LOW-RANK MATRIX COMPLETION AGAINST MISSING ROWS AND COLUMNS WITH SEPARABLE 2-D SPARSITY PRIORS

Jiaoru Yang†, *Lei You*†, *Kun Li*‡, *and Jingyu Yang**†

† School of Electrical and Information Engineering, Tianjin University, Tianjin 300072, China
‡ School of Computer Science and Technology, Tianjin University, Tianjin 300072, China

ABSTRACT

Most existing matrix completion approaches assume that entries of matrices are missing at random, which could be violated in practical applications. This paper proposes a novel matrix completion method equipped with Joint Priors of LOw-rank and Separable 2-D Sparsity (JPLOSS) to complete missing rows and columns besides random missing. The underlying matrix is regularized by a low-rank prior, and its rows and columns are regularized by a row and a column dictionary, respectively. An reweighting scheme is incorporated into both the low-rank and sparsity terms to promote the low-rankness and sparseness simultaneously. The proposed model is effectively solved by an alternating direction method under the augmented Lagrangian multiplier framework. Experiments on both synthetic data and real images demonstrate the effectiveness and superiority of the proposed model in completing matrices with missing rows and columns compared with state-of-the-art matrix completion approaches.

Index Terms— Matrix completion, low-rank matrix approximation, sparse representation, inpainting

1. INTRODUCTION

Recovering an unknown matrix from a sampling of its entries has attracted considerable attention recently [1–5]. This problem occurs in many areas of computer vision and machine learning, such as image inpainting [6], recommender systems [7], and background modeling [8,9]. It is an ill-posed problem to complete a matrix from a subset of its entries. A commonly adopted assumption is that the latent matrix is low-rank or approximately low-rank so that its low-rank approximation can be more easily obtained. Specifically, Candès *et al.* [10] further demonstrated that most low-rank matrices can be recovered exactly from an incomplete set of entries under some mild conditions via convex programming.

Numerous models and algorithms in terms of low-rank matrix completion have been proposed over the recent years, such as singular value thresholding (SVT) [1], augmented lagrangian multiplier method (ALM) [2], accelerated proximate gradient algorithm (APG) [11] and iteratively reweighted nuclear norm (IRNN) [4], which provide strong theoretical guarantees for accurate matrix completion under quite general conditions. In particular, the nuclear norm is reweighted iteratively to enhance low-rankness of a matrix [4,12], inspired by reweighted ℓ_1 norm minimization for sparsity enhancement [13]. However, all of them and most existing completion models assume that missing entries are distributed randomly and each row or column should have some values observed, which can be breaked in many practical applications. Common examples include bursty packet loss in image transmission, scribbling lines in image inpainting and missing traces in seismic data interpolation. Given these issues, Yang *et al.* [14] put forward a matrix completion method with double priors to recover structurally-incomplete matrices (ReLaSP). The ReLaSP model is extended to recover matrices degraded by highly-structural missing and various types of noise in [15]. However, these methods fail to recover matrices with missing rows and columns due to insufficient priors.

In this paper, we propose a novel matrix completion model with Joint Priors of LOw-rank and Separable 2-D Sparsity (JPLOSS) to recover low-rank matrices against missing rows and columns besides random missing. Our model is based on the key observation that rows or columns in a matrix are strongly correlated and these signals have sparse representations under some dictionaries. Different from the plain nuclear norm minimization methods which only minimize the rank of matrices, our JPLOSS model exerts joint priors of low-rank and separable 2-D sparsity on the latent matrix to remedy the insufficiency of low-rank prior in regularizing missing rows and columns. Rows and columns of incomplete matrices are regularized by separable 2-D sparsity priors in view of the intra-row and intra-column correlations. A reweighting scheme is further utilized to boost low-rankness and sparseness by reweighting the low-rank and sparsity terms iteratively. To solve the JPLOSS model, we derive an alternating direction method under the augmented Lagrangian multiplier (ALM-ADM) framework. Experiments on both synthetic data and real images demonstrated that JPLOSS achieves consistently accurate completion of matrices with missing rows and columns than many state-of-the-art methods.

This work was supported by the National Natural Science Foundation of China under Grant 61372084 and Grant 61571322.

