Using 2D ARMA-GARCH for ultrasound images denoising

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***Abstract*—This paper deals with speckle reduction of ultrasound (US) images withing a framework of 2D ARMA-GARCH modeling for wavelet coefficients of log transformed US images and using a class of generalized method of moments (GMM) estimators with interesting asymptotic properties. The choice of a 2D ARMA-GARCH process has been well justified by authors in [9]. An estimation of clean wavelet image coefficients is derived by a Minimum Mean Square Error (MMSE). To prove the performance of the proposed approach, obtained results are compared with various image denoising methods.**

I. Introduction

The use of ultrasound imaging is well established in the field of medical diagnostic technology. The principal problem faced by them is the noise introduced due to the consequence of the constructive and destructive coherent summation of ultrasound echoes. This phenomenon is termed as speckle noise. It is common to laser, SAR, and sonar imagery [1, 2, 3]. Therefore, several techniques for suppressing image noise have been developed [4, 5, 6, 7, 8]. In this work, we are interested in techniques based on multi-scale decomposition of the image in space frequency domain, making it a powerful method to distinguish image information from the noisy data. In this context, traditional denoising methods include Markov-Random field that serves as a prior in the image denoising and restauration. The Gauss Markov Random Field (GMRF) is the most popular method in image analysis, anomaly detection and image denoising. However, research on statistical properties of images wavelet coefficients have shown that the marginal distribution of wavelet coefficients are highly kurtotic, and can be described using suitable heavy-tailed distributions [9]. So in general, recent studies have shown that some crucial spatial data we come across in digital signal and image processing are neither linear nor Gaussian. Hence, the modeling of this type of data by spatial non-linear models has become an appealing and popular tool for investigating both spatiality and non-Gaussianity patterns. Indeed, the authors in [10] introduced a two dimensional GARCH model for cluster modeling and anomaly detection. They have also shown that the two-dimensional GARCH model generalizes the causal Gauss Markov Random Field (GMRF). It is largely used in cluster modeling with the disadvantage of having a constant conditional variance through the space which makes the use of a GARCH cluster modeling better than the use of a GMRF one. Next the authors in [11] shown that the sub-band decomposition of SAR images has signicantly non-gaussian statistics that are best described by the 2D-GARCH regression model and its variants. Historically, introduction of the Autoregressive Conditional Heteroscedasticity (ARCH) model in the famous paper of [12] was a natural starting point in modeling the time-varying behavior of the volatility in financial time series. This model allows the variance to depend on the past of the random process. Numerous variants and extensions of this model have been proposed. Generalized ARCH (GARCH) model is the main natural extension of this model. The link has been performed in a way that is similar to the passage from the AR model to the ARMA one. The processing of spatial interaction (dependence) and spatial structure (heterogeneity) in practice may be modeled by some random fields $(x_t)_{t\in z^d}$. Hence, various nonlinear spatial models have been introduced in the literature. General spatial models with infinite interactions have been proposed in [13]. The authors in [14] have studied some statistical questions for some spatial bilinear models. Parallely, some contributions have been made in applied fields. Recently, the authors in [15] have proposed a two-dimensionally indexed Random Coefficients Autoregressive models (2D-RCA) to capture the space-varying behavior of the volatility in textured images.

The paper is organized as follows: Section 2 is dedicated to the GMM application to the 2D ARMA-GARCH model. In Section 3, we present our ultrasound imaging algorithm that should achieve both noise reduction and feature preservation. The experimental results of the proposed method and other image denoising methods are presented and compared in Section 4. Finally, a conclusion is given in Section 5.

II. Parameter estimation of 2D ARMA-GARCH model of first order

Consider the observations $\{y_t, t \in S]\mathbf{0},\mathbf{N}]\}$, where $\mathbf{N} = (N_1, N_2)$ and $S[\mathbf{1},\mathbf{N}] := \{(l,m) \in Z^2, 1 \le l \le N_1, 1 \le m \le N_2\}$ by arranging its terms by the lexico-graphic order. Our goal is to estimate the parameters of 2D ARMA-GARCH model given by the equation:

