IMAGE SUPER-RESOLUTION VIA DEEP DILATED CONVOLUTIONAL NETWORKS

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ABSTRACT

Deep learning techniques have been successfully applied in single image super-resolution (SR). Recently, researches have shown that increasing the depth of network can significantly improve SR performance. Very deep networks for SR achieved a large improvement than former methods. However, simply increasing depths basically introduce more parameters and this lead to cumbersome computational cost. In this paper, we present a general and effective method to accelerate very deep networks for single image SR. Our method is based on dilated convolution operation, which support exponential expansion of the receptive field without increasing filter size. With the help of dilated convolution, shallow networks can achieve large receptive field and exploit contextual information in an efficient way. Based on a very deep network, we propose a 12 layers dilated convolutional network for SR (DCNSR). While accelerating 2x speed, our shallow network achieves better performance than original deep networks and shows state-of-the-art reconstructed results.

Index Terms— Super-Resolution, Deep Networks, Dilated Convolution, Acceleration

1. INTRODUCTION

As a classical problem in computer vision, single image superresolution (SR) aims at recovering a visually pleasing high-resolution (HR) image from a given low-resolution (LR) one. Since multiple HR image patches could correspond to the same LR image patches, SR is an inherently ambiguous and highly ill-posed problem. To address this problem, a large number of single image SR methods have been proposed, such as interpolation-based methods [1, 2, 3], reconstruction-based methods [4, 5, 6] and learning-based methods [7, 8, 9, 10, 11, 12, 13, 14, 15]. Among them, learning-based methods have attracted more attention from the community recently. Through learning a mapping function from corresponding pairs of LR-HR image patches, learning-based methods delivered superior performance in image SR. More recently, inspired by the great success achieved by deep learning, deep convolutional neural networks (CNNs) have been successfully used for image SR and obtained large improvements in accuracy [9, 10, 11, 12, 13, 14].

To the best of our knowledge, Dong *et al.* [9] first proposed super-resolution convolutional neural network (SRCNN) and demonstrated that CNNs can be used to learn a mapping from LR to HR space in an end-to-end manner. Lately, in order to investigate whether domain expertise can be used to design better deep network

architectures, Wang et al. [10] proposed sparse coding based network (SCN). Based on a set of sparse coding sub-networks, they successfully added sparse prior into deep network and their SCN outperforms SRCNN with a smaller model size. Furthermore, deeper structures have been explored in [11, 12]. Inspired by VGG-net used for ImageNet classification [16], Kim et al. [11] proposed a 20 weight layers CNN termed VDSR and obtained a significant improvement in accuracy. Benefited from larger image contextual information and stronger learning capacity, deeper network significantly boosts performance in image SR. However, increasing depth basically introduces more parameters and this will cause two problems. First, expensive computation will slow down the speed of reconstruction. Second, more training data is required to prevent over-fitting. To handle the problem of increasing parameters while keeping performance, Kim et al [12] used a deeply-recursive convolutional network (DRCN). Nonetheless, the computation of DRCN is as costly as VDSR. Therefore, the speed of SR is still slow for real time application.

Recently, there are a number of CNN acceleration studies. However, the exploration of accelerating deep networks for SR is very limited. The general procedure of SR consists of two decoupled steps: upsampling the original LR image to the desired size using bicubic interpolation; taking the upscaled LR image as input and reconstructing the HR image by learned mapping function. Because of the first step, the computation cost grows quadratically with the spatial size of the HR image. Thus, handling the upsampling operation at the end of network is an effective way for acceleration. Inspired by this idea, Shi et al. [13] proposed a sub-pixel convolution layer to store the information of HR image into smaller size feature maps with multiple channels. Similarly, Dong et al. [14] adopted deconvolutional layer to replace the bicubic interpolation. Besides, they re-design the original SRCNN structure into a more compact one. Their new fast SRCNN achieves a significant speed up. However, much domain knowledge and cumbersome experiments are needed for designing a faster architecture. A simple and adaptive method to accelerate standard CNN networks for image SR is still absent.