*Corresponding Author: yjy@tju.edu.cn

2. MATRIX COMPLETION VIA JPLOSS

2.1. Proposed JPLOSS Model

Let $\mathbf{D}$ denote an incomplete observation matrix which contains both structural (entire row and column missing) and random missing pixels and $\mathbf{\Omega}$ represent known entries in $\mathbf{D}$. The completed matrix is $\mathbf{A}$ and a low-rank prior is exploited to regularize $\mathbf{A}$ based on inter-row/-column correlations. As structural missing component may destroy low-rank structure of images, we introduce separable 2-D sparsity priors that each column/row of $\mathbf{A}$ has sparse representation under column/row dictionary $\mathbf{\Phi}_c$/$\mathbf{\Phi}_r$ with corresponding coefficient matrix $\mathbf{B}$/$\mathbf{C}$ when there is entire-row/column missing. $\mathbf{E}$ is an error matrix representing missing image corruption and missing entries in $\mathbf{D}$ are set as zeros. We further equip our model with a reweighting strategy to improve the sparsity and low-rank recovery. Then, the proposed model is formulated as:

$$\begin{aligned} \min\ & \operatorname{tr}\left(\mathbf{W}_a \circ \mathbf{\Sigma}\right) + \gamma_B \|\mathbf{W}_b \circ \mathbf{B}\|_1 + \gamma_C \|\mathbf{W}_c \circ \mathbf{C}\|_1 \\ \text{s.t.}\ & \mathbf{A} = \mathbf{\Phi}_c \mathbf{B}, \\ & \mathbf{A}^\top = \mathbf{\Phi}_r \mathbf{C}, \\ & \mathbf{A} + \mathbf{E} = \mathbf{D}, \mathcal{P}_{\mathbf{\Omega}}\left(\mathbf{E}\right) = 0, \end{aligned} \tag{1}$$

where $\operatorname{tr}(\cdot)$ represents the trace of a matrix, "$\circ$" denotes element-wise multiplication of two matrices, and $\|\cdot\|_1$ is ℓ_1 norm of a matrix. $\mathbf{\Sigma} := \operatorname{diag}\left([\sigma_1, \sigma_2, \ldots, \sigma_n]\right)$ is a diagonal matrix composed of singular values of $\mathbf{A}$ in a non-increasing order. $\mathbf{W}_a$, $\mathbf{W}_b$ and $\mathbf{W}_c$ signify the weighting matrices for weighted low-rank term and separable sparsity terms, respectively. γ_B and γ_C are regularization coefficients. $\mathcal{P}_{\mathbf{\Omega}}(\cdot)$ stands for the projection operator onto $\mathbf{\Omega}$.

2.2. Algorithm for JPLOSS Model

In the proposed JPLOSS model, we minimize the ℓ_1 norm instead of the non-convex ℓ_0 norm. A reweighting scheme is used to counteract the influence of the signal magnitude on the ℓ_1 penalty function [13]. As suggested in [13], the weights should be inversely proportional to the true signal magnitudes, i.e., small coefficients are discouraged by large weights, while large coefficients are encouraged by small weights. Likewise, the reweighted nuclear norm is an approximate of the rank function, and the weights on singular values are determined in a similar way [12]. Hence, the weighting matrices $\mathbf{W}_a$, $\mathbf{W}_b$ and $\mathbf{W}_c$ are updated iteratively according to the magnitudes of current estimated singular values $\mathbf{\Sigma}^{(l)}$, coefficient matrix $\mathbf{B}^{(l)}$ and $\mathbf{C}^{(l)}$, using the inverse proportion rule [13].

Given weighting matrices $\mathbf{W}_a$, $\mathbf{W}_b$ and $\mathbf{W}_c$, problem (1) is essentially a convex optimization problem that can be solved by augmented lagrangian method (ALM). Thus, we exploit the ALM method to convert the constrained optimization problem (1) into an unconstrained one with the following partial augmented lagrangian function:

$$\begin{aligned} & L_{\mu_1,\mu_2,\mu_3}\left(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{E}, \mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Y}_3\right) \\ & \quad = \operatorname{tr}\left(\mathbf{W}_a \circ \mathbf{\Sigma}\right) + \gamma_B \|\mathbf{W}_b \circ \mathbf{B}\|_1 + \gamma_C \|\mathbf{W}_c \circ \mathbf{C}\|_1 \\ & \qquad + \langle \mathbf{Y}_1, \mathbf{A} - \mathbf{\Phi}_c \mathbf{B} \rangle + \frac{\mu_1}{2} ||\mathbf{A} - \mathbf{\Phi}_c \mathbf{B}||_{\mathrm{F}}^2 \\ & \qquad + \langle \mathbf{Y}_2, \mathbf{A}^\top - \mathbf{\Phi}_r \mathbf{C} \rangle + \frac{\mu_2}{2} ||\mathbf{A}^\top - \mathbf{\Phi}_r \mathbf{C}||_{\mathrm{F}}^2 \\ & \qquad + \langle \mathbf{Y}_3, \mathbf{D} - \mathbf{A} - \mathbf{E} \rangle + \frac{\mu_3}{2} ||\mathbf{D} - \mathbf{A} - \mathbf{E}||_{\mathrm{F}}^2, \end{aligned} \tag{2}$$

where $\mathbf{Y}_1$, $\mathbf{Y}_2$ and $\mathbf{Y}_3$ denote the Lagrange multiplier matrices. μ_1, μ_2 and μ_3 are penalty factors. $\langle \cdot, \cdot \rangle$ represents the matrix inner product and $\|\cdot\|_{\mathrm{F}}$ signifies the matrix Frobenius norm.

