsymbol{\pi}_{1:b_j}^{(j)}| / (b_i b_j)^{\frac{1}{2}}, \quad (3)$$

where $|\boldsymbol{\pi}_{1:b_i}^{(i)} \cap \boldsymbol{\pi}_{1:b_j}^{(j)}|$ denotes the number of the common elements between $\boldsymbol{\pi}_{1:b_i}^{(i)}$ and $\boldsymbol{\pi}_{1:b_j}^{(j)}$. b_i and b_j are the numbers of the elements taken into account for hypotheses θ_i and θ_j , respectively. Then, we set $S(v_i, v_j) = f(\theta_i, \theta_j)$.

Intuitively, if two model hypotheses θ_i and θ_j share more common top-ranked elements in their preference lists, they will achieve a higher similarity, and vice versa. As shown in Fig. 1, the similarities between the model hypotheses generated from the inliers of the same model are high. Otherwise, the similarities are relatively low.

Next, we evaluate each vertex v_j by assigning a weighting score w_{v_j} as [13]:

$$w_{v_j} = \frac{1}{N_j} \sum_{i=1}^{N_j} \frac{K(r_i^j / h_j)}{\delta_j h_j}, \quad (4)$$

where δ_j and N_j are the inlier scale and the number of inliers of hypothesis θ_j , respectively; h_j denotes the bandwidth of the hypothesis θ_j ; r_i^j denotes the residual measured from the model hypothesis θ_j to the data point x_i ; and $K(r_i^j / h_j)$ is a kernel function (such as the Epanechnikov kernel) used to

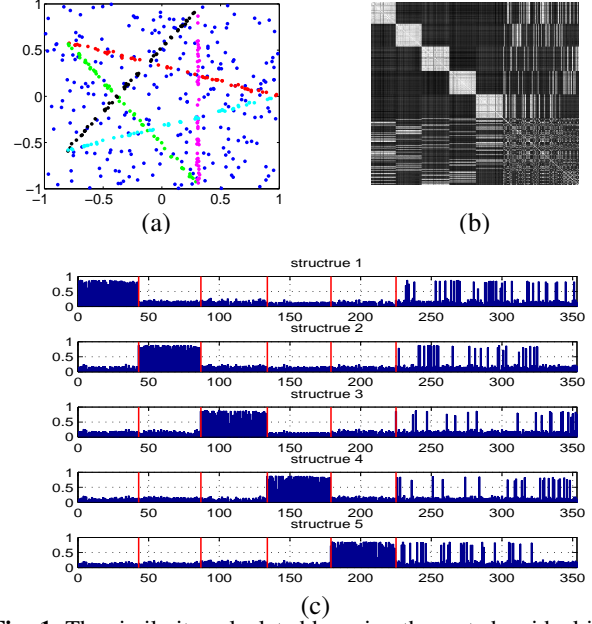


Fig. 1: The similarity calculated by using the sorted residual information for the five star data. (a) The original data. (b) The corresponding matrix C of size 353×353 , representing the similarities between model hypotheses. (c) Average values of rows in matrix C organized by structure membership, red vertical lines reveal the group boundary.

calculate the density of data points around the hypothesis θ_j .

Thus, a vertex will achieve a high weighting score when the corresponding model hypothesis is generated from the inliers belonging to a true structure. To improve the computational efficiency, we propose to prune some “bad” vertices, i.e., vertices with low weighting scores. As in [20], we use the entropy thresholding method to automatically select significant vertices.

2.2. Weighted Median Shift on Graphs

In the previous subsection, we have constructed a graph $G = (\mathcal{V}, \mathcal{E})$. Next, we introduce a weighted median-shift method to seek modes on the graph. For an arbitrary vertex v_j , let $U_j \subset \mathcal{V}$ be a set of vertices whose similarities to a vertex v_j are higher than a certain threshold th , i.e., $\forall v_i \in U_j, S(v_i, v_j) \geq th$. Then, the vertices in U_j form the subgraph G_j of the graph G . In [21], the median concept is extended to the graph domain:

$$\bar{g} = \operatorname{argmin}_{g \in G} \sum_{g_i \in G} d(g_i, g), \quad (5)$$

where $G = \{g_1, g_2, \dots, g_n\}$. $d(g_i, g)$ is the graph edit distance of g_i and g , and \bar{g} is the median graph of the graph G . It has an intuition that the median graph of G is the subgraph which possesses the minimum sum of distances to all subgraphs in G . Similarly, we define the median vertex of the graph G_j as:

$$\bar{v}_j = \operatorname{argmax}_{v \in U_j} \sum_{v_i \in U_j} S(v_i, v), \quad (6)$$

