

# DCT/DWT BLIND MULTIPLICATIVE WATERMARKING THROUGH STUDENT-T DISTRIBUTION

*Antonis Mairgiotis*<sup>\*#</sup>

*Lisimachos P. Kondi*<sup>#</sup>

*Yongyi Yang*<sup>&</sup>

<sup>#</sup> University of Ioannina, Ioannina, Greece

<sup>\*</sup> Technological Educational Institute of Thessaly, Larissa, Greece

<sup>&</sup> Illinois Institute of Technology, Chicago IL, USA

## ABSTRACT

In this work, which addresses issues related to the efficient hiding of watermark information in the transform domain, we propose to model the transform coefficients with the Student-t distribution through the multiplicative rule of embedding. Based on the observation that the statistical distribution of the transform coefficients has heavy tailed behavior, we design a new class of watermark detectors following the multiplicative rule of embedding. We present experimental results that compare the proposed method with known state-of-the-art multiplicative watermark detectors and demonstrate its effectiveness in terms of sensitivity and robustness.

**Index Terms**—multiplicative image watermarking, Student-t, DWT, DCT

## 1. INTRODUCTION

Towards copyright protection and authentication of data, digital image watermarking has been an attractive solution. The main idea is to hide the known data –the watermark– into the digital medium –the host data– in order to provide a kind of data protection [1]–[3]. In previous years, various methods have been proposed in the image watermarking literature, based on statistical decision theory [22]. Generally, based on the embedding/detection domain these methods are classified as: spatial domain [4]–[7] and transform domain [7]–[11]. Common transforms that are selected for image watermarking are Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT) and Discrete Wavelet Transform (DWT).

Working in the transform domain, DWT has been proven a popular choice [9], [11], [12]. The energy compaction property exhibited in the transform domain suggests that the distortions introduced by the watermark into a set of transform coefficients will be spread over all components in the spatial domain so as the change in the pixel values is less significant. Depending on the embedding rule used in a watermarking system, the secret information is often embedded in an additive [4]–[14] or multiplicative way [15]–[20].

Many detectors rely on the Gaussian distribution of the data [8], [22], [23]. Although well justified, it has been proven suboptimal in transform based watermarking. This kind of distribution assumption on the host signal's characteristics does not hold in practice [5]. Various previous works [8]–[11] show that the distribution of the transform coefficients is far from Gaussian since it contains outliers, which cause the distribution to be heavy tailed. Thus, following the additive rule of embedding, the correlator detector is optimal only when coefficients follow Gaussian distribution [2], [8]. But, following the multiplicative rule of embedding, the correlator detector fails to achieve optimal performance [15]. Working in environments like the DFT domain, the correlator still works, but it is optimal only when the magnitudes follow e.g. the exponential distribution [15].

An improvement toward better detectors asks for more accurate statistical models. From a statistical detection theory viewpoint, the original data modeling is very important and its influence on detector sensitivity is a crucial point [22], [23]. Thus, various pdfs like Gaussian [4], [5], Generalized Gaussian Distribution (GGD) [8], Laplacian [16], modified Gauss-Hermite [14], Student-t [13] and Bessel-K form density [12], have been proposed for the DCT or DWT domain for the additive watermarking problem.

Hernandez et al. in their work [8], proposed an extension of the Gaussian distribution, known as Generalized Gaussian distribution [16]. Based on this distribution, they modeled the 8x8 DCT coefficients, providing a new detector structure which outperformed the correlator detector. The same statistical model was used in the work of Cheng and Huang [7], where they proposed a new DWT based watermark detector based on GGD. Using the DFT domain, a very good candidate was Weibull distribution, for which Barni et al. [19] proposed an optimum detector. Trying to exploit the heavy-tailed distribution of DWT subband coefficients, Kwitt et al. [11] proposed the Cauchy member of SaS (Symmetric alpha Stable) family distributions. Recently, Bian and Liang [12] proposed the application of Bessel-K form density and derived appropriate watermark detectors for the additive problem.

In the multiplicative case, robust optimum detectors have been proposed for all the aforementioned domains. In the work of Cheng and Huang [15], an investigation on robust optimum detection of multiplicative watermarks has been proposed. Another distribution that has been used in multiplicative watermarking in DWT was the Laplacian distribution in the work of Ng [16]. This distribution is a special case of generalized Gaussian and optimality issues can be found in the work of [20]. The Cauchy probability model has been proposed in the framework of DCT domain multiplicative watermarking by Yin et al. [17].