Instead of solving Eq. (2) directly, we exploit alternating direction method (ADM) to estimate $\mathbf{B}$, $\mathbf{C}$, $\mathbf{A}$, $\mathbf{E}$, $\mathbf{Y}_1$, $\mathbf{Y}_2$ and $\mathbf{Y}_3$ alternately:

$$\begin{cases} \mathbf{B}^{k+1} = \arg\min\limits_{\mathbf{B}} \{L_{\mu_1^k,\mu_2^k,\mu_3^k}\left(\mathbf{A}^k, \mathbf{B}, \mathbf{C}^k, \mathbf{E}^k, \mathbf{Y}_1^k, \mathbf{Y}_2^k, \mathbf{Y}_3^k\right)\}, \\ \mathbf{C}^{k+1} = \arg\min\limits_{\mathbf{C}} \{L_{\mu_1^k,\mu_2^k,\mu_3^k}\left(\mathbf{A}^k, \mathbf{B}^{k+1}, \mathbf{C}, \mathbf{E}^k, \mathbf{Y}_1^k, \mathbf{Y}_2^k, \mathbf{Y}_3^k\right)\}, \\ \mathbf{A}^{k+1} = \arg\min\limits_{\mathbf{A}} \{L_{\mu_1^k,\mu_2^k,\mu_3^k}\left(\mathbf{A}, \mathbf{B}^{k+1}, \mathbf{C}^{k+1}, \mathbf{E}^k, \mathbf{Y}_1^k, \mathbf{Y}_2^k, \mathbf{Y}_3^k\right)\}, \\ \mathbf{E}^{k+1} = \arg\min\limits_{\mathcal{P}_{\mathbf{\Omega}}(\mathbf{E})=0} \{L_{\mu_1^k,\mu_2^k,\mu_3^k}\left(\mathbf{A}^{k+1}, \mathbf{B}^{k+1}, \mathbf{C}^{k+1}, \mathbf{E}, \mathbf{Y}_1^k, \mathbf{Y}_2^k, \mathbf{Y}_3^k\right)\}, \\ {\mathbf{Y}_1}^{k+1} = \mathbf{Y}_1^k + \mu_1^k \left(\mathbf{A}^{k+1} - \mathbf{\Phi}_c \mathbf{B}^{k+1}\right), \\ {\mathbf{Y}_2}^{k+1} = \mathbf{Y}_2^k + \mu_2^k \left({\mathbf{A}^{k+1}}^\top - \mathbf{\Phi}_r \mathbf{C}^{k+1}\right), \\ {\mathbf{Y}_3}^{k+1} = \mathbf{Y}_3^k + \mu_3^k \left(\mathbf{D} - \mathbf{A}^{k+1} - \mathbf{E}^{k+1}\right), \\ \mu_1^{k+1} = \rho_1 \mu_1^k, \mu_2^{k+1} = \rho_2 \mu_2^k, \mu_3^{k+1} = \rho_3 \mu_3^k. \end{cases} \tag{3}$$

As there is no closed-form solution of $\mathbf{B}$-subproblem in Eq. (3), we adopt the accelerated proximal gradient (APG) algorithm [11] to achieve an approximate solution which is derived as:

$$\begin{cases} \mathbf{U}_{j+1} = \mathbf{Z}_j - \frac{\mu_1^k}{L_f} \mathbf{\Phi}_c^\top \left(\mathbf{\Phi}_c \mathbf{Z}_j - \frac{1}{\mu_1^k} \mathbf{Y}_1^k - \mathbf{A}^k\right), \\ \mathbf{B}_{j+1}^k = \operatorname{soft}\left(\mathbf{U}_{j+1}, \frac{\gamma_B}{L_f} \mathbf{W}_b\right), \\ t_{j+1} = \frac{1+\sqrt{4t^2+1}}{2}, \\ \mathbf{Z}_{j+1} = \mathbf{B}_{j+1}^k + \frac{t_j - 1}{t_{j+1}} (\mathbf{B}_{j+1}^k - \mathbf{B}_j^k), \end{cases} \tag{4}$$