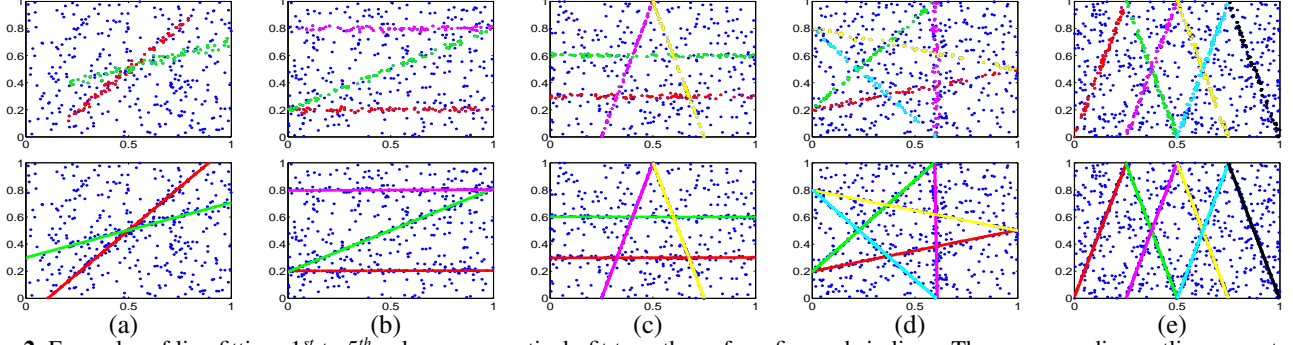


Fig. 2: Examples of line fitting. 1st to 5th columns respectively fit two, three, four, five and six lines. The corresponding outlier percentages are respectively 83%, 85%, 88%, 91% and 92%. The first row is the original data, and the second row is the fitting results obtained by WMSG. We do not show the results obtained by the competing methods here due to the space limit.

which calculates the median vertex of G by using the similarity measure, rather than distance. In Equation (6), all vertices in G_j are assigned the same weight (i.e., 1), which is not reasonable since some vertices are significantly more important than others. And some bad vertices may be chosen as the median vertices when they cluster together, which has a negative effect for the following shifting process. Therefore, we define the weighted median vertex of G_j by the following equation:

$$\bar{v}_{w_j} = \operatorname{argmax}_{v \in U_j} \sum_{v_i \in U_j} S(v_i, v) w_{v_i}, \quad (7)$$

which combines the weighting score and similarity measure to avoid choosing bad vertices as the median vertices.

In fact, the term $\sum_{v_i \in U_j} S(v_i, v_j) w_{v_i}$ can be regarded as the local authority score (LAS) of the vertex v_j [22]. Then, we compute the LASs of all vertices and shift the vertex v_j towards its weighted median vertex \bar{v}_{w_j} only if $LAS_{\bar{v}_{w_j}} > LAS_{v_j}$. Specifically, let \hat{v}_j be the vertex that v_j shifts to, then

$$\hat{v}_j = \begin{cases} \bar{v}_{w_j}, & \text{if } LAS_{\bar{v}_{w_j}} > LAS_{v_j}, \\ v_j, & \text{otherwise.} \end{cases} \quad (8)$$

Since vertices shift towards the direction of local authority ascending, they will finally converge to stable vertices, i.e., authority modes. Similar to the median-shift method [18], the weighted median-shift method is easy to trap into the local optimum. In other words, redundant models may exist. In this paper, we solve this problem by applying the Mutual Information Theory (MIT) [13]. Since the mutual information of two modes will be large if they correspond to the same structure, we fuse the modes with large mutual information.