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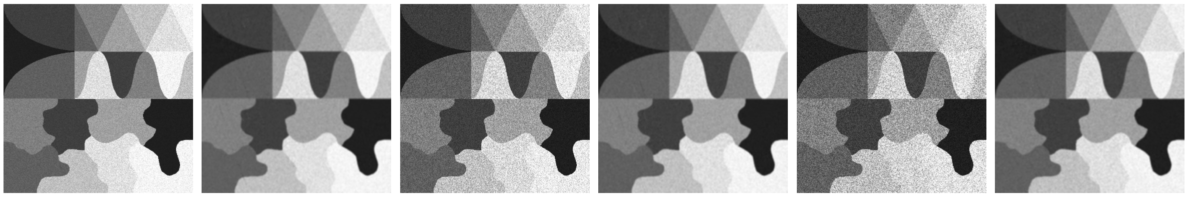


(a) (b) (c) (d) (e) (f)

Fig. 1. Restoration of synthetic images: with different speckle variance. (a, c, e) synthetic noisy images with$\sigma^2 = 0.01$, $\sigma^2 = 0.04$ and $\sigma^2 = 0.08$ respectively. (b, d, f) Filtered images by the proposed method.

$$
\begin{gathered}
y(i,j) = ay(i,j-1)+by(i-1,j)+cy(i-1,j-1)+\varepsilon(i,j),\\
\varepsilon(i,j) = \sqrt{h(i,j)}\eta(i,j),\\
h(i,j) = \omega + \alpha_1\varepsilon^2(i,j-1) + \alpha_2\varepsilon^2(i-1,j) + \alpha_3\varepsilon^2\\
(i-1,j-1)+\beta_1 h(i,j-1)+\beta_2 h(i-1,j)+\beta_3 h(i-1,j-1).
\end{gathered}
\tag{2.1}
$$

Where $\{\eta(i,j);(i,j) \in Z^2\}$ is an independent and identically distributed sequence of random variables with mean zero and variance 1.

The vector parameter is $\theta = (a,b,c,\omega,\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\beta_3)^{'} = (\underline{\alpha}^{'},\underline{\varphi}^{'})^{'}$.

For this purpose, we rewrite the Model (2.1) as a linear regression model, i.e.,

$$y(i,j) = \underline{x}(i,j)^{'}\underline{\alpha} + \varepsilon(i,j). \tag{2.2}$$

Where $\underline{x}(i,j) = (y(i,j-1),y(i-1,j),y(i-1,j-1))^{'}$ and $\varepsilon(i,j)$ represents the residual process. Various estimation procedures for θ have been discussed. Indeed, for 1D ARMA-GARCH models, authors in [16] have considered quasi-maximum likelihood estimation (QMLE) of θ. They have showed that the QML estimator of $\underline{\theta}$ is strongly consistent and asymptotically normal under the assumption that $E(\eta(i,j)^4)$ and $E(\varepsilon(i,j)^4)$ are finite. For the 2D case, authors in [10] have proposed the method of maximum likelihood but not from an asymptotic point of view. Authors in [17] have proposed the Gaussian maximum likelihood estimator (GMLE) in the context of a general form of spatial ARMA processes that includes the 2D ARMA-GARCH process as a particular case. They showed that the GMLE is consistent and also asymptotically distribution-free in the sense that the limit distribution is normal, unbiased and with a variance depending only on the autocorrelation function. In this paper, we attempt to introduce a general class of strongly consistent and asymptotically normal estimators for $\underline{\theta}$ with optimal asymptotic variance. The estimation procedure is a two-step procedure inspired from [18] to estimate the parameters of a random coefficient regression model. The first step is to estimate $\underline{\alpha}$ and the second one is to estimate $\underline{\varphi}$.

For the parameter $\underline{\alpha}$, the GMM estimator is given by

$$\widehat{\underline{\alpha}}_{\mathbf{N}} = (\sum_{(i,j)\in S[\mathbf{1},\mathbf{N}]} \underline{x}(i,j)\underline{x}(i,j)^{'})^{-1} (\sum_{(i,j)\in S[\mathbf{1},\mathbf{N}]} \underline{x}(i,j)\varepsilon(i,j)). \tag{2.3}$$

For the parameter $\underline{\varphi}$, the GMM estimator is given by

$$\widehat{\underline{\varphi}}_{\mathbf{N}} = (\sum_{(i,j)\in S[\mathbf{1},\mathbf{N}]} \widehat{\underline{U}}(i,j)\widehat{\underline{U}}(i,j)^{'})^{-1} \sum_{(i,j)\in S[\mathbf{1},\mathbf{N}]} \widehat{\underline{U}}(i,j)\widehat{\varepsilon}(i,j)^2. \tag{2.4}$$

Where $\widehat{\underline{U}}(i,j) = (1,\widehat{\varepsilon}(i,j-1),\widehat{\varepsilon}(i-1,j),\widehat{\varepsilon}(i-1,j-1),\widehat{h}(i,j-1),\widehat{h}(i-1,j),\widehat{h}(i-1,j-1))^{'}$ with $\widehat{\varepsilon}(i,j) = y(i,j) - \underline{x}(i,j)^{'}\widehat{\underline{\alpha}}_{\mathbf{N}}$ and $\widehat{h}(i,j) = \frac{\widehat{\varepsilon}(i,j)^2}{\eta(i,j)^2}$.

