# CORRELATION PRESERVING ON GRAPHS FOR IMAGE DENOISING

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## ABSTRACT

In this paper, we propose a novel dictionary-driven image denoising method based on correlation preserving on graphs. To overcome the drawbacks of the instable and unreliable correlations among a set of learned basis vectors, two effective regularized strategies are employed in our coding process. Specifically, a graph-based regularizer is built to preserve the global similarity, which can adaptively capture both geometric structures and discriminative features of textured patches. In particular, edge weights in the graph are obtained by seeking a nonnegative low-rank construction. Besides, a locality constraint is designed to automatically preserve not only spatial neighborhood information but also internal consistency present in noisy patches while learning an overcomplete dictionary. Experimental results show that our method achieves state-of-the-art denoising results in terms of both PSNR and subjective visual quality.

***Index Terms—*** Dictionary learning, image denoising, graph Laplacian, locality preserving, low-rank.

## 1. INTRODUCTION

The objective of image denoising task is to recover clear image from a noisy measurement while preserving its main informative features such as the edges and textures. The noise introduced during the image acquisition process is generally assumed to be an additive zero-mean white and homogeneous Gaussian distribution [1]. The estimate for unknown original image is actually an ill-posed inverse linear problem due to the inadequate constraints [2]. To derive closed-form solutions, certain prior information of images need be utilized to regularize the recovery process. It is proved natural signals and images have essentially sparse representations [3]. Hence, the sparsity in an analytical transform domain or learned dictionary can be fully exploited in this denoising problem.

Many successful denoising methods have been proposed base on the correlations which reliably exist in spatial domain, transform domain or sparse coding [4]. A pioneer work called as non-local mean method (NLM) is to remove the noise by averaging the pixels with the spatial self-similarity correlation between patches from different locations [5]. According to the non-local and redundant correlations in image patches, famous benchmark BM3D [6] groups structurally similar patches to form 3D stack and then performs the collaborative filtering in DCT or Haar wavelet transforms. Recent advances for image denoising rely on coding correlation of image patches which can be sparsely represented by linear combination of basis vectors from an over-complete dictionary [7]-[15]. Representative methods include learned simultaneous sparse coding (LSSC) [7], clustering based sparse representations (CSR) [8] and its non-local improved version NCSR [9], K-clustering with singular value decomposition (K-SVD) [10] and its variants [11]. Besides, the combination of sparse models and low rank completion of data matrix is used to denoise the image [16]. Through performance analysis for these methods, the strong denoising capability roots in the utilization of the multi-level correlations which mainly contain the geometric similarity in external patches and the coding consistency of similar features in internal patches [4]. However, current denoising methods based on dictionary learning have some limitations to further improve because similarity of image patches might be damaged in sparse coding process. Converting high-dimension data to vector format [17] will lose high local correlations among the neighboring pixels. Moreover, small patch sizes decrease complexity of textures [18], and thus non-local structural similarity will be affected. In parallel, when the dictionary is adaptively learned from the image itself using the patches containing high noise levels, the correlations of nonzero coefficients will not correctly reflect the stable properties of feature space and result in amount of artifacts. Although some later works [16],[19] have made efforts to tackle these problems, the optimal denoising gain is still not achieved.

In this work, we address these issues and propose a novel denoising method on graphs, which makes a full exploitation of structure and feature subspaces in the image. Leveraging on the low-rank representation (LRR) model, we design an adaptive graph Laplacian (AGL) model as the regularizer to preserve global similarity correlations and enhance dissimilarity with the noise. Besides, the locality constraint (LC) coding is used to preserve the consistence in internal basis sets. By incorporating these preserving terms, the strong noise can be effectively removed.