3. EXPERIMENTAL RESULTS

In this section, we compare the performance of the proposed WMSG with KF [8], AKSWH [13], T-linkage [9], MSH [16] and GMSF [15]. We test all the six fitting methods on the line fitting, homography and fundamental matrix estimation tasks. For each competing method, we first use the proximity sampling [23] to obtain 2000 model hypotheses for line fitting (Sec. 3.1) and 10000 model hypotheses for homography

Table 1: The fitting errors (in percentage) for line fitting on five datasets (the best results are boldfaced)

		M1	M2	M3	M4	M5	M6
2 lines	Std.	0.46	0.00	0.19	0.00	0.42	0.00
	Avg.	0.33	0.02	0.07	0.01	0.23	0.01
3 lines	Std.	0.14	0.00	0.14	0.01	0.00	0.00
	Avg.	0.03	0.02	0.04	0.04	0.01	0.01
4 lines	Std.	0.14	0.00	0.00	0.00	0.35	0.00
	Avg.	0.03	0.00	0.01	0.01	0.86	0.00
5 lines	Std.	0.66	0.00	0.69	0.01	0.14	0.00
	Avg.	0.35	0.02	0.28	0.07	0.03	0.01
6 lines	Std.	0.14	0.50	0.52	0.20	0.39	0.00
	Avg.	0.03	0.43	0.19	0.05	0.19	0.01

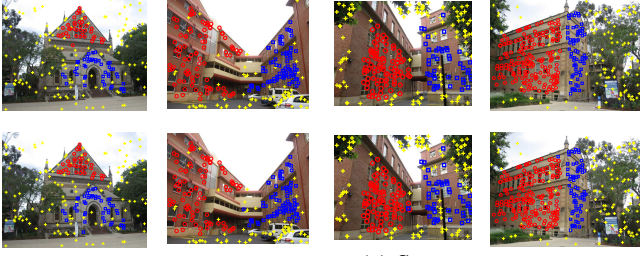
(M1-KF; M2-AKSWH; M3-T-linkage; M4-MSH; M5-GMSF; M6-WMSG.)

and fundamental matrix estimation (Sec. 3.2). The line fitting errors are computed as [8] and the homography and fundamental matrix fitting errors are computed as [9].

3.1. Line Fitting

We evaluate the six fitting methods on five challenging data for line fitting (see Table 1). We repeat the experiments 50 times and report the average fitting errors and standard deviations. The fitting results corresponding to the average fitting errors obtained by the proposed method are shown in Fig. 2.

From Fig. 2 and Table 1, we can see that: (1) For the “two lines” data, MSH and WMSG can achieve the lowest average errors. AKSWH, MSH and WMSG obtain the stablest results because they work in the parameter space. In contrast, KF, T-linkage and GMSF achieve relatively bad results. (2) For the “three lines” data, the three lines intersect at two points. GMSF and WMSG achieve the lowest average errors, while AKSWH, GMSF and WMSG achieve the lowest standard deviations. (3) For the “four lines” data, all methods except for GMSF achieve low average errors and standard deviations. (4) For the “five lines” data, the five lines are seriously intersected, thus causing the bad performances for KF and T-linkage. This is because that KF and T-linkage are the clustering-based methods which cluster the data points. However, the data points lying at the intersections naturally belong



(a) Elderhalla (b) Nese (c) Sene (d) Oldclassicswing
Fig. 3: Homography based segmentation on four image pairs. The first row is the original images and the second row is the fitting results obtained by WMSG.

Table 2: The fitting errors (in percentage) for homography matrix fitting on four datasets (the best results are boldfaced)

		M1	M2	M3	M4	M5	M6
D1	Std.	4.59	0.47	0.61	3.24	1.58	0.00
	Avg.	16.13	1.07	1.73	1.40	1.86	0.93
D2	Std.	2.77	7.12	8.23	0.35	13.29	0.56
	Avg.	39.24	3.10	1.73	0.59	5.91	0.38
D3	Std.	5.14	0.33	0.54	0.23	5.46	0.23
	Avg.	25.06	0.58	1.42	0.51	1.46	0.93
D4	Std.	4.16	2.60	0.86	1.92	4.03	0.09
	Avg.	31.91	1.15	3.53	1.20	1.97	0.53

(M1-KF; M2-AKSWH; M3-T-linkage; M4-MSH; M5-GMSF; M6-WMSG; D1-Elderhalla; D2-Nese; D3-Sene; D4-Oldclassicswing.)