In this work, we propose a simple yet effective way to accelerate very deep convolutional networks for SR without losing performance. In [11], they compared the SR performance of different networks with depth ranging from 5 to 20 and indicated that deeper networks produced better performance because of large receptive field and high nonlinearities. Besides this, we further argue that the size of receptive filed is more important than the depth of network. In SR problem, high receptive filed means more contextual information used for reconstruction. While holding the same size of receptive filed, networks with different depths will produce similar reconstructed results. So we introduce dilated convolution operation into the framework of deep learning based SR methods. Compared to normal convolution, dilated convolution support exponential expan-

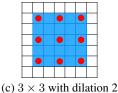
ICIP 2017

This work is supported by the National Natural Science Foundation of China (Grant No. 61403376 and 61370039), the Beijing Nature Science Foundation (Grant No. 4162064) and the Open Research Fund of Hunan Provincial Key Laboratory of Network Investigational Technology (Grant No. 2015HNWLFZ055).

sion of the receptive field without increasing filter size. Therefore, shallower networks with less parameters can obtain the same size of receptive field as very deep networks. With the help of dilated convolution, we show a 12 layers CNN can produce better results than VDSR, with nearly half parameters and computations.







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Fig. 1. Dilated convolution operation supports larger receptive filed than traditional convolution. The receptive field of (c) is the same as (b), but filter size in (c) is only 3×3 .

Specifically, the details and contributions of this work are mainly in three aspects:

- We firstly introduce dilated convolution into deep learning based SR methods and proposed a state-of-the-art dilated convolutional network for SR (DCNSR).
- We demonstrate receptive filed is a significant important factor in SR task. While keeping the same size of receptive field, networks with different depths will produce similar results.
- With the help of dilated convolution, DCNSR yields better performance with less parameters and faster speed. The idea of network design can be easily applied into existing SR architectures and this strategy benefits other accelerating approaches.

In the following, we will first review the related work of VDSR. Then, in Section 3 we will describe dilated convolution in detail and present the new DCNSR. Section 4 shows our implementation details and experiments, in which we compare the performance of our method with the state-of-the-art approaches. Finally, we conclude this paper in Section 5.

2. REVIEW OF VERY DEEP CONVOLUTIONAL NETWORKS FOR SR

Inspired by the successful of deep and thin network in image classification task [16], Kim *et al.* [11] designed a very deep convolutional networks for SR (VDSR). Compared to 3 weight layers in SRCNN [9], VDSR used 20 layers and achieved significant improvement. All layers except the first and the last are of the same type: 64 filters of the size $3\times 3\times 64$. Both of the first and last layer consisted of a single filter of size $3\times 3\times 64$. Rectified linear unit (ReLu) is used as activation function. The receptive filed size of VDSR is 41×41 , which is much large than SRCNN (13×13 in SRCNN). With larger receptive filed size, VDSR can use more context information to predict image details.

In addition, for speeding up training convergence, they suggested a residual learning network structure. Instead of reconstructing HR results directly, they used VDSR to learn the residual of HR and LR images. The advantage of residual learning can be explained in two sides. For SR problem, LR images and HR images are largely similar. Modelling the difference between HR and LR images can reserve the information of LR images more effectively. This idea is

similar to former classical SR methods [7] which aimed at recovering the high-frequency information of HR images. For deep neural networks, residual learning make very deep networks easier to optimize [17]. With the help of this strategy and high learning rate, they trained VDSR over 80 epochs and achieved a new state-of-the-art performance in single image SR. Unfortunately, increasing depth basically introduces more parameters and this will cause to slow down the speed of reconstruction. In this work, we mainly focus on this problem, and propose the new DCNSR model.

3. THE PROPOSED MODEL

In this section, we first introduce dilated convolution operation. Then, we describe the importance of receptive field in CNN based single image SR. Due to these analyses, we proposed DCNSR by introducing dilated convolution operation into VDSR, and ensure the receptive field is same with VDSR.