where $\operatorname{soft}(\cdot,\cdot)$ is the soft-thresholding function [16] applying on the matrix element-wisely and t_j is a positive sequence with $t_1 = 1$. A backtracking line-search strategy [17] is used to generate a scalar sequence $\{L_j\}$ that approximates L_f. In like manner, we approximate $\mathbf{C}$ as follows:

$$\begin{cases} \widetilde{\mathbf{U}}_{j+1} = \widetilde{\mathbf{Z}}_j - \frac{\mu_2^k}{L_f} \mathbf{\Phi}_r^\top \left(\mathbf{\Phi}_r \widetilde{\mathbf{Z}}_j - \frac{1}{\mu_2^k} \mathbf{Y}_2^k - {\mathbf{A}^k}^\top\right), \\ \mathbf{C}_{j+1}^k = \operatorname{soft}\left(\widetilde{\mathbf{U}}_{j+1}, \frac{\gamma_C}{L_f} \mathbf{W}_c\right), \\ \widetilde{t}_{j+1} = \frac{1+\sqrt{4\widetilde{t}^2+1}}{2}, \\ \widetilde{\mathbf{Z}}_{j+1} = \mathbf{C}_{j+1}^k + \frac{\widetilde{t}_j - 1}{\widetilde{t}_{j+1}} (\mathbf{C}_{j+1}^k - \mathbf{C}_j^k). \end{cases} \tag{5}$$

The solution of $\mathbf{A}^{k+1}$ is obtained by utilizing singular value thresholding (SVT) method given by:

$$\begin{cases} \left(\mathbf{H}^k, \mathbf{\Sigma}^k, \mathbf{V}^k\right) = \operatorname{svd}\left(\mathbf{Q}^k\right), \\ \mathbf{Q}^k = \frac{\left(\mathbf{Y}_3^k - \mathbf{Y}_1^k - \mathbf{Y}_2^{k\top} + \mu_1^k \mathbf{\Phi}_c \mathbf{B}^{k+1} + \mu_2^k \mathbf{C}^{k+1\top} \mathbf{\Phi}_r^\top + \mu_3^k \mathbf{D} - \mu_3^k \mathbf{E}^k\right)}{\mu_1^k + \mu_2^k + \mu_3^k}, \\ \mathbf{A}^{k+1} = \mathbf{H}^k \operatorname{soft}\left(\mathbf{\Sigma}^k, \frac{1}{\mu_1^k + \mu_2^k + \mu_3^k} \mathbf{W}_a\right) \mathbf{V}^{k\top}, \end{cases} \tag{6}$$

where $\operatorname{svd}(\cdot)$ denotes the singular value decomposition of a matrix. The solution of $\mathbf{E}^{k+1}$ consists of two parts: in the observation space $\mathbf{\Omega}$, entry values of $\mathbf{E}$ are considered to be zeros, while in the complementary space $\bar{\mathbf{\Omega}}$ of $\mathbf{\Omega}$, the first derivation is employed to derive the closed-form of $\mathbf{E}$. Hence,

$$\mathbf{E}^{k+1} = \mathcal{P}_{\mathbf{\Omega}}(0) + \mathcal{P}_{\bar{\mathbf{\Omega}}}\left(\mathbf{D} - \mathbf{A} + \frac{\mathbf{Y}_3}{\mu_3}\right). \tag{7}$$

3. EXPERIMENTAL RESULTS

In this section, we conduct several experiments on both synthetic data and real images to validate the effectiveness of our JPLOSS algorithm for matrix completion against missing rows and columns. All the experiments are implemented under following parameter settings: $\varepsilon = 0.001$, $\rho_1 = \rho_2 = \rho_3 = 1.1$, $\mu_1 = \mu_2 = \mu_3 = 0.5/\max(\sigma(\mathbf{D}))$, where $\sigma(\mathbf{D})$ denotes the singular values of $\mathbf{D}$. The convergence error is set to 1×10^{-4}. Besides, $\gamma_B = \gamma_C = 0.1$ for synthetic data and $\gamma_B = \gamma_C = 0.01$ for real images. The relative error (RE) is used to measure the completion results of synthetic data: $RE = \|\mathbf{A}^* - \mathbf{A}\|_F / \|\mathbf{A}\|_F$, where $\mathbf{A}$ is the groundtruth and $\mathbf{A}^*$ is the recovery result. The performance of real image restoration is measured by peak signal-to-noise ratio (PSNR) and structural similarity (SSIM) [18] for objective and subjective evaluation, respectively.