III. Image denoising algorithm

In ultrasound images, the speckle which is a multiplicative noise is generally more difficult to remove than additive noise, because its intensity varies with the image intensity. Multiplicative noise model is given by:

$$y = x \cdot \eta. \tag{3.1}$$

where the noisy image y is the product of the original image x, and the speckle noise "η". In most applications involving multiplicative noise, the noise content is assumed to be stationary with unitary mean and unknown noise variance σ_η^2. A logarithmic transformation is applied to convert multiplicative noise into an additive noise [22]. The noise component "η" is then given by:

$$\ln y = \ln x + \ln \eta. \tag{3.2}$$

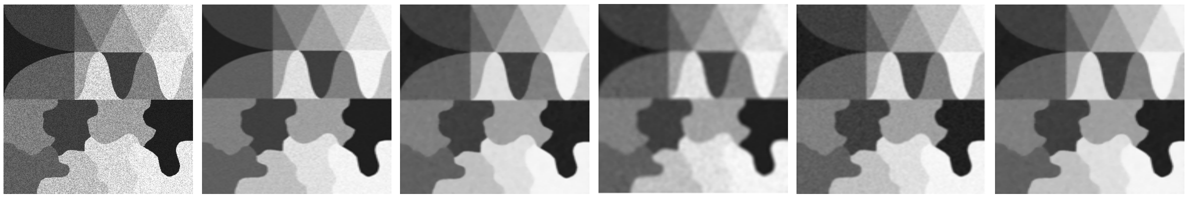
There is three steps in our image denoising procedure:

- First, the Discrete Wavelet Transform (DWT) is applied to $\ln(y)$, to obtain the sub-bands at different scales and orientations. We represent the (DWT) of $\ln(y)$, $\ln(x)$ and $\ln(\eta)$ by Y^s, X^s and N^s respectively. So, from (3.2) we get

$$Y^s = X^s + N^s$$

- Second, a 2D-ARMA-GARCH model of first order defined by equation (2.1) is used to model the wavelet coefficients of each level (except for the lowpass residual band) i.e., for each level, a 2D-ARMA-GARCH model of first order is selected for 3 sub-bands by computing the estimators of each model parameter as described in section 2. Now for each sub-band, we consider an estimator $\widehat{X}^s$ for X^s which minimizes the expected distortion given $\sigma^2_{X^s}$ and the noisy wavelet coefficients Y^s. In particular, we use the MMSE estimator given by:

$$\widehat{X}^s_{ij} = \frac{\sigma^2_{X^s_{ij}}}{\sigma^2_{X^s_{ij}} + \sigma^2_{N^s}} Y^s_{ij} = \frac{\sigma^2_{Y^s_{ij}} + \sigma^2_{N^s}}{\sigma^2_{Y^s_{ij}}} Y^s_{ij}$$



(a) (b) (c) (d) (e) (f)

Fig. 2. Speckle removal algorithms Comparison. (a) Synthetic Noisy image with $\sigma^2 = 0.02$ (b) Filtred image with ATV algorithm. (c) restored images with AD method. (d) de-noised images with bilateral filter. (e) Filtred images with a 2D-GARCH model .(f) Restored images by the proposed method.

where $\sigma^2_{Y^s_{ij}} = h_{ij}$ is to denote the conditional variance of Y^s_{ij} that should be computed from the stationary 2D ARMA-GARCH process of first order with the estimated parameters and $\sigma^2_{N^s}$ is the input noise variance. Notice that in some applications of image denoising $\sigma^2_{N^s}$ is known. As it is not the case in our situation, we use relation proposed in [24] for computing the input noise variance. For further details about the method one can consult [9], section 2. - Finally, we perform the inverse DWT to reconstruct the logarithmic of the denoised image. Then it is subjected to an exponential transformation which is the inverse logarithmic operation, that yields the denoised image.