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## 2. CORRELATION PRESERVING SPARSE CODING

### 2.1. AGL Regularized Coding

The AGL-based sparse coding scheme can explicitly take into account the manifold structure of image data. Based on spectral graph theory, AGL is adopted as a smooth operator to preserve the manifold structure. The obtained representations vary smoothly along the geodesics of the data manifold. Then the basis vector can encode the intrinsic structures embedded in new space according to the manifold assumption [20]. Given a set of data point $\mathbf{Y} \in \mathbb{R}^{d \times n}$, we construct a nearest neighbor graph $G$ with $n$ vertices denoting all data points. Let $\mathbf{W} \in \mathbb{R}^{n \times n}$ be a weight matrix of $G$. The similarity can be measured by edge weights and thus the affinity between the vertices is preserved. That is, geometrical structure of data space is described by AGL. Mathematically, this relationship is formulated by the following regularization function:

$$R(z) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (z_i - z_j) \mathbf{W}_{ij} = \mathbf{Tr}(\mathbf{ZLZ}^T) \tag{1}$$

where $z_i$ and $z_j$ are the corresponding mappings of two points $y_i$ and $y_j$ to construct a sparse coefficient matrix $\mathbf{Z}$. $\mathbf{L} = \Lambda - \mathbf{W}$ is the Laplacian matrix. $\Lambda$ is a diagonal matrix whose diagonal elements are the sum of elements $\{\mathbf{W}_{ij}\}$ in $\mathbf{W}$.

To take advantage of spectral properties, the matrix $\mathbf{L}$ is further normalized. Applying the fast symmetry preserving matrix balancing procedure [21] to $\mathbf{W}$ yields the doubly stochastic filtering matrix $\mathbf{K}$ and returns a diagonal scaling matrix $\mathbf{C}$. Then the normalized matrix $\overline{\mathbf{L}}$ is defined as

$$\overline{\mathbf{L}} = \mathbf{I} - \mathbf{K} = \mathbf{I} - \mathbf{C}^{-1/2} \mathbf{W} \mathbf{C}^{-1/2} \tag{2}$$

The matrix $\overline{\mathbf{L}}$ is symmetric and positive semi-definite, which is directly interpreted as a data-adaptive Laplacian filter with the expected behavior. Compared with un-normalized graph Laplacian, $\overline{\mathbf{L}}$ can provide better performance.

### 2.2. Edge Weight Computation Using LRR

The LRR model is based on the assumption that data points are approximately sampled from low-dimensional subspaces. For a set of data samples, LRR finds the lowest rank representation of all data. It has been shown that LRR is efficient in exploring low-dimensional subspace structures embedded in data. Let the matrix $\mathbf{Y} \in \mathbb{R}^{d \times n}$ be sampled from independent subspaces. Then each column can be represented by linear combination of bases in the dictionary $\mathbf{A}$. By imposing the most sparsity and lowest rank constraints, the coefficient matrix $\mathbf{Z}$ can be reconstructed by solving the following optimization problem

$$\begin{cases} \min\limits_{Z,E} \|\mathbf{Z}\|_* + \beta \|\mathbf{Z}\|_1 + \alpha \|E\|_1 \\ s.t. \ \mathbf{Y} = \mathbf{AZ} + E, \quad Z \geq 0 \end{cases} \tag{3}$$

where $\|\mathbf{Z}\|_*$ is the nuclear norm defined as the sum of all singular values of $\mathbf{Z}$, and $\|E\|_1$ is the $l_1$-norm of noise term. $\beta > 0$ is a parameter to balance between the low-rankness and sparsity. The parameter $\alpha > 0$ is used to balance the effect of noise, which is set empirically. The LRR can extract the global structures of data $\mathbf{Y}$, while the sparsity can capture the local relevance of each data vector. The problem (3) can be solved by the fast method adopted in [22]. This alternating direction method uses less auxiliary variables and no matrix inversions, and hence it can convergence fast to the minimum solution.