In this work we investigate the statistical characterization of marginal distribution of the wavelet detail subband coefficients (Fig.1) and the distribution of DCT coefficients, based on Student's t distribution [13] and the consequent study of its application in the multiplicative watermarking problem.

This paper is organized as follows. The multiplicative watermarking problem is briefly explained in Section 2. The proposed statistical model of the DWT/DCT coefficients and the derived test statistic are given in Sections 3 and 4 respectively. The models of comparison are given in Section 5, whereas the numerical results are shown in Section 6. The work is concluded in Section 7.

## 2. MULTIPLICATIVE WATERMARKING PROBLEM AND PROPOSED TEST STATISTIC

Let  $\mathbf{x}=[x_1, x_2, \dots, x_N]^T$  be the  $N$ -sequence transform coefficients of the original image in vector notation, whereas the watermark signal that is embedded is defined by  $\mathbf{w}=[w_1, w_2, \dots, w_N]^T$ . The watermark is generated based on a pseudo-random sequence taking the values  $\{+1, -1\}$  with equal probabilities. Based on these definitions the commonly used multiplicative embedding rule [1]–[3] is defined by [22]:

$$y_i = x_i (1 + \gamma w_i), \quad i=1, \dots, N \quad (1)$$

where  $\mathbf{y}=[y_1, y_2, \dots, y_N]^T$  is the sequence of watermarked data in vector format and  $\gamma$  is the known parameter that controls the strength of the watermark, providing a tuning tool between robustness and imperceptibility of the watermark [1]–[3]. In order to avoid high degrees of distortion a common way of embedding is following an adaptive embedding procedure like the multiplicative rule. It is well known that the multiplicative watermark detection problem can be formulated as a binary hypothesis problem where the observation is the possibility that watermarked data can be viewed as a noisy environment in which we seek for the presence or absence of the hidden information-watermark [22]:

$$\begin{aligned} H_0 : \mathbf{y} &= \mathbf{x} \\ H_1 : \mathbf{y} &= \mathbf{x}(1 + \gamma \mathbf{w}) \end{aligned} \quad (2)$$

where  $H_0$  is the null hypothesis and  $H_1$  is the alternative one. In order to define a decision rule we resort to the LRT (likelihood ratio test):

$$l(\mathbf{y}) = \frac{p_y(\mathbf{y} | H_1)}{p_y(\mathbf{y} | H_0)} \quad (3)$$

where  $p_y(\mathbf{y} | H_0)$ ,  $p_y(\mathbf{y} | H_1)$  are the conditional pdfs under the two hypotheses. In addition, assuming that the transform coefficients are i.i.d. (independent and identical distributed) with pdf  $p_x(x_i)$ , we can express the likelihood ratio test as:

$$l(\mathbf{y}) = \frac{\prod_{i=1}^N \frac{1}{|1 + \gamma w_i|} p_x\left(\frac{y_i}{1 + \gamma w_i}\right)}{\prod_{i=1}^N p_x(y_i)} \quad (4)$$

The i.i.d. assumption is a mechanism that has been proved effective over the years [1]–[14]. Even though real images are non-stationary processes and cannot be realized as i.i.d. signals, they can be regarded as decorrelated signals. Nevertheless, for many years the tractability of i.i.d. models and the low complexity of the obtained solutions are a good motivation to consider this class of models in extent.

## 3. PROPOSED STATISTICAL MODEL

It is well known that coefficients of DWT detail/DCT coefficients are symmetric, with mean value close to zero obeying leptokurtotic marginal densities (e.g. densities with heavy tailed characteristics). The Student-t distribution is defined as [13], [21]:

$$St(x_i | 0, \lambda, \nu) = \frac{\Gamma(\nu + 1/2)}{\Gamma(\nu/2)} \left( \frac{\lambda}{\pi \nu} \right)^{1/2} \left( 1 + \frac{\lambda}{\nu} x_i^2 \right)^{-\frac{\nu+1}{2}}, \quad i=1, \dots, N \quad (5)$$

where  $\nu$  is called “degrees of freedom” and is responsible for the heavy tailed shape of the distribution. Note that, depending on the value of  $\nu$  we can have either the Cauchy distribution ( $\nu \rightarrow 1$ ) or the Gaussian distribution ( $\nu > 10$ ).