3.1. Dilated Convolution

Let $f: \mathbb{Z}^2 \to \mathbb{R}$ be a discrete function. Let $\Omega_r = [-r, r]^2 \cap \mathbb{Z}^2$ and let $k: \Omega_r \to \mathbb{R}$ be a discrete filter of size $(2r+1)^2$. The discrete (full) convolution operator * can be defined as

$$(f * k)(\mathbf{p}) = \sum_{\mathbf{s} + \mathbf{t} = \mathbf{p}} f(\mathbf{s}) k(\mathbf{t}). \tag{1}$$

Recently, Yu et al. [18] generalized this operator and proposed a new one named dilated convolution:

$$(f *_{l} k)(\mathbf{p}) = \sum_{\mathbf{s}+l\mathbf{t}=\mathbf{p}} f(\mathbf{s})k(\mathbf{t}). \tag{2}$$

where l is a dilation factor and $*_l$ is referred as a dilated convolution or an l-dilated convolution. The familiar full convolution * can be regarded as a specific version of l-dilated convolution when l=1.

Compared to traditional convolution, the dilated convolution operation can apply the same filter at different ranges using different dilation factors. Thus, it can support exponential expansion of the receptive filed without increasing filters size or network depth. For example, Fig. 1 illustrates the receptive field of different filter size and dilation. As shown in this figure, the receptive field of 2-dilated convolution is same with 5×5 convolution, while its parameters are same with the 3×3 convolution.

3.2. The Importance of Receptive Field

Yu et al. [18] had demonstrated the effectiveness of dilated convolution for dense prediction. Similarly, we argue that image SR can benefit from dilated convolution operation since receptive filed is also an important factor in SR problem. In the task of SR, the size of receptive field means the amount of contextual information that can be exploited to infer high-frequency components. As SR is a highly ill-posed problem, collecting and analyzing more context can afford the network more clues to predict image details. Thus, we consider networks with different depths but the same size of receptive field will produce similar reconstructed results. There are three strategies to raise the size of receptive field: (1) holding on filters size, adding more weight layers; (2) using large filters and (3) replacing traditional convolution by l-dilated convolution, where l > 1. In order to show the importance of receptive filed and the effectiveness of dilated convolution, strategy (2) and (3) are used to get large receptive field. While holding the same size of receptive field, we compare the performances of different network settings.

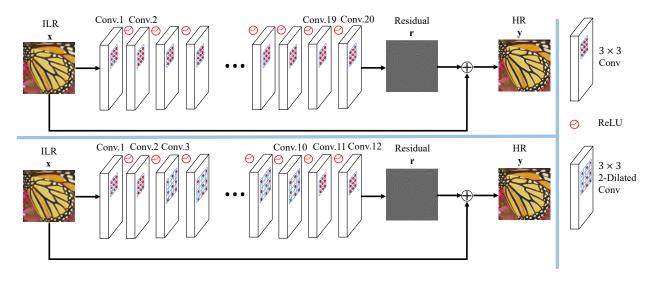


Fig. 2. Network architectures of VDSR (top) and VDSR_12_Dilated (bottom). VDSR has 20 layers, and VDSR_12_Dilated has 12 layers.

Table 1. Network architecture of VDSR_12_Dilated. Benefiting from dilated convolution operation, VDSR_12_Dilated can achieve 41×41 receptive field with 12 layers and 3×3 filters.

Layer	1	2	3	4	5	6	7	8	9	10	11	12
Convolution	3×3	3×3	3×3	3×3	3×3	3×3	3×3	3×3	3×3	3×3	3×3	3×3
Dilation	1	1	2	2	2	2	2	2	2	2	1	1
Receptive Filed	3×3	5×5	9×9	13×13	17×17	21×21	25×25	29×29	33×33	37×37	39×39	41×41
Output Channels	64	64	64	64	64	64	64	64	64	64	64	1

3.3. DCNSR Model

VDSR achieves the new state-of-the-art and it does not need specific network structure design, therefore, we choose it as our baseline network. Holding on the same size of receptive field as VDSR (41 \times 41), we design five networks with different filters size or dilations for comparison. (1) VDSR_20 is the baseline 20 layers network from [11]. The filter size of all layers is 3 \times 3. (2) VDSR_12 is a 12 layers network with filter size 3 \times 3 and 5 \times 5. (3) VDSR_12_Dilated is also a 12 layers network but with filter size 3 \times 3. All layers with filter size 5 \times 5 in (2) are replaced by 3 \times 3 filters and 2-dilated convolution. (4) VDSR_10 and (5) VDSR_10_Dilated are similar to (2) and (3) respectively. Fig. 2 illustrates the difference between VDSR and VDSR_12_Dilated in detail. Table 1 explains how a 12 layers network achieves 41 \times 41 receptive filed.