Given an appropriately designed dictionary $\mathbf{A}$, the optimal solution $\mathbf{Z}^*$ of low-rank recovery can accurately reveal some underlying correlations of data. The *ij*-th element of $\mathbf{Z}^*$ reflects the similarity between the samples $y_i$ and $y_j$. The sparse constraint ensures that the graph derived from $\mathbf{Z}^*$ is sparse. The low rank guarantees that the coefficients of samples are highly correlated in the same subspace, so $\mathbf{Z}^*$ can capture the global structures of the whole data. Based on these low-rank characteristics, after obtaining the optimal coefficient matrix $\mathbf{Z}^*$, the graph weight $\mathbf{W}$ is derived from it. The matrix $\mathbf{W}$ is defined as follows:

$$\mathbf{W} = (\mathbf{Z}^* + (\mathbf{Z}^*)^T) / 2 \tag{4}$$

In practice, for preventing the noise to affect the dependencies of graph adjacent structures [22], the coefficients in the weight matrix are set zeros under the given threshold $\mathbf{T}$. Let the parameter $\sigma$ be stand deviation of Gaussian noise. The region size is set as $L \times L$. The value of $\mathbf{T}$ is computed as follows:

$$\mathbf{T} = \sigma \sqrt{2 \log L^2} \tag{5}$$

### 2.3. LC-based Coding

This coding scheme utilizes the LC to project local features into the manifold space where their geometric structures are more easily identified. Hence, learned dictionary can best reconstruct an image while preserving locality correlations. Let $\mathbf{F} = [f_1, f_2, ..., f_N]$ denote the local descriptors of an image, which can be converted into a set of sparse codes $\mathbf{E} = [e_1, e_2, ..., e_N]$. Given a trained dictionary $\mathbf{B}$, the corresponding locality-constrained coding can find the coefficient $e_i$ for each feature $f_i \in \mathbb{R}^k$ by minimizing an objective function, which is given by

$$\begin{cases} \min\limits_{E} \sum_{i=1}^{N} \|f_i - \mathbf{B} e_i\|_2^2 + \gamma \|\varphi_i \odot e_i\|_2^2 \\ s.t. \quad \mathbf{1}^T e_i = 1, \quad \forall i \end{cases} \tag{6}$$

where the symbol $\odot$ denotes the element-wise multiplication. $\varphi_i \in \mathbb{R}^M$ is the locality adaptor that weights each basis vector proportional to its similarity to the input descriptor $f_i$. Finally, the locality adaptor $\varphi_i$ is formulated as

$$\varphi_i = \exp\left(\frac{dist(f_i, \mathbf{B})}{\delta}\right) \tag{7}$$

Here, $dist(f_i, \mathbf{B}) = [dist(f_i, \mathbf{b}_1), ..., dist(f_i, \mathbf{b}_M)]^T$, and $dist(f_i, \mathbf{b}_j)$ is the Euclidean distance between $f_i$ and $\mathbf{b}_j$. Here $\delta$ is used for adjusting the weights of decay speed for the locality adaptor. The constraint $\mathbf{1}^T e_i = 1$ ensures shift-invariant requirement.

The LC-based coding has several favorable properties that can achieve less reconstruction errors in problem (6). It has been suggested that the locality is more important than sparsity because the locality must lead to sparsity but not necessary vice versa [23]. Due to the over-completeness of dictionary, the coding process might select quite different bases for similar patches to favor sparsity, thus losing correlations between codes. On the other hand, the explicit locality adaptor by $l_2$-norm can ensure that similar patches will have similar codes [24].