IV. Experimental results

In this section, the proposed method is applied on both synthetic images corrupted by a multiplicative speckle noise and some representative real ultrasound medical images. We use Daubechies (Db4) with two levels of decomposition and 2D-ARMA-GARCH of first order. Moreover, the denoising ability, effectiveness and efficiency of the proposed algorithm are compared with some classical relevant filters such as bilateral filter [19], ATV (Additive Total Variation) filter [20], AD (Anisotropic Diffusion) filter [21] and the approach proposed in [9].

A. Experiments on synthetic images

In the first set of experiments, synthetic images have been used in (Fig.1) to test the performance of the proposed algorithm with different speckle variance (0.01, 0.04 and 0.08). To investigate the speckle suppression and edge preservation performance of the proposed method, some quantitative measures are computed such as MSE, PSNR, MSSIM, SC, NAE, FOM and Q index [23, 25].

B. Performance measurement metrics

In order to verify the performance of 2D ARMA-GARCH proposed algorithm it has been tested on a benchmark image corrupted by a multiplicative noise. The image y is produced from an image x as $y = x \cdot \eta$, where η is the speckle noise with variance $\sigma^2 = 0.02$. We evaluated the similarity measure by using the following two indices SC and MSSIM. Structural content (SC) is the measure of image similarity based on small regions of the image containing signicant low level structural information. The large value of SC means that image is of poor quality. Pratt's figure of merit (FOM) is an edge preservation index [23]. The value of FOM will be close to 1, if the edges are well preserved. In addition, The PSNR (peak signal to noise ratio) reflects the level of speckle noise. Accordingly, the quality of the restored image is in direct proportion to the value of PSNR , so the larger PSNR shows that the denoising algorithm ability is strong. The simplest and most widely used full reference quality metric is the mean squared error (MSE) computed by averaging the squared intensity differences of restored and reference image pixels. The smaller MSE means the better ability to reduce the speckle noise.

C. Results and comparison

The performance of the proposed method has been studied on synthetic image corrupted by a multiplicative speckle noise with different standard deviation $\sigma^2 = 0.01, \sigma^2 = 0.04$ and $\sigma^2 = 0.08$. Fig. 1 allows the visual quality evaluation of the resultant images produced by the proposed filter. We observe that the proposed method performs better in smooth regions providing better visual quality. Even if the image is overly noisy, the proposed method can improve the image quality. After plotting SC and NAE for different speckle standard deviation $\sigma^2 = 0.01, 0.03, ..., 0.19$, it has been noticed that normalized absolute error (NAE) values versus speckle variance are small and very close. There is no marked difference between them (Similarly in the plot of the structural content (SC)); this means that the proposed method have a strong ability in speckle reduction whatever the speckle level applied. Table 1 recapitulates PSNR, FOM, MSSIM, MSE, NAE, SC, Q index for a synthetic image. Table values confirm the visual observation. In these assessment frameworks, the performance of the PSNR, MSE and NAE values for the 2D ARMA-GARCH method have shown remarked superiorities over those of other algorithms. This means that our method has a strong ability to reduce noise. Yet, there is not much difference between our method and ATV filter [20] in MSSIM, NAE and Q index. This means that the two methods can both maintain the details of the image feature similarity and a high quality for restored images. The proposed method has a better edge retention capacity because it has the best values of FOM metric. Specifically, the PSNR, FOM, MSSIM, MSE, NAE, SC, Q index values of the restored image by 2D ARMA-GARCH method are higher than that of the ATV, AD, 2D-ARMA-GARCH model with maximum of Likelihood (ML) estimator proposed in [9] and bilateral filters in most tests. This means that the proposed method has reducee the speckle

TABLE I
QUALITY METRICS FOR THE SYNTHETIC IMAGE APPLIED WITH SEVERAL METHODS

	$PSNR$	FOM	$MSSIM$	MSE	NAE	SC	Q
ATV	67.9417	0.6858	0.6166	0.0027	0.0435	1.0469	0.9884
AD	72.2819	0.5858	0.6097	0.0038	0.0587	1.0814	0.9803
$Bilateral$	68.6892	0.4538	0.2119	0.0880	0.1398	1.0490	0.9930
$GMM-2D-GARCH$	75.2263	0.7578	0.6192	0.0019	0.0422	1.0415	0.9966
$ML-2D-GARCH$	73.7190	0.7069	0.5212	0.0028	0.1147	1.2477	0.9868