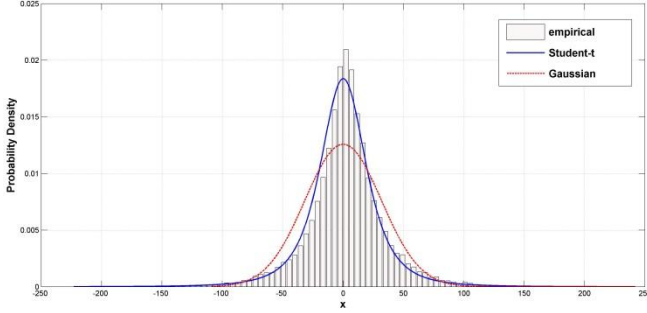
The ability of the distribution to capture the whole range of the tail's decay behavior (from a slow decay to a more rapid one), results in a robust model with respect to heavy tailed data description. The  $\lambda$  parameter is called a “precision” parameter and is in essence a scale parameter. The proposed distribution may be obtained as a compound distribution, derived from the normal and the gamma distributions [21]. Thus, the application of a Student-t distribution consists of a two-level data generation process. Assuming that we have the i.i.d. random variable  $\tau_i$  that follows Gamma pdf:

$$p(\tau_i) = \text{Gamma}(\nu/2, \nu/2) \quad (6)$$

then the wavelet coefficient  $x_i$  is drawn from a Gaussian distribution with zero mean and precision parameter  $\lambda \tau_i$ :

$$p(x_i | \tau_i) = N(0, (\lambda \tau_i)^{-1}) \quad (7)$$

In order to estimate the values of the parameters of our interest we resort to the known EM (Expectation Maximization) algorithm [13].



**Fig. 1** Probability density functions of the empirical data, the Student-t model and the Gaussian model applied in the second level of the wavelet transform for *Lena* image (horizontal details).

#### 4. PROPOSED TEST STATISTIC

Taking the logarithm of the likelihood ratio test of Eq. (4), and using the conditional Student-t pdfs, under the two hypotheses, the decision rule becomes:

$$l(\mathbf{y}) = \sum_{i=1}^N (\nu + 1) \log \left\{ \frac{(1 + \gamma w_i)^2 (\nu + \lambda y_i^2)}{\nu (1 + \gamma w_i)^2 + \lambda y_i^2} \right\}. \quad (8)$$

In either domain we work, by using Eq. (8), we employ  $N$  coefficients in total. Working in the DWT domain, if we need to apply the same test in more subbands, then we can follow a “subband adaptive” rule, where the test statistic takes the form:

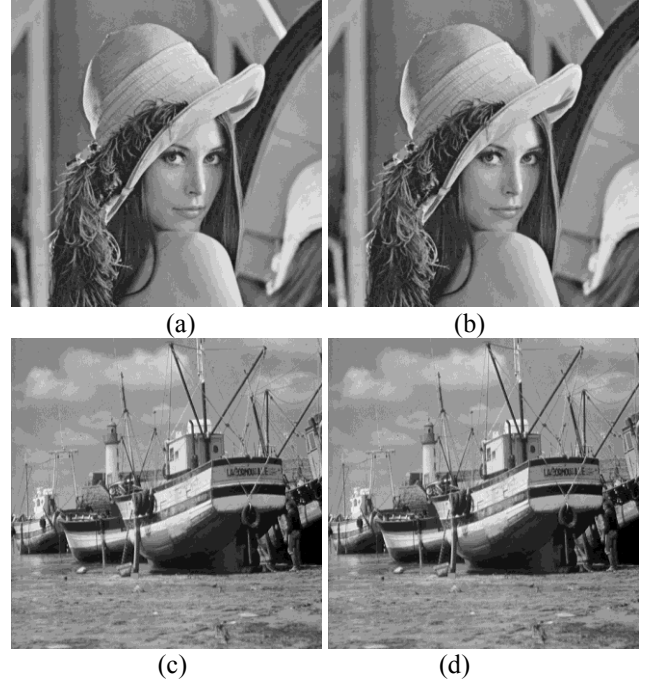
$$l(\mathbf{y}) = \sum_{k=1}^K (\nu_k + 1) \sum_{i=1}^{N_k} \log \left\{ \frac{(1 + \gamma w_i)^2 (\nu_k + \lambda_k y_i^2)}{\nu_k (1 + \gamma w_i)^2 + \lambda_k y_i^2} \right\} \quad (9)$$

Then, parameter  $k$  denotes the specific subband and parameter  $K$  the total number of subbands. Thus, in Eq. (9) we employ  $N_k$  coefficients for every subband we use.

