SKELLAM DISTRIBUTION BASED ADAPTIVE TWO-STAGE NON-LOCAL METHODS FOR PHOTON-LIMITED POISSON NOISY IMAGE RECONSTRUCTION

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ABSTRACT

Two-stage non-local methods represented by the Poisson non-local means (PNLM) method [1] and the non-local principal component analysis (NLPCA) [2] method perform well for photon-limited Poisson image reconstruction, where patch-similarity computation in the non-local reconstruction stage is guided and affected by a pre-reconstructed image obtained at the first stage. In this paper, we propose a new method to provide a better pre-reconstructed image with low computational cost. Firstly, we propose an adaptive method for fitting the linear relationship between image intensity and Poisson parameter; Secondly, we obtain an initial estimated image according to this relationship. Lastly, we obtain new pre-reconstructed image by adjusting the initial estimation according to Skellam distribution. Numerical experiments show that our method can provide a better pre-reconstructed image, and therefore can improve the performance of the PNLM method and reduce the computational cost of the NLPCA method efficiently.

***Index Terms*—**Poisson image, Skellam distribution, two-stage strategy, pre-reconstructed image, non-local method

1. INTRODUCTION

Reconstruction of photon-limited Poisson image is urgent demand and particularly challenging in many application fields such as security monitoring, astronomy and medical imaging. Compared with widely used traditional methods such as variance-stabilizing transformation based methods [3-4] and empirical Bayesian based regularization methods [5-6], two-stage non-local methods represented by the PNLM method and the NLPCA method perform better for the photon-limited Poisson image reconstruction.

As we all know, accurate computation of similarities between non-local patches is crucial for any non-local method. However, information loss and structural damage in photon-limited Poisson noisy image lead to great difficulties for accurate computation of the non-local similarities. To remedy this, both of the PNLM method and the NLPCA method utilize a two-stage strategy as follows: at the first stage, a pre-reconstructed image is obtained; at the second stage, non-local method is used for image reconstruction, where the computation of the similarities between non-local patches is guided and affected by the obtained pre-reconstructed image. The PNLM method utilizes simple averaging filter at the first stage, and therefore some important structures such as edges will be blurred which leads to inaccurate similarity computation. The NLPCA method combines Poisson distribution based PCA and sparse Poisson intensity estimation method in a non-local estimation framework to obtain a better pre-estimated image, but the computational cost is very high.

From the perspective of statistics, reconstruction of Poisson image aims to estimate the parameters such as mean and variance of Poisson distribution. It has been shown that the intensity of pixels are linearly related to these Poisson parameters [7] which has been used for Poisson noise removal in [8]. Motivated by this property, we propose to obtain a pre-reconstructed image according to this linear relationship. However, this linear relationship should be estimated by using the pixels in homogeneous patches. In [7-8], homogeneous patches are chosen manually which causes difficulties in practice.

In this paper, we aim to provide a better pre-reconstructed image which can give better guidance for the similarities computation at the second reconstruction stage. Firstly, we propose an adaptive method for fitting the linear relationship between the intensity of pixels and Poisson parameters by choosing the homogeneous patches adaptively. Secondly, we obtain an initial estimated image based on this relationship according to the intensity of the observed image. Lastly, we obtain a pre-reconstructed image by adjusting the initial estimation obtained according to the Skellam distribution based acceptance range of intensity difference.

The rest of this paper is organized as follows: In Section2, we describe the PNLM method and the NLPCA method briefly; In Section3, we introduce our adaptive method in detail; Numerical experiments are given in Section4 to demonstrate the efficiency of our method, and finally, the discussion and conclusion are provided Section 5.

2. TWO-STAGE NON-LOCAL METHODS

To simplify, let y_s be the value of the Poisson noisy image at site s and x_s its underlying noise-free value, which satisfies

$$p\left(y_s \left| x_s\right.\right) = \frac{e^{-x_s} x_s^{y_s}}{y_s!} . \quad (1)$$

Poisson noisy image reconstruction aims to estimate the noise-free image x from the observed image y .