3.1. Experiments on Synthetic Data

We generate the low-rank matrix $\mathbf{A}$ as $\mathbf{A} = \mathbf{\Phi}_c \mathbf{\Lambda} \mathbf{\Phi}_r{}^\top$, where $\mathbf{\Phi}_c$ and $\mathbf{\Phi}_r$ are two randomly generated dictionaries by the Matlab command *randn*, and $\mathbf{\Lambda}$ is a diagonal matrix whose number of nonzero diagonal elements determines the rank of synthetic matrix $\mathbf{A}$. Our method is compared with four state-of-the-art methods, i.e., SVT [1], IALM [2], IRNN [4] as well as ReLaSP [14] on three synthetic data differs in matrix size (m), matrix rank ($rank$), and coefficient sparseness (spa) under a series of different combinations of total missing rate ($total\_mr$), ratio of entire row missing among total missing (row_mr), ratio of entire column missing among total missing (col_mr), and ratio of random missing among total missing ($rand\_mr$). In particular, we conduct ReLaSP successively to first complete the missing rows and then recover the missing columns by inputing the transpose of the output matrix of previous step for fair comparison.

Table 1 presents numerical results of matrix completion by SVT [1], IALM [2], IRNN [4], ReLaSP [14] and the proposed method. JPLOSS-0 and JPLOSS-2 represent our JPLOSS method without reweighting strategy and within two reweighting iterations, respectively, which is same for RelaSP-2. As can be seen, our method achieves the best completion results with much smaller relative error in almost all the cases compared with other methods. SVT [1], IALM [2] and IRNN [4] can recover matrices with low to middle random missing rates, however, only IRNN [4] is able to complete matrices against high random missing rates. Furthermore, all of these three methods fail to complete matrices with both random and structural missing since they merely employ the low-rank prior which cannot provide sufficient regularization on entire row and column missing. Thanks to the separable 2-D sparsity priors on rows and columns of matrices, our method is capable of completing matrices accurately against both missing rows and columns simultaneously and achieving smaller relative error than ReLaSP [14] which essentially aims to recovering missing rows in almost all the configurations. JPLOSS-2 outperforms JPLOSS-0 for most cases especially at high missing rates, confirming the effectiveness of reweighting of low-rankness and sparsity.

Table 1. Relative error comparison for matrix completion on synthetic data in different configurations.

Data	total_mr	25%					50%					75%				
	row_mr	0%	20%	0%	20%	30%	0%	20%	0%	20%	30%	0%	20%	0%	20%	30%
	col_mr	0%	0%	20%	20%	30%	0%	0%	20%	20%	30%	0%	0%	20%	20%	30%
	rand_mr	100%	80%	80%	60%	40%	100%	80%	80%	60%	40%	100%	80%	80%	60%	40%
	SVT [1]	2.98E-04	0.2183	0.2209	0.3123	0.3869	1.58E-04	0.4207	0.3065	0.4516	0.5183	0.4265	0.4973	0.4921	0.5895	0.6064
	IALM [2]	2.83E-04	0.2203	0.2209	0.5295	0.5805	1.36E-04	0.5238	0.5341	0.6194	0.6432	0.4385	0.5748	0.5720	0.6540	0.6889
m=200	IRNN [4]	1.43E-04	0.1743	0.2147	0.3128	0.3725	1.72E-04	0.3596	0.3465	0.4428	0.5799	2.88E-04	0.2769	0.3991	0.4708	0.6358
rank=12	ReLaSP-2 [14]	6.43E-05	6.31E-05	6.76E-05	7.12E-05	**7.06E-05**	1.16E-04	1.14E-04	1.13E-04	1.11E-04	1.17E-04	1.74E-04	1.60E-04	1.69E-04	1.67E-04	1.48E-04
spa=3%	JPLOSS-0	**4.74E-05**	**6.05E-05**	**6.16E-05**	9.61E-05	9.85E-05	1.79E-04	9.79E-05	9.85E-05	1.13E-04	1.40E-04	2.22E-04	1.39E-04	1.30E-04	1.40E-04	1.67E-04
	JPLOSS-2	5.16E-05	5.91E-05	5.93E-05	**6.62E-05**	7.58E-05	**7.53E-05**	**8.22E-05**	**7.64E-05**	**8.44E-05**	**9.43E-05**	**9.53E-05**	**9.52E-05**	**9.98E-05**	**1.00E-04**	**9.87E-05**
	SVT [1]	1.25E-04	0.2287	0.2263	0.3081	0.3773	1.80E-04	0.3391	0.3266	0.4250	0.5166	0.5620	0.5958	0.6425	0.6618	0.6665
	IALM [2]	7.62E-05	0.2276	0.2263	0.3081	0.5850	1.88E-04	0.5656	0.5374	0.5991	0.6548	0.4635	0.5767	0.6744	0.6763	0.7335
m=300	IRNN [4]	1.05E-04	0.2855	0.1774	0.3194	0.4098	1.39E-04	0.2781	0.3029	0.4265	0.5775	1.95E-04	0.3655	0.4104	0.4946	0.6245
rank=24	ReLaSP-2 [14]	5.97E-05	6.44E-05	6.43E-05	**6.59E-05**	**6.75E-05**	1.09E-04	1.14E-04	1.10E-04	1.06E-04	1.10E-04	1.51E-04	1.61E-04	1.55E-04	1.55E-04	1.53E-04
spa=4%	JPLOSS-0	**4.91E-05**	**5.78E-05**	**5.74E-05**	6.72E-05	6.86E-05	**6.94E-05**	**7.82E-05**	7.43E-05	9.29E-05	1.29E-04	1.01E-04	1.04E-04	1.19E-04	1.18E-04	1.31E-04
	JPLOSS-2	5.06E-05	6.05E-05	5.94E-05	6.67E-05	7.03E-05	7.23E-05	7.98E-05	**7.24E-05**	**7.98E-05**	**8.52E-05**	**9.15E-05**	**9.17E-05**	**8.67E-05**	**9.99E-05**	**1.13E-04**
	SVT [1]	0.0284	0.2468	0.2039	0.3332	0.3584	0.0612	0.2938	0.3321	0.4531	0.5388	0.1703	0.4136	0.3968	0.5184	0.6115
	IALM [2]	0.1651	0.2930	0.2574	0.3672	0.3881	0.1653	0.3303	0.3650	0.4749	0.5558	0.1663	0.3997	0.3975	0.5212	0.6533
m=400	IRNN [4]	7.27E-05	0.2452	0.2021	0.3321	0.3575	1.32E-04	0.2908	0.3289	0.4518	0.5381	1.71E-04	0.3690	0.3668	0.4994	0.6038
rank=40	ReLaSP-2 [14]	5.93E-05	6.14E-05	6.30E-05	6.62E-05	**7.02E-05**	1.04E-04	1.07E-04	1.07E-04	1.06E-04	1.14E-04	1.46E-04	1.55E-04	1.56E-04	1.49E-04	1.49E-04
spa=5%	JPLOSS-0	5.05E-05	**5.55E-05**	**5.92E-05**	**6.51E-05**	8.85E-05	7.36E-05	8.35E-05	8.26E-05	1.00E-04	1.03E-04	9.76E-05	9.84E-05	1.02E-04	1.11E-04	1.15E-04
	JPLOSS-2	**5.00E-05**	5.57E-05	6.03E-05	6.86E-05	7.38E-05	**7.15E-05**	**7.74E-05**	**7.94E-05**	**7.90E-05**	**9.55E-05**	**8.96E-05**	**8.50E-05**	**8.67E-05**	**1.07E-04**	**1.12E-04**