#### 5. EMBEDDING

In this work, we resort to Monte Carlo experiments, through “random watermark” experiments. Thus, 1000 pseudorandomly generated watermarks are added to the two known standard test images as depicted in Fig. 2 (*Lena* and *Boat* of size 512x512) at every run. Working in the DCT domain, we transform the image by 8x8 block-wise DCT. Then, we select low- and mid- frequency coefficients as host coefficients for watermark embedding. More specifically, we embed the watermark information in a total of 13 coefficients from the third coefficient in a zig-zag order. Thus, we have 53248 identically distributed coefficients in total. Following the inverse DCT, we have the watermarked image.

Working in the DWT domain, we apply a two-level wavelet transform using the db8 wavelet [24] and we embed the hidden information using the multiplicative rule of Eq. (2) in the second level’s detail subbands trying to balance between robustness and perceptual invisibility. The watermarked image is then obtained by applying the inverse wavelet transform.



**Fig. 2** Two known images (watermarking in DWT domain): (a, b) *Lena*, original and watermarked, PSNR=54.48 dB, (c, d) *Boat*, original and watermarked, PSNR=56.14 dB.

#### 6. MODELS OF COMPARISON

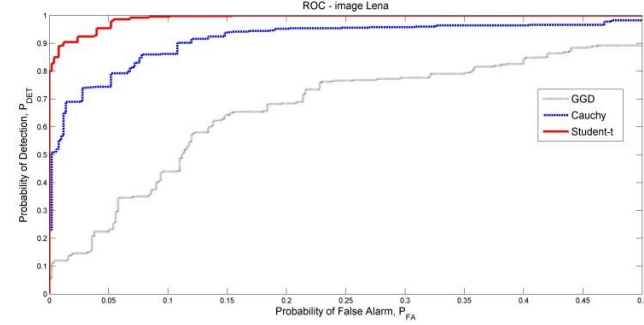
Generalized Gaussian Density (GGD) is a well-established model for wavelet detail subband coefficients and has been applied in the multiplicative watermarking problem with very good results [15]. In addition, we will compare our method with the alpha-stable family and more specifically with the Cauchy member, since it is the only non-Gaussian alpha-stable distribution with a closed form probability density function [17]. The modeling of the DWT coefficients using the Laplacian pdf and the consequent multiplicative detector is also a choice that is used for comparison reasons [16]. Though the Laplacian is a special form of the GGD distribution, we use it here for better comparison between the proposed classes of detectors. The Laplacian model has been investigated in the work of Ng and Garg in [16].

#### 7. NUMERICAL EXPERIMENTS

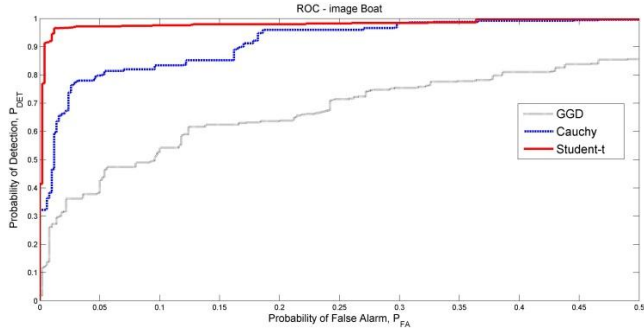
*DCT domain:* For low watermark to document ratios (WDRs) [13], in Fig. 3 and Fig. 4, we can observe that the proposed detector has superior detection performance compared with the other detectors. To draw valid

conclusions about the robustness property, we also run many experiments under intentional or unintentional attacks. In Fig. 5 the performance under median filtering with size equal to 3x3 is depicted using the ROC curves only for the image *Lena*. It is again obvious that the proposed detector has better performance when we model the DCT coefficients using the Student-t distribution

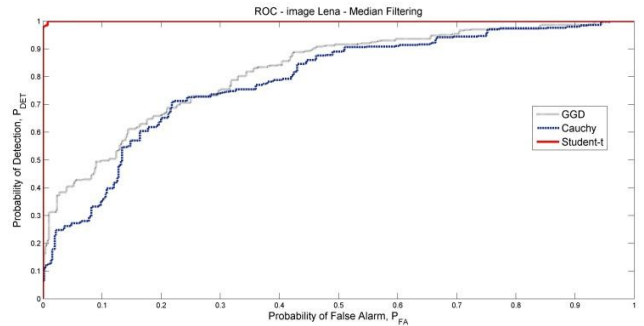
**DWT domain:** According to the experimental results, in Fig. 6 we observe that the performance of the proposed detector is at the same high level with the case of GGD and Laplacian based detectors. Considering the case of image Boat in Fig. 7, we observe that the performance of the proposed detector is still the same, maintaining the same high performance level. In Figs. 8 and 9, considering the performance of the proposed detector under JPEG attack with quality factor of 10%, it is obvious that the proposed detector retains the ability to detect watermark better than the other detectors for the same WDR value.