Based on the traditional non-local means [9] method for Gaussian noise removal, Deledalle et.al [1] proposed an improved Poisson non-local means (PNLM) method to estimate x_s as follows

$$\hat{x}_s = PNLM(y) = \frac{\sum_{t\in N(s)} w_{s,t} \cdot y_t}{\sum_{t\in N(s)} w_{s,t}} , \quad (2)$$

where $N(s)$ is the window centered on s and $w_{s,t}$ is a data-driven weight depending on the similarity between pixels with indexes s and t defined by

$$w_{s,t} = \exp\left(-\frac{\sum_b f(y_{s+b}, y_{t+b})}{\alpha} - \frac{\sum_b g(\tilde{x}_{s+b}, \tilde{x}_{t+b})}{\beta} \right) \quad (3)$$

where α and β are filtering parameters, f and g are two similarity criteria suitable respectively to compare noisy data y and pre-estimated data $\tilde{x}$ [1]. The PNLM method is a two-stage method, where pre-reconstructed image $\tilde{x}$ should be estimated at the first stage, which is obtained by using averaging filter in [1]. Averaging filter is a good technique to estimate the mean of image intensities in homogeneous image patch, but will cause blurring at the edges, which will lead to inaccurate computation of the non-local similarity coefficient $w_{s,t}$ and reduce the performance of non-local reconstruction at the second stage.

Another representative two-stage method for Poisson noisy image reconstruction is the NLPCA method proposed in [2]. The NLPCA method creates a collection of patches of the observed image y and a pre-estimated image u firstly and groupings the vectorized noisy patches of y into K groups according to the similarities between each pair of patches of the image u . For each patch group $Y_k^u \in R^{M_k \times M}$, where M_k is the number of the similar patches of size $\sqrt{M} \times \sqrt{M}$ in the k-th group, we minimize the following loss function:

$$\left(U_k^*, V_k^*\right) = \arg\min_{U,V} \left\{ \sum_{i=1}^{M_k} \sum_{j=1}^{M} \exp\left(UV\right)_{i,j} - (Y_k^u)_{i,j} \left(UV\right)_{i,j} \right\} \quad (4)$$

and then obtain the reconstructed patch group as

$$\hat{F}_k = \exp\left(U_k^* V_k^*\right) , \quad (5)$$

where $(\cdot)_{ij}$ denotes the j-th pixel in the i-th patch. After averaging the pixels in different overlapping reconstructed patches, we can obtain the final reconstructed image $\hat{x}$.

In the NLPCA method, grouping of similar patches of the observed Poisson image y is guided by a pre-estimated image $\tilde{x}$. To improve the performance, two-stage strategy is introduced in the NLPCA method as follows: at the first stage, we choose $u = y$ in (5) and obtain a pre-reconstructed image $\tilde{x}$; at the second stage, we choose $u = \tilde{x}$ in (5) and compute the final reconstructed image $\hat{x}$. Compared to the PNLM method, the NLPCA method can obtain better pre-reconstructed image, but the computational cost is very high.

3. DESCRIPTION OF THE PROPOSED METHOD

To improve the performance of two-stage non-local method such as the PNLM method and the NLPCA method, we aims to obtain better pre-reconstructed image with low computational cost in this paper

3.1. Adaptive Linear Relationship Fitting

It has been shown that the intensity of pixels are linearly related to the parameters such as mean and variance of Poisson distribution, which can be formulated as follows [7]

$$\mu_s = F\left(y\right) = a \cdot y_s + b \quad (6)$$

where y_s and μ_s are the intensity and the approximation of the mean of Poisson distribution at the pixel with index s , a and b are the fitting parameters. If $N(s)$ is a homogeneous patch centered on pixel s , the mean of Poisson distribution at the pixel with index s can be approximated by the local mean defined as follows

$$m_s = \frac{1}{|N(s)|^2} \sum_{t\in N(s)} y_t . \quad (7)$$

Using the data set $\left\{(y_s, m_s)\right\}$ in homogeneous patches, the fitting parameters a and b can be optimized by using least-square method. In [7], homogeneous patches are chosen manually, but we propose a method to choose the homogeneous patches adaptively in this paper.