(a) (b) (c) (d) (e) (f) (g)

Fig. 1. completion results of *Image 4* with 50% total missing. (a) GroundTruth; (b) corrupted; (c) FaLRTC [3] (15.46/0.3582); (d) DA-based [19] (25.91/0.8866); (e) OMP [20] (24.71/0.8286); (f) ReLaSP [14] (29.56/0.8778) and (g) Ours (**32.23/0.9289**). Partial details in red rectangles are enlarged for better visual inspection displayed in the second row.

3.2. Experiments on Real Images

As natural images can be regarded as approximately low rank matrices and may be partially corrupted due to coding or transmission, it's quite naturally to recover the missing information by applying the low-rank matrix minimization algorithms. We conduct image completion experiments on the Berkeley Segmentation Dataset (BSDS500) [21], comparing our JPLOSS with four methods, i.e., Fast Low Rank Tensor Completion (FaLRTC) [3], deterministic annealing method (DA-based) [19], sparse representation using OMP [20] and ReLaSP [14]. Two fixed global column and row dictionaries of size 100×400 are generated by online dictionary learning method [22] on Kodak image set [23]. The input missing image is divided into 100×100 patches to be completed in a sliding way on each patch. The row missing ratio and column missing ratio are fixed at 20%. Results of the four methods are generated by the provided codes, and the optimal parameters are tuned for fair comparison.