## 3. PROPOSED DENOISING METHOD

### 3.1. Denoising Formulation

Consider that the observed image $\mathbf{Y}$ is corrupted by the white Gaussian noise $\mathbf{v}$ with distribution $N(0, \sigma^2)$. This degradation process is modeled by $\mathbf{Y}=\mathbf{X}+\mathbf{v}$. The clear image vector $\mathbf{X}\in\mathbb{R}^N$ is estimated by using so-called correlation preserving sparse coding (CPSC) model. Note that the AGL regularizer and the LC are incorporated for constructing the CPSC model by means of the complementary combination. Through seeking for the optimal sparse representation of image data different from the non-sparse noise, the denoised image $\mathbf{X}$ can be obtained. Based on the unified regularization, our CPSC model for the whole-image denoising problem is formulated as

$$(\mathbf{X},\mathbf{D},\mathbf{S})=\arg\min_{\mathbf{X},\mathbf{D},\mathbf{S}}\left\{\begin{array}{l}\|\mathbf{Y}-\mathbf{X}\|_2^2+\mu\sum_{i=1}^{M}\|\mathbf{D}s_i-\mathbf{U}_i\mathbf{X}\|_2^2\\+\rho\mathbf{Tr}(\mathbf{S}\bar{\mathbf{L}}\mathbf{S}^T)+\lambda\sum_{i=1}^{M}\|\varphi_i\odot s_i\|_2^2\end{array}\right\} \quad (8)$$

where the matrix $\mathbf{U}_i\in\mathbb{R}^{r\times N}$ is used to extract the $i$-th patch from the image. Considering both the computational complexity and the utilization of repetition patterns, the image is divided into fully overlapping small patches to deal with. The operator $\mathbf{U}_i\mathbf{X}$ denotes an image patch of size $\sqrt{r}\times\sqrt{r}$ pixels extracted from the clear image $\mathbf{X}$ at location $i$. $\mathbf{S}=[s_1,s_2,\ldots,s_M]\in\mathbb{R}^{k\times M}$ is the set of sparse coefficients. The estimate of an image patch can be sparsely represented by a linear combination of spare coefficients $\{s_i\in\mathbb{R}^k\}$ over a trained dictionary $\mathbf{D}\in\mathbb{R}^{r\times k}$. The regularization parameters $\mu>0$, $\rho>0$ and $\lambda>0$ are tuned to empirically control the constrained weights, respectively. The matrix $\bar{\mathbf{L}}$ is computed from Eqs. (2)-(4). According to Eqs. (6) and (7), the weight vector $\varphi_i$ is obtained. The regularization terms in Eq. (8) can ensure the convergence to a globally optimal solution.

### 3.2. Optimization for CPSC

With the alternating iteration strategy, the optimization for the CPSC model can be divided into two main stages: updating the dictionary $\mathbf{D}$ stage while fixing the $\mathbf{S}$; and updating the sparse coefficients $\mathbf{S}$ stage while fixing the $\mathbf{D}$; until convergence. Then, we obtain the desired clear image $\mathbf{X}$.

When $\mathbf{S}$ is assumed known, the dictionary atoms $\{d_i\}$ are updated by solving the problem (9). The K-SVD algorithm in [11] can be directly used to train and update the dictionary.

$$\mathbf{D}=\arg\min_{\mathbf{D}}\left\{\mu\sum_{i=1}^{M}\|\mathbf{D}s_i-\mathbf{U}_i\mathbf{X}\|_2^2\right\}\ s\ t\ \ \|d_i\|_2^2\le 1 \quad (9)$$

The problem (10) is convex by fixing the dictionary $\mathbf{D}$. Thus, the global minimum can be achieved. We update each vector $s_i$ individually, while holding other vectors $\{s_j\}_{j\neq i}$ the constants. The problem (10) can be solved by following the feature-sign search algorithm in [20]. It updates the solutions, and linearly searches for the optimal sparse coefficients. The new $s_i$ is further normalized by shift-invariant constraint $\mathbf{1}^T s_i=1$.

$$\mathbf{S}=\arg\min_{\mathbf{S}}\left\{\begin{array}{l}\mu\sum_{i=1}^{M}\|\mathbf{D}s_i-\mathbf{U}_i\mathbf{X}\|_2^2+\rho\sum_{i,j=1}^{M}\bar{L}_{ij}s_i^Ts_j\\+\lambda\sum_{i=1}^{M}\|\varphi_i\odot s_i\|_2^2\end{array}\right\} \quad (10)$$