According to (7), we can obtain a local mean image m with m_s the value at the pixel s . Then, we compute the gradient magnitude map as follows

$$G(s) = \sqrt{m_h^2(s) + m_v^2(s)} \quad (8)$$

where $m_h(s)$ and $m_v(s)$ are the magnitudes of derivatives at the pixel s in horizontal and vertical directions, respectively.

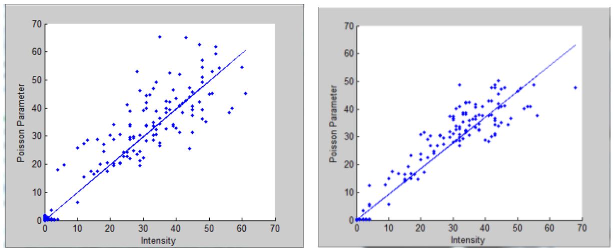
If

$$\max_{t\in N(s)} G(t) < T \quad (9)$$

with $T \approx 0$ a positive threshold, then we consider that $N(s)$ is a homogeneous patch and m_s is a reasonable estimation of the mean of the Poisson distribution at the pixel s . According to (9), we can choose the homogeneous patches in the image adaptively.

In Figure 1, we show the linear relationship fitting results obtained by using our method and the method proposed in [7] for the Saturn image showed in Figure 2.

Obviously, our method can fit the linear relationship better. From the perspective of statistics, reconstruction of Poisson image aims to estimate the mean of Poisson distribution. Using the equation (6), we can obtain a reasonable estimation of the original intensities at the pixels in the non-homogeneous regions, and therefore we obtain an initial estimated image μ according to the observed image y.



Method in [7] Our method

Figure 1: Comparison of the method [7] and our method

3.2. Skellam Distribution based Intensity Adjustment

The generative model of Poisson noisy image (e.g the equation (1)) implies that even the intensities of two pixels in the noise-free image are same, the noisy intensities may still be different. However, according to (7), different y_s must lead to different μ_s, which obviously does not conform to the generative model. To remedy this, we propose to adjust μ_s according to the Skellam distribution, which is use to descript the distribution of the difference between two independent Poisson random variables.

The probability mass function of a Skellam distribution can be written as [10]:

$$f\left(k;\mu_1,\mu_2\right)=e^{-(\mu_1+\mu_2)}\left(\mu_1/\mu_2\right)^{k/2}\cdot I_{|k|}\left(2\sqrt{\mu_1\mu_2}\right), \quad (10)$$

where μ_1 and μ_2 are the means of two Poisson distributions respectively, k is the difference between two Poisson random variables and $I_k\left(z\right)$ denotes the modified Bessel function of the first kind.

In [7], Skellam distribution based intensity difference acceptance range is utilized to statistically test if an intensity difference comes from same or different signals. Inspired by this work, we propose to adjust μ_s according to the intensity difference acceptance range defined by

$$I_s=\arg\max_{I} A_s\left(I\right) \;\; s.t. \;\; A_s\left(I\right)\le 1-\delta \quad (11)$$

where $A_s\left(I\right)=\sum_{k=-I}^{I} e^{-2\mu_s}\cdot I_{|k|}\left(2\mu_s\right)$, u_s is obtained by the equation (6) and δ is the error rate used to control confi-dence level.

For any pixel $s\in N(r)$ with $N(r)$ the patch centered on a reference pixel, we adjust μ_s as follows

$$\tilde{x}_s=\begin{cases}\mu_R, & if\left|y_s-y_r\right|\le I_r\\ \mu_s, & if\left|y_s-y_r\right|> I_r\end{cases} \quad (12)$$

Here, we need to choose some reference pixels manually such that the union of the $N(r)$ covers the whole image. The adjusted image $\tilde{x}$ is the pre-reconstructed image used for the second reconstruction stage of the PNLM method and the NLPCA method in this paper. Then, we obtain the Skellam distribution based PNLM (S-PMLM) method and the Skellam distribution based NLPCA (S-NLPCA) method, which can be summarized as follows:

Algorithm: **S-PNLM and S-NLPCA**

Input: Observed image y, and parameters T and δ :

First stage:

Setp1. Obtain a local mean image m according to (7);

Step2. Compute the gradient magnitude map G and choose the homogeneous patches according to the equations (8)-(9);

Step3. Obtain the linear relationship (6) according to the data set $\{(y_s,m_s)\}$ in homogeneous patches;

Step4. Compute the initial estimated image μ according to the linear relationship (6);

Step5. Compute the intensity difference acceptance range I_s according to the equation (11);

Step6. Adjust the initial estimated image μ according to (12) and obtain the pre-reconstructed image $\tilde{x}$;

Second stage:

S-PNLM: Reconstruct the image according to (2);

S-NLPCA: Reconstruct the image according to (4)-(5) with $u=\tilde{x}$ in the equation (4).

Output: reconstructed image $\hat{x}$.

4. NUMERICAL EXPERIMENTS

In this paper, to demonstrate the efficiency of our method, we compare our method with the most related methods: the PNLM method [1] and the NLPCA method [2]. The test images are showed in the Figure 2. For each tested image, we simulate the Poisson noisy image by scaling the original image to four different images with peak values of pixels as 0.1, 0.2, 0.5 and 10 firstly, and then corrupting them with Poisson noise.

(a) Saturn (b) House (c) Lena (d) Peppers

Figure2 Original images used in our experiments.

The results are obtained by running the Matlab codes on an Intel(R) Core(TM) i3-2310M CPU (2.20 GHz, 2.20 GHz) computer with RAM of 4.00 GB.

The Table 1 shows the comparison of objective index PSNR between the PNLM method and the proposed S-PNLM method. In Table 2, we show the PSNR comparison of the NLPCA method and the S-NLPCA method. Moreover, we also illustrate the average CPU time cost in the Table 2. The results showed in the Table1 and Table2 demonstrate that our method can improve the PSNR better. The CPU time of our method is only half of the CPU time of the NLPCA method.

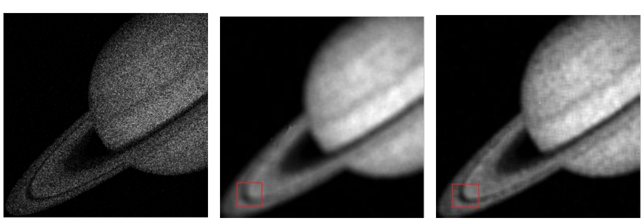
Table 1 PSNR Comparison of PNLM and S-PNLM

Peak	Method	Saturn	House	Lena	Peppers
0.1	PNLM	17.86	12.70	13.77	13.26
	S-PNLM	**18.66**	**15.54**	**14.75**	**14.33**
0.2	PNLM	19.12	15.84	15.68	15.60
	S-PNLM	**20.41**	**16.36**	**16.63**	**16.28**
0.5	PNLM	21.56	18.15	18.10	17.82
	S-PNLM	**21.93**	**18.74**	18.10	17.42
10	PNLM	29.56	26.70	25.44	25.26
	S-PNLM	**31.23**	**26.95**	**25.51**	**25.30**

Table 2 Comparison of NLPCA and S-NLPCA

Method	Saturn	House	Lena	Peppers	Ave. Time
		Peak=0.1			
NLPCA	18.51	15.60	14.79	14.62	77s
S-NLPCA	**18.81**	**15.64**	**14.84**	**14.67**	**31s**
		Peak=0.2			
NLPCA	20.62	**17.03**	16.64	16.34	87s
S-NLPCA	**20.73**	16.92	**16.85**	**16.45**	**32s**
		Peak=0.5			
NLPCA	23.16	19.68	19.08	18.72	74s
S-NLPCA	**23.26**	**20.01**	**19.13**	**18.85**	**34s**
		Peak=10			
NLPCA	30.56	24.91	22.63	**21.40**	77s
S-NLPCA	**30.62**	**25.13**	**22.78**	21.39	**35s**