Table 2. Quantitative Results (PSNR(dB)/SSIM) of image completion with different missing rates

Image	total_mr	5%	10%	20%	30%	50%
1	FaLRTC [3]	22.09/0.8770	19.04/0.7737	15.92/0.6126	14.36/0.4789	12.46/0.2982
	DA-based [19]	34.22/0.9751	30.70/0.9464	28.15/0.8995	25.15/0.8516	22.73/0.7369
	OMP [20]	28.09/0.7967	27.05/0.7476	25.83/0.6858	24.56/0.6355	22.37/0.5447
	ReLaSP [14]	37.28/0.9820	33.91/0.9607	30.50/0.9165	27.53/0.8543	24.09/0.7192
	JPLOSS	**39.22/0.9880**	**36.23/0.9744**	**33.30/0.9490**	**30.83/0.9151**	**27.91/0.8362**
2	FaLRTC [3]	23.74/0.8809	19.46/0.7732	17.27/0.5962	15.15/0.4804	13.06/0.3078
	DA-based [19]	35.68/0.9742	33.26/0.9483	31.00/0.9054	29.47/0.8913	25.80/0.7878
	OMP [20]	28.36/0.8444	27.62/0.8170	26.43/0.7773	25.41/0.7397	23.17/0.6742
	ReLaSP [14]	38.99/0.9890	35.08/0.9760	31.71/0.9517	29.34/0.9172	26.02/0.8288
	JPLOSS	**39.39/0.9899**	**35.65/0.9780**	**32.59/0.9583**	**30.42/0.9272**	**27.37/0.8572**
3	FaLRTC [3]	23.49/0.8719	20.26/0.7529	17.14/0.5966	15.16/0.4667	13.41/0.2404
	DA-based [19]	35.17/0.9808	32.69/0.9601	30.21/0.9315	28.60/0.9022	25.99/0.8297
	OMP [20]	30.96/0.8986	30.09/0.8789	29.16/0.8559	27.89/0.8229	25.75/0.7714
	ReLaSP [14]	41.07/0.9923	37.50/0.9832	34.56/0.9652	31.63/0.9395	28.99/0.8813
	JPLOSS	**41.50/0.9927**	**37.83/0.9842**	**35.43/0.9691**	**32.99/0.9503**	**30.43/0.9037**
4	FaLRTC [3]	25.90/0.9029	22.65/0.7995	19.01/0.6579	18.48/0.5327	15.46/0.3582
	DA-based [19]	34.75/0.9780	33.39/0.9753	29.30/0.9506	32.51/0.9398	25.91/0.8866
	OMP [20]	33.38/0.9357	32.18/0.9208	29.86/0.8992	28.42/0.8791	24.71/0.8286
	ReLaSP [14]	43.31/0.9940	39.71/0.9842	35.70/0.9647	33.70/0.9427	29.56/0.8778
	JPLOSS	**43.96/0.9952**	**40.79/0.9889**	**37.32/0.9769**	**35.69/0.9653**	**32.23/0.9289**
AVE.	FaLRTC [3]	23.81/0.8832	20.35/0.7748	17.33/0.6158	15.79/0.4897	13.60/0.3012
	DA-based [19]	34.75/0.9770	33.39/0.9575	29.30/0.9217	32.51/0.8963	25.91/0.8103
	OMP [20]	30.20/0.8689	29.23/0.8411	27.82/0.8045	26.57/0.7693	24.00/0.7047
	ReLaSP [14]	40.16/0.9893	36.55/0.9760	33.12/0.9495	30.55/0.9134	27.17/0.8268
	JPLOSS	**41.02/0.9915**	**37.63/0.9814**	**34.66/0.9633**	**34.48/0.9395**	**29.49/0.8815**

The quantitative results of image completion are presented in Table 2. Our method achieves the best performance consistently for all the test cases. The PSNR/SSIM value of FaLRTC is relatively lower due to the failure of completing entire row and column missing. Our method outperforms the DA-based method, OMP and ReLaSP by 5.22 dB, 7.89 dB and 1.95 dB, respectively. Fig. 1 shows the qualitative visual results with partial details enlarged. As can be seen, the recovered result by FaLRTC still leaves missing rows and columns incomplete. The completed image by the DA-based method suffers from line-like artifacts, meanwhile, the result tends to be a little fuzzy recovered by OMP. ReLaSP cannot fill in the missing rows and columns simultaneously which desires for more complex implementation and more computation time. However, our proposed method completes both structural and random missing accurately thanks to the joint priors of low-rank and separable 2-D sparsity.

4. CONCLUSION

We have presented a novel method called JPLOSS for matrix completion against missing rows and columns. In addition to regularizing the latent matrix by a low-rank prior, we have further introduced separable 2-D sparsity priors to regularize its rows and columns under row and column dictionaries exploiting the intra-row and intra-column correlations. Both the low-rank and sparsity priors are reweighted by a inverse proportion rule on the fly to enhance the low-rankness and sparsity, respectively. An efficient reweighted ALM-ADM algorithm has been derived to solve JPLOSS model with element-wise weights. Experimental results on both synthetic data and real images demonstrate the superiority of the proposed method for matrix completion against missing rows and columns. Future work includes: 1) deriving theoretical bound for matrix completion against structural missing, 2) training adaptive dictionaries for better image completion performance.