Given all sparse codes and the dictionary, a closed-form solution has the quadratic expression. The globally denoised image can be obtained as the following form:

$$\mathbf{X}=(\mathbf{I}+\mu\sum_{i=1}^{M}\mathbf{U}_i^T\mathbf{U}_i)^{-1}(\mathbf{Y}+\mu\sum_{i=1}^{M}\mathbf{U}_i^T\mathbf{D}s_i). \quad (11)$$

where $\mathbf{I}$ is the identity matrix. The matrix $\mathbf{U}_i^T$ returns a clean patch to its original location. Finally, the detailed algorithm procedure of CPSC denoising is summarized in **Algorithm 1**.

**Algorithm 1** The Proposed CPSC Denoising Algorithm.

**Input:** The noisy image $\mathbf{Y}$; Maximum number of iterations $J$; The regularization parameters $\mu$, $\rho$, $\lambda$; The dictionary size ***r*** and ***k***; The weight parameters $\alpha$, $\beta$, $\delta$.

**Output:** The optimal solutions ($\mathbf{X}$, $\mathbf{D}$, $\mathbf{S}$).

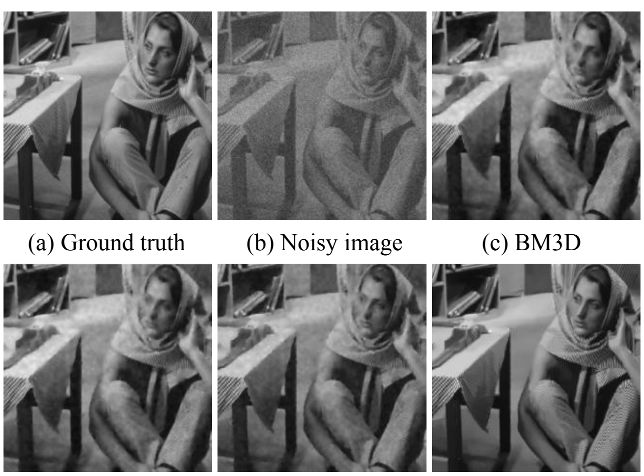
**Initialization:** $J=100$; $\mu=1.2$; $\rho=0.5$; $\lambda=0.3$; $r=64$; $k=256$; $\alpha=10$; $\beta=0.2$; $\delta=80$; $\mathbf{X}=\mathbf{Y}$.

1: **Set** $\mathbf{D}$ as overcomplete DCT dictionary; **Compute** the mean intensity of each patch; **Subtract** the mean intensity.
2: **Obtain** the optimal $\mathbf{Z}^*$ using the LADMAP method in [22]; Compute the weight matrix $\mathbf{W}$.
3: **Compute** the filtering matrix $\mathbf{K}$ by applying fast procedure [21] to $\mathbf{W}$; Normalize the matrix $\mathbf{L}$ to obtain $\bar{\mathbf{L}}$.
4: **Repeat** times:
5: **Update** the dictionary $\mathbf{D}$ stage: perform K-SVD scheme.
6: **Update** the coefficient $\mathbf{S}$ stage: search for optimal sparse coefficients using the fast feature-sign procedure.
7: **End** the loop when it satisfies the stopping condition.
8: **Compute** the clear image $\mathbf{X}$.

## 4. EXPERIMENTAL RESULTS

In this section, we have evaluated denoising performance of the proposed CPSC method. Moreover, we fully compared our method with other three state-of-the-art denoising methods, including BM3D [6], EPLL [12], NSCR [9]. In the experiments, seven natural images are carefully selected for the testing, and their noisy versions are simulated by adding the independent white Gaussian noise with the varying deviation level $10 \leq \sigma \leq 60$. For all the dictionary learning methods, we extract image patches of the same size 8×8 with a sliding distance of 1, and randomly select image patches from the training sets. As for other main parameters of our method, we set them following the initial values illustrated in **Algorithm 1**.