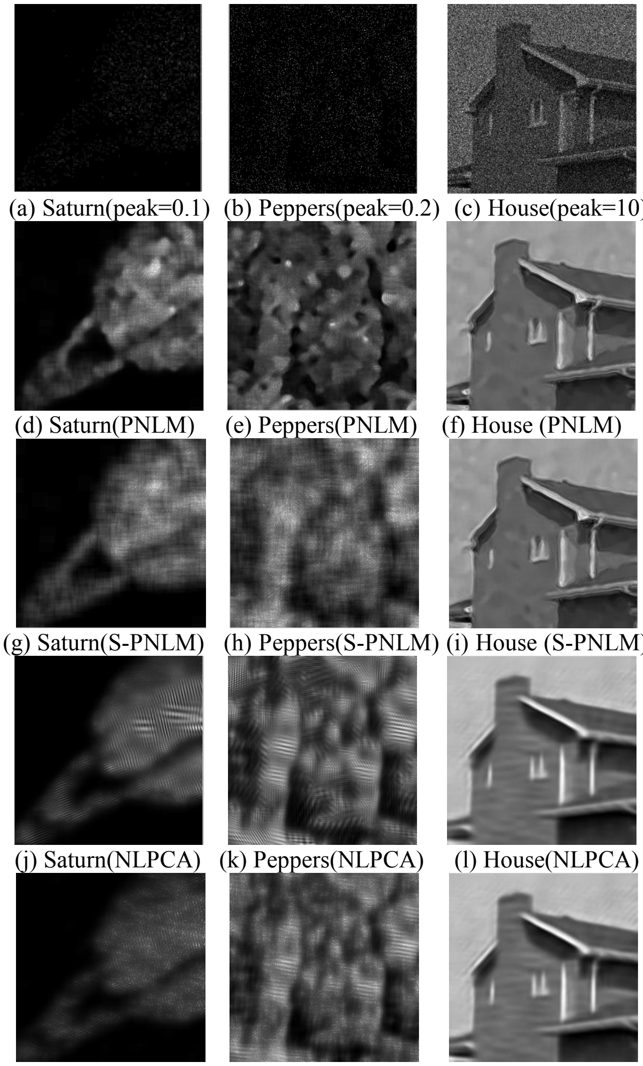
In Figure 3, we show the pre-reconstructed images used in the PNLM method and S-PNLM method. The results show that our method can preserve edges well. It is the reason that our method can improve the PSNR much better at the second reconstruction stage.



(a)Poisson image (b)PNLM (c) Our Method

Figure 3 Comparison of pre-reconstructed images (Peak = 10).

In the Figure 4, we show the reconstructed images obtained by these methods. The results show that the S-PNLM method can preserve edges much better than the PNLM method (see the edges of the circle around the Saturn). Moreover, due to inaccurate similarity computation, the PNLM method causes blocky effect and the NLPCA method causes undesirable stripe structures (See Figure 4(e) and Figure 4(k)), while our improved method can achieve better results.



(a) Saturn(peak=0.1) (b) Peppers(peak=0.2) (c) House(peak=10)
(d) Saturn(PNLM) (e) Peppers(PNLM) (f) House (PNLM)
(g) Saturn(S-PNLM) (h) Peppers(S-PNLM) (i) House (S-PNLM)
(j) Saturn(NLPCA) (k) Peppers(NLPCA) (l) House(NLPCA)
(m)Saturn(SNLPCA) (n)Peppers(SNLPCA) (p)House(SNLPCA)

Figure4 Comparison of reconstructed images

5. CONCLUSIONS

In this paper, we propose a new adaptive method to provide a better pre-reconstructed image with low computational cost for the two-stage non-local methods for Poisson noisy image reconstruction. Our method has three advantages: (i) homogeneous patches are chosen adaptively,and our method can fit the linear relationship between the intensity of pixels and Poisson parameters better. (ii) our method can provide a better pre-reconstructed image in which the edges can be preserved well, which is important for improving the performance of reconstruction. (iii) our method has low computational cost which is important in practice.

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