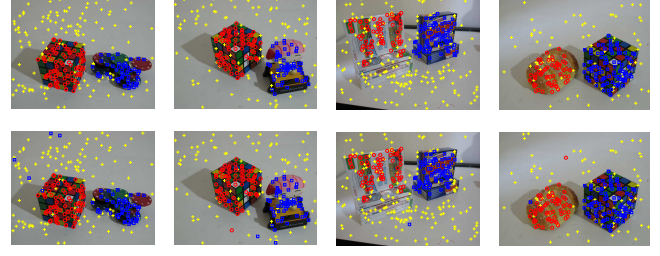
to all the lines crossing them. But KF and T-linkage assign them to only one line, which affects the fitting results. (5) For the “six lines” data, only WMSG achieves the lowest average error and standard deviation which are much lower than the results obtained by the other five competing methods.

3.2. Homography and Fundamental Matrix Estimation

For the homography and fundamental matrix estimation problems, we use four real image pairs from the AdelaideRMF dataset [24] for each task. We also repeat each experiment 50 times and report the standard deviations and the average fitting errors in Table 2 and Table 3. The fitting results corresponding to the average fitting errors of the proposed method are shown in Fig. 3 and Fig. 4.

From Fig 3 and Table 2, we can see that WMSG obtains relatively good results for the task of homography estimation, achieving the lowest average fitting errors and the lowest standard deviations in three out of the four data respectively. MSH achieves the lowest average fitting errors once and the lowest standard deviations twice. KF achieves bad results since it is sensitive to the step size used to calculate the ordered residual kernel. AKSWH, T-linkage and GMSF achieve high standard deviations, which indicates that the performances of these methods are not very stable.

From Fig 4 and Table 3, we can see that WMSG achieves the lowest average fitting errors on all the four image pairs for the fundamental matrix estimation problem. Both T-linkage and MSH achieve the lowest standard deviations twice. In



(a) Cubechips (b) Cubetoy (c) Gamebiscuit (d) Breadcube
Fig. 4: Two-view based motion segmentation on four image pairs. The first row is the original images and the second row is the fitting results obtained by WMSG.

Table 3: The fitting errors (in percentage) for fundamental matrix fitting on four datasets (the best results are boldfaced)

		M1	M2	M3	M4	M5	M6
D1	Std.	4.06	12.94	0.70	1.00	4.27	0.88
	Avg.	7.17	5.30	3.23	3.74	7.62	3.07
D2	Std.	3.00	3.88	8.17	0.75	4.48	0.78
	Avg.	35.01	4.98	3.94	2.31	5.08	2.27
D3	Std.	4.98	12.01	0.83	7.15	5.72	4.14
	Avg.	13.59	14.38	3.74	5.05	6.85	2.58
D4	Std.	5.11	8.35	7.84	0.76	8.54	1.15
	Avg.	12.99	6.51	3.99	2.92	7.31	2.79

(M1-KF; M2-AKSWH; M3-T-linkage; M4-MSH; M5-GMSF; M6-WMSG; D1-Cubechips; D2-Cubetoy; D3-Gamebiscuit; D4-Breadcube.)

contrast, KF, AKSWH and GMSF obtain relatively bad results. Since KF works in a paradigm: removing outliers and clustering the remaining inliers, even one outlier which is wrongly remained as an inlier may have negative influence on the final fitting results.

4. CONCLUSIONS

In this paper, we propose to fit models by seeking modes on a graph, where each vertex represents a model hypothesis and each edge represents the similarity between two model hypotheses. Based on the graph G , we propose a novel mode-seeking algorithm (WMSG) which iteratively seeks the weighted median vertices in the vertex sets. Vertices are finally shifted to the authority modes, which are selected as the candidate model hypotheses. And the resulting modes are fused by using the MIT. Experiments on both synthetic data and real images show that the proposed method outperforms several state-of-the-art fitting methods.

5. ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grants U1605252, 61472334 and 61571379, by Research Grant of the University of Macau no. MYRG2014-00009-FST, by the Natural Science Foundation of Fujian Province of China under Grant 2017J01127 and by the Fundamental Research Funds for the Central Universities under Grant 20720170045.

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