4. EXPERIMENTAL RESULTS

In this section, we evaluate the performance of our method on several datasets. We first describe datasets used for training and testing. Next, we describe implementation details. Finally, we evaluate different dilated convolution based SR models, and compare our method with several state-of-the-art SR methods.

4.1. Datasets

Training Dataset: The 91-image dataset from Yang *et al.* [7] is widely used as the training set in learning-based SR methods. However, 91 images are not enough for deep network to gain best performance because of the increasing parameters and complexity of deep model. Therefore, following Kim *et al.* [11], we used 200 images

from Berkeley Segmentation Dataset [19] additionally. Thus 291 images are used for benchmark with other methods. Data augmentation technique is used to getting more training data. Scale augmentation used in [11] is also adopted in our training. So we can handle multiple scales SR task with only one model.

Notedly, for results in Section 4.3, we only used 91 images to train networks fast, so performances can be slightly different between Section 4.3 and Section 4.4.

Testing Dataset: For a fair comparison, we use Set5 [20], Set14 [21] and BSD100 [19] for testing. The original images are first down-sampled by bicubic interpolation and then up-sampled to desired size to generate LR-HR image pairs for both training and evaluation. Zero padding is applied in all convolutional layers to keep the size of output maps as the same as input.

4.2. Implementation Details

The network structure and parameters setting are described in Table 2. We use Adam [22] with a mini-batch size of 64. β_1 and weight dacay are set to 0.9 and 0.0001, respectively. Following [11], we initialize the weights by the method described in He *et al.* [23]. All experiments are trained over 80 epochs (11698 iterations with batch size 64) with a learning rete of 10^{-4} . Gradient clipping used in [11] is not necessary in our training since we use Adam instead of SGD.

In testing, we only process the luminance channel with our method. After reconstructing, we shave the image border in the same way as [9] for objective evaluations to ensure fair comparison. In our implementation, all the experiments are implemented using the caffe package [24] on a GTX 980Ti GPU. Since dilated convolution operation has not been supported by CuDNN [25], all the

Table 2. Performance of	different network settings.	All the five networks have 4	41×41 receptive field and scale factor is 2.

	VDSR_20	VDSR_12	VDSR_12_Dilated	VDSR_10	VDSR_10_Dilated
First Part	Conv(3,64,1,D1)	Conv(3,64,1,D1)	Conv(3,64,1,D1)	Conv(3,64,1,D1)	Conv(3,64,1,D1)
				Conv(3,64,64,D1)-	Conv(3,64,64,D1)-
		Conv(3,64,64,D1)-	Conv(3,64,64,D1)-	Conv(5,64,64,D1)-	Conv(3,64,64,D2)-
Mid Part	18Conv(3,64,64,D1)	8Conv(5,64,64,D1)	8Conv(3,64,64,D2)	4Conv(7,64,64,D1)	4Conv(3,64,64,D3)
		-Conv(3,64,64,D1)	-Conv(3,64,64,D1)	-Conv(5,64,64,D1)	-Conv(3,64,64,D2)
				-Conv(3,64,64,D1)	-Conv(3,64,64,D1)
Last Part	Conv(3,1,64,D1)	Conv(3,1,64,D1)	Conv(3,1,64,D1)	Conv(3,1,64,D1)	Conv(3,1,64,D1)
PSNR(Set5)	37.46 dB	37.45 dB	37.44 dB	37.40 dB	37.38 dB
PSNR(Set14)	32.83 dB	32.80 dB	32.87 dB	32.65 dB	32.80 dB
PSNR(BSD100)	31.65 dB	31.62 dB	31.72 dB	31.45 dB	31.65 dB
Parameters	665921	895873	370368	1082496	296064
Speedup	1.0×	1/13.2×	2.0×	1/10.4×	2.6×

Table 3. PSNR and SSIM comparison on three test datasets among different SR methods.