The images named as Barbara and Boat are corrupted by the Gaussian noise with the stand deviations $\sigma = 20$ and $\sigma = 50$, respectively. As shown in Fig. 1, our method can remove almost all the noise and achieve the best subjective result among four methods. Due to the similarity correlation preservation of the AGL regularization, the informative structures in the denoised image are very close to original image while generating few visual artifacts. For the high noise level, it can be clearly seen from Fig. 2 that our denoising result contains both sharper edges and smoother regions. By encoding the LC for local feature preservation, our method can reliably exclude the noise from candidate patches, in which strong classifying capability ensures the elegant visual effect.

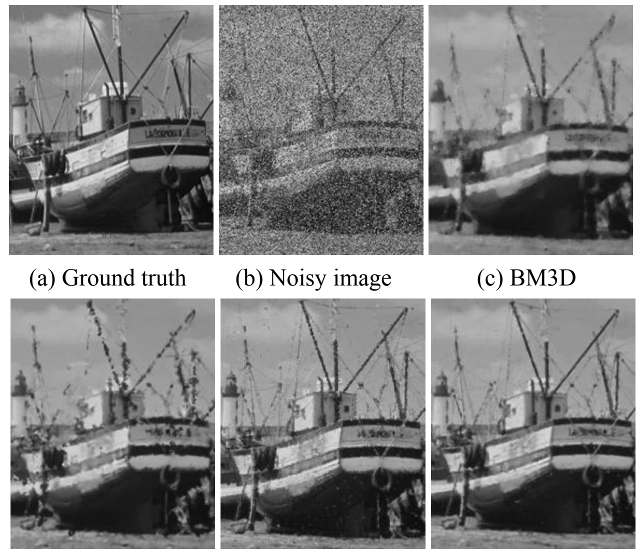


(a) Ground truth (b) Noisy image (c) BM3D

(d) EPLL (e) NSCR (f) Our

**Fig. 1**. Denoising results for image ***Barbara***.

To evaluate the objective quality of the denoised images, the values of peak signal-to-noise (PSNR) are computed. The results are shown in Table I. For each image and at each noise level, the highest PSNRs are highlighted in bold. It has shown that the proposed method invariably performs best on all test images. Furthermore, the average statistics for the denoised performance testified that CPSC model can preserve intrinsic correlations well for the various textures, and is capable of discriminating with the noise. Meanwhile, numerical scheme can efficiently ensure the convergence of optimal solutions within 100 iterations.



(a) Ground truth (b) Noisy image (c) BM3D

(d) EPLL (e) NSCR (f) Our

**Fig. 2**. Denoising results for image ***Boat***.

**Table 1.** The comparisons of PSNRs (in dB).

| Image | BM3D | EPLL | NSCR | Proposed |
|---|---|---|---|---|
| Monarch | 30.58 | 30.31 | 31.18 | **32.08** |
| Barbara | 29.28 | 29.45 | 29.87 | **31.04** |
| Wheel | 25.86 | 25.12 | 26.32 | **27.54** |
| Airplane | 27.73 | 28.54 | 27.97 | **29.36** |
| Lena | 28.89 | 29.15 | 29.50 | **30.61** |
| House | 26.62 | 26.99 | 27.32 | **27.89** |
| Boat | 28.53 | 28.67 | 29.38 | **29.65** |
| Average | 28.21 | 28.32 | 28.79 | **29.74** |

## 5. CONCLUSIONS

In this paper, we have presented an effective and efficient noise removal method based on the unified CPSC model. We utilize the structured space and the mapped feature space as two complementary strategies to address such an ill-posed inverse problem. By integrating LRR information into the AGL model, global similarity correlation is exploited and encoded better to separate original image data from the noise. Then the LC coding is applied to further preserve the feature correlations over learned codes to remove the residual noise. Experimental results show that the performance of the proposed denoising method is quite competitive with state-of-the-art denoising methods, even much better for the strong noise.

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