5. REFERENCES

[1] Jianfeng Cai, Emmanuel J Candes, and Zuowei Shen, "A singular value thresholding algorithm for matrix completion," *Siam Journal on Optimization*, vol. 20, no. 4, pp. 1956–1982, 2010.

[2] Zhouchen Lin, Minming Chen, and Yi Ma, "The augmented lagrange multiplier method for exact recovery of corrupted low-rank matrices," *arXiv preprint arXiv:1009.5055*, 2010.

[3] Ji Liu, Przemyslaw Musialski, Peter Wonka, and Jieping Ye, "Tensor completion for estimating missing values in visual data," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 35, no. 1, pp. 208–220, 2013.

[4] C. Lu, J. Tang, S. Yan, and Z. Lin, "Generalized nonconvex nonsmooth low-rank minimization," pp. 4130–4137, 2014.

[5] T. H. Oh, Y. W. Tai, J. C. Bazin, and H. Kim, "Partial sum minimization of singular values in robust pca: Algorithm and applications," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 38, no. 4, pp. 744–758, 2016.

[6] Nikos Komodakis, "Image completion using global optimization," in *Computer Vision and Pattern Recognition, 2006 IEEE Computer Society Conference on*, 2006, pp. 442–452.

[7] Harald Steck, "training and testing of recommender systems on data missing not at random,," in *ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Washington, Dc, Usa, July*, 2010, pp. 713–722.

[8] Jingyu Yang, Xin Sun, Xinchen Ye, and Kun Li, "Background extraction from video sequences via motion-assisted matrix completion," pp. 2437–2441, 2014.

[9] Xinchen Ye, Jingyu Yang, Xin Sun, and Kun Li, "Foregroundcbackground separation from video clips via motion-assisted matrix restoration," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 25, no. 11, pp. 1–1, 2015.

[10] Emmanuel J Candès and Benjamin Recht, "Exact matrix completion via convex optimization," *Foundations of Computational mathematics*, vol. 9, no. 6, pp. 717–772, 2009.

[11] Kim-Chuan Toh and Sangwoon Yun, "An accelerated proximal gradient algorithm for nuclear norm regularized linear least squares problems," *Pacific Journal of Optimization*, vol. 6, no. 615-640, pp. 15, 2010.

[12] Yigang Peng, Jinli Suo, Qionghai Dai, and Wenli Xu, "Reweighted low-rank matrix recovery and its application in image restoration," *IEEE Trans. on Cybernetics*, vol. 44, no. 12, pp. 2418–2430, 2014.

[13] Emmanuel J. Candes, Michael B. Wakin, and Stephen P. Boyd, "Enhancing sparsity by reweighted ℓ_1 minimization," *Journal of Fourier Analysis and Applications*, vol. 14, no. 5-6, pp. 877–905, 2008.

[14] Jingyu Yang, Xuemeng Yang, and Xinchen Ye, "Completion of structurally-incomplete matrices with reweighted low-rank and sparsity priors," in *IEEE International Conference on Acoustics, Speech and Signal Processing*, 2016.

[15] Jingyu Yang, Xuemeng Yang, Xinchen Ye, and Chunping Hou, "Reconstruction of structurally-incomplete matrices with reweighted low-rank and sparsity priors," *IEEE Transactions on Image Processing*, 2016.

[16] D. L. Donoho, "De-noising by soft-thresholding," *IEEE Transactions on Information Theory*, vol. 41, no. 3, pp. 613–627, 1995.

[17] Amir Beck and Marc Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," *Siam Journal on Imaging Sciences*, vol. 2, no. 1, pp. 183–202, 2009.

[18] Zhou Wang, Alan C Bovik, Hamid R Sheikh, and Eero P Simoncelli, "Image quality assessment: from error visibility to structural similarity," *IEEE Trans. on Image Processing*, vol. 13, no. 4, pp. 600–612, 2004.

[19] Xin Li, "Image recovery via hybrid sparse representations: A deterministic annealing approach," *IEEE Journal of Selected Topics in Signal Processing*, vol. 5, no. 5, pp. 953–962, 2011.

[20] http://spams devel.gforge.inria.fr/, "Sparse modeling software," .

[21] Pablo Arbelez, Michael Maire, Charless Fowlkes, and Jitendra Malik, "Contour detection and hierarchical image segmentation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 33, no. 5, pp. 898–916, 2010.

[22] Julien Mairal, Francis Bach, Jean Ponce, and Guillermo Sapiro, "Online dictionary learning for sparse coding," in *Proceedings of the 26th Annual International Conference on Machine Learning*. ACM, 2009, pp. 689–696.

[23] http://r0k.us/graphics/kodak/, "Kodak lossless true color image suite," .