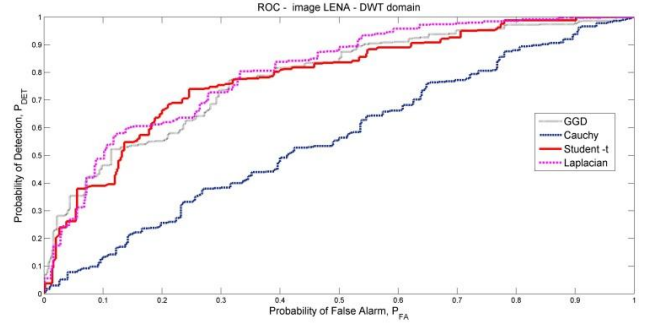
**Fig. 3** Detection performance comparison in the DCT domain– Image Lena, WDR=-63.9 dB.



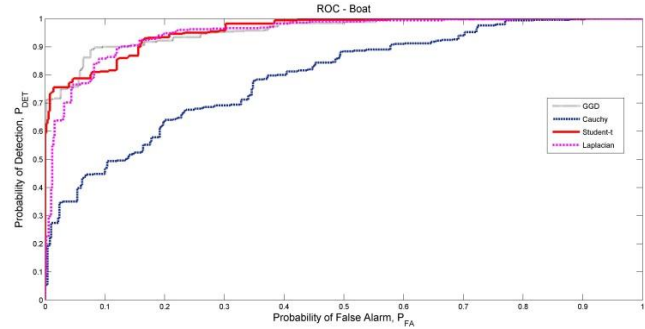
**Fig. 4** Detection performance comparison in the DCT domain – Image Boat, WDR=-63.6dB.



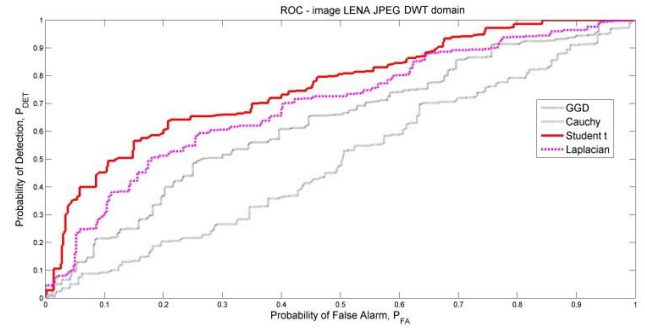
**Fig. 5** Detection performance comparison under median filtering with window size equal to 3x3 in the DCT domain– Image Lena, WDR=-53.5 dB.



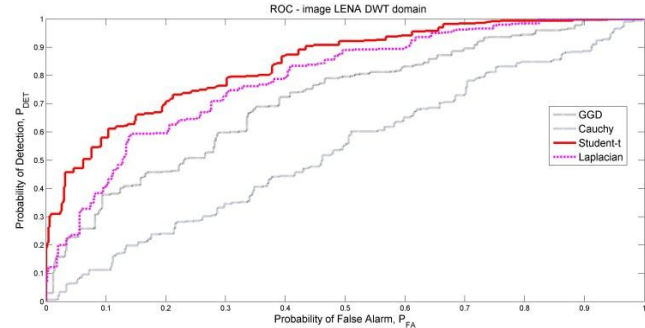
**Fig. 6** Detection performance comparison in the DWT domain– Image Lena, WDR=-57.02 dB.



**Fig. 7** Detection performance comparison in the DWT domain– Image Boat, WDR=-57.3 dB.



**Fig. 8** Detection performance comparison in the DWT domain under JPEG attack– Image Lena, WDR=-55.9 dB.



**Fig. 9** ROC curves for the detection performance comparison in the DWT domain under JPEG attack– Image Boat, WDR=-55.8 dB.

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