Dataset	Scale	Bicubic	A+	SRCNN	CSCN	VDSR_20	DCNSR	
		PSNR/SSIM	PSNR/SSIM	PSNR/SSIM	PSNR/SSIM	PSNR/SSIM	PSNR/SSIM	
Set5	×2	33.66/0.9299	36.57/0.9545	36.48/0.9542	36.93/0.9552	37.58/0.9591	37.46/0.9585	
	×3	30.39/0.8682	32.67/0.9093	32.57/0.9090	33.10/0.9144	33.68/0.9218	33.74/0.9219	
	$\times 4$	28.42/0.8104	30.36/0.8617	30.31/0.8628	30.86/0.8732	33.33/0.8828	31.37/0.8831	
Set14	×2	30.23/0.8688	32.47/0.9063	32.45/0.9067	32.56/0.9074	33.00/0.9125	32.91/0.9116	
	×3	27.54/0.7742	29.29/0.8203	29.30/0.8215	29.41/0.8231	29.75/0.8305	29.76/0.8312	
	$\times 4$	26.00/0.7027	27.47/0.7514	27.50/0.7513	27.64/0.7578	27.95/0.7647	27.99/0.7661	
BSD100	×2	29.56/0.8431	30.77/0.8756	31.36/0.8879	31.40/0.8884	31.86/0.8956	31.81/0.8947	
	×3	27.21/0.7385	28.18/0.7791	28.41/0.7863	28.50/0.7875	28.80 /0.7964	28.80/0.7972	
	×4	25.96/0.6675	26.74/0.7065	26.90/0.7103	27.03/0.7161	27.24/0.7230	27.26/0.7241	

experiments are tested without CuDNN acceleration for comparing pure computational loads.

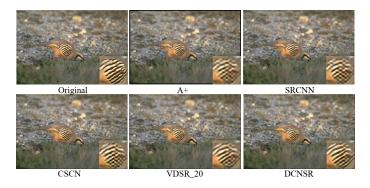


Fig. 3. The '8023' image from BSD100 dataset ($4 \times$ upscaling).

4.3. Evaluation of Different Dilated Convolution Based Models

In Table 2, we give the architectures and performances of these 5 networks. All these networks expect VDSR_10 achieve similar PSNR performance. This is corresponding to our assumption that different networks with the same size of receptive filed will produce similar results. With the same depths, dilated networks (3) and (5) show better performance, less parameters and faster speed than (2) and (4). This is caused by the drawback of large filter size. While filters with bigger size can obtain large receptive field, they also brings more noisy into the network learning procedure. In addition, compared to 3×3 filters, there are much redundance in learned 5×5 and 7×7 kernels because the values in these big kernels are highly correlated. Lastly, we find that the speed of (2) and (4) are much

slower than (3) and (5), even slower than (1). To sum up, receptive field is an important factor in SR task and dilated convolution is a better techniques for achieving large receptive field.

4.4. Comparisons with State-of-the-Art Methods

Since VDSR_12_Dilated achieves the best result with fast speed, we compare VDSR_12_Dilated with other state-of-the-art SR methods and named it as dilated convolutional network for SR (DCN-SR). Compared methods are A+[8], SRCNN [26], CSCN [10] and our VDSR_20 implementation. The implementations are all from the publicly available codes provided by the authors. In Table 3, we provide a summary of quantitative evaluation on testing datasets. Our DCNSR yields the highest average PSNR and SSIM in all these datasets. In Fig. 3, the reconstructed images of our DSNSR is much sharper and clearer than other results. Reconstructed results obtained with DCNSR are available online for all three datasets¹.

5. CONCLUSION

In this paper, we introduce dilated convolution to accelerate the speed of very deep networks for SR. We first show that receptive field is an important factor in image SR. Networks with different depths but the same receptive field will produce similar HR results. Second, we propose dilated convolution to replace full convolution operation. Dilated convolution operation is a much better technique for gathering large receptive field. Based on a 20 layers very deep network, we design five different networks setting and show the effectiveness of dilated convolution operation both in performance and speed. Without specific network design, our strategy is a general method for deep learning based SR acceleration and it benefits other accelerating approaches.

¹https://drive.google.com/open?id=0ByMcIJq3Oj8peGplamJQNjM1TlU

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