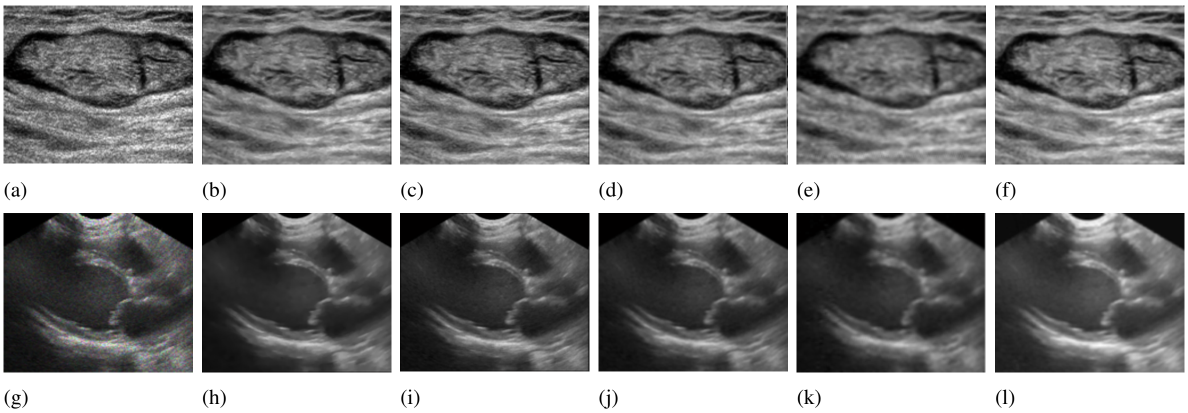


Fig. 3. Speckle removal algorithms Comparison on real US images.(a, g) Real ultrasound medical images. (b, h) Filtred images with ATV algorithm. (c, i) restored images with AD method. (d, j) de-noised images with bilateral filter.(e, k) Filtred images with a 2D-GARCH model. (f, l) restored images by the proposed method.

without blurring or distorting the image structures.

Moreover, the results of the GMM-2D-ARMA-GARCH and other popular speckle removal methods given by Fig. 2 (Speckle removal algorithms Comparison) have showed the superiority of the proposed algorithm against ATV, AD, ML-2D-ARMA-GARCH and bilateral filters in terms of speckle suppression and edge preservation. During these experiments, the simulated noisy images are corrupted by speckle noise Fig. 2(a). The ATV filter Fig. 2(b) removed the noise and preserved the edges. The AD method reduces noise and it has a low retention of discontinuities Fig. 2(c). The ML-2D-ARMA-GARCH filter reduces speckle and preserves image features. While the bilateral filter despeckles well Fig. 2(d) but blurs the image edges. The proposed method out performs the four tested methods, because it is the most effective to suppress speckle and maintain image features and edges. It provides a the best quality metrics. Besides, it is faster than ATV and ML-2D-ARMA-GARCH methods and lower than AD and bilateral filters for an image of size 257×257 pixel. The proposed algorithm takes 4.3789s, and it is the faster one for an image of size 150×150 pixels. It takes 1.45081s. In regards to the number of iterations our method yield the good results in only one iteration.

D. Experiment on real ultrasound images

Experiments have been also conducted using various ultrasound images. Practically, clean reference images are not available for ultrasound images. Hence, the performance of the proposed method in the case of real US images is compared from a qualitative point of view. Fig. 3 shows the noisy images and the corresponding despeckled images obtained by the proposed method and the other methods. The restored images using AD and ML-2D-ARMA-GARCH algorithms are rather weak compared to the other methods. Good results are given by the ATV filter as observed in Fig. 3(b, h). Both of bilateral filter Fig. 3(d, j) and the proposed method Fig. 3(f, l) reduce noise while keeping the amount of structures on the image. Bilateral filter and AD model operate better than ATV and ML-2D-ARMA-GARCH method which presents some artifacts. Finally, the proposed method can not only eliminate the multiplicative noise in the image including its edges, but also it preserves the details well.

V. Conclusion

In this work a novel method for denoising ultrasound medical images using a GMM estimation and 2D-ARMA-GARCH model is proposed. First a wavelet transform is applied to the image algorithm. then we propose to employ the 2D ARMA-GARCH model to capture the two-dimensional heteroscedasticity of wavelet subands. Experimental results have been compared to other popular speckle reduction methods in terms of speckle reduction, image quality, edge and feature preservation performance. The proposed method takes into consideration the true statistics of the signal and noise components. It is also effective in removing speckle and presents competitive results on synthetic and real ultrasound medical images compared to established methods in